A Numerical Study of Atangana-Baleanu and Caputo-Fabrizio for MHD Flow Problem over a Vertical Hot Stretching Sheet with Variable Viscosity and Thermal Conductivity

Dipen Saikia, Utpal Kumar Saha, G. C. Hazarika

***Abstract*— A numerical investigation has been made to study the effects of variable viscosity and thermal conductivity over a vertical hot stretching sheet by using Atangana-Baleanu (AB) and Caputo-Fabrizio (CF) fractional derivatives. As the viscosity and thermal conductivity of a fluid are dependent on temperature, these properties are considered as a variable. We have also considered radiation and chemical reaction. The governing partial differential equations along with the boundary conditions are made dimensionless using suitable similarity transformations so that physical parameters appear in the equations and interpretations on these parameters can be done suitably. The equations so obtained are discritized using ordinary finite difference scheme and we solved the discritized equations numerically adopting a method based on the Gauss-Seidel iteration scheme. Numerical techniques are used to find the values from AB and CF formulae for fractional derivatives on time. The effects of various parameters involved in the problem viz., viscosity parameter, thermal conductivity parameter, magnetic field parameter, radiation parameter, Schmidt number, prandlt number, chemical reaction parameter etc. on velocity, temperature, and concentration distribution at the plate have been shown graphically. The coefficient of skin-friction, heat transfer rate, and Sherwood number are also obtained and presented in tabular form. The effects of each parameter are prominent. A comparison has been given on AB and CF methods in tabular form. It is observed that both the methods agreed well.**

***Index Terms*- AB fractional, CF fractional, free convection, velocity, MHD fluid, variable viscosity, variable thermal conductivity.**

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I. Introduction

THE boundary layer flow and heat transfer of a viscous fluid over flat surfaces have been investigated in several technological processes such as hot rolling, metal extrusion, continuous stretching of plastic films and glass fiber, polymer extrusion, wires drawing and metal spinning. So the study of two dimensional boundary layer flows over a stretching sheet has gained much interest. Coupled heat and mass transfer popularly known as double diffusion, has so many important applications in the field of science and engineering, such as chemical catalytic reactor and processes, underground disposal of nuclear wastes, migration of moisture through the air contained in fibrous insulation, spreading of chemical pollutants through water-saturated soil, diffusion of medicine in blood veins, filtration etc. Application of the magnetic field to the fluid flow gives utmost importance in the fields of meteorology, astrophysics, cosmic fluid dynamics, geophysics, solar physics etc. Magnetohydrodynamic flow has applications in the motion of the earth's core also.

Due to the technological importance, studies of free convection flow of viscous incompressible fluid past a semi-infinite or infinite vertical plate have been done by many researchers. Uwanta et al. [14] studied about MHD fluid flow over a vertical plate with Dufour and Sorret effects. Kumar et al. [10] and Uddin et al. [13] are investigated the effects of thermal diffusion and chemical reaction on unsteady fluid flow over a vertical porous plate with a heat source. Effects of thermal radiation and chemical reaction on free convection flow past a moving vertical plate were analyzed by Hemalatha and Reddy [5]. Soundalgekar [11] observed the effects of free convection on the flow past an infinite vertical oscillating plate. Elabashbeshy [4] analyzed double diffusion of an MHD flow along with a vertical plate with variable surface tension and concentration. Zubi [15] studied MHD heat and mass transfer over a permeable vertical plate with a chemical reaction.

