# Adaptive Stabilization of Fractional-order Energy Supply-demand System with Dead-zone Nonlinear Inputs

Xiaomin Tian, Zhong Yang, Zhen Yang

***Abstract*—In this paper, the adaptive stabilization of fractional-order energy supply-demand system with unknown parameters is investigated. We assume that the controlled system is perturbed by external disturbances and model uncertainties, the bounds of both model uncertainties and external disturbances to be unknown in advance. Moreover, the effects of dead-zone nonlinear inputs are taken into account. In order to deal with these unknown parameters, some fractional-order adaption laws are given. Finally, simulation results are presented to verify the effectiveness and robustness of the proposed control strategy.**

***Index Terms*—adaptive stabilization, fractional-order energy supply-demand system, dead-zone nonlinear input.**

## I. Introduction

IN recent years, there has been an increasing attention on energy issues, it has been known that providing energy supply-demand security gives the most important necessity for the energy security and has a vital role for the energy and economic progress[1]. Hence, stabilizing the energy supply-demand system to achieve its well development is a major strategic significance for ensuring energy security.

Recently, studying fractional-order system has become an active research area. In particular, control and synchronization of the fractional order chaotic systems have attracted much attention from various scientific fields. Some methods have been proposed to achieve chaos synchronization in fractional order chaotic systems. Such as nonlinear feedback control [2], nonlinear state observer control [3], active control [4], adaptive control [5-7], etc.

However, all control methods in the abovementioned works are derived based on the ideal assumption of the control inputs, actually, the dead-zone nonlinearity is often encountered in various engineering systems, which can cause unpredictable and undesirable motions in systems. Thus, it is urgent to consider the effects of the dead-zone inputs. On the other hand, almost of all control scheme in existing literature are focus on the stability analysis of fractional-order systems based on traditional Lyapunov theory, the application of fractional-order Lyapunov stability theory is still an challenging problem and very few articles are dedicated to this problem.

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Furthermore, in most of the existing research results, system parameters are assumed to be known in advance to design the desired controller, while, the stabilization of fractional-order system with unknown parameters and dead-zone nonlinear input are not considered simultaneously. As a matter of fact, there always exist parameter mismatches and distortions in physical world, so, it is much more attractive and challenging to realize the stabilization of fractional-order chaotic system with unknown parameters.

Motivated by the above discussion, the main goal of this paper is to propose a new adaptive control strategy to realize the stabilization of fractional-order energy supply-demand system with unknown parameters and dead-zone nonlinear inputs, the bounds of both model uncertainties and external disturbances are unknown in advance. The structure of this paper is organized as follows. In section 2, relevant definitions, lemmas are given. Main results are presented in section 3. Simulation results are shown in section 4. Finally, conclusion is included in section 5.

## II. Preliminaries

The Riemann-Liouville, Caputo definition are main definitions of fractional calculus.

**Definition 1** The $\alpha$ th-order Riemann-Liouville fractional integration of function $f(t)$ is given by

$$ {}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{1} $$

where $\Gamma(\cdot)$ is the Gamma function.

**Definition 2** For $n-1 < \alpha \le n$, $n \in R$, the Riemann-Liouville fractional derivative of order $\alpha$ of the function $f(t)$ is defined as

$$ \begin{aligned} {}_{t_0}D_t^{\alpha} f(t) &= \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \\ &= \frac{d^n}{dt^n} I^{n-\alpha} f(t) \end{aligned} \tag{2} $$

**Definition 3** The $\alpha$ th-order Caputo fractional derivative of function $f(t)$ is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m} f(t), & \alpha = m \end{cases} \tag{3} $$

where $m$ is the smallest integer number, larger than $\alpha$.

**Lemma 1** (see [8, 9]) Consider the autonomous system

$$ D^{\alpha} x = Ax \qquad or \qquad D^{\alpha} x = f(x) \tag{4} $$

where $\alpha \in (0, 1]$ is the fractional order and $x = [x_1, x_2, ..., x_n]^T$ is the state variable. $A \in R^{n \times n}$ is a constant

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matrix. If there is a real symmetric positive definite matrix P such that the inequation $J = x^T P D^\alpha x \le 0$ always holds for any states, then system (4) is asymptotically stable.

For the detailed application of the above lemma in fractional-order chaotic systems, the reader can refer to Refs. [8-12].

