Numerical Solution for Heat Transfer of Non – Newtonian Second – Grade Fluid Flow over Stretching Sheet via Successive Linearization Method

Faisal Salah and Mubarak H. Elhafian

Abstract— **The main purpose of this paper is to obtain the numerical solutions for the MHD flow of heat transfer of incompressible second grade fluid on a stretching sheet channel. The governing partial differential equations are converted into ordinary differential equation by using a similarity transformation. The nonlinear equation governing the flow problem is modeled and then solved numerically by means of a successive linearization method (SLM). The numerical results are derived in tables for comparisons. The important result of this comparison is to show the high precision of the SLM in solving system of nonlinear differential equations. The solutions take into account the behavior of Newtonian and non-Newtonian fluids. Graphical outcomes of various non-Newtonian parameters such as mixed convection parameter, Hartman, Deborah and Prandtl numbers on the flow, field are discussed and analyzed. Besides this the present results have been tested and compared with the available published results in a limiting manner and an excellent agreement is found.**

Index Terms— **Second grade fluid, successive linearization, stretching sheet channel.**

I. INTRODUCTION

In the recent years, a great deal of interest has been gained for fluids applications. Some fluids are not easy expressed by particular constitutive relationship between shear rates and stress, which is totally different than the viscous fluids [1] and [2]. These fluids including many home items namely, toiletries, paints, cosmetics certain oils, shampoo, jams, soups etc have different features and they are denoted by non-Newtonian fluids. In general, the categorization of non-Newtonian fluid models is given under three class which are named the integral, differential, and rate types[3]-[8]. In the present study, the main interest is to discuss the heat transfer flow of magnetohydrodynamic (MHD) second grade fluid over stretching sheet. The effects of the stretching sheet on the fluid flow is attracted attentions of several scientist, thus a big number of research was achieved. The most important industrial to enhance the production and ductility of parts with big precision is shaping. The study of MHD fluid flow on stretching sheet can be implemented to extrusion casting, drawing, plastic films, polymer, hot rolling and several engineering applications. Following the advances in this field, the researchers in this field always try to improve the accuracy by using different methods on behaviour of fluid. One of these methods was used in this field is the application of magnetohydrodynamic flow. This application is known as MHD. MHD is the study of the interaction of electro conducting fluids with phenomena of electromagnetic. The flow of MHD fluid in the presence of magnetic field is very important in many regions of applied science, engineering and technology such as MHD pumps and MHD power generation. Due to this fact many researchers are still contributing in the field of MHD fluids mechanics [9] – [12]. Another important application of nanoparticles in base fluid, which is seek to improving behaviour of fluid and madding optimal use of the changes. Due to various engineering issues and different boundary conditions, intensive research has been achieved in this field, which is summarized briefly. Because of various boundary conditions and different engineering situations, Waqas et al. [13] discussed the stratified flow of nonliquid with heat generation in a linear stretchable surface. Ghadikolaei et al. [14] analyzed the flow and heat transfer of second grade fluid on an stretching sheet channel. The study of heat transfer with mixed convection flow of nonliquid that passed through a stretching perpendicular plate with the presence of three various types of nanoparticles, Cu, Al_2O_3 and TiO_2 to analyzes various thermal conductivity of the nonliquid and the velocity of nanoparticles and the research on the Nusselt number was found out by Xinhuisi et al. [15]. Many available publish work in this filed that are listed in Refs. [16] – [19].

The most phenomena in the field of engineering and science that occur are nonlinear. With this nonlinearity the equations become more difficult to handle and solve. Some of these nonlinear equations can be solved by using approximate analytical methods such as Homotopy analysis method (HAM) proposed by S. liao [20] and S. liao [21], Homotopy Perturbation method (HPM)[22] it was found by Ji-Huan [23] and Adomain decomposition method (ADM) (Q. Esmaili et al. [24], O. D Makinde et al. [25] and O. D Makinde [25]. However, some of these equations are solved via traditional numerical techniques such as finite difference method, shooting method and Keller box method, Runge-Kutta. Recently some studies have presented a new method called Successive Linearization Method (SLM). The approach of SLM is based on transforming an ordinary

Manuscript received November 15, 2018; revised May 22, 2019. F. Salah with the Department of Mathematics, College of Science & Arts, Rabigh, King Abdul-Aziz University,21911, Saudi Arabia and Department of Mathematics, Faculty of Science, University of Kordofan,51111 Elobid, Sudan, e-mail: faisal19999@yahoo.com. Mubarak H. Elhafian with the Department of Mathematics, College of Science & Arts, Rabigh, King Abdul-Aziz University,21911, Saudi Arabia and Department of Mathematics , e-mail: hafian10@yahoo.com.

nonlinear differential equation into an iterative scheme made up of linear equations which are then solved using numerical approaches. The (S L M) is a very simple and robust method that gives very accurate solutions to the nonlinear equations. This method has been applied successfully in many nonlinear problems in sciences and engineering, such as the MHD flows of non-Newtonian fluids and heat transfer over a stretching sheet [27], viscoelastic squeezing flow between two parallel plates [28], two dimensional laminar flow between two moving porous walls [29] and convective heat transfer for MHD boundary layer with pressure gradient [30] and [31]. Therefore, the effectiveness, validity, accuracy and flexibility of the SLM are verified among of all these successful applications.