The effects of heat source/sink in thermal convection are significant where a higher temperature difference exists between the surface and the fluid in contact with the surface. Buoyancy effects on MHD free convection flow in the presence of heat source/sink were studied by Baag *et al.* [3] and Princely [9]. Effects of chemical reaction on MHD flow over a moving vertical plate in the presence of heat sources were analyzed by Tripathy *et al.* [12]. An unsteady MHD free convective flow past a vertical plate with thermal diffusion and heat source was investigated by Ahmed *et al.*[1]. Nadeem et al. [8] made comparative study on generalized Casson fluid model using CF and AB fractional derivatives with chemical reaction and heat generation and presented their results graphically. Fractional calculus has become a burning topic in research due to two reasons/weaknesses: problem of the singular kernel with locality and problem of the non-singular kernel with non-locality. In order to avoid the problem of the singular kernel, Michele Caputo and Mauro Fabrizio proposed a fractional derivative by employing an exponential function [7]. Indeed, the claim of a singular kernel for the fractional derivative operator is not based on their observations; even they suggested their fractional derivative operator is appropriate for various physical problems. Atangana *et al.* [2] employed the time-fractional Caputo–Fabrizio derivative on the advection-diffusion equation for tracing out the fundamental solutions using the Laplace transform for the fractional diffusion phenomenon. Lai, Kulacki [6] discussed the effect of variable viscosity on convective heat and mass transfer along a vertical surface in standard porous media.

In this paper, we investigate the effects of variable viscosity and thermal conductivity of viscous incompressible fluid flow in the presence of a uniform magnetic field over a moving vertical plate. The governing partial differential equations along with the boundary conditions are made dimensionless by using some suitable non-dimensional parameters. The non-dimensional governing equations with the non-dimensional boundary conditions are discretized with ordinary finite-difference kernel solved numerically with the help of AB fractional derivative and CF fractional derivative method by developing suitable programming code in MATLAB. Comparisons of the results obtained by both the methods are shown in tabular form.

II. FORMULATION OF THE PROBLEM

Let us consider an unsteady incompressible fluid for free convective flow which occupies the space above a moving plate in the $\bar{x}\bar{y}$ plane, and the plate is normal to the $\bar{y}$ axis. Initially, $\bar{T}_\infty$ is the temperature and $\bar{C}_\infty$ is the concentration level to the plate whereas plate as well as fluid is kept at rest. At $t = 0^+$, the double diffusion from the plate to the fluid have gained the temperature $\bar{T}_w$, and concentration level near the plate is $\bar{C}_w$. A uniform magnetic field with strength B_0 is applied in the transverse direction of the flow. The condition of incompressibility is obviously satisfied for such type of flow. Keeping the usual Boussinesq approximation in mind, the governing boundary layer equations are:

Equation of Continuity:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

Equation of conservation of momentum:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \vartheta \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\rho}\frac{\partial \mu}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\sigma B_0^2 \bar{u}}{\rho} + g\beta_T(\bar{T} - \bar{T}_\infty) + g\beta_C(\bar{C} - \bar{C}_\infty) \tag{2}$$

Equation of conservation of energy:

$$\rho C_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{\partial \lambda}{\partial \bar{y}}.\frac{\partial \bar{T}}{\partial \bar{y}} + \lambda\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\bar{J}^2}{\sigma} - \frac{\partial \bar{q}_r}{\partial \bar{y}} \tag{3}$$

Equation of concentration:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = \frac{\partial D}{\partial \bar{y}}\frac{\partial \bar{C}}{\partial \bar{y}} + D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - Ch.\bar{C} \tag{4}$$

The initial boundary conditions are:

