A Novel TOPSIS-MABAC Method for Multi-attribute Decision Making with Interval Neutrosophic Set

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***Abstract*—Interval neutrosophic set is a useful tool to describe the indeterminate, inconsistent, and incomplete information. This paper presents the application of the new TOPSIS-MABAC model with interval neutrosophic number in multi-attribute decision making problem. In this model, the combined weight of attributes is obtained based on TOPSIS method while the best alternatives by MABAC method. Firstly, some definitions of INS are given in this paper. Secondly, the objective attribute weights are determined by TOPSIS method, and then a combined attribute weight is proposed. Finally an extended MABAC method is developed to rank the alternatives in multi-attribute decision making problem and two illustrative examples are given to demonstrate the practicality and effectiveness of this new method.**

***Index Terms*—Interval Neutrosophic Set, TOPSIS, MABAC, MADM, combined weight.**

I. INTRODUCTION

MULTI-ATTRIBUTE decision making (MADM) problem [1] is an important part of modern decision science. It is widely used in engineering, economy, management and many other fields, such as investment decision-making, project evaluation, etc. Because of the fuzziness of human thinking and the complexity and uncertainty of objective things, it is difficult for a decision maker to express the evaluation value of an attribute with a crisp value. For this reason, fuzzy value is a better choice to describe these fuzzy information.

Fuzzy set (FS) is characterized by membership function and was firstly proposed by Zadeh [2], it has been regarded as a very useful tool to describe fuzzy information. On this basis, Atanassov [3], [4] proposed the intuitionistic fuzzy set (IFS) with membership function and non-membership function, and used it to solve some decision problems, then some aggregation operators based on these were proposed by Xu [5], [6] and some method for MADM with IFS were proposed in [7], [8]. Furthermore, Atanassov and Gargov [4], [9] extended the membership function and non-membership function to interval numbers and proposed interval-value intuitionistic fuzzy set (IVIFS). But IFS and IVIFS can only deal with incomplete information, but not uncertain and inconsistent information.

Therefore, Smarandache [10], [11] firstly proposed Neutrosophic Set (NS), however NS was mainly put forward from a philosophical viewpoint, which is difficult to be applied in the field of science and engineering. So Wang et al. [12] proposed Single-valued Neutrosophic Set (SVN-S) with the corresponding properties and operation rules. Furthermore, Ye [13] proposed the correlation coefficient and weighted correlation coefficient for SVNS, and then proposed single-valued neutrosophic cross-entropy for multi-criteria decision-making (MCDM) in [14]. In fact, sometimes the degree of truth, indeterminacy and falsity can not be defined exactly in the real number but denoted by several possible interval value. Similar to IVIFS, Wang et al. [15] proposed Interval Neutrosophic Set (INS) and gave the set-theoretic operators of INS. Based on the Hamming distance and the Euclidean distance of the INS, Ye [16] proposed a similarity measure of INS and then proposed a MADM method by using it. Aggregation operator plays an important role in MADM problem, because it can fuse multiple values into a single comprehensive value. To use the advantages of Einstein operations and generalized weighted average operator, a generalized simplified neutrosophic number Einstein weighed aggregation(GSNNEWA) operator is proposed in [17]. Then an extended single-valued neutrosophic normalized weighted Bonferroni mean(SVNNWBM) aggregation operator based on Einstein operations is proposed by Yang [18]. Meanwhile some multi-valued neutrosophic linguistic power operators are proposed to aggregate the multi-valued neutrosophic linguistic information in [19], such as multi-valued neutrosophic linguistic power weighted average(MVNLPWA) operator and the multi-valued neutrosophic linguistic power weighted geometric (MVNLPWG) operator.