## III. Main Results

In this section, a robust adaptive control law and parameter adaption laws are proposed to achieve the stabilization of fractional-order energy supply-demand system considering dead-zone nonlinear input.

The dynamical model of a fractional-order energy supply-demand system with model uncertainties and external disturbances is presented

$$\begin{aligned} D^\alpha x_1 &= a_1 x_1(1-\frac{x_1}{M}) - a_2(x_2+x_3) - d_3 x_4 \\ &\quad +\Delta f_1(x) + d_1(t) + \phi_1(u_1(t)) \\ D^\alpha x_2 &= -b_1 x_2 - b_2 x_3 + b_3 x_1[N+(x_1-x_3)] \\ &\quad +\Delta f_2(x) + d_2(t) + \phi_2(u_2(t)) \\ D^\alpha x_3 &= c_1 x_3(c_2 x_3 - c_3) + \Delta f_3(x) + d_3(t) + \phi_3(u_3(t)) \\ D^\alpha x_4 &= d_1 x_1 - d_2 x_4 + \Delta f_4(x) + d_4(t) + \phi_4(u_4(t)) \end{aligned} \quad (5)$$

where $\alpha \in (0,1]$ is fractional order of the system, $x = [x_1, x_2, x_3, x_4]^T$ is the state vector of system, $\Delta f_i(x)$ is model uncertainty, $d_i(t)$ is external disturbance, $\phi_i(u_i(t))$ is dead-zone nonlinear input, and satisfying

$$\phi_i(u_i(t)) = \begin{cases} (u_i(t)-u_{+i})\phi_{+i}(u_i(t)), & u_i(t) > u_{+i} \\ 0, & u_{-i} \le u_i(t) \le u_{+i} \\ (u_i(t)-u_{-i})\phi_{-i}(u_i(t)), & u_i(t) < u_{-i} \end{cases} \quad (6)$$

where $i = 1,2,3,4$. $\phi_{+i}(\cdot)$ and $\phi_{-i}(\cdot)$ are nonlinear continuous functions of $u_i(t)$, and $u_{+i}$, $u_{-i}$ are given constants. Besides, the nonlinear input $\phi_i(u_i(t))$ outside of the dead-band has gain reduction tolerance $\rho_{+i}$, $\rho_{-i}$, they satisfy the following inequalities

$$\begin{cases} (u_i(t)-u_{+i})\phi_i(u_i(t)) \ge \rho_{+i}(u_i(t)-u_{+i})^2, u_i(t) > u_{+i} \\ (u_i(t)-u_{-i})\phi_i(u_i(t)) \ge \rho_{-i}(u_i(t)-u_{-i})^2, u_i(t) < u_{-i} \end{cases} \quad (7)$$

where $\rho_{+i}, \rho_{-i}$ are nonzero positive constants.

Our goal in this paper is to design a robust controller to realize the adaptive stabilization of system (5) with unknown parameters and dead-zone nonlinear inputs. Before we introduce the control method, two assumptions are first given.

**Assumption 1** We assume that the parameters $a_1$, $a_2$, $b_1$, $b_2$, $b_3$, $c_1$, $c_2$, $c_3$, $d_1$, $d_2$, $d_3$, $M$ and $N$ to be unknown in advance. Letting $\theta_1 = [a_1, a_1/M, a_2, d_3]^T$, $\theta_2 = [b_1, b_2, b_3 N, b_3]^T$, $\theta_3 = [c_1 c_2, c_1 c_3]^T$, $\theta_4 = [d_1, d_2]^T$ as the vector of unknown parameters for the first, second, third and fourth equation of the system (5), respectively.

**Assumption 2** In general, it is assumed that the model uncertainties $\Delta f_i(x) \in R$ and external disturbances $d_i(t) \in R$ are bounded by

$$|\Delta f_i(x)| \le \beta_i, \quad |d_i(t)| \le \gamma_i, \quad i = 1,2,3,4 \quad (8)$$

where $\beta_i$ and $\gamma_i$ are unknown positive constants.

Next, we will design an adaptive controller to achieve the stabilization for fractional-order energy supply-demand system.