Presently a new investigation on heat transfer of an incompressible second grade fluid on a stretching sheet channel is discussed. The governing equations of second grade with MHD are utilized. The numerical solution to the resulting nonlinear problem is computed by using the SLM approach. The embedded flow parameters are discussed and illustrated graphically.

II. Mathematical Formulation of the problem

A. Flow analysis

Here we considering the two – dimensional steady laminar flow of an incompressible MHD second grade fluid, which is past a flat sheet coincide with the plane $y=0$, confining the flow to $y>0$. Along x - axis there is two opposite and equal forces are applied. Due to this the wall is stretched and reserving the origin fixed. Under the constant and boundary layer assumptions, the continuity, constitutive equation of second grade fluid and energy equation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\left[\frac{\partial^2 u}{\partial y^2}+\gamma\left(\begin{array}{l}\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)\\ +-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\\ +v\frac{\partial^3 u}{\partial y^3}\end{array}\right)\right] \tag{2}$$

$$-\frac{\sigma}{\rho}B_0^2 u+g\beta_T\left(T-T_\infty\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{k}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where (u,v) are the components of velocity in (x,y) directions, $\upsilon\left(=\frac{\mu}{\rho}\right)$ the kinematic viscosity , μ is the dynamic viscosity, γ is the retardation time, ρ density of fluid , σ is the electric conductivity, B_0 is the uniform magnetic fluid , g is the gravitational acceleration, β_T the coefficient of thermal expansion, T is temperature of fluid, $\alpha\left(=\frac{k}{\rho c}\right)$ the thermal diffusivity, k the fluid thermal conductivity , ρc the fluid capacity heat and c_p the specific heat.

The relevant boundary conditions are defined as

$$u=u_w=cx,\ v=0 \text{ at } y=0,\ c>0 \tag{4}$$

$$u\to 0,\ \frac{\partial u}{\partial y}\to 0 \text{ as } y\to\infty, \tag{5}$$

$$\begin{array}{l}T=T_w\left(=T_\infty+Ax^s\right) \text{ at}\\ y=0,\ T\to T_\infty \quad \text{as } y\to\infty\end{array} \tag{6}$$

Where c is the stretching rate, T_w ,T_∞ are constants and s is the parameter wall temperature.

B. Transformation

Introducing the following dimensionless variables

$$u=cxf'(\eta),\ v=-(c\upsilon)^{\frac{1}{2}}f(\eta),$$

$$\eta=\left(\frac{c}{\upsilon}\right)^{\frac{1}{2}},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$Ec=\left(\frac{c^2}{Acp}\right). \tag{7}$$

Utilizing equation (7), equation (1) is satisfied automatically and equations (2) and (3) characterize to the following problems statement

$$f'''+ff''-f'^2+\beta\left(2ff''-(f'')^2-ff^{iv}\right)-M^2f'+\lambda\theta=0 \tag{8}$$

$$\theta''+\Pr f\theta'-s\Pr f'\theta=-\Pr Ec\left(f''\right)^2 \tag{9}$$

Clearly that all solutions for equation (9) are in similar type when $s=2$. If we neglected the dissipative heat, then equation (9) takes the simpler form

$$\theta''+\Pr f\theta'-2\Pr f'\theta=0 \tag{10}$$

Here $\beta(=\gamma c)$ are Deborah number, $M\left(=\frac{\sigma B_0^2}{c\rho}\right)$ is the Hartman number, $\lambda\left(=\frac{Gr_x}{\mathrm{Re}_x^2}\right)$ is the mixed convection parameter, $\Pr\left(=\frac{\upsilon}{\alpha}\right)$ is the Prandtl number and $Ec\left(=\frac{c^2}{Acp}\right)$ is the Eckert number.

The related boundary conditions

$$f=0,\ f'=1 \quad at \quad \eta=0, \tag{11}$$

$$f' \to 0, f'' \to 0 \ as \ \eta \to \infty. \tag{12}$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0\ . \tag{13}$$

III. Solution of the problem

A. Procedure of computational

Here successive linearization method (SLM) [27] – [30] is implemented to obtain the numerical solutions for nonlinear system (8) and (10) corresponding to the boundary condition Eq. (11) – (13).

For SLM solution we select the initial guesses functions

$f(\eta)$ and $\theta(\eta)$ in the form

$$f(\eta) = f_i(\eta) + \sum_{m=0}^{i-1} F_m(\eta)\ ,$$

$$\theta(\eta) = \theta_i(\eta) + \sum_{m=0}^{i-1} \theta_m(\eta)\ . \tag{14}$$

Here the two functions $f_i(\eta)$ and $\theta_i(\eta)$ are representative unknown functions. $F_m(\eta),\ m \geq 1$, $\theta_m(\eta),\ m \geq 1$ are successive approximation which are obtained by recursively solving the linear part of the equation that results from substituting Eq. (14) in the governing equations. The mean idea of SLM that the assumption of unknown function $f_i(\eta)$ and $\theta_i(\eta)$ are very small when i becomes larger, therefore, the nonlinear terms in $f_i(\eta)$ and $\theta_i(\eta)$ and their derivatives are considered to be smaller and thus neglected. The intimal guess functions $F_o(\eta)$, $\theta_o(\eta)$ which are selected to satisfy the boundary conditions

$$F_0(\eta) = 0\ , F_0'(\eta) = 1 \quad at \quad \eta = 0,$$
$$F_0'(\eta) \to 0\ ,\ F_0''(\eta) \to 0 \quad at \quad \eta \to \infty,$$
$$\theta_o(0) = 1,\ \ \theta_o(\infty) \to 0 \tag{15}$$

which are taken to be in the form

$$F_0(\eta) = \left(1 - e^{-\eta}\right) \text{and}\ \theta_0(\eta) = e^{-\eta}. \tag{16}$$