Actually, there are many methods for MADM problem under neutrosophic environment, including TODIM [20], [21], [22], [23], TOPSIS [24], [25], [26], [27], VIKOR [28], ELECTRE [29] and so on. TOPSIS method which was proposed by Hwang and Yoon [30] and MABAC method which was proposed by Pamucar D and Cirovic Gare [31] are two useful tools for dealing with MADM problem. They have straightforward calculation process, systematic process and reasonable logic, which demonstrate the basis of decision-making. Recently many researchers have extended these methods to neutrosophic set. Chi and Liu [32] used the maximizing deviation method under interval neutrosophic environment to determinate the

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attribute weight and proposed an extended TOPSIS method with INS. Peng and Dai [33] proposed the combined weights based on the gray system method and used single-valued neutrosophic TOPSIS method and MABAC method for MADM problem while in [34] used the entropy method and linear weighted comprehensive method to obtain the combined weight, and then presented MABAC approach for MADM problem under interval neutrosophic environment.

Considering that different attribute weights may cause different ranking results of alternatives, some researchers focus on the determination of attribute weights. Ji et al. [35] proposed a way to determine the weights named the mean-squared deviation weight method with single-valued neutrosophic linguistic set. Based on the entropy of NS, Biswas et al. [36] determined the unknown attribute weights by using information entropy method to find the best alternative for MADM problem while [37] by a deviation model. Furthermore Tan et al. [38] proposed a method based on the entropy of NSs to determine the weights and used the single-valued neutrosophic VIKOR method to deal with group decision making problem. Above all, we can know that information entropy method is a significative tool for determining the attribute weight.

In this paper, we propose the TOPSIS-MABAC method, which is a combined method under interval neutrosophic environment for solving MADM problem. The specific arrangements of this article are structured as follows. In section 2, we briefly introduce some concepts and definitions of INS. In Section 3, we propose TOPSIS method to determine the objective attribute weights and the combined weights, and then use MABAC method to obtain the best alternative. In Section 4, we give two examples to illustrate the application of proposed method. In Section 5, we make a conclusion for this paper.

II. PRELIMINARIES OF NEUTROSOPHIC

In this section, some concepts and definitions of NS and INS are introduced.

A. Neutrosophic set

Definition 1: [10] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$ and $F_A(x) : X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$,$I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2: [10] The complement of a neutrosophic set A is denoted by A^C and is defined as $T_{A^C}(x) = \{1^+\} \ominus T_A(x)$, $I_{A^C}(x) = \{1^+\} \ominus I_A(x)$, $F_{A^C}(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 3: [10] A neutrosophic set A is contained in the other neutrosophic set B: $A \subseteq B$ if and only if

$$\inf T_A(x) \le \inf T_B(x), \sup T_A(x) \le \sup T_B(x)$$

$$\inf I_A(x) \ge \inf I_B(x), \sup I_A(x) \ge \sup I_B(x)$$

$$\inf F_A(x) \ge \inf F_B(x), \sup F_A(x) \ge \sup F_B(x)$$

for every $x \in X$.

Because NS is difficult to apply in real applications, so Wang et al. [15] developed INS.

B. Interval neutrosophic set

Definition 4: [15] Let X be a space of points(objects) with generic elements in X denoted by x. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, then A can be denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \,|x \in X\}$, where $T_A(x) = \left[T_A^L(x), T_A^U(x)\right]$, $I_A(x) = \left[I_A^L(x), I_A^U(x)\right]$, $F_A(x) = \left[F_A^L(x), F_A^U(x)\right] \subseteq [0, 1]$ for every x in X, and $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

For convenience, we refer to $A = \langle T_A, I_A, F_A\rangle = \left\langle \left[T_A^L, T_A^U\right], \left[I_A^L, I_A^U\right], \left[F_A^L, F_A^U\right]\right\rangle$ as an interval neutrosophic number (INN), which is a basic unit of INS.

Definition 5: [15] The complement of an INS A is denoted by A^C and is defined as

$$A^C = \langle F_A(x), [1 - \sup I_{A(x)}, 1 - \inf I_{A(x)}], T_A(x)\rangle \quad (1)$$