**Theorem 1** Consider the fractional-order energy supply-demand system (5), if the controller is designed as

$$u_i(t) = \begin{cases} -\delta_i \xi_i sgn(x_i) + u_{-i}, & x_i > 0 \\ 0, & x_i = 0 \\ -\delta_i \xi_i sgn(x_i) + u_{+i}, & x_i < 0 \end{cases} \quad (9)$$

where $i = 1,2,3,4$. $sgn(\cdot)$ is the sign function, $\delta_i = \rho_i^{-1}$, $\rho_i = min\{\rho_{-i}, \rho_{+i}\}$, and $\xi_i = |\hat{\theta}_i|^T |f_i(x)| + \hat{\beta}_i + \hat{\gamma}_i + k_i$, in which, $\hat{\theta}_i$, $\hat{\beta}_i$ and $\hat{\gamma}_i$ are estimation of $\theta_i$, $\beta_i$ and $\gamma_i$, respectively, $k_i > 0$ is a constant gain. $f_1(x) = [x_1, -x_1^2, -(x_2 + x_3), -x_4]^T$, $f_2(x) = [-x_2, -x_3, x_1, x_1(x_1 - x_3)]^T$, $f_3(x) = [x_3^2, -x_3]^T$, $f_4(x) = [x_1, -x_4]^T$.

The parametric update laws are selected as

$$\begin{aligned} D^\alpha \hat{\theta}_1 &= f_1(x) x_1 = [x_1^2, -x_1^3, -x_1(x_2+x_3), -x_1 x_4]^T \\ D^\alpha \hat{\theta}_2 &= f_2(x) x_2 = [-x_2^2, -x_2 x_3, x_1 x_2, x_1 x_2(x_1 - x_3)]^T \\ D^\alpha \hat{\theta}_3 &= f_3(x) x_3 = [x_3^3, -x_3^2]^T \\ D^\alpha \hat{\theta}_4 &= f_4(x) x_4 = [x_1 x_4, -x_4^2]^T \end{aligned} \quad (10)$$

besides, $\hat{\beta}_i$ and $\hat{\gamma}_i$ are updated by

$$\begin{aligned} D^\alpha \hat{\beta}_i &= \mu_i |x_i| \\ D^\alpha \hat{\gamma}_i &= \eta_i |x_i| \end{aligned} \quad (11)$$

in which, $\mu_i$ and $\eta_i$ are positive constants.

Then the adaptive stabilization of fractional-order energy supply-demand system (5) can be achieved.

**Proof** According to Lemma 1, we denote a new state variable as $X^T = [x^T, \widetilde{\theta}^T, \widetilde{\beta}^T, \widetilde{\gamma}^T]$, which is a row vector, where

$$\begin{aligned} x^T &= [x_1, x_2, x_3, x_4] \\ \widetilde{\theta}^T &= [\widetilde{\theta}_1^T, \widetilde{\theta}_2^T, \widetilde{\theta}_3^T, \widetilde{\theta}_4^T], \quad \widetilde{\theta}_i^T = \hat{\theta}_i^T - \theta_i^T \\ \widetilde{\beta}^T &= [\widetilde{\beta}_1, \widetilde{\beta}_2, \widetilde{\beta}_3, \widetilde{\beta}_4], \quad \widetilde{\beta}_i = \hat{\beta}_i - \beta_i \\ \widetilde{\gamma}^T &= [\widetilde{\gamma}_1, \widetilde{\gamma}_2, \widetilde{\gamma}_3, \widetilde{\gamma}_4], \quad \widetilde{\gamma}_i = \hat{\gamma}_i - \gamma_i \end{aligned} \quad (12)$$

Choose a real symmetric positive finite matrix P in the form of

$$P = diag\left(I_4, I_4, I_4, I_2, I_2, \frac{1}{\mu_1}, \frac{1}{\mu_2}, \frac{1}{\mu_3}, \frac{1}{\mu_4}, \frac{1}{\eta_1}, \frac{1}{\eta_2}, \frac{1}{\eta_3}, \frac{1}{\eta_4}\right) \quad (13)$$

where $I$ is unit matrix, and its subscript represents dimension, then we can construct a function to prove the stability of the closed-loop system, written as

