# The Fractional Relationship between Viscosity and Surface Tension on Lubricating Oils

Endang Rusyaman, Kankan Parmikanti, Diah Chaerani and Sudradjat Supian

*Abstract*—**Viscosity is a measure which represents the magnitude of a moving fluid. Meanwhile, the surface tension of a fluid is the stretched elastic tendency of the fluid due to the attraction force between the molecules. Previous studies are interested in the fluid mechanics problem presenting the relationship between the viscosity and surface tension in both linear and exponential models. The aim of this study is to present the the discourse about the fractional relationship in the form of fractional differential equation model based on the empirical data of the measurement of surface tension and viscosity of lubricating oil in the laboratory. To find solution of fractional differential equation, Laplace transform and Mittag-Lefler function are used. The output of this research is a proposed fractional model and some graphs showing the relationship between them.**

*Index Terms*—**viscosity, surface-tension, fluid, relationship, fractional.**

## I. Introduction

VISCOSITY is a measure of fluid that states the magnitude of friction between molecules in a fluid. The viscous the fluid the greater the friction in the fluid, so the more difficult the liquid to flow and an object increasingly difficult to move in viscous fluid. In the liquid, viscosity occurs is due to the force of cohesion while in the gas is the result of collisions between molecules. The measure of the viscosity of a fluid is called the viscosity coefficient with notation $\eta$ and the unit is $Nsm$ or Pascal seconds. If an object moves with velocity $v$ in a fluid with a coefficient of viscosity $\eta$, it means that the object will experience a frictional force equal to

$$F_{frc} = k\eta v$$

where $k$ is a constant that depends on the geometric shape of the object. For a spherical object, $k = 6\pi r$ so that the frictional force becomes

$$F_{frc} = 6\pi r\eta v.$$

On the other hand, the surface tension of a fluid is a fluid tendency to stretch, so that the surface is covered by a membrane caused by cohesion. Surface tension is the force on the surface of each unit of length. For a fluid with one surface,

$$\gamma = \frac{F}{d}$$

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where $\gamma$ is the surface tension with the unit $N/m$, $F$ is the force with the unit $N$, and $d$ is the surface length with the unit $m$.

Previous study proposes that there is no relationship between viscosity and the surface tension because surface tension is about steady state while viscosity is about movement [1]. However, based on the empirical data, other studies agree that there is a relationship between both of them. Starting from Pelofsky [2] which provides an empirical relationship between the natural logarithm of surface tension and the inverse of viscosity in a large variety of liquids with the formula of

$$\gamma = A \exp\left(-\frac{B}{\eta}\right)$$

where $A$ and $B$ are constants. According to the study, the model is applicable in both organic and inorganic solutions of pure and mixed components. The Pelofsky formula is then challenged by Schonhorn [3] which argues that it can only be applied in a limited temperature range and will fail at the critical temperature as viscosity is finite while the surface tension is zero. He extends the formula by introducing the concept of vapour viscosity $\eta_v$ and liquid viscosity $\eta_l$ into the following equation:

$$\gamma = A \exp\left(-\frac{B}{\eta_l - \eta_v}\right).$$

The equation appears to be valid for a variety of liquids over the entire range of temperature including critical temperature to the temperature at which the viscosity becomes infinite.

Schonhorn equation, however, is not successful in correlating the temperature where $\eta \to \infty$ gand surface tension at the melting point. Thus, empirical equation has been developed by Ahmari and Amiri [4]:

$$\gamma = A \cdot \frac{Tc - T}{Tc - Tm}\left(1 - \frac{B}{\eta}\right)$$

where $A$ and $B$ are constant while $Tc$ and $Tm$ are critical and melting temperatures respectively. The result of their study indicates that correlation between surface tension and viscosity can be valid in the absence of any surface active agent and capillarity.

Furthermore, A.J. Queimada *et al*, generalized relation between surface tension and viscosity by doing a study on pure and mixed $n$-alkanes [5], [6], whereas for saturated normal fluids, a correlation containing four adjustable coefficients for every fluid were obtained by fitting 200 data points. The search for the model of the relationship between viscosity and surface tension are then continued by several studies [7], [8], [9], [10]. Among them, there are Zheng, Tian and

Mulero [11] who propose the following equation

$$\ln \eta = A + \frac{B}{\gamma^n + C}.$$

Some studies even correlate it with the density level [12].

With regard to the problem of fractional calculus, Mahmood *et al* [13] uses fractional derivative model to find exact analytic solutions for the unsteady flow of a non-Newtonian fluid between two cylinders. The problem on fractional differential equation, as well as fractional viscoelastic fluids were also studied previously [14], [15]. More specifically, there is a Viscoelastic Fluids study that used only fractional derivatives, without differential equations [16], [17], [18], [19], while Hayat *et al* and Khan *et al* discussed periodic unidirectional flow of viscoelastic fluids and potential vortex with the fractional Maxwell model [20], [21]. The next development conducted by Podlubny *et al* [22] uses a Partial Fractional Differential Equation as a method to model the relationship between viscosity and tension surface and continued by Yoon *et al* [23] which applied the equation in a case of viscoelasticity.