Therefore, beginning from the initial guess, the subsequent solution F_i and θ_i are calculated by successively solving the linearized from the equation which is obtained by substituting Eq. (14) in the governing equations (8) and (10). Then we arrive at the linearized equations to be solved are

$$\begin{aligned} &a_{1,i-1}F_i^{iv} + a_{2,i-1}F_i''' + a_{3,i-1}F_i'' \\ &\quad + a_{4,i-1}F_i' + a_{5,i-1}F_i + \lambda\theta_i = r_{1,i-1} \end{aligned} \tag{17}$$

$$\begin{aligned} &b_{1,i-1}F_i' + b_{2,i-1}F_i + \theta_i'' \\ &\quad + b_{3,i-1}\theta_i' + b_{4,i-1}\theta_i = r_{2,i-1} \end{aligned} \tag{18}$$

Subject to the boundary conditions

$$F_i(0) = \theta_i(\infty) = 0,\ \ F_i'(0) = \theta_i(0) = 1 \tag{19}$$

where the coefficients parameters $a_{k,i-1},\ b_{h,i-1}\ (k = 1,2,3,4,5), (h = 1,2,3,4)$ and $r_{j,i-1},\ j = 1,2.$ are defined as $a_{1,i-1} = -\beta\sum_{m=0}^{i-1} F_m$,

$$a_{2,i-1} = 1 + 2\beta\sum_{m=0}^{i-1} F_m'\ ,$$

$$a_{3,i-1} = \sum_{m=0}^{i-1} F_m - 2\beta\sum_{m=0}^{i-1} F_m''\ ,$$

$$a_{4,i-1} = -2\sum_{m=0}^{i-1} F_m' - M^2 + 2\beta\sum_{m=0}^{i-1} F_m'''\ ,$$

$$a_{5,i-1} = \sum_{m=0}^{i-1} F_m'' - \beta\sum_{m=0}^{i-1} F_m^{iv}$$

and

$$\begin{aligned} r_{1,i-1} &= -\sum_{m=0}^{i-1} F_m''' - \sum_{m=0}^{i-1} F_m \sum_{m=0}^{i-1} F_m'' + \left(\sum_{m=0}^{i-1} F_m'\right)^2 \\ &-\beta\left[2\sum_{m=0}^{i-1} F_m'\sum_{m=0}^{i-1} F_m'' - \left(\sum_{m=0}^{i-1} F_m''\right)^2 - \sum_{m=0}^{i-1} F_m\sum_{m=0}^{i-1} F_m^{iv}\right] \\ &+M^2\sum_{m=0}^{i-1} F_m' - \lambda\sum_{m=0}^{i-1}\theta_m . \end{aligned} \tag{20}$$

$$b_{1,i-1} = -2\,\Pr\sum_{m=0}^{i-1}\theta_m\ , \qquad b_{2,i-1} = \Pr\sum_{m=0}^{i-1}\theta_m'\ ,$$

$$b_{3,i-1} = \Pr\sum_{m=0}^{i-1} F_m\ ,\quad b_{4,i-1} = -2\Pr\sum_{m=0}^{i-1} F_m'$$

$$r_{2,i-1} = -\sum_{m=0}^{i-1}\theta_m'' - \Pr\sum_{m=0}^{i-1} F_m\sum_{m=0}^{i-1}\theta_m' + \Pr\sum_{m=0}^{i-1} F_m'\sum_{m=0}^{i-1}\theta_m \tag{21}$$

When we solve Eqs.(8) and (10) iteratively, the solution for F_i and θ_i has been obtained and finally after K iterations the solution $f(\eta)$ and $\theta(\eta)$ can be written as $f(\eta) \approx \sum_{m=0}^{K} F_m(\eta), \theta(\eta) \approx \sum_{m=0}^{K}\theta_m(\eta)$. In order to apply SLM firstly transform the domain solution from $[0,\infty)$ to $[-1,1]$. SLM is based on the Chebyshev spectral collection method. This method is depending on the Chebyshev polynomials defined on the interval $[-1,1]$. Thus, by using the truncation of domain approach where the problem is solved in the interval $[0, L]$ where L is scaling parameter used to impose the boundary condition at infinity. Thus, this can be obtained via the transformation

$$\frac{\eta}{L}=\frac{\xi+1}{2}, \quad -1\le\xi\le 1. \tag{22}$$

By using the Gauss-Lobatto collocation points we can discretize the domain $[-1,1]$ as follows

$$\xi=\cos\frac{\pi j}{N},\ F_i\approx\sum_{k=0}^{N}F_i\left(\xi_k\right)T\left(\xi_j\right),\quad j=0,1,\dots N \tag{23}$$

where N is the number of collection points and T_k is the k^{th} Chebyshev polynomial given by $T_k\left(\xi\right)=\cos\left[k\cos^{-1}\left(\xi\right)\right]$.