$$J = X^T P D^\alpha X \quad (14)$$

that is

$$\begin{aligned} J = {} & x_1 D^\alpha x_1 + x_2 D^\alpha x_2 + x_3 D^\alpha x_3 + x_4 D^\alpha x_4 \\ & + \widetilde{\theta}_1^T D^\alpha \widetilde{\theta}_1 + \widetilde{\theta}_2^T D^\alpha \widetilde{\theta}_2 + \widetilde{\theta}_3^T D^\alpha \widetilde{\theta}_3 + \widetilde{\theta}_4^T D^\alpha \widetilde{\theta}_4 \\ & + \frac{1}{\mu_1}\widetilde{\beta}_1 D^\alpha \widetilde{\beta}_1 + \frac{1}{\mu_2}\widetilde{\beta}_2 D^\alpha \widetilde{\beta}_2 + \frac{1}{\mu_3}\widetilde{\beta}_3 D^\alpha \widetilde{\beta}_3 + \frac{1}{\mu_4}\widetilde{\beta}_4 D^\alpha \widetilde{\beta}_4 \\ & + \frac{1}{\eta_1}\widetilde{\gamma}_1 D^\alpha \widetilde{\gamma}_1 + \frac{1}{\eta_2}\widetilde{\gamma}_2 D^\alpha \widetilde{\gamma}_2 + \frac{1}{\eta_3}\widetilde{\gamma}_3 D^\alpha \widetilde{\gamma}_3 + \frac{1}{\eta_4}\widetilde{\gamma}_4 D^\alpha \widetilde{\gamma}_4 \end{aligned} \quad (15)$$

Inserting (5) into (15), it yields

$$
\begin{aligned}
J = {} & x_1\Big[a_1x_1(1-x_1/M)-a_2(x_2+x_3)-d_3x_4+\Delta f_1(x) \\
& +d_1(t)+\phi_1(u_1)\Big]+x_2\Big[-b_1x_2-b_2x_3+b_3x_1\big(N \\
& +(x_1-x_3)\big)+\Delta f_2(x)+d_2(t)+\phi_2(u_2)\Big] \\
& +x_3\Big[c_1x_3(c_2x_3-c_3)+\Delta f_3(x)+d_3(t)+\phi_3(u_3)\Big] \\
& +x_4\Big[d_1x_1-d_2x_4+\Delta f_4(x)+d_4(t)+\phi_4(u_4)\Big] \\
& +\sum_{i=1}^{4}\Big((\hat\theta_i-\theta_i)^TD^\alpha\hat\theta_i+\frac{1}{\mu_i}(\hat\beta_i-\beta_i)D^\alpha\hat\beta_i \\
& +\frac{1}{\eta_i}(\hat\gamma_i-\gamma_i)D^\alpha\hat\gamma_i\Big) \qquad (16)
\end{aligned}
$$

According to (10) and (11), we know that

$$
\begin{aligned}
\theta_1^TD^\alpha\hat\theta_1 &= a_1x_1^2-\frac{a_1}{M}x_1^3-a_2x_1(x_2+x_3)-d_3x_1x_4 \\
\theta_2^TD^\alpha\hat\theta_2 &= -b_1x_2^2-b_2x_2x_3+b_3Nx_1x_2+b_3x_1x_2(x_1-x_3) \\
\theta_3^TD^\alpha\hat\theta_3 &= c_1c_2x_3^3-c_1c_3x_3^2 \\
\theta_4^TD^\alpha\hat\theta_4 &= d_1x_1x_4-d_2x_4^2 \qquad (17)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{1}{\mu_i}\beta_iD^\alpha\hat\beta_i &= \beta_i|x_i| \\
\frac{1}{\eta_i}\gamma_iD^\alpha\hat\gamma_i &= \gamma_i|x_i| \qquad (18)
\end{aligned}
$$

then substituting (17) and (18) into (16), it yields

$$
\begin{aligned}
J = {} & x_1\Delta f_1(x)+x_1d_1(t)+x_1\phi_1(u_1)+x_2\Delta f_2(x) \\
& +x_2d_2(t)+x_2\phi_2(u_2)+x_3\Delta f_3(x)+x_3d_3(t) \\
& +x_3\phi_3(u_3)+x_4\Delta f_4(x)+x_4d_4(t)+x_4\phi_4(u_4) \\
& +\sum_{i=1}^{4}\Big(\hat\theta_i^TD^\alpha\hat\theta_i+\frac{1}{\mu_i}\hat\beta_iD^\alpha\hat\beta_i+\frac{1}{\eta_i}\hat\gamma_iD^\alpha\hat\gamma_i\Big) \\
& -\sum_{i=1}^{4}\big(\beta_i|x_i|+\gamma_i|x_i|\big) \qquad (19)
\end{aligned}
$$