## II. Basic Theory

In this section, we present some basic theories that support the subject matter, such as fractional derivatives, fractional differential equations, and Mittag-Lefler functions.

### A. Fractional Derivative

Fractional derivative is derivative of a function with fractional numbers order. At least, there are three prominent definition regarding this concept, namely Grunwald-Letnikov, Riemann-Liouville, and Caputo. Two of them are presented below. First, the definition of Fractional derivatives based on Grunwald-Letnikov.

**Definition 1** Fractional derivatives of $f(x)$ with $\alpha$-order at interval $[a,b]$ are

$$D_x^\alpha f(x) = \lim_{h\to 0} \frac{1}{h^n} \sum_{i=0}^{n} (-1)^i \frac{\Gamma(\alpha+1)}{\Gamma(i+1)\Gamma(\alpha-i+1)} f(x-ih)$$

where $n = \lfloor \frac{b-a}{h} \rfloor$.

On the other hand, based on Riemann-Liouville, definition of fractional derivative is as follows.

**Definition 2** Fractional derivative of $f(x)$ with order $\alpha$ around $x = a$ is

$${}_aD_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^n \int_a^x f(t)(x-t)^{-(\alpha-n+1)} dt$$

with $n-1 <= \alpha <= n$ or $n-1 = \lfloor \alpha \rfloor$.

From several definitions presented, it can be obtained that fractional derivative of $f(x) = x^p$ with order $\alpha$ is

$$D_x^\alpha x^p = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)} x^{p-\alpha}. \quad (1)$$

### B. Fractional Differential Equation

Fractional differential equation is differential equation with fractional order $\alpha_i \in \boldsymbol{Q}$. General form of fractional differential equation is

$$a_1 y^{(\alpha_1)} + a_2 y^{(\alpha_2)} + ... + a_n y^{(\alpha_n)} = f(t)$$

where $a_i$ are real constans, $\alpha_i$ fractional numbers for every $i$, and $y = y(t)$ as a solution function. More specifically, at the last section of this paper, it will be shown that the problem of the relationship between surface tension and viscosity can be expressed in form

$$\gamma^{(\alpha)} + a\gamma = be^{k\eta}$$

where $\gamma$ represents surface tension and $\eta$ represents viscosity.

### C. Mittag-Leffler Function

The Mittag-Leffler function is a function that has an important role in fractional calculus, which can be used to find solutions of the fractional differential equations resulting from Laplace transforms. This function has two parameters, and is very flexible, so it can be transformed into an exponent function, trigonometric function, or other functions depending on the parameter values.

**Definition 3** The Mittag-Leffler function with two parameters $\alpha$ and $\beta$ is defined as follows [22]

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, (\alpha > 0, \beta > 0). \quad (2)$$

Another form of Mittag-Leffler function is $\epsilon_k(t, \lambda; \alpha, \beta)$. This function is used to solve fractional differential equations which is a particular case of the Mittag-Leffler function. The function is defined as

$$\epsilon_k(t, \lambda; \alpha, \beta) = t^{\alpha k + \beta - 1} E_{\alpha,\beta}^{(k)}(\lambda t^\alpha), k = 0, 1, 2, ... \quad (3)$$

with $E_{\alpha,\beta}^{(k)}(z)$ is $k$-order derivatives of Mittag-Leffler function in (2) where

$$E_{\alpha,\beta}^{(k)}(z) = \sum_{i=0}^{\infty} \frac{(i+k)! z^i}{i! \Gamma(\alpha i + \alpha k + \beta)}, k = 0, 1, 2, ....$$

Laplace transformation of Mittag-Leffler function is [22]

$$L[\epsilon_k(t, \pm\lambda; \alpha, \beta)] = \frac{k! s^{\alpha-\beta}}{(s^\alpha \mp \lambda)^{k+1}}. \quad (4)$$

## III. Main Result

For the needs of this research, the samples of 25 types of lubricants from various brands and various types had been taken, then their viscosity and surface tension in the laboratory were measured. The results are presented in the Table I.

As a preliminary, from this data we then obtained some of the following regression models presented in Table II, where $\gamma$ represents surface tension and $\eta$ represents viscosity.

According to those regression results, two model predictions are taken which are fractional models. The first one is a fractional derivative model, and the other one is a fractional differential equation model.