The derivatives of the variable at the collocation points are in the form

$$\frac{d^rF_i}{d\eta^r}=\sum_{k=0}^{N}\boldsymbol{D}_{kj}^{r}F_i\left(\xi_k\right),\qquad j=0,1,\dots N$$

$$\frac{d^r\theta_i}{d\eta^r}=\sum_{k=0}^{N}\boldsymbol{D}_{kj}^{r}\theta_i\left(\xi_k\right),\qquad j=0,1,\dots N \tag{24}$$

where r is the order of differentiation and $\boldsymbol{D}=\frac{2}{L}D$ with D is the Chebyshev spectral differentiation matrix. Substituting Eqs. (22) to (24) into Eqs. (17) and (18) we arrive at the matrix equation

$$\boldsymbol{A}_{i-1}\boldsymbol{X}_i=\boldsymbol{R}_{i-1} \tag{25}$$

$$\boldsymbol{A}_{i-1}=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix},\quad \boldsymbol{x}_{i-1}=\begin{bmatrix}F_i\\ \theta_i\end{bmatrix},\quad \boldsymbol{R}_{i-1}=\begin{bmatrix}r_{1,i-1}\\ r_{2,i-1}\end{bmatrix}$$

where

$$\boldsymbol{A}_{11}=a_{1,i-1}\boldsymbol{D}^4+a_{2,i-1}\boldsymbol{D}^3+a_{3,i-1}\boldsymbol{D}^2+a_{4,i-1}\boldsymbol{D}+a_{5,i-1}I,$$

$$\boldsymbol{A}_{12}=\lambda I,$$

$$\boldsymbol{A}_{21}=b_{1,i-1}\boldsymbol{D}+b_{2,i-1}I,$$

$$\boldsymbol{A}_{22}=\boldsymbol{D}^2+b_{3,i-1}\boldsymbol{D}+b_{4,i-1}I.$$

Following the above procedure, we can obtain the solution as $\boldsymbol{X}_i=\boldsymbol{A}^{-1}{}_{i-1}\boldsymbol{R}_{i-1}$. (26)

B. Convergence analysis

TABLE. I

The convergence for numerical values of $-f''(0)$ and $-\theta'(0)$ for different order of approximation when $M=0.50,\ \beta=0.01, \mathrm{Pr}=1$ and $\lambda=0.20$.

Order of approximation	$-f''(0)$	$-\theta'(0)$
1	1.0064665467	1.3322397230
5	1.0187454298	1.3280607773
10	1.0247104681	1.3256387760
20	1.0280770572	1.3242611294
30	1.0286148699	1.3240443457
50	1.0287099250	1.3240066355
70	1.0287119648	1.3240058514
90	1.0287120018	1.3240058381
95	1.0287120022	1.3240058380
100	1.0287120022	1.3240058380
120	1.0287120022	1.3240058380
130	1.0287120022	1.3240058380
140	1.0287120022	1.3240058380
150	1.0287120022	1.3240058380

TABLE. II

The numerical values of $f(\eta)$ and $f'(\eta)$ when, $M=.5, \lambda=.2, \mathrm{Pr}=1$ for different values of β.

β	η	$f(\eta)$	$f'(\eta)$
.01	0	0	1
	0.1	0.095034	0.902154
	0.2	0.180727	0.813783
	0.5	0.390635	0.596779
	1	0.623521	0.355247
	2	0.844509	0.125790
	3	0.922864	0.044742
	4	0.950821	0.016019
	5	0.960862	0.005781
.03	0	0	1
	0.1	0.095076	0.902974
	0.2	0.180886	0.815254
	0.5	0.391436	0.599424
	1	0.625799	0.358283
	2	0.849359	0.127768
	3	0.929141	0.045701
	4	0.957749	0.016429
	5	0.968059	0.005944

TABLE. III

The numerical values of $\theta(\eta)$ and $-\theta'(\eta)$ when, $M = .5, \lambda = .2, \Pr = 1$ for different values of β.

β	η	$\theta(\eta)$	$-\theta'(\eta)$
0.01	0	1	1.324005
	.1	0.877045	1.139906
	.2	0.771105	0.984194
	.5	0.531029	0.643729
	1	0.296179	0.333857
	2	0.101442	0.104412
	3	0.037087	0.036705
	4	0.013899	0.013551
	5	0.005259	0.005099
0.03	0	1	1.326444
	.1	0.876804	1.142276
	.2	0.770635	0.986394
	.5	0.530007	0.645174
	1	0.294738	0.334171
	2	0.100222	0.103914
	3	0.036364	0.036265
	4	0.013525	0.013288
	5	0.005079	0.004963

C. Numerical scheme testing

The aim here is to test our numerical results and compare with published works results in the literature as limiting cases situations. Thus, we compare the present results with the available results in reference [6] and [14] It is found that our results are in excellent agreement with those of [6] and [14] as shown in tables IV and V.

TABLE. V

Comparison of numerical values of $f(\eta)$ with Ref:[14] when, $M = 0 = \lambda = 0, \Pr = 1$ and $\beta = 0.01$.

β	η	Ref:[14]	Present work
.01	0	0	0
	0.1	0.095199	0.095194
	0.2	0.181400	0.181338
	0.5	0.394050	0.393892
	1	0.633463	0.633460
	2	0.866679	0.867642
	3	0.952228	0.954211
	4	0.983566	0.986229
	5	-	0.998059

TABLE. IV

Comparison of numerical values of $-\theta'(\eta)$ with Ref:[6] when, $M = 0 = \lambda = 0, \Pr = 1$ and $\beta = 0.01$.