knowing from (8), that is

$$
\begin{aligned}
J \le {} & \beta_1|x_1|+\gamma_1|x_1|+x_1\phi_1(u_1)+\beta_2|x_2| \\
& +\gamma_2|x_2|+x_2\phi_2(u_2)+\beta_3|x_3|+\gamma_3|x_3| \\
& +x_3\phi_3(u_3)+\beta_4|x_4|+\gamma_4|x_4|+x_4\phi_4(u_4) \\
& +\sum_{i=1}^{4}\Big(\hat\theta_i^TD^\alpha\hat\theta_i+\frac{1}{\mu_i}\hat\beta_iD^\alpha\hat\beta_i+\frac{1}{\eta_i}\hat\gamma_iD^\alpha\hat\gamma_i\Big) \\
& -\sum_{i=1}^{4}\big(\beta_i|x_i|+\gamma_i|x_i|\big) \\
= {} & x_1\phi_1(u_1)+x_2\phi_2(u_2)+x_3\phi_3(u_3)+x_4\phi_4(u_4) \\
& +\sum_{i=1}^{4}\Big(\hat\theta_i^TD^\alpha\hat\theta_i+\frac{1}{\mu_i}\hat\beta_iD^\alpha\hat\beta_i+\frac{1}{\eta_i}\hat\gamma_iD^\alpha\hat\gamma_i\Big) \\
\le {} & x_1\phi_1(u_1)+x_2\phi_2(u_2)+x_3\phi_3(u_3)+x_4\phi_4(u_4) \\
& +\sum_{i=1}^{4}\Big(|\hat\theta_i|^T|D^\alpha\hat\theta_i|+\frac{1}{\mu_i}|\hat\beta_i||D^\alpha\hat\beta_i|+\frac{1}{\eta_i}|\hat\gamma_i||D^\alpha\hat\gamma_i|\Big) \\
= {} & x_1\phi_1(u_1)+x_2\phi_2(u_2)+x_3\phi_3(u_3)+x_4\phi_4(u_4) \\
& +|\hat a_1||x_1^2|+|\hat a_1/\hat M||x_1^3|+|\hat a_2||x_1(x_2+x_3)| \\
& +|\hat d_3||x_1x_4|+|\hat b_1||x_2^2|+|\hat b_2||x_2x_3|+|\hat b_3\hat N||x_1x_2| \\
& +|\hat b_3||x_1x_2(x_1-x_3)|+|\hat c_1\hat c_2||x_3^3|+|\hat c_1\hat c_3||x_3^2| \\
& +|\hat d_1||x_1x_4|+|\hat d_2||x_4^2|+|\hat\beta_1||x_1|+|\hat\gamma_1||x_1| \\
& +|\hat\beta_2||x_2|+|\hat\gamma_2||x_2|+|\hat\beta_3||x_3|+|\hat\gamma_3||x_3| \\
& +|\hat\beta_4||x_4|+|\hat\gamma_4||x_4| \\
= {} & x_1\phi_1(u_1)+x_2\phi_2(u_2)+x_3\phi_3(u_3)+x_4\phi_4(u_4) \\
& +|\hat\theta_1|^T|f_1(x)||x_1|+|\hat\theta_2|^T|f_2(x)||x_2| \\
& +|\hat\theta_3|^T|f_3(x)||x_3|+|\hat\theta_4|^T|f_4(x)||x_4| \\
& +|\hat\beta_1||x_1|+|\hat\gamma_1||x_1|+|\hat\beta_2||x_2|+|\hat\gamma_2||x_2| \\
& +|\hat\beta_3||x_3|+|\hat\gamma_3||x_3|+|\hat\beta_4||x_4|+|\hat\gamma_4||x_4| \qquad (20)
\end{aligned}
$$