$$\begin{aligned}
&\bar{t} \le 0 : \bar{u} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \quad \forall y \\
&\bar{t} \succ 0 : \bar{u} = U_0\cos(\varpi\bar{t}),\ \bar{T} = \bar{T}_\infty + (\bar{T}_w - \bar{T}_\infty)A\bar{t}, \\
&\qquad \bar{C} = \bar{C}_\infty + (\bar{C}_w - \bar{C}_\infty)A\bar{t} \text{ at } \bar{y} = 0 \\
&\bar{t} \succ 0 : \bar{u} \to 0,\ \bar{T} \to \bar{T}_\infty,\ \bar{C} \to \bar{C}_\infty \text{ at } \bar{y} \to \infty
\end{aligned} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the fluid velocities in $\bar{x}$ and $\bar{y}$ directions respectively, ϑ is the kinematic viscosity, ρ is the fluid density, μ is the viscosity of the fluid, σ is the electrical conductivity, g is the acceleration due to gravity, β_T is the coefficient of volume expansion for heat transfer, β_C is the coefficient of volume expansion for mass transfer, $\bar{T}$ is the temperature and $\bar{T}_\infty$ is the temperature at free stream of the fluid, $\bar{C}$ is the concentration and $\bar{C}_\infty$ is the concentration at free stream of the fluid, λ is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, $\bar{q}_r$ is the radiative heat flux, D is the mass diffusivity, U_0 is the velocity of the plate, $\bar{C}_w$ is the species concentration at the surface of the plate and A is a constant, $\bar{J}$ is the electric current density. By Ohm's law $\bar{J} = \sigma[\vec{E} + \vec{q} \times \vec{B}]$, $\vec{q}$ be fluid velocity at a particular point, $\vec{B} = B_0\hat{j}$ be the applied magnetic field. Here no electric field is applied for which $\vec{E} = 0$, Ch is the chemical reaction rate of the species concentration.

The last two terms of eqn. (2) represent the thermal and concentration buoyancy effects respectively. In eqn. (3) the last two terms denote the heat absorption and thermal radiation effects respectively. Also, the last term of eqn. (4) represents the chemical reaction effect.

By the Rosseland approximation, the radiative heat flux is given by

$$\bar{q}_r = -\frac{4\bar{\sigma}}{3\bar{a}}\frac{\partial \bar{T}^4}{\partial \bar{y}} \tag{6}$$

where $\bar{a}$ is the mean absorption coefficient and $\bar{\sigma}$ is the Stefan – Boltzmann constant.

Assuming that the temperature differences within the flow is such that the term $\bar{T}^4$ may be expressed as a linear function of the temperature, we expand $\bar{T}^4$ in Taylor's series about $\bar{T}_\infty$ as follows:

$$\bar{T}^4 = \bar{T}_\infty^{\,4} + 4\bar{T}_\infty^{\,3}\left(\bar{T} - \bar{T}_\infty\right) + 6\bar{T}_\infty^{\,2}\left(\bar{T} - \bar{T}_\infty\right)^2 + ...$$

By neglecting the higher order terms beyond 1st degree in $\bar{T} - \bar{T}_\infty$ we have $\bar{T}^4 = 4\bar{T}_\infty^{\,3}\bar{T} - 3\bar{T}_\infty^{\,4}$ (7)

Using equations (6) and (7), eqn. (3) reduces to

$$\rho C_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{\partial \lambda}{\partial \bar{y}}.\frac{\partial \bar{T}}{\partial \bar{y}} + \lambda\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + Q_0\left(\bar{T} - \bar{T}_\infty\right) - \frac{16\bar{\sigma}T_\infty^{\,3}}{3\bar{a}}\frac{\partial^2 \bar{T}^4}{\partial \bar{y}^2} \tag{8}$$

Let us introduced the below mentioned dimensionless quantities:

$$x = \frac{U_0\bar{x}}{\vartheta_\infty},\quad y = \frac{U_0\bar{y}}{\vartheta_\infty},\quad u = \frac{\bar{u}}{U_0},\quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty},$$
$$v = \frac{\bar{v}}{U_0},\ \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty},\quad t = \frac{U_0^{\,2}\bar{t}}{\vartheta_\infty},\quad \omega = \frac{\bar{\omega}\vartheta_\infty}{U_0^{\,2}},$$
$$\xi = \frac{\vartheta_\infty Ch}{U_0^{\,2}},\ A = \frac{U_0^{\,2}}{\vartheta_\infty}. \tag{9}$$

The viscosity and thermal conductivity of the fluid are assumed to be inverse linear function of temperature [6] as follows:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left\{1 + \gamma\left(\bar{T} - \bar{T}_\infty\right)\right\} \tag{10}$$
$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left\{1 + \delta\left(\bar{T} - \bar{T}_\infty\right)\right\} \tag{11}$$

where γ and δ are constants which depend on the thermal property of the fluid.