Definition 6: [15] Let A and B be two INNs and $\lambda > 0$. The operators for INNs are defined as following:

$$\begin{aligned}(1) A \oplus B = \langle\ & \left[T_A^L + T_B^L - T_A^L \cdot T_B^L, T_A^U + T_B^U - T_A^U \cdot T_B^U\right], \\ & \left[I_A^L \cdot I_B^L, I_A^U \cdot I_B^U\right], \\ & \left[F_A^L \cdot F_B^L, F_A^U \cdot F_B^U\right]\ \rangle\end{aligned} \quad (2)$$

$$\begin{aligned}(2) A \otimes B = \langle\ & \left[T_A^L \cdot T_B^L, T_A^U \cdot T_B^U\right], \\ & \left[I_A^L + I_B^L - I_A^L \cdot I_B^L, I_A^U + I_B^U - I_A^U \cdot I_B^U\right], \\ & \left[F_A^L + F_B^L - F_A^L \cdot F_B^L, F_A^U + F_B^U - F_A^U \cdot F_B^U\right]\ \rangle\end{aligned} \quad (3)$$

$$\begin{aligned}(3) \lambda A = \langle\ & \left[1 - \left(1 - T_A^L\right)^\lambda, 1 - \left(1 - T_A^U\right)^\lambda\right], \\ & \left[\left(I_A^L\right)^\lambda, \left(I_A^U\right)^\lambda\right], \\ & \left[\left(F_A^L\right)^\lambda, \left(F_A^U\right)^\lambda\right]\ \rangle\end{aligned} \quad (4)$$

$$\begin{aligned}(4) (A)^\lambda = \langle\ & \left[\left(T_A^L\right)^\lambda, \left(T_A^U\right)^\lambda\right], \\ & \left[1 - \left(1 - I_A^L\right)^\lambda, 1 - \left(1 - I_A^U\right)^\lambda\right], \\ & \left[1 - \left(1 - F_A^L\right)^\lambda, 1 - \left(1 - F_A^U\right)^\lambda\right]\ \rangle\end{aligned} \quad (5)$$

Definition 7: [16] Let A and B be two INNs, then the normalized Euclidean distance between A and B is

$$\begin{aligned} d(A, B) = \Big\{ & \frac{1}{6}\left[\left(T_A^L - T_B^L\right)^2 + \left(T_A^U - T_B^U\right)^2\right. \\ & + \left(I_A^L - I_B^L\right)^2 + \left(I_A^U - I_B^U\right)^2 \\ & \left. + \left(F_A^L - F_B^L\right)^2 + \left(F_A^U - F_B^U\right)^2\right]\Big\}^{\frac{1}{2}} \end{aligned} \quad (6)$$

Definition 8: [39] Let A be an INN, a score function $S(A)$ of A is:

$$S(A) = \frac{4 + T_A^L - I_A^L - F_A^L + T_A^U - T_A^U - T_A^U}{6} \quad (7)$$

Definition 9: [39] Let A be an INN, an accuracy function $H(A)$ of A is:

$$H(A) = \frac{\left(T_A^L + T_A^U\right) - \left(F_A^L + F_A^U\right)}{2} \quad (8)$$

Definition 10: [39] Let A and B be two INNs, $S(A)$ and $S(B)$ be the score functions, $H(A)$ and $H(B)$ be the accuracy functions, then if $S(A) < S(B)$, then $A < B$; if $S(A) = S(B)$, then

(1) if $H(A) = H(B)$, then $A = B$;

(2) if $H(A) < H(B)$, then $A < B$.

III. TOPSIS-MABAC METHOD FOR INTERVAL NEUTROSOPHIC MADM PROBLEM

In this section, we study the TOPSIS-MABAC method under interval neutrosophic environment.

Let $A = \{A_1, A_2, ..., A_m\}$ be a set of alternatives, $C = \{C_1, C_2, ..., C_n\}$ be a series of attributes, and $w = \{w_1, w_2, \cdots, w_n\}$ be the subjective weight of the attributes, w_j is the weight of the jth attribute where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The INN

$$a_{ij} = \left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right] \right\rangle$$

is the evaluated value of A_i under C_j, then the decision matrix A $= (a_{ij})_{m\times n}$ is obtained.

To get the optimal alternative(s), we propose the TOPSIS-MABAC method with INN.

