Dynamic Behaviors a Single Species Stage Structure Model with Density Dependent Birth Rate and Non-selective Harvesting in a Partial Closure

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***Abstract*—A single species stage structure model with density dependent birth rate and non-selective harvesting in a partial closure is proposed and studied in this paper. By constructing some suitable Lyapunove function, sufficient conditions which ensure the global stability of the boundary equilibrium and positive equilibrium are obtained, respectively. Our results indicates that once the system admits a unique positive equilibrium, it is globally stable. Also, as a direct corollary of our result, we show that the condition which ensure the local stability of the boundary equilibrium of the system is enough to ensure its globally asymptotically stable. Our results supplement and complement some known results.**

***Index Terms*—Stage structure, density dependent birth rate, global stability.**

I. Introduction

DURING the last decade, many scholars investigated the dynamic behaviors of the ecosystem, see [1]-[42] and the references cited therein. Such topics as the extinction, persistent and stability are extensively investigated. It is well known that many species take several stage throughout their life, and to modeling such kind of phenomenon, many scholars ([1]-[21],[41]-[42]) proposed the stage structured population system, and some interesting results were obtained.

As far as the stage structured model is concerned, two different ideas were applied in establishing the corresponding modelling. The first one is to assume that the species needs time to grow up, and this leads to the delayed model. For example, Chen, Wang, Lin et al[3] studied the persistent property of the following stage-structured predator-prey model

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= r_1x_2(t) - d_{11}x_1(t) - r_1e^{-d_{11}\tau_1}x_2(t-\tau_1),\\
\frac{dx_2(t)}{dt} &= r_1e^{-d_{11}\tau_1}x_2(t-\tau_1) - d_{12}x_2(t)\\
&\quad -b_1x_2^2(t) - c_1x_2(t)y_2(t),\\
\frac{dy_1(t)}{dt} &= r_2y_2(t) - d_{22}y_1(t) - r_2e^{-d_{22}\tau_2}y_2(t-\tau_2),\\
\frac{dy_2(t)}{dt} &= r_2e^{-d_{22}\tau_2}y_2(t-\tau_2) - d_{21}y_2(t)\\
&\quad -b_2y_2^2(t) + c_2y_2(t)x_2(t).
\end{aligned}$$

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They obtained a set of sufficient conditions which ensure the globally asymptotically stable of the positive equilibrium. Chen et al[2] investigated the persistent and extinction property of the above system, they showed that the extinction of the prey species did not always follows the extinction of the predator species. For more works on stage structured model incorporating time delay, one could refer to [1]-[14], [42] and the references cited therein.

Another way to constructing the stage structured ecosystem is to assume that there are proportional number of immature species becomes mature species. In this case, the single species stage structured system takes the following form.

$$\begin{aligned}
\frac{dx_1}{dt} &= \alpha x_2(t) - \beta x_1(t) - \delta_1 x_1(t),\\
\frac{dx_2}{dt} &= \beta x_1(t) - \delta_2 x_2(t) - \gamma x_2^2(t),
\end{aligned}$$

Recently, based on this model, Xiao and Lei[17] proposed the following single species stage structure system incorporating partial closure for the populations and non-selective harvesting:

$$\begin{aligned}
\frac{dx_1}{dt} &= \alpha x_2 - \beta x_1 - \delta_1 x_1 - q_1Emx_1,\\
\frac{dx_2}{dt} &= \beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2Emx_2.
\end{aligned}$$

The authors showed that the birth rate of the immature species and the fraction of the stocks for the harvesting plays crucial role on the dynamic behaviors of the system. For more works on this direction, one could refer to [15]-[17],[21]-[22],[41] and the references cited therein.

It brings to our attention that to this day, though there are many papers investigated the dynamic behaviors of the stage structured ecosystem, all of those papers ([1]-[18],[20]-[22]) assumed that the birth rate of the species is proportion to the mature species, and only Liang and Zhou[19] considered the stage structured system with nonlinear birth rate. Indeed, they proposed the following single species model

$$\begin{aligned}
\dot{x}(t) &= \frac{p}{q+x}y - d_1x - \delta x,\\
\dot{y}(t) &= \delta x - d_2y - Ey,
\end{aligned} \tag{1.1}$$

where x, y are the density of the immature and mature species at time t, respectively. $\frac{p}{q+x}$ is the birth rate of the immature species, d_1 is the death rate of the immature species, and δ is the transform rate of the immature species to mature species, d_2 is the death rate of the mature species, E is the harvesting effort.

The system may admits two non-negative equilibrium $A(0,0)$ and $B(x^*,y^*)$.