We define two parameters as $\theta_r = \frac{\bar{T}_r - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}$ called viscosity parameter and $\theta_c = \frac{\bar{T}_c - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}$ called thermal Conductivity parameter.

Using these two parameters in (10) and (11), we have the viscosity and thermal conductivity respectively as

$$\mu = -\frac{\mu_\infty\theta_r}{\theta - \theta_r}\ ,\ \lambda = -\frac{\lambda_\infty\theta_c}{\theta - \theta_c} \tag{12}$$

Using the transformations (9) and (12), the non-dimensional forms of (2), (3) and (4) are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\theta_r}{\theta - \theta_r}\frac{\partial^2 u}{\partial y^2} + \frac{\theta_r}{(\theta - \theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + Gr\theta + Gr_m\phi \tag{13}$$

$$\Pr\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \left(\frac{4}{3}Kr - \frac{\theta_c}{\theta - \theta_c}\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{\theta_c}{(\theta - \theta_c)^2}\left(\frac{\partial \theta}{\partial y}\right)^2 - \left(\frac{\theta_r}{\theta - \theta_r}\right)\Pr.Ec\left(\frac{\partial u}{\partial y}\right)^2 + MEc.u^2 \tag{14}$$

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{\theta_r}{(\theta - \theta_r)^2}\cdot\frac{1}{Sc}\frac{\partial \theta}{\partial y}\cdot\frac{\partial \phi}{\partial y} - \frac{\theta_r}{Sc(\theta - \theta_r)}\frac{\partial^2 \phi}{\partial y^2} - \xi(\phi + N_c) \tag{15}$$

The related initial and boundary conditions are reduced to the form:

$$t \le 0 : u = 0,\ v = 0,\ \theta = 0,\ \phi = 0 \quad \forall y$$
$$t \succ 0 : u = \cos(\omega t),\ v = 0,\ \theta = t,\ \phi = t \ \text{ at } y = 0$$
$$t \succ 0 : u \to 0,\ v \to 0, \theta \to 0,\ \phi \to 0 \ \text{ at } y \to \infty \tag{16}$$

where $Ec = \frac{U_o^{\,2}}{\bar{T}_w C_p}$ is the Eckert number,

$M = \frac{\sigma B_0^{\,2}\vartheta_\infty}{\rho U_0^{\,2}}$ is the Magnetic field parameter,

$So = D\frac{\bar{T}_w - \bar{T}_\infty}{\vartheta_\infty\left(\bar{C}_w - \bar{C}_\infty\right)}$ is the Soret number,

$Sc = \frac{\vartheta_\infty}{D}$ is the Schmidt number of fluid,

$N_c = \frac{\bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}$ is the concentration difference parameter,

$Kr = \frac{16\bar{a}\,\vartheta_\infty\bar{\sigma}\bar{T}_\infty^{\,2}}{3\lambda_\infty U_0^{\,2}}$ is the radiation parameter,

$Gr = \frac{\vartheta_\infty g\beta_T\left(\bar{T}_w - \bar{T}_\infty\right)}{U_0^{\,2}}$ is the Grashof Number,

$Gr_m = \frac{\vartheta_\infty g\beta_C\left(\bar{C}_w - \bar{C}_\infty\right)}{U_0^{\,2}}$ is the Concentration buoyancy parameter and $\Pr = \frac{\rho\vartheta_\infty C_p}{\lambda_\infty}$ is the Prandlt number.