β	η	Ref: [6]	Present work
.01	0	1.334735	1.334733
	0.1	1.150410	1.150382
	0.2	0.993973	0.994026
	0.5	0.650461	0.650523
	1	0.335684	0.335643
	2	0.102150	0.102133
	3	-	0.034583
	4	-	0.012274
	5	0.004444	0.004441
	10	2×10^{-5}	0.000029

IV. RESULTS AND DISCUSSION

This section concerns with the graphical illustrations obtained by using successive linearization method for velocity, temperature profiles. These profiles show the variations of embedded flow parameters in the solution expressions for heat transfer analysis for an incompressible MHD flow of second grade fluid on a stretching sheet channel. The physical interpretation of the problem has been discussed in Figures 1 – 8. These figures are plotted in order to illustrate such variations. Here The graphs have been determined for the MHD heat transfer flow of steady second grade fluid over stretching sheet. Figure 1 is prepared to see the effects of applied magnetic field (Hartman number) M on the velocity profile. Keeping $\beta, \Pr, \lambda$ fixed and varying M, it is seen that the velocity profile decreases when the magnetic field parameter M become larger . From physical side we observe that when we increasing the values of M, the flow on velocity profile of $f'(\eta)$ decreases, in fact this is due to the effects of the transverse magnetic field on the electrically conducting fluid which gives rise to a resistive type Lorentz force which tends to slow down the motion of the fluid. Figure 2 shows that for strong imposed magnetic force this lead to larger temperature. This is due to fact that for strong magnetic fore, the Lorenz force becomes dominant and then the temperature of the liquid increased. Figure 3 shows the effects of the mixed convection parameter λ on the velocity profile when $M, \beta, \Pr$ are fixed. It is worth noticing that by increasing the parameter λ reveals that buoyancy because of augments of gravity which boosts on the velocity $f'(\eta)$. Besides that, the thickness of boundary layer for large λ is also getting higher. In Figure 4 we show that for larger λ, this would lead to increase in the temperature profile (this is much related to decrease in the boundary layer thickness). Figure 5 is plotted to show the for variation of Prandtl number $\Pr$ on $f(\eta)$. It is notice that from Figure 5, Prandtl number $\Pr$ has same effect on $f'(\eta)$ same as temperature in Figure 6. Figure 6 is sketched for the variation of Prandtl number $\Pr$ on

$\theta(\eta)$. It is noted that for lager $\Pr$,the thermal field is lower and then this reduce the temperature. In fact law Prandtl number $\Pr$ assist fluid with higher thermal conductivity and this create thicker thermal boundary layer than that for lager $\Pr$ .. Finally, Figs.7-8 shows the effects of viscosity parameter β on velocity and temperature profiles over the sheet. In fact this parameter has dual behavior in terms of temperature and velocity. As we know, by increasing in β reduces viscosity of fluid, so we see that the effect is very small for both profiles. Moreover the viscous case is recovered by putting $\beta = 0$.

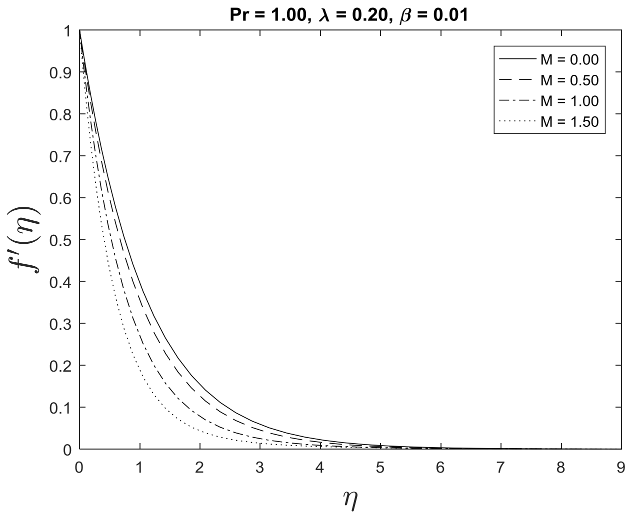

Figure 1: Effects of Hartman number M for $f'(\eta)$.

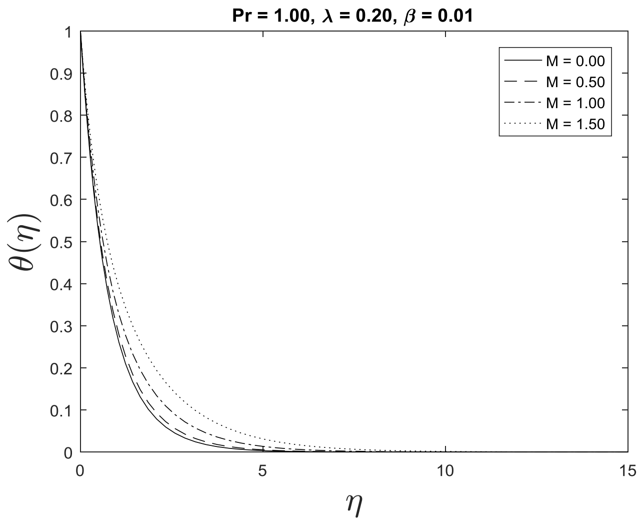

Figure 2: Effects of Hartman number M for $\theta(\eta)$.