Step 1: Normalization of the decision matrix. That is, normalized the matrix $A = (a_{ij})_{m\times n}$ into $R = (r_{ij})_{m\times n} = \left(\left\langle \left[T_{ij}^{NL}, T_{ij}^{NU},\right], \left[I_{ij}^{NL}, I_{ij}^{NU},\right], \left[F_{ij}^{NL}, F_{ij}^{NU},\right] \right\rangle\right)_{m\times n}$, where

$$r_{ij} = \begin{cases} a_{ij}, C_j \text{ is the benefit type attribute} \\ a_{ij}^c, C_j \text{ is the cost type attribute} \end{cases} \tag{9}$$

and a_{ij}^c is defined by definition 5.

Step 2: Calculating the combined weight. Although the decision makers determine the subjective weights w_j of these attributes, it is not enough and reasonable to use only subjective weight. So a combined weight method is proposed.

According to the normalized decision matrix, we can define the positive ideal solution (PIS) and negative ideal solution (NIS) as following:

$$PIS = R^+ = \left(R_1^+, R_2^+, \cdots, R_n^+\right) \tag{10}$$

$$NIS = R^- = \left(R_1^-, R_2^-, \cdots, R_n^-\right) \tag{11}$$

where

$$R_j^+ = \begin{pmatrix} \left[\max_i T_{ij}^{NL}, \max_i T_{ij}^{NU}\right] \\ \left[\min_i I_{ij}^{NL}, \min_i I_{ij}^{NU}\right] \\ \left[\min_i F_{ij}^{NL}, \min_i F_{ij}^{NU}\right] \end{pmatrix} \tag{12}$$

and

$$R_j^- = \begin{pmatrix} \left[\min_i T_{ij}^{NL}, \min_i T_{ij}^{NU},\right] \\ \left[\max_i I_{ij}^{NL}, \max_i I_{ij}^{NU},\right] \\ \left[\max_i F_{ij}^{NL}, \max_i F_{ij}^{NU},\right] \end{pmatrix} \tag{13}$$

for $j = 1, 2, \cdots, n$.

(1) Determination of objective weight vector $\omega^o = (\omega_1, \omega_2, \cdots, \omega_n)$.

For the PIS, the closer distance between A_i and R^+, the better A_i is. So we assume that the weighted distance between A_i and R^+ under C_j is $e_i^+(\omega) = \sum_{j=1}^{n} d\left(r_{ij}, R_j^+\right)\omega_j^+$. So

$$e^+(\omega) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(r_{ij}, R_j^+\right)\omega_j^+$$

represents the sum of the weighted distance between all the alternatives and PIS. Therefore, we can establish the model as follows:

$$\begin{cases} \min \quad e^+(\omega) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(r_{ij}, R_j^+\right)\omega_j^+ \\ \text{s.t.} \quad \sum_{j=1}^{n}\omega_j^+ = 1, \omega_j^+ > 0, j = 1, 2, \cdots, n \end{cases}$$

By constructing the Lagrange function, we can get

$$\omega_j^+ = \frac{\sum_{i=1}^{m} d\left(r_{ij}, R_j^+\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m} d\left(r_{ij}, R_j^+\right)} \tag{14}$$

Similarly we can get

$$\omega_j^- = \frac{\sum_{i=1}^{m} d\left(r_{ij}, R_j^-\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m} d\left(r_{ij}, R_j^-\right)}. \tag{15}$$

So the objective weight is $\omega^o = (\omega_1, \omega_2, \cdots, \omega_n)$, where $\omega_j = \frac{1}{2}\left(\omega_j^+ + \omega_j^-\right)$.

(2) Determination of combined weight vector $\overline{\omega} = (\overline{\omega}_1, \overline{\omega}_2, \cdots, \overline{\omega}_n)$, where $\overline{\omega}_j = \lambda\omega_j + (1-\lambda)w_j$, for $j = 1, 2, \cdots, n$, and $0 \le \lambda \le 1$.