When $x_i > 0$, by surveying (6), (7) and (9), it is apparent that $u_i(t) < u_{-i}$

$$
\begin{aligned}
(u_i(t)-u_{-i})\phi(u_i(t)) &= -\delta_i\xi_i sgn(x_i)\phi(u_i(t)) \\
&\ge \rho_{-i}\delta_i^2\xi_i^2 sgn^2(x_i) \\
&\ge \rho_i\delta_i^2\xi_i^2 sgn^2(x_i) \qquad (21)
\end{aligned}
$$

since $\delta_i = \rho_i^{-1}$, $\xi_i > 0$, according to above inequality, we have

$$-sgn(x_i)\phi_i(u_i(t)) \ge \xi_i sgn^2(x_i) \qquad (22)$$

multiply both sides of inequality (22) by $|x_i|$, and according to $sgn^2(x_i)=1$, $|x_i|sgn(x_i)=x_i$, we have

$$x_i\phi_i(u_i(t)) \le -\xi_i|x_i| \qquad (23)$$

when $x_i < 0$, through similar derivation, the inequality (23) still holds. Substituting the inequality (23) into (20), it yields

$$
\begin{aligned}
J \le {} & -\xi_1|x_1|-\xi_2|x_2|-\xi_3|x_3|-\xi_4|x_4| \\
& +|\hat\theta_1|^T|f_1(x)||x_1|+|\hat\theta_2|^T|f_2(x)||x_2| \\
& +|\hat\theta_3|^T|f_3(x)||x_3|+|\hat\theta_4|^T|f_4(x)||x_4|+|\hat\beta_1||x_1| \\
& +|\hat\gamma_1||x_1|+|\hat\beta_2||x_2|+|\hat\gamma_2||x_2|+|\hat\beta_3||x_3| \\
& +|\hat\gamma_3||x_3|+|\hat\beta_4||x_4|+|\hat\gamma_4||x_4| \\
= {} & -\big(|\hat\theta_1|^T|f_1(x)|+\hat\beta_1+\hat\gamma_1+k_1\big)|x_1| \\
& -\big(|\hat\theta_2|^T|f_2(x)|+\hat\beta_2+\hat\gamma_2+k_2\big)|x_2| \\
& -\big(|\hat\theta_3|^T|f_3(x)|+\hat\beta_3+\hat\gamma_3+k_3\big)|x_3| \\
& -\big(|\hat\theta_4|^T|f_4(x)|+\hat\beta_4+\hat\gamma_4+k_4\big)|x_4| \\
& +|\hat\theta_1|^T|f_1(x)||x_1|+|\hat\theta_2|^T|f_2(x)||x_2| \\
& +|\hat\theta_3|^T|f_3(x)||x_3|+|\hat\theta_4|^T|f_4(x)||x_4|+|\hat\beta_1||x_1| \\
& +|\hat\gamma_1||x_1|+|\hat\beta_2||x_2|+|\hat\gamma_2||x_2|+|\hat\beta_3||x_3| \\
& +|\hat\gamma_3||x_3|+|\hat\beta_4||x_4|+|\hat\gamma_4||x_4| \\
= {} & -k_1|x_1|-k_2|x_2|-k_3|x_3|-k_4|x_4| \\
\le {} & -k||x|| < 0 \qquad (24)
\end{aligned}
$$

where $k = min\{k_1,k_2,k_3,k_4\} > 0$, according to Lemma 1, system (5) is asymptotically stable. Consequently, the adaptive stabilization of fractional-order energy supply-demand system with unknown parameters and dead-zone nonlinear inputs is achieved. Therefore, the proof is completed.

## IV. Simulation Results

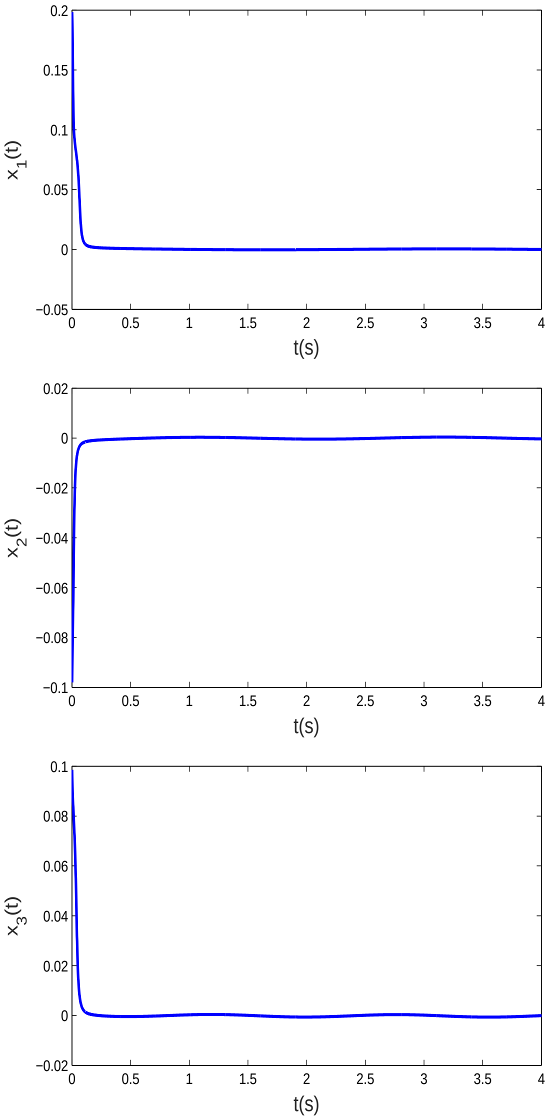
To validate the efficiency and feasibility of the proposed control strategy, an example is given. For the controlled system (5), the following model uncertainties and external disturbances are considered in this simulation.