A. Atangana-Baleanu Fractional Derivatives

In order to generate the AB fractional model, we replace governing partial differential equations with respect to time by the AB fractional operator of the order $0 \prec \alpha \prec 1$, Eqns. (13)-(15) become

$$AB\left(\frac{\partial^{\alpha} u(y,t)}{\partial t^{\alpha}}\right) = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\theta_r}{\theta-\theta_r}\frac{\partial^2 u}{\partial y^2} + \frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + Gr\theta + Gr_m\phi \tag{17}$$

$$AB\left(\frac{\partial^{\alpha}\theta(y,t)}{\partial t^{\alpha}}\right) = -u\frac{\partial\theta}{\partial x} - v\frac{\partial\theta}{\partial y} + \frac{1}{\Pr}\left(\frac{4}{3}Kr - \frac{\theta_c}{\theta-\theta_c}\right)\frac{\partial^2\theta}{\partial y^2} + \frac{1}{\Pr}\frac{\theta_c}{(\theta-\theta_c)^2}\left(\frac{\partial\theta}{\partial y}\right)^2 - \left(\frac{\theta_r}{\theta-\theta_r}\right).Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{M.EC}{\Pr}\theta \tag{18}$$

$$AB\left(\frac{\partial^{\alpha}\phi(y,t)}{\partial t^{\alpha}}\right) = -u\frac{\partial\phi}{\partial x} - v\frac{\partial\phi}{\partial y} + \frac{\theta_r}{(\theta-\theta_r)^2}.\frac{1}{Sc}\frac{\partial\theta}{\partial y}.\frac{\partial\phi}{\partial y} - \frac{\theta_r}{Sc(\theta-\theta_r)}\frac{\partial^2\phi}{\partial y^2} + So\frac{\partial^2\theta}{\partial y^2} + \xi(\phi - N_c) \tag{19}$$

where, $\frac{\partial^{\alpha}u(y,t)}{\partial t^{\alpha}}$ is the AB fractional operator of order α defined as

$$AB\left(\frac{\partial^{\alpha}u(y,t)}{\partial t^{\alpha}}\right) = \frac{1}{1-\alpha}\int_0^t u'(y,t)E_{\alpha}\left(\frac{-\alpha(z-t)}{1-\alpha}\right)dt \tag{20}$$

Where $E_{\alpha}\left(-t^{\alpha}\right) = \sum_{m=0}^{\infty}\frac{(-t)^{\alpha m}}{\Gamma(1+\alpha m)}$ is the Mittage – Leffler function.

B. Caputo- Fabrizio Fractional Derivatives

In order to generate the CF fractional model, we replace governing partial differential equations with respect to time by the CF fractional operator of the order $0 \prec \beta \prec 1$, Eqns. (13)-(15) become

$$CF\left(\frac{\partial^{\beta}u(y,t)}{\partial t^{\beta}}\right) = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\theta_r}{\theta-\theta_r}\frac{\partial^2 u}{\partial y^2} + \frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} - Mu + Gr\theta + Gr_m\phi \tag{21}$$

$$CF\left(\frac{\partial^{\beta}\theta(y,t)}{\partial t^{\beta}}\right) = -u\frac{\partial\theta}{\partial x} - v\frac{\partial\theta}{\partial y} + \frac{1}{\Pr}\left(\frac{4}{3}Kr - \frac{\theta_c}{\theta-\theta_c}\right)\frac{\partial^2\theta}{\partial y^2} + \frac{1}{\Pr}\frac{\theta_c}{(\theta-\theta_c)^2}\left(\frac{\partial\theta}{\partial y}\right)^2 - \left(\frac{\theta_r}{\theta-\theta_r}\right).Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{M.EC}{\Pr}\theta \tag{22}$$

$$CF\left(\frac{\partial^{\beta}\phi(y,t)}{\partial t^{\beta}}\right) = -u\frac{\partial\phi}{\partial x} - v\frac{\partial\phi}{\partial y} + \frac{\theta_r}{(\theta-\theta_r)^2}.\frac{1}{Sc}\frac{\partial\theta}{\partial y}.\frac{\partial\phi}{\partial y} - \frac{\theta_r}{Sc(\theta-\theta_r)}\frac{\partial^2\phi}{\partial y^2} + So\frac{\partial^2\theta}{\partial y^2} + \xi(\phi - N_c) \tag{23}$$

where, $\frac{\partial^{\beta}u(y,t)}{\partial t^{\beta}}$ is the CF fractional operator of order β defined as

$$CF\left(\frac{\partial^{\beta}u(y,t)}{\partial t^{\beta}}\right) = \frac{1}{1-\beta}\int_0^t u'(y,t)Exp\left(\frac{-\beta(z-t)}{1-\beta}\right)dt \tag{24}$$