Step 3: Calculation of the weighted normalized decision matrix matrix $V = (v_{ij})_{m\times n}$, where $v_{ij} = \left\langle \left[\overline{T}_{ij}^L, \overline{T}_{ij}^U\right], \left[\overline{I}_{ij}^L, \overline{I}_{ij}^U\right], \left[\overline{F}_{ij}^L, \overline{F}_{ij}^U\right] \right\rangle$, for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, and

$$\begin{cases} \left[\overline{T}_{ij}^L, \overline{T}_{ij}^U\right] = \left[1 - \left(1 - T_{ij}^{NL}\right)^{\overline{\omega}_j}, 1 - \left(1 - T_{ij}^{NU}\right)^{\overline{\omega}_j}\right] \\ \left[\overline{I}_{ij}^L, \overline{I}_{ij}^U\right] = \left[\left(I_{ij}^{NL}\right)^{\overline{\omega}_j}, \left(I_{ij}^{NU}\right)^{\overline{\omega}_j}\right] \\ \left[\tilde{F}_{ij}^L, \overline{F}_{ij}^U\right] = \left[\left(F_{ij}^{NL}\right)^{\overline{\omega}_j}, \left(F_{ij}^{NU}\right)^{\overline{\omega}_j}\right] \end{cases} \tag{16}$$

Step 4: Determination of the border approximation area matrix $G = (g_j)_{1\times n}$, where

$$\begin{aligned} g_j = \prod_{i=1}^{m}(v_{ij})^{\frac{1}{m}} = \langle & \left[\prod_{i=1}^{m}\left(\overline{T}_{ij}^L\right)^{\frac{1}{m}}, \prod_{i=1}^{m}\left(\overline{T}_{ij}^U\right)^{\frac{1}{m}}\right], \\ & \left[1 - \prod_{i=1}^{m}\left(1 - \overline{I}_{ij}^L\right)^{\frac{1}{m}}, 1 - \prod_{i=1}^{m}\left(1 - \overline{I}_{ij}^U\right)^{\frac{1}{m}}\right], \\ & \left[1 - \prod_{i=1}^{m}\left(1 - \overline{F}_{ij}^L\right)^{\frac{1}{m}}, 1 - \prod_{i=1}^{m}\left(1 - \overline{F}_{ij}^U\right)^{\frac{1}{m}}\right] \rangle \end{aligned} \tag{17}$$

Step 5: Computing the distance matrix $D = (d_{ij})_{m\times n}$, where

$$d_{ij} = \begin{cases} d\left(v_{ij}, g_j\right) & \text{if } v_{ij} > g_j \\ 0 & \text{if } v_{ij} = g_j \\ -d\left(v_{ij}, g_j\right) & \text{if } v_{ij} < g_j \end{cases} \tag{18}$$

The distance measure d_{ij} is defined in Eq.(6), and the method for comparing v_{ij} and g_j is defined in definition 11.

Step 6: Calculating the Q_i, where $Q_i = \sum_{j=1}^{n} d_{ij}$.

Step 7: Ranking all alternatives according to the value of Q_i. The larger the value of Q_i, the better the alternative A_i is.

IV. NUMERICAL EXAMPLE

A. Example 1

In this section, two examples about the investment selection of a company is given to illustrate the feasibility of the proposed method.

Considering the decision-making problem adapted from [16]. There is an investment company which wants to invest a number of money in a best option. There are four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company should take a decision according to the following three attributes : (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; (3) C_3 is the environmental impact analysis, where C_1 and C_2 are benefit type attributes, and C_3 is a cost type attribute. The subjective weight vector of the attribute is given by $w = (0.35, 0.25, 0.4)$. The four possible alternatives are to be evaluated under the above three attributes by the form of INNs, as shown in the following interval neutrosophic decision matrix A:

$$A = \begin{bmatrix} \langle[0.4,0.5][0.2,0.3][0.3,0.4]\rangle & \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle & \langle[0.7,0.9][0.2,0.3][0.4,0.5]\rangle \\ \langle[0.6,0.7][0.1,0.2][0.2,0.3]\rangle & \langle[0.6,0.7][0.1,0.2][0.2,0.3]\rangle & \langle[0.3,0.6][0.3,0.5][0.8,0.9]\rangle \\ \langle[0.3,0.6][0.2,0.3][0.3,0.4]\rangle & \langle[0.5,0.6][0.2,0.3][0.3,0.4]\rangle & \langle[0.4,0.5][0.2,0.4][0.7,0.9]\rangle \\ \langle[0.7,0.8][0.0,0.1][0.1,0.2]\rangle & \langle[0.6,0.7][0.1,0.2][0.1,0.3]\rangle & \langle[0.6,0.7][0.3,0.4][0.8,0.9]\rangle \end{bmatrix} \tag{19}$$

Then we use the proposed method to obtain the best alternative(s).