By analysing the sign of the characteristic root of Jacobian matrix, the authors obtained the following results.

Theorem A. *If* $p < \dfrac{q(d_2+E)(d_1+\delta)}{\delta}$ *holds, then* $O(0,0)$ *is locally asymptotically stable.*

By constructing the suitable Dulac function, the authors obtained the following result.

Theorem B. *If* $p > \dfrac{q(d_2+E)(d_1+\delta)}{\delta}$ *holds, then the unique positive equilibrium* $B(x^*,y^*)$ *is globally asymptotically stable.*

From the expression of Theorem A and B, two interesting issue is proposed:

(1) Is it possible for us to investigate the global stability property of the boundary equilibrium?

(2) It is well known that Dulac criterion could only be applied to the two dimensional system, and could not be applied to the system more then two dimension. However, if we further consider the influence of the predator or competition or mutualism, etc, and try to propose a stage structured multispecies ecosystem, the system may become three or four dimension, and it becomes impossible to investigate the dynamic behaviors by using the technique of Liang and Zhou[24]. Hence, it is necessary to give a new proof of the Theorem B, and the method of the new proof could be applied to higher dimensional system.

Now let's consider the following example.

Example 1.1. Consider the following system

$$\begin{aligned} \dot{x}(t) &= \frac{1}{1+x}y - x - x, \\ \dot{y}(t) &= x - y - y, \end{aligned} \tag{1.2}$$

Here, we assume that $p=q=d_1=\delta=E=d_2=1$. One could easily check that $p=1<4=\dfrac{q(d_2+E)(d_1+\delta)}{\delta}$ holds, hence, from Theorem A, one could see that $O(0,0)$ is locally asymptotically stable. However, numeric simulation (Fig. 1 and 2) shows that $O(0,0)$ is globally asymptotically stable.

From Example 1.1, we have the following conjecture.

Conjecture. *If* $p < \dfrac{q(d_2+E)(d_1+\delta)}{\delta}$ *holds, then* $O(0,0)$ *is globally asymptotically stable.*

On the other hand, as was pointed out by Chakraborty, Das and Kar[36], the study of resource-management including fisheries, forestry and wildlife management has great importance, it is necessary to harvest the population but harvesting should be regulated, such that both the ecological sustain ability and conservation of the species can be implemented in a long run. Chakraborty, Das and Kar[36] proposed the following predator-prey model incorporating partial closer

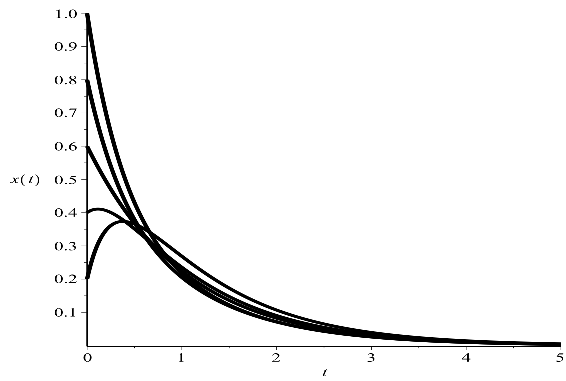

Fig. 1. Dynamic behaviors of the system (1.2), the initial condition $(x(0),y(0)) = (1,0.3),(0.8,0.5),(0.6,1),(0.4,1.4)$ and $(0.2,2)$, respectively.

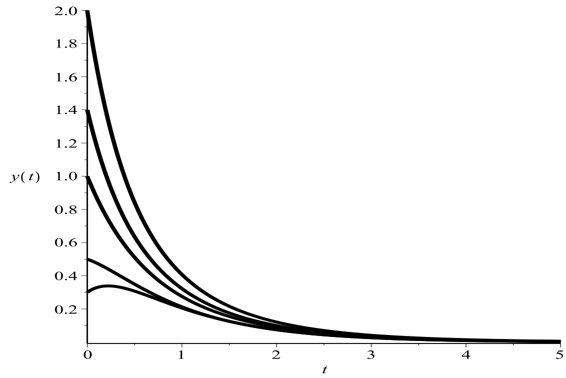

Fig. 2. Dynamic behaviors of the system (1.2), the initial condition $(x(0),y(0)) = (1,0.3),(0.8,0.5),(0.6,1),(0.4,1.4)$ and $(0.2,2)$, respectively.

and non-selective harvesting

$$\begin{aligned} \frac{dx}{dt} &= rx\Big(1-\frac{x}{K}\Big) - \frac{\alpha xy}{a+bx+cy} - q_1 mEx, \\ \frac{dy}{dt} &= sy\Big(1-\frac{y}{L}\Big) + \frac{\beta xy}{a+bx+cy} - q_2 mEx. \end{aligned}$$