$$\begin{aligned}\Delta f_1(x)+d_1(t) &= 0.01sin(2x_1)+0.02cos(2t)\\ \Delta f_2(x)+d_2(t) &= 0.02sin(3x_2)-0.02cos(3t)\\ \Delta f_3(x)+d_3(t) &= -0.01cos(4x_3)-0.025sin(4t)\\ \Delta f_4(x)+d_4(t) &= 0.025sin(3x_4)+0.015sin(4t) \quad (25)\end{aligned}$$

Letting $\alpha = 0.95$, the initial values are set as $x(0) = [0.2, -0.1, 0.1, -0.2]^T$, the positive constants $k_1 = k_2 = k_3 = k_4 = 10$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 2$, $\eta_1 = \eta_2 = \eta_3 = \eta_4 = 4$, all initial values of estimate parameters are set as 0.1, the dead-zone nonlinear inputs are selected as follows

$$\phi_i(u_i(t))=\begin{cases}(u_i(t)-1.5)(1-0.5cos(u_i(t))), & u_i(t)>1.5\\ 0, & -0.5\le u_i(t)\le 1.5\\ (u_i(t)+0.5)(0.7-0.5sin(u_i(t))), & u_i(t)<-0.5\end{cases} \quad (26)$$

correspondingly, parameters $\rho_{+i} = 0.5$, $\rho_{-i} = 0.2$, $\rho_i = min\{\rho_{-i}, \rho_{+i}\} = 0.2$, $\delta_i = 5$, $i = 1, 2, 3, 4$, according to (9), the appropriate controller can be designed. We can get the state trajectories of the controlled system, given in Fig.1



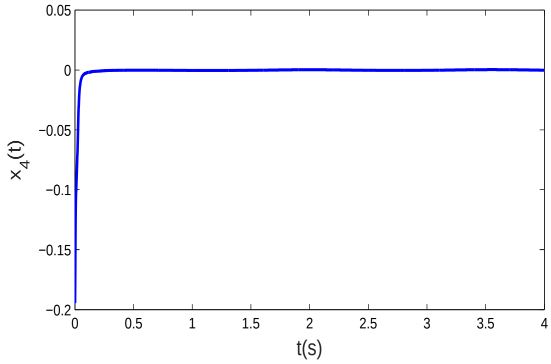
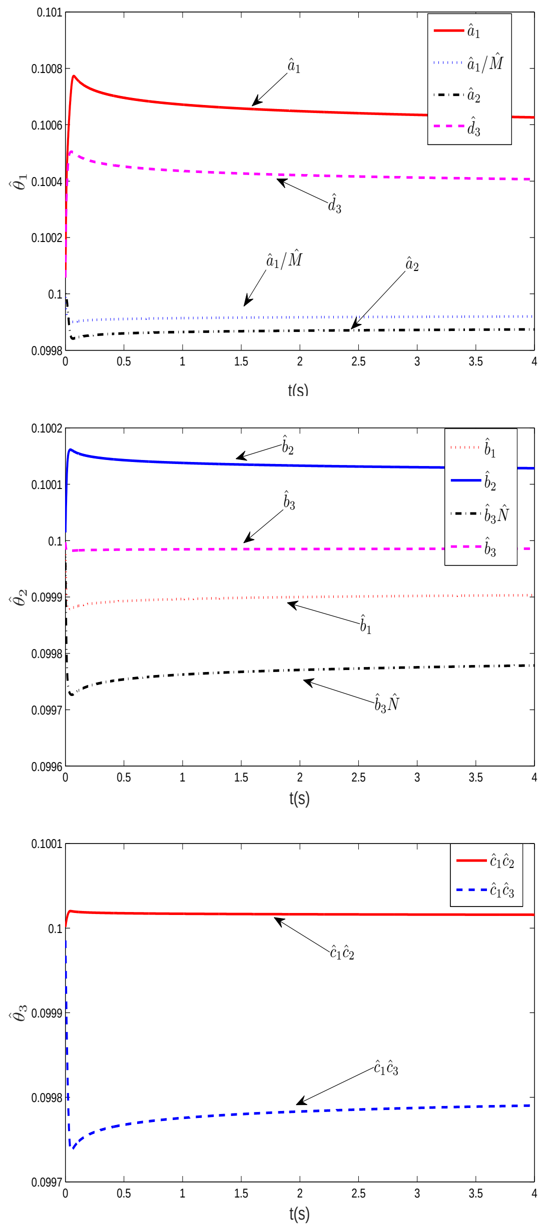