Step 1: Normalized the decision matrix. According the Eq.(9), we can get the normalized matrix:

$$R = \begin{bmatrix} \langle[0.4,0.5][0.2,0.3][0.3,0.4]\rangle & \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle & \langle[0.4,0.5][0.7,0.8][0.7,0.9]\rangle \\ \langle[0.6,0.7][0.1,0.2][0.2,0.3]\rangle & \langle[0.6,0.7][0.1,0.2][0.2,0.3]\rangle & \langle[0.8,0.9][0.5,0.7][0.3,0.6]\rangle \\ \langle[0.3,0.6][0.2,0.3][0.3,0.4]\rangle & \langle[0.5,0.6][0.2,0.3][0.3,0.4]\rangle & \langle[0.7,0.9][0.6,0.8][0.4,0.5]\rangle \\ \langle[0.7,0.8][0.0,0.1][0.1,0.2]\rangle & \langle[0.6,0.7][0.1,0.2][0.1,0.3]\rangle & \langle[0.8,0.9][0.6,0.7][0.6,0.7]\rangle \end{bmatrix} \tag{20}$$

Step 2: Calculating the combined weight. According to the Eq.(12) and Eq.(13), we can get $R^+ = \left(R_1^+, R_2^+, R_3^+\right)$ and $R^- = \left(R_1^-, R_2^-, R_3^-\right)$, where

$$R_1^+ = \langle[0.7, 0.8], [0.0, 0.1], [0.1, 0.2]\rangle$$

$$R_2^+ = \langle[0.6, 0.7], [0.1, 0.2], [0.1, 0.3]\rangle$$

$$R_3^+ = \langle[0.8, 0.9], [0.5, 0.7], [0.3, 0.5]\rangle$$

and

$$R_1^- = \langle[0.3, 0.5], [0.2, 0.3], [0.3, 0.4]\rangle$$

$$R_2^- = \langle[0.4, 0.6], [0.2, 0.3], [0.3, 0.4]\rangle$$

$$R_3^- = \langle[0.4, 0.5], [0.7, 0.8], [0.7, 0.9]\rangle$$

According to Eq.(14) and Eq.(15), we can get

$$\omega_1^+ = 0.4, \omega_2^+ = 0.2, \omega_3^+ = 0.4$$

$$\omega_1^- = 0.3, \omega_2^- = 0.2, \omega_3^- = 0.5$$

Then the objective weight is obtained and denoted as $\omega^\circ = (0.35, 0.2, 0.45)$. In this section, we set $\lambda = 0.7$. So the combined weight is $\overline{\omega} = (0.35, 0.215, 0.435)$.

Step 3: Calculation of the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$. According to Eq.(16), we can get the weighted normalized decision matrix as follows:

$$V = \begin{bmatrix} \langle[0.1637,0.2154][0.5693,0.6561][0.6561,0.7256]\rangle & \langle[0.1040,0.1788][0.6095,0.7719][0.7075,0.8212]\rangle & \langle[0.1993,0.2603][0.8563,0.9075][0.8563,0.9552]\rangle \\ \langle[0.2744,0.3439][0.4467,0.5693][0.5693,0.6561]\rangle & \langle[0.1788,0.2281][0.6095,0.7075][0.7075,0.7719]\rangle & \langle[0.5035,0.6327][0.7397,0.8563][0.5923,0.8007]\rangle \\ \langle[0.1174,0.2744][0.5693,0.6561][0.6561,0.7256]\rangle & \langle[0.1385,0.1788][0.7075,0.7719][0.7719,0.8212]\rangle & \langle[0.4077,0.6327][0.8007,0.9075][0.6713,0.7397]\rangle \\ \langle[0.3439,0.4307][0.0000,0.4467][0.4467,0.5693]\rangle & \langle[0.1788,0.2281][0.6095,0.7075][0.6095,0.7719]\rangle & \langle[0.5035,0.6327][0.8007,0.8563][0.8007,0.8563]\rangle \end{bmatrix} \tag{21}$$