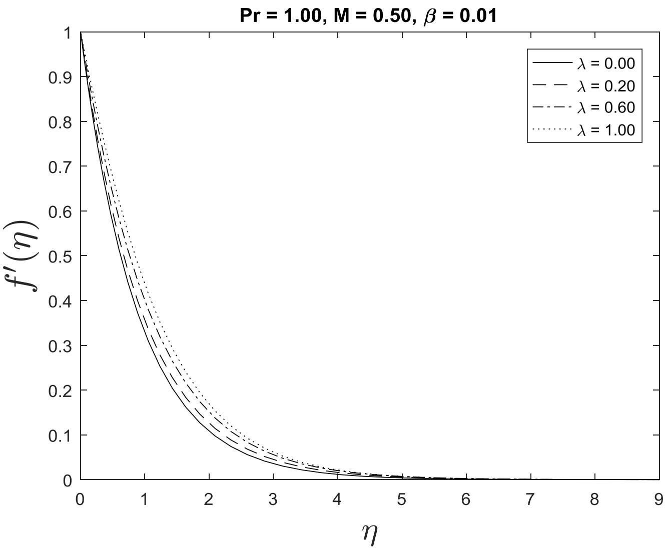

Figure 3: Effects of mixed convection parameter λ for $f'(\eta)$.

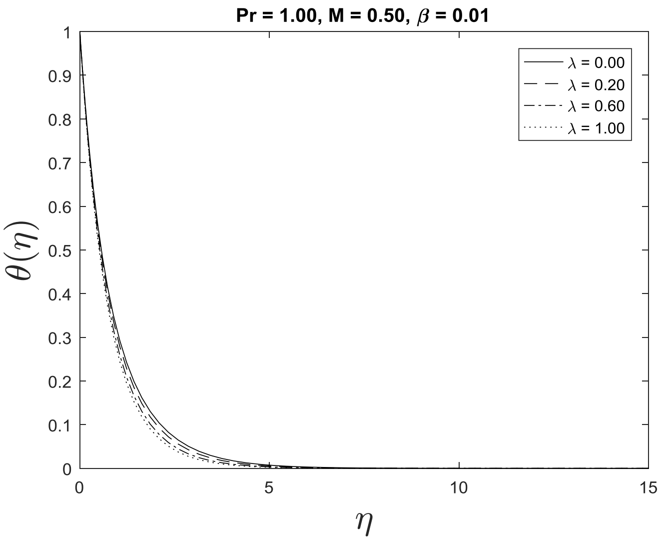

Figure 4: Effects of mixed convection parameter λ for $\theta(\eta)$.

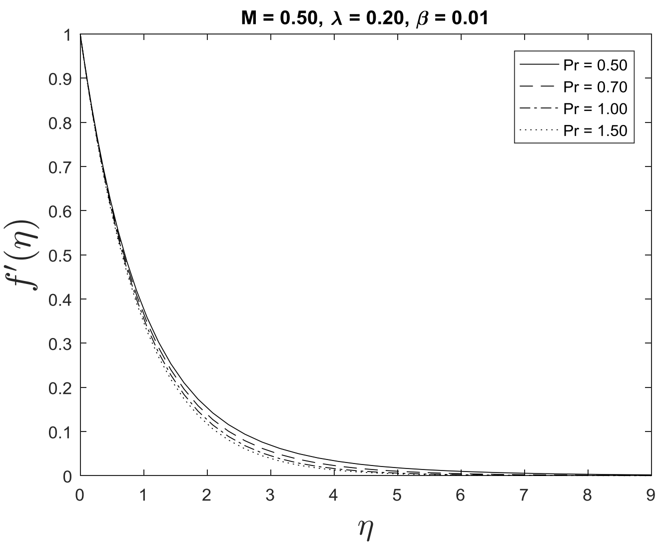

Figure 5: Effects of Prandtl number $\Pr$ for $f'(\eta)$.

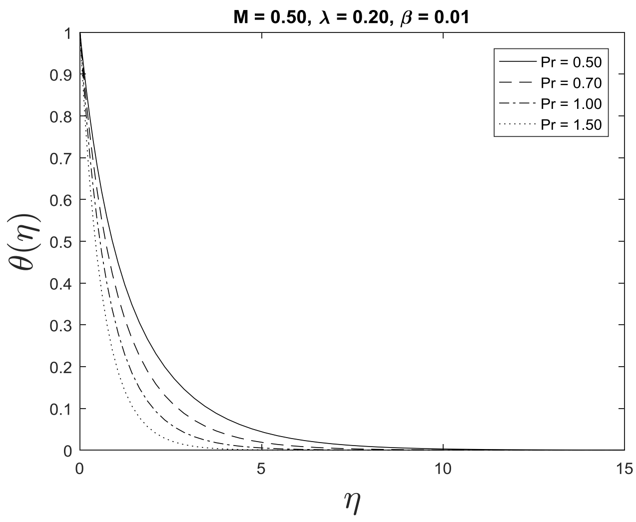

Figure 6: Effects of Prandtl number $\Pr$ for $\theta(\eta)$.