Fig. 1: State trajectories of system (5) with controller activated

From Fig. 1, it can be seen that all state trajectories of the controlled system (5) converge to zero asymptotically. The time response of estimate parameters vectors $\hat{\theta}_i$, $i = 1, 2, 3, 4$ are displayed in Fig.2, respectively. It is clear that all unknown parameters are successfully identified.



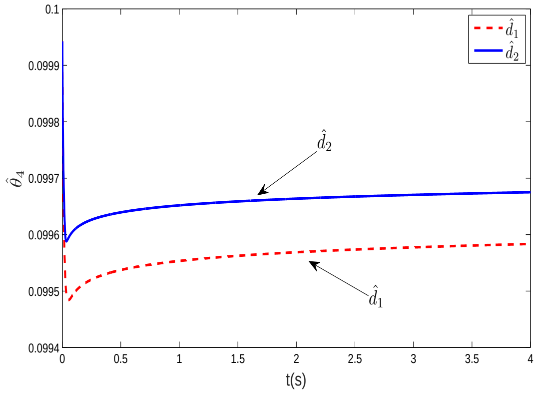



Fig. 2: Time response of estimate parameters in system (5)

Furthermore, the estimation about system uncertainties are shown in Fig.3. From Fig.3, it is obviously that all bounds of uncertainties are confirmed.

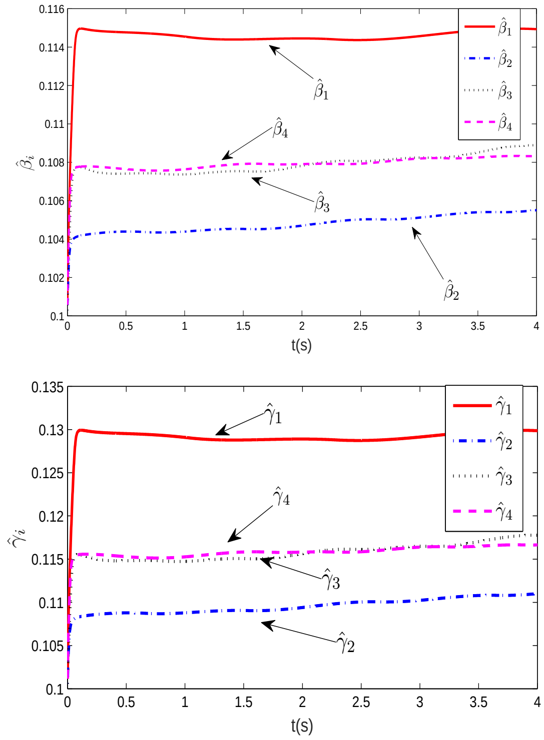



Fig. 3: Estimation of system uncertainties in system (5)

## V. Conclusion

This paper researched the problem of stabilizing uncertain fractional-order energy supply-demand system with dead-zone input nonlinearity. It is assumed that the system parameters and the bounds of uncertainties are fully unknown in advance. A new stability theory is applied to prove the asymptotic stability of fractional-order system. Simulation results verified the proposed control method is effective and feasible.

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