Step 4: Calculating the border approximation area matrix $G = (g_j)_{1\times n}$. According to Eq.(17), we can get:

$$g_1 = \langle[0.2064, 0.3059], [0.4340, 0.5903], [0.5903, 0.6750]\rangle$$

$$g_2 = \langle[0.1465, 0.2020], [0.6367, 0.7417], [0.7045, 0.7980]\rangle$$

$$g_3 = \langle[0.3788, 0.5067], [0.8037, 0.8847], [0.7511, 0.8648]\rangle$$

Step 5: Calculating the distance matrix D.

$$D = \begin{bmatrix} -0.0812 & -0.0275 & -0.1387 \\ 0.0353 & 0.0311 & 0.1046 \\ -0.0800 & -0.0410 & 0.0809 \\ 0.2142 & 0.0472 & 0.0761 \end{bmatrix} \tag{22}$$

Step 6: Calculating $Q_i = \sum_{j=1}^{3} d_j,\ i = 1, 2, \cdots, 4$. We can get

$$Q_1 = -0.2474, Q_2 = 0.1710, Q_3 = -0.0431, Q_4 = 0.3375$$

According to the value of Q_i, we can get $A_4 > A_2 > A_3 > A_1$, so A_4 is the best alternative.

B. Example 2

A venture capital firm wants to choose an innovating enterprise to invest, where $A = \{A_1, A_2, A_3\}$ are three enterprises and $C = \{C_1, C_2, C_3\}$ are three attributes of each. Here C_1, C_2 and C_3 represent team management, industrys outlook, and enterprise competitiveness, respectively, where C_1 is a cost type attribute, and C_2, C_3 are benefit type attributes. The subjective weight is $w = (0.35, 0.4, 0.25)$. Assume that the decision matrix which is given by decision maker is:

$$A = \begin{bmatrix} \langle[0.2,0.3][0.3,0.4][0.2,0.5]\rangle & \langle[0.3,0.6][0.4,0.5][0.3,0.4]\rangle & \langle[0.5,0.6][0.3,0.4][0.2,0.4]\rangle \\ \langle[0.4,0.5][0.2,0.3][0.3,0.4]\rangle & \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle & \langle[0.7,0.9][0.2,0.3][0.4,0.5]\rangle \\ \langle[0.7,0.8][0.1,0.2][0.2,0.3]\rangle & \langle[0.6,0.7][0.2,0.4][0.1,0.3]\rangle & \langle[0.6,0.7][0.3,0.4][0.8,0.9]\rangle \end{bmatrix} \tag{23}$$

Then we use the proposed method to obtain the best alternative(s).

Step 1: Normalized the decision matrix. According the Eq.(9), we can get the normalized matrix:

$$R = \left[\begin{array}{l} \langle[0.2,0.5][0.6,0.7][0.2,0.3]\rangle \\ \langle[0.3,0.4][0.7,0.8][0.4,0.5]\rangle \\ \langle[0.2,0.3][0.8,0.9][0.7,0.8]\rangle \end{array} \right.$$
$$\begin{array}{l} \langle[0.3,0.6][0.4,0.5][0.3,0.4]\rangle \\ \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle \\ \langle[0.6,0.7][0.2,0.4][0.1,0.3]\rangle \end{array}$$
$$\left. \begin{array}{l} \langle[0.5,0.6][0.3,0.4][0.2,0.4]\rangle \\ \langle[0.7,0.9][0.2,0.3][0.4,0.5]\rangle \\ \langle[0.6,0.7][0.3,0.4][0.8,0.9]\rangle \end{array} \right] \quad (24)$$