### A. Fractional Derivatives Model

From the power regression presented in Table II, it can be predicted fractional derivative model as follows:

$$\gamma = aD^\alpha \eta + b \quad (5)$$

where $\gamma$ is the surface tension, $\eta$ is the viscosity, $\alpha$ is the fractional order, and $a$, $b$ are the real constants.

TABLE I
VISCOSITY AND SURFACE-TENSION ON ROOM TEMPERATURE $20°C$

| No | Brand | Type | Viscosity | Surface-tension |
|---|---|---|---|---|
| 1 | Brand-1 | SAE 40 | 296 | 19.22 |
| 2 | Brand-1 | SAE 90 | 406 | 19.00 |
| 3 | Brand-1 | B 40 | 342 | 19.25 |
| 4 | Brand-1 | 40 | 260 | 18.98 |
| 5 | Brand-1 | 20W-50 | 339 | 19.12 |
| 6 | Brand-2 | 10W-40 | 193 | 18.68 |
| 7 | Brand-2 | 15W-40 | 257 | 18.94 |
| 8 | Brand-2 | 20W-50 | 367 | 19.13 |
| 9 | Brand-3 | 0W-20 | 50 | 18.14 |
| 10 | Brand-3 | 15W-50 | 277 | 18.68 |
| 11 | Brand-3 | 10W-40 | 189 | 18.58 |
| 12 | Brand-4 | 20W-40 | 363 | 19.03 |
| 13 | Brand-4 | 15W-40 | 246 | 18.68 |
| 14 | Brand-5 | 15W-40 | 225 | 18.77 |
| 15 | Brand-5 | 10W-40 | 184 | 18.80 |
| 16 | Brand-5 | 20W-50 | 336 | 19.29 |
| 17 | Brand-6 | 40 | 350 | 19.25 |
| 18 | Brand-6 | 15W-40 | 230 | 19.01 |
| 19 | Brand-7 | 10W-40 | 183 | 18.74 |
| 20 | Brand-7 | 15W-40 | 207 | 18.95 |
| 21 | Brand-8 | 15W-40 | 234 | 18.88 |
| 22 | Brand-8 | 40 | 361 | 19.19 |
| 23 | Brand-8 | 15W-40 | 229 | 18.97 |
| 24 | Brand-9 | 15W-40 | 205 | 18.58 |
| 25 | Brand-10 | 20W-50 | 269 | 18.74 |

TABLE II
REGRESSION MODEL OF RELATION OF VISCOSITY AND SURFACE-TENSION

| No | Type Regression | Model | R-square |
|---|---|---|---|
| 1 | Linear | $\gamma$ = 0.0027$\eta$+18.182 | 0.6745 |
| 2 | Power | $\gamma$ = 16.128 $\eta^{0.0288}$ | 0.6986 |
| 3 | Exponential | $\gamma$ = 18.189 exp 0.0001$\eta$ | 0.6753 |
| 4 | Logaritmic | $\gamma$ = 0.5393 ln $\eta$ + 15.932 | 0.6928 |

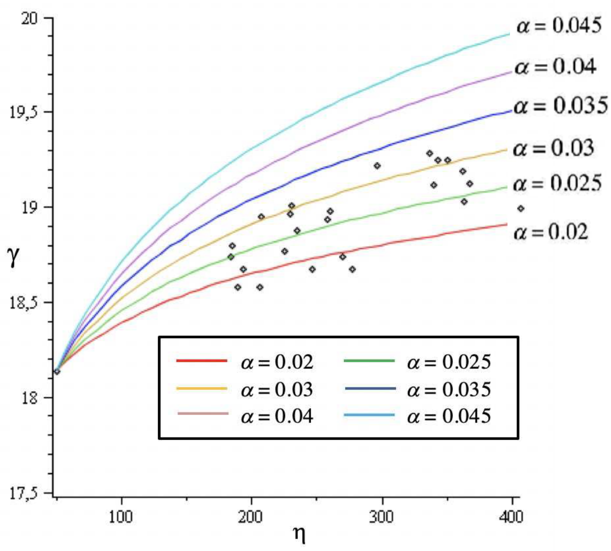


Fig. 1. Graphs of Fractional Derivative Model

Based on the scatter plot of data in Table I, Table II, and the graphs in Figure 1, the fractional derivative model is obtained

$$\gamma = 16.223 D^{0.975}\eta. \tag{6}$$

where $D^{0.975}\eta$ means that "fractional derivative of $\eta$ with order 0.975".

Finally, using equation (1), we get the solution

$$\gamma = 16.45\eta^{0.025}.$$

with green graph in Figure 1. Figure 1 also shows graphs with various numbers of $\alpha$.

Selection of green graph in Figure 1 as the best fractional derivative model is also supported by the value of Mean Square Error (MSE) which is smaller than the other models as shown in Table III.