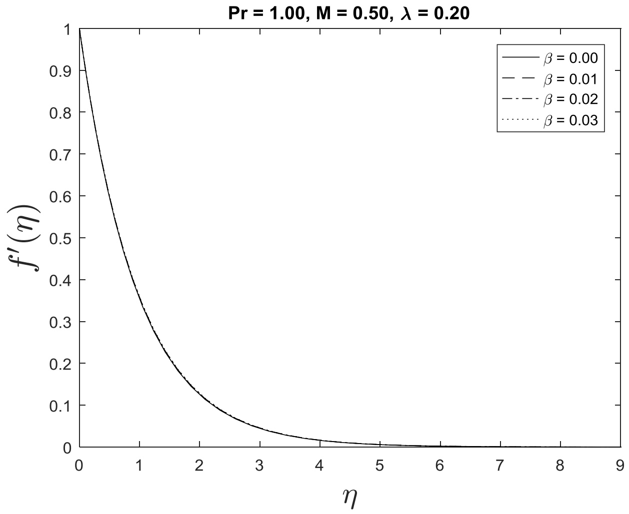

Figure 7: Effects of Deborah number β for $f'(\eta)$.

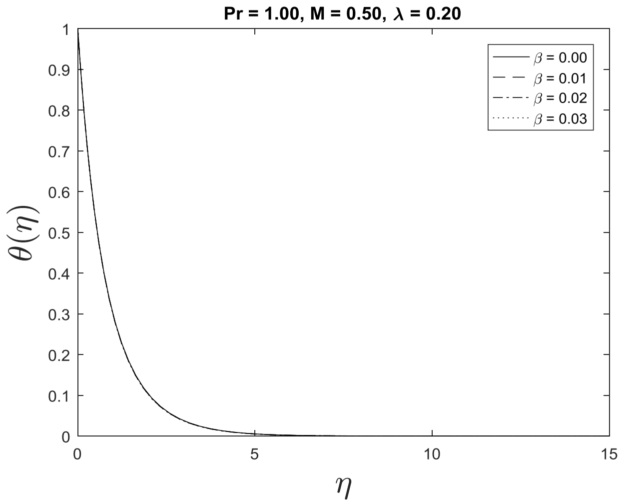

Figure 8: Effects of Deborah number β for $\theta(\eta)$.

V. CONCLUSION

In this research, the problem of MHD heat transfer of an incompressible second grade fluid on a stretching sheet channel is solved numerically. The numerical solutions are well established by SLM. The influence of various parameters is shown through different graphs. The present results have been tested and compared with the available published results in Ref: [6] and Ref: [14], in a limiting situation shown in tables IV - V and an excellent agreement is found. The mean results from solving theses equations have been summarized as follows:

1- The effects of M on velocity and temperature is quite opposite.
2- The variation of Prandtl number $\Pr$ on velocity and temperature is similar.
3- By increasing M and $\Pr$, the velocity and temperature fields decrease.
4- The variation of M and λ on velocity and temperature are similar.
5- The results corresponding to viscous fluid can be obtained by choosing $\beta = 0$.

REFERENCES

[1] R. Ellahi, T Hayat, T Javed, S Asghar, “On the analytic solution of nonlinear flow problem involving Oldroyd 8-constant fluid,” Mathematical and Computer Modelling, vol. 48, no. 7-8, pp. 1191-1200, 2008.

[2] T. Hayat, Masood Khan, and M. Ayub, “Couette and Poiseuille flows of an Oldroyd 6-constant fluid with magnetic field,” Journal of mathematical analysis and applications. vol. 298, no. 1, pp. 225-244, 2004.

[3] C. Fetecau, Sharat C. Prasad, and K. R. Rajagopal, “A note on the flow induced by a constantly accelerating plate in an Oldroyd-B fluid,” Applied mathematical modelling, vol. 31, no. 4, pp. 647-654, 2007.

[4] F. Salah, Z. Abdul Aziz, C. Ching, D Ling, “On accelerated MHD flows of second grade fluid in a porous medium and rotating frame,” IAENG International Journal of Applied Mathematics, vol. 43, no. 3, pp. 106-113, 2013.

[5] T. Hayat, C. Fetecau, and M. Sajid, “Analytic solution for MHD transient rotating flow of a second grade fluid in a porous space,” Nonlinear Analysis: Real World Applications, vol. 9, no. 4, pp. 1619-1627, 2008.

[6] R. Cortell, “A note on flow and heat transfer of a viscoelastic fluid over a stretching sheet,” International Journal of Non-Linear Mechanics, vol. 41, no. 1, pp. 78-85, 2006.

[7] Z.A. Aziz, F. Salah, L. Dennis, “On Accelerated Flow for MHD Generalized Burgers’ Fluid in a Porous Medium and Rotating Frame,” IAENG International Journal of Applied Mathematics, vol. 41, no. 3, pp. 199–205, 2011.

[8] A. N. Kashif, Z.A. Aziz, F. Salah , K. K. Viswanathan, “Convective Heat Transfer in the Boundary Layer Flow of a Maxwell Fluid Over a Flat Plate Using an Approximation Technique in the Presence of Pressure Gradient, “ Engineering Letters, vol. 26, no. 1, pp. 14-22, 2018.

[9] T. Hayat, M. Awais, and S. Asghar, “Radiative effects in a three-dimensional flow of MHD Eyring-Powell fluid,” Journal of the Egyptian Mathematical Society, vol. 21, no. 3, pp. 379-384, 2013.