Step 2: Calculating the combined weight. According to Eq.(12) and Eq.(13), we can get $R^+ = \left(R_1^+, R_2^+, R_3^+\right)$ and $R^- = \left(R_1^-, R_2^-, R_3^-\right)$, where

$$R_1^+ = \langle[0.3,0.5],[0.6,0.7],[0.2,0.3]\rangle$$
$$R_2^+ = \langle[0.6,0.7],[0.1,0.3],[0.1,0.3]\rangle$$
$$R_3^+ = \langle[0.7,0.9],[0.2,0.3],[0.2,0.4]\rangle$$

and

$$R_1^- = \langle[0.2,0.3],[0.8,0.9],[0.7,0.8]\rangle$$
$$R_2^- = \langle[0.3,0.6],[0.4,0.5],[0.3,0.4]\rangle$$
$$R_3^- = \langle[0.5,0.6],[0.3,0.4],[0.8,0.9]\rangle$$

According to Eq.(14) and Eq.(15), we can get

$$\omega_1^+ = 0.34, \omega_2^+ = 0.26, \omega_3^+ = 0.40$$
$$\omega_1^- = 0.34, \omega_2^- = 0.23, \omega_3^- = 0.43$$

Then the objective weight is obtained and denoted as $\omega^\circ = (0.34, 0.245, 0.415)$. In this section, we set $\lambda = 0.7$. So the combined weight is $\overline{\omega} = (0.343, 0.2915, 0.3655)$. For convenience of calculation, we denote the combined weight as $\overline{\omega} = (0.35, 0.3, 0.35)$.

Step 3: Calculation of the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$. According to Eq.(16), we can get the weighted normalized decision matrix as follows:

$$V = \left[\begin{array}{l} \langle[0.0751,0.2154][0.8362,0.8826][0.5693,0.6561]\rangle \\ \langle[0.1174,0.1637][0.8826,0.9249][0.7256,0.7846]\rangle \\ \langle[0.0751,0.1174][0.9249,0.9638][0.8826,0.9249]\rangle \end{array} \right.$$
$$\begin{array}{l} \langle[0.1015,0.2403][0.7597,0.8123][0.6968,0.7597]\rangle \\ \langle[0.1421,0.2403][0.5012,0.6968][0.6170,0.7597]\rangle \\ \langle[0.2403,0.3032][0.6170,0.7597][0.5012,0.6968]\rangle \end{array}$$
$$\left. \begin{array}{l} \langle[0.2154,0.2744][0.6561,0.7256][0.5693,0.7256]\rangle \\ \langle[0.3439,0.5533][0.5693,0.6561][0.7256,0.7846]\rangle \\ \langle[0.2744,0.3493][0.6561,0.7256][0.9249,0.9638]\rangle \end{array} \right] \quad (25)$$

Step 4: Calculating the border approximation area matrix $G = (g_j)_{1\times n}$. According to Eq.(17), we can get:

$$g_1 = \langle[0.0872,0.1606],[0.8870,0.9316],[0.7597,0.8827]\rangle$$
$$g_2 = \langle[0.1513,0.2597],[0.6419,0.7608],[0.6131,0.7403]\rangle$$
$$g_3 = \langle[0.2728,0.3737],[0.6293,0.7042],[0.7929,0.8711]\rangle$$

Step 5: Calculate the distance matrix D.

$$D = \begin{bmatrix} 0.1263 & -0.0668 & 0.1194 \\ 0.0443 & 0.0642 & 0.0959 \\ -0.0597 & 0.0644 & -0.0684 \end{bmatrix} \quad (26)$$

Step 6: Calculating $Q_i = \sum_{j=1}^{3} d_j$, $i = 1,2,3$. We can get

$$Q_1 = 0.1789, Q_2 = 0.2044, Q_3 = -0.0637$$

According the value of Q_i, we can get $A_2 > A_1 > A_3$, so A_2 is the best alternative.

V. Conclusion

The aim of this paper is to introduce a new approach for MADM with interval neutrosophic set. At the beginning of this article, we briefly introduce some concepts and definitions of INS, and then propose TOPSIS method for determining the objective attribute weights and the combined weights. Next, in order to get the best alternative(s), we combine the MABAC method with the combined weights. Finally, we give two examples to illustrate the application of the proposed method. From the results we can see that this new method is useful for multi-attribute decision making problem, and the determination of the combined weight enriches and develops the method of multi-attribute decision making problem.

In the further, we shall develop more methods for decision making problems under neutrosophic environment and apply them into different fields, such as venture capital, pattern recognition and comprehensive evaluation.

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