Since then, many scholars ([17], [36]-[37]) investigated the dynamic behaviours of the nonselective harvesting ecosystem incorporating partial closure. However, to this day, only Xiao and Lei[17] proposed and studied the stability property of the stage structured model incorporating partial closure. Still no scholar propose and study the stage structured model with nonlinear birth rate and non-selective harvesting in a partial closure. This motivated us to propose the following model:

$$\begin{aligned} \dot{x}(t) &= \frac{p}{q+x}y - d_1 x - \delta x - q_1 Emx, \\ \dot{y}(t) &= \delta x - d_2 y - q_2 Emy, \end{aligned} \tag{1.3}$$

where $p,q,d_1,E,m,\delta,d_2,$ are all positive constants, and q_1, q_2 are nonnegative constants, m is a nonnegative constants which satisfies $0 \le m \le 1$. Obviously, if we take $m=1, q_1=0, q_2=1$, then system (1.3) will reduce to system (1.1). Hence our model can be seen as the generalization of the model (1.1). However, there are some essential differences between model (1.1) and (1.3):

(1) We assume that both immature and mature species could be harvested, while in model (1.1), the author only assume that the mature species could be harvested;

(2) We assume that there are some partial of closure where species could not be harvested, which is described by parameter m.

By direct computation, one could easily see that the model always admits boundary equilibrium $O(0,0)$, also, if

$$p > \frac{q(d_2 + Emq_2)(d_1 + \delta + Emq_1)}{\delta} \tag{1.4}$$

holds, then system (1.3) admits the unique positive equilibrium $E(x^*, y^*)$, where

$$\begin{aligned} x^* &= \frac{H}{(d_2 + Emq_2)(d_1 + \delta + Emq_1)}, \\ y^* &= \frac{\delta H}{(d_2 + Emq_2)^2(d_1 + \delta + Emq_1)}. \end{aligned} \tag{1.5}$$

where $H = \delta p - q(d_2 + Emq_2)(d_1 + \delta + Emq_1)$.

The aim of this paper is to investigate the dynamic behaviors of the system (1.3). More precisely, we will investigate the stability property of the boundary equilibrium in the next section and the global stability property of the positive equilibrium in Section 3, respectively. As a corollary, we will prove the conjecture, and to give a new proof of the global stability of the positive equilibrium of the system (1.1). We end this paper by a briefly discussion.

II. Global stability of the boundary equilibrium

Concerned with the global stability of the boundary equilibrium $O(0,0)$, we have the following result:

Theorem 2.1. *Assume that*

$$p < \frac{q(d_2 + Emq_2)(d_1 + \delta + Emq_1)}{\delta} \tag{2.1}$$

holds, then the boundary equilibrium $O(0,0)$ of system (1.3) is globally stable.

Proof. Let's consider the Lyapunov function

$$V_1(x,y) = x + \frac{d_1 + \delta + q_1 Em}{\delta} y. \tag{2.2}$$

One could easily see that the function V is zero at the boundary equilibrium $O(0,0)$ and is positive for all other positive values of x, y. The time derivative of $V(x,y)$ along the trajectories of (1.3) is

$$\begin{aligned} & D^+V_1(t) \\ = {} & \frac{p}{q+x} y - d_1 x - \delta x - q_1 Emx \\ & + \frac{d_1 + \delta + q_1 Em}{\delta} \Big(\delta x - d_2 y - q_2 Emy \Big) \\ = {} & \frac{p}{q+x} y - \frac{d_1 + \delta + q_1 Em}{\delta} \Big(d_2 + q_2 Em \Big) y \\ \leq {} & \frac{p}{q} y - \frac{d_1 + \delta + q_1 Em}{\delta} \Big(d_2 + q_2 Em \Big) y \\ = {} & \Big(\frac{p}{q} - \frac{d_1 + \delta + q_1 Em}{\delta} \Big(d_2 + q_2 Em \Big) \Big) y. \end{aligned} \tag{2.3}$$

It then follows from

$$p < \frac{q(d_2 + Emq_2)(d_1 + \delta + Emq_1)}{\delta}$$

that $D^+V_1(t) < 0$ strictly for all $x, y > 0$ except the boundary equilibrium $A(0,0)$, where $D^+V_1(t) = 0$. Thus, $V_1(x,y)$ satisfies Lyapunov's asymptotic stability theorem ([21]), and the boundary equilibrium $A(0,0)$ of system (1.1) is globally asymptotically stable.

This ends the proof of Theorem 2.1.