[10] M. Y. Malik, A. Hussain, and S. Nadeem, “Boundary layer flow of an Eyring–Powell model fluid due to a stretching cylinder with variable viscosity,” Scientia Iranica, vol. 20, no. 2, pp. 313-321, 2013.

[11] M. Hussain, T. Hayat, S. Asghar, C. Fetecau, “Oscillatory flows of second grade fluid in a porous space,” Nonlinear Analysis: Real World Applications, vol. 11, no. 4, pp. 2403-2414, 2010.

[12] M. Husain, T. Hayat, C. Fetecau, S. Asghar, “On accelerated flows of an Oldroyd-B fluid in a porous medium,” Nonlinear Analysis: Real World Applications, vol. 9, no. 4, pp. 1394-1408, 2008.

[13] M. Waqas, M. Khan, T. Hayat, A. Alsaedi, “Stratified flow of an Oldroyd-B nanoliquid with heat generation,” Results in Physics, vol. 7, pp. 2489-2496, 2017.

[14] SS. Ghadikolaei, K. Hosseinzadeh, M. Yassari, “Analytical and numerical solution of non-Newtonian second-grade fluid flow on a stretching sheet,” Thermal Science and Engineering Progress, vol. 5, pp. 309-316, 2018.

[15] X. Si, H Li, L. Zheng, Y. Shen, X. Zhang, “A mixed convection flow and heat transfer of pseudo-plastic power law nanofluids past a stretching vertical plate,” International Journal of Heat and Mass Transfer, vol. 105, pp. 350-358, 2017.

[16] H. Zargartalebi, M. Ghalambaz, A. Noghrehabadi, “Stagnation-point heat transfer of nanofluids toward stretching sheets with variable thermo-physical properties,” Advanced Powder Technology, vol. 26, no. 3, pp. 819-829, 2015.

[17] A M. Megahed, “Variable fluid properties and variable heat flux effects on the flow and heat transfer in a non-Newtonian Maxwell fluid over an unsteady stretching sheet with slip velocity,” Chinese Physics B, vol. 22, no. 9, pp. 094701, 2013.

[18] K. Sadeghy, H. Hajibeygi, SM. Taghavi, “Stagnation-point flow of upper-convected Maxwell fluids,” International Journal of Non-Linear Mechanics, vol. 41, no. 10, pp. 1242-1247, 2006.

[19] S. Mukhopadhyay, “Heat transfer analysis of the unsteady flow of a Maxwell fluid over a stretching surface in the presence of a heat source/sink,” Chinese Physics Letters, vol. 29, no. 5, pp, 054703, 2012.

[20] S. J. Liao, “The proposed homotopy analysis technique for the solution of nonlinear problems,” Diss. Ph. D. Thesis, Shanghai Jiao Tong University, 1992.

[21] S.J. Liao, “On the homotopy analysis method for nonlinear problems,” Applied Mathematics and Computation. vol. 147, no. 2 , pp. 499-513, 2004.

[22] A. Cheniguel, H. Benaddi, “The He Homotopy Perturbation Method for Heat-like Equation with Variable Coefficients and Non Local Conditions,” Proceedings of the International MultiConference of Engineers and Computer Scientists, vol. I, Hong Kong, 2018.

[23] JH. He, “Homotopy perturbation technique,” Computer methods in applied mechanics and engineering, vol. 178, no. 3-4, pp. 257-262, 1999.

[24] Q. Esmaili, A. Ramiar, E. Alizadeh, DD. Ganji, “An approximation of the analytical solution of the Jeffery–Hamel flow by decomposition method,” Physics Letters A, vol. 372, no. 19, pp. 3434-3439, 2008.

[25] O. D. Makinde, PY. Mhone, “Hermite–Padé approximation approach to MHD Jeffery–Hamel flows,”Applied Mathematics and Computation, vol. 181, no. 2, pp. 966-972, 2006.

[26] O. D. Makinde, “Effect of arbitrary magnetic Reynolds number on MHD flows in convergent-divergent channels,” International Journal of Numerical Methods for Heat & Fluid Flow, vol. 18, no. 6, pp. 697-707, 2008.

[27] S. Shateyi, SS. Motsa, “Variable viscosity on magnetohydrodynamic fluid flow and heat transfer over an unsteady stretching surface with Hall effect,” Boundary Value Problems, vol. 2010, no. 1, pp. 257-568, 2010.

[28] Z. Makukula, SS. Motsa, P. Sibanda, “On a new solution for the viscoelastic squeezing flow between two parallel plates,” Journal of Advanced Research in Applied Mathematics, vol. 2, no. 4, pp. 31-38, 2010.

[29] ZG. Makukula, P. Sibanda, SS. Motsa, “A novel numerical technique for two-dimensional laminar flow between two moving porous walls,” Mathematical problems in Engineering, vol. 2010, 2010.

[30] MAM. Ahmed, ME. Mohammed,AA. Khidir, “On linearization method to MHD boundary layer convective heat transfer with low pressure gradient,” Propulsion and Power Research. vol. 4, no. 2, pp. 105-113, 2015.

[31] M.S. ABEL, JV. TAWADE, MM. NANDEPPANAVAR, “MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet,” Meccanica, vol. 47, no. 2 , pp. 385-393, 2012.