### B. Fractional Differential Equation (FDE) Model

Based on exponential regression in Table II, the fractional differential equation model can be predicted as follows:

$$\gamma^{(\alpha)} + a\gamma = be^{k\eta} \tag{7}$$

with $0 < \alpha < 1$ and initial conditions $\gamma(0) = 0$.

Solution of this FDE are as follows.

Using the Laplace transforms on both sides and the linearity properties of this transformation, from equation (7) is obtained

$$s^{\alpha}F(s) + aF(s) = \frac{b}{s-k},$$

so that

$$F(s) = \frac{\frac{b}{s-k}}{s^{\alpha}+a} = \sum_{n=0}^{\infty}\frac{\frac{bk^n}{s^{n+1}}}{s^{\alpha}+a}.$$

Furthermore, by using Laplace inverse to $F(s)$, we obtained the function of solution in the form of Mittag-Lefler function as follows:

$$\gamma(\eta) = \sum_{n=0}^{\infty} bL^{-1}\left(\frac{0!s^{\alpha-(\alpha+n+1)}}{(s^{\alpha}+a)^{0+1}}k^n\right).$$

By using equation (4) obtained

$$\gamma(\eta) = \sum_{n=0}^{\infty} bk^n \epsilon_0(\eta, -a; \alpha, (\alpha+n+1)).$$

Thus based on (3), the solution of the fractional differential equation (7) in the form of the Mittag-Lefler function is

$$\gamma(\eta) = \sum_{n=0}^{\infty} bk^n \eta^{\alpha+n} E_{\alpha,\alpha+n+1}(-a\eta^{\alpha})$$

or can be written in form

$$\gamma(\eta) = \sum_{n=0}^{\infty}\sum_{i=0}^{\infty} bk^n\eta^{\alpha+n}\frac{(-a\eta^{\alpha})^i}{\Gamma(\alpha i+\alpha+n+1)}. \tag{8}$$

Based on the scatter plot of data in Table I, can be shown some possible solutions as seen in Figure 2.

Thus, based on the result of observation of solution function in Figure 2 and matching with Table I, the most optimum model is the blue graph with the fractional differential equation model:

$$\gamma^{(\alpha)} + a\gamma = be^{k\eta}$$

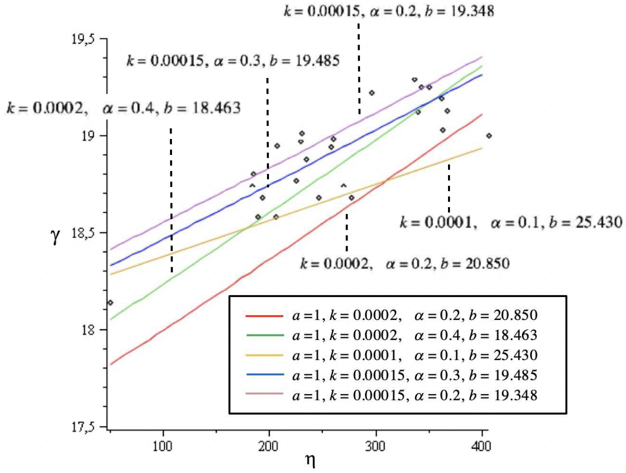


Fig. 2. Graphs of fractional differential equation model

where $a = 1$, $b = 19.485$, $k = 0.00015$, and $\alpha = 0.3$, or explicitly, can be expressed as

$$\gamma^{(0.3)} + \gamma = 19.485e^{0.00015\eta}.$$

Selection of blue graph in Figure 2 as the best fractional differential equation model is also supported by the value of Mean Square Error (MSE) which is smaller than the other models as shown in Table III.

TABLE III
MSE COMPARISON FOR DIFFERENT MODELS

| No | Graph in | MSE |
|---|---|---|
| 1 | Fig. 1 Red Graph | 0.056258 |
| 2 | Fig. 1 Green Graph | 0.022719 |
| 3 | Fig. 1 Yellow Graph | 0.038015 |
| 4 | Fig. 2 Green Graph | 0.033479 |
| 5 | Fig. 2 Blue Graph | 0.023443 |
| 6 | Fig. 2 Purple Graph | 0.034969 |

## IV. Conclusion

Based on the research that has been conducted using the measurement of surface tension and viscosity of various brand of lubricating oils marketed, we found that there is an empirical relationship between surface tension and viscocity. This relationship can be expressed in the regression model or fractional relationship, i.e., the fractional derivative model and the fractional differential equation model. Regardless of the size of the MSE value, the existence of the solution of the fractional differential equation model is assured.

## Acknowledgment

The authors would like to thank the Rector of Universitas Padjadjaran and Director of Directorate of Research and Community Service Universitas Padjadjaran who gave funding for dissemination of this paper through Riset Program Desentralisasi dan Kompetitif Nasional 2018 with contract number: 1084/UN6.D/LT/2018.

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