As a direct corollary of Theorem 2.1, now let's consider the stability property of $A(0,0)$ of the system (1.1), we have

Corollary 2.1. *Assume that* $p < \dfrac{q(d_2 + E)(d_1 + \delta)}{\delta}$ *holds, then $O(0,0)$ of system (1.1) is globally stable.*

Remark 2.1. Corollary 2.1 gives the positive answer to the Conjecture.

III. Global stability of the positive equilibrium

Concerned with the stability property of the positive equilibrium $E(x^*, y^*)$, we have the following result.

Theorem 3.1. *Assume that*

$$p > \frac{q(d_2 + Emq_2)(d_1 + \delta + Emq_1)}{\delta} \tag{3.1}$$

holds, then system (1.3) admits the unique positive equilibrium $E(x^*, y^*)$, which is globally stable.

Proof. Let $E(x^*, y^*)$ be the positive equilibrium of the system (1.3), obviously, $B(x^*, y^*)$ satisfies

$$\begin{aligned} \frac{p}{q + x^*} y^* - d_1 x^* - \delta x^* - q_1 Emx^* &= 0, \\ \delta x^* - d_2 y^* - q_2 Emy^* &= 0, \end{aligned} \tag{3.2}$$

Let's consider the Lyapunov function

$$\begin{aligned} V_2(x,y) = {} & K_1 \Big(x - x^* - x^* \ln \frac{x}{x^*} \Big) \\ & + K_2 \Big(y - y^* - y^* \ln \frac{y}{y^*} \Big). \end{aligned}$$

One could easily see that the function V_2 is zero at the positive equilibrium $E(x^*, y^*)$ and is positive for all other positive values of x, y. The time derivative of $V_2(x,y)$ along the trajectories of (1.3) is

$$\begin{aligned} & D^+V_2(t) \\ = \; & K_1\frac{x-x^*}{x}\Big(\frac{p}{q+x}y - d_1x - \delta x - q_1Emx\Big) \\ & +K_2\frac{y-y^*}{y}\Big(\delta x - d_2y - q_2Emy\Big) \\ = \; & K_1\frac{x-x^*}{x}\Big(\frac{p}{q+x}y - \frac{p}{q+x^*}y \\ & +\frac{p}{q+x^*}y - d_1x - \delta x - q_1Emx\Big) \\ & +K_2\frac{y-y^*}{y}\Big(\delta x - d_2y - q_2Emy\Big) \\ = \; & -K_1\frac{yp}{x(q+x)(q+x^*)}(x-x^*)^2 \\ & +K_1\frac{x-x^*}{x}\Big(\frac{p}{(q+x^*)x^*}yx^* \\ & -d_1x - \delta x - q_1Emx\Big) \\ & +K_2\frac{y-y^*}{y}\Big(\delta x - d_2y - q_2Emy\Big) \\ = \; & -K_1\frac{yp}{x(q+x)(q+x^*)}(x-x^*)^2 \\ & +K_1\frac{x-x^*}{x}\Big(\frac{p}{(q+x^*)x^*}(yx^*-y^*x) \\ & +\frac{p}{(q+x^*)x^*}y^*x - d_1x \\ & -\delta x - q_1Emx\Big) \\ & +K_2\frac{y-y^*}{y}\Big(\frac{\delta}{y^*}(xy^*-yx^*) \\ & +\delta\frac{yx^*}{y^*} - d_2y - q_2Emy\Big) \end{aligned} \tag{3.3}$$

From the first equation of (2.1), we have

$$\begin{aligned} \frac{p}{q+x^*}y^* &= (d_1+\delta+q_1Em)x^*, \\ \delta x^* &= (d_2+q_2Em)y^*. \end{aligned} \tag{3.4}$$

Substituting (3.4) into (3.3), leads to

$$\begin{aligned} & D^+V_2(t) \\ = \; & -K_1\frac{yp}{x(q+x)(q+x^*)}(x-x^*)^2 \\ & +K_1\frac{x-x^*}{x}\Big(\frac{p}{(q+x^*)x^*}\times \\ & \Big(y(x^*-x)+x(y-y^*)\Big)\Big) \\ & +K_2\frac{y-y^*}{y}\frac{\delta}{y^*}\Big(x(y^*-y)+y(x-x^*)\Big) \\ = \; & -K_1\frac{yp}{x(q+x)(q+x^*)}(x-x^*)^2 \\ & -K_1\frac{py}{x(q+x^*)x^*}(x-x^*)^2 \\ & -K_2\frac{\delta x}{y^*y}(y-y^*)^2 \\ & +\Big[\frac{K_1p}{x^*(q+x^*)}+\frac{K_2\delta}{y^*}\Big](x-x^*)(y-y^*). \end{aligned} \tag{3.5}$$

Now let's take $K_2 = 1, K_1 = \dfrac{\delta}{y^*}\dfrac{x^*(q+x^*)}{p}$, then

$$\begin{aligned} & D^+V_2(t) \\ = \; & -\frac{\delta}{y^*}\frac{x^*(q+x^*)}{p}\frac{yp}{x(q+x)(q+x^*)}(x-x^*)^2 \\ & -\frac{\delta}{y^*}\frac{x^*(q+x^*)}{p}\frac{py}{x(q+x^*)x^*}(x-x^*)^2 \\ & -\frac{\delta x}{y^*y}(y-y^*)^2 \\ & +\Big[\frac{\delta}{y^*}\frac{x^*(q+x^*)}{p}\frac{p}{x^*(q+x^*)}+\frac{\delta}{y^*}\Big]\times \\ & (x-x^*)(y-y^*) \end{aligned} \tag{3.6}$$

$$\begin{aligned} = \; & -\frac{\delta x^*y}{y^*x(q+x)}(x-x^*)^2 - \frac{\delta y}{y^*x}(x-x^*)^2 \\ & +\frac{2\delta}{y^*}(x-x^*)(y-y^*) - \frac{\delta x}{y^*y}(y-y^*)^2 \\ = \; & -\frac{\delta x^*y}{y^*x(q+x)}(x-x^*)^2 \\ & -\frac{\delta}{y^*}\Big[\sqrt{\frac{y}{x}}(x-x^*) - \sqrt{\frac{x}{y}}(x-x^*)\Big]^2. \end{aligned}$$

Therefore, $D^+V_2(t) < 0$ strictly for all $x, y > 0$ except the positive equilibrium $B(x^*, y^*)$, where $D^+V_2(t) = 0$. Thus, $V_2(x,y)$ satisfies Lyapunov's asymptotic stability theorem, and the positive equilibrium $B(x^*, y^*)$ of system (1.1) is globally asymptotically stable.

This ends the proof of Theorem 3.1.

As a direct corollary of Theorem 3.1, concerned with the global stability of the positive equilibrium $B(x^*, y^*)$ of system (1.1), we have the following result.

Corollary 3.1. *Assume that*

$$p > \frac{q(d_2+E)(d_1+\delta)}{\delta}$$

holds, then the unique positive equilibrium $B(x^*, y^*)$ is globally asymptotically stable.

Remark 3.1. As was pointed out in section 1, Liang and Zhou[19] had proved Theorem B by using the Dulac criterion, here, we prove the Theorem 3.1 by constructing some suitable Lyapunov function. Noting that system (1.1) is the special case of system (1.3), hence we can say that we have proved the Theorem B by using the differential method, and this method maybe applied to higher dimension ecological modelling, we will attempt to do more in-depth research in the future.

IV. Conclusion

Liang and Zhou[19] proposed the system (1.1). By using the Dulac criterion, they could obtained the sufficient conditions, which ensure the global asymptotical stability of the positive equilibrium, however, they could only investigated the local stability property of the boundary equilibrium. In this paper, we first propose a conjecture, that is, the conditions which ensure the local stability of the boundary equilibrium is enough to ensure its globally stability. Next, stimulated by the works of Chakraborty, Das and Kar[20] and Xiao and Lei[17], we argued that to this day, still no scholars consider the stage structured system with nonlinear birth rate and non-selective harvesting in partial closure. We propose the system (1.3), which can be seen as the generalization of the system (1.1).

By constructing some suitable Lyapunov functions, we investigate the stability property of the boundary equilibrium and positive equilibrium of the system (1.3).

Theorem 2.1 gives condition which ensure the global stability of the boundary equilibrium. As a corollary of Theorem 2.1, we could obtain the corollary 2.1. Compared with the corollary 2.1 and Theorem A, we have the following result.

Conclusion A. The condition which ensure the local stability of the positive and boundary equilibria of the system (1.1) is enough to ensure its' global stability.

On the other hand, by constructing some suitable Lyapunov function, also, by some technical analysis, we give the sufficient conditions which ensure the global stability of the positive equilibrium of the system (1.3). From the expression of the positive equilibrium (see (1.4)), we could see that the conditions which ensure the existence of the positive equilibrium is the same as that the conditions which ensure the global stability of the positive equilibrium. Therefore, we can draw the conclusion.

Conclusion B. Once the system (1.3) admits a unique positive equilibrium, it is globally asymptotically stable.

V. Declarations

Competing interests

The authors declare that there is no conflict of interests.

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Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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