C. Numerical Solution

Solutions of equations (17) – (20) or (21) – (24) are obtained by using ordinary finite difference scheme. Discritization is performed using the following formulae:

$$\frac{\partial u}{\partial t} = \frac{u_{i+1,j,k} - u_{i,j,k}}{\Delta t},$$

$$\frac{\partial u}{\partial x} = \frac{u_{i,j+1,k} - u_{i,j,k}}{\Delta x}$$

$$\frac{\partial u}{\partial y} = \frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta y},$$

$$\frac{\partial u}{\partial x} = \frac{u_{i,j+1,k} - u_{i,j,k}}{\Delta x}$$

$$\frac{\partial\phi}{\partial y} = \frac{\phi_{i,j,k+1} - \phi_{i,j,k}}{\Delta y},$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{\Delta y^2},$$

$$\frac{\partial^2\theta}{\partial y^2} = \frac{\theta_{i,j,k+1} - 2\theta_{i,j,k} + \theta_{i,j,k-1}}{\Delta y^2},$$

$$\frac{\partial^2\phi}{\partial y^2} = \frac{\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}}{\Delta y^2} \text{ etc.}$$

The fractional derivatives given by (20) or (24) are calculated using numerical integration. Finally the set of equation (17) – (19) or (21) – (23) together with boundary condition (16) completely discritized and the discritized equations are solved by using an iterative method based on Gauss- seidel scheme.

The boundary condition corresponding to (16) becomes

$$t \le 0 : u_{i,j,k} = 0,\ v_{i,j,k} = 0,\ \theta_{i,j,k} = 0,\ \phi_{i,j,k} = 0 \quad \forall i, j, k$$

$$t \succ 0 : u_{i,j,1} = \cos(\omega t),\ v_{i,j,1} = 0,\ \theta_{i,j,1} = t,\ \phi_{i,j,1} = t \quad \text{since } k = 1 \text{ when } y = 0$$

$$t \succ 0 : u_{i,j,N} \to 0,\ v_{i,j,N} \to 0,\ \theta_{i,j,N} \to 0\ \phi_{i,j,N} \to 0 \quad \text{since } y \to \infty \text{ means } k \to N$$

D. *Important Physical Parameters*

Three important physical parameters for the present problem are

(a) *Coefficient of skin friction:*

The viscous drag at the plate per unit area in the direction of the plate velocity is given by the Newton's law of viscosity in the form

$$\tau = \mu \frac{\partial \bar{u}}{\partial \bar{y}}\bigg)_{\bar{y}=0} = \frac{\mu U_0^{\,2}}{\vartheta_\infty} \frac{\partial u}{\partial y}\bigg)_{y=0}$$

The non-dimensional skin-friction at the plate in the direction of the free stream is given by,

$$C_f = \frac{\tau}{\rho U_0^{\,2}}$$

$$= -\frac{\theta_r}{\theta - \theta_r} . \frac{\partial u}{\partial y}\bigg)_{y=0} = -\frac{\theta_r}{t - \theta_r}\left(\frac{u_{i,j,2} - \cos(\omega t)}{\Delta y}\right)$$

(b) Nusselt number:

The Nusselt number measures the rate of heat transfer at the plate. The heat flux q from the plate to the fluid is given by the Fourier's law of heat conduction in the form

$$q = -\lambda \frac{\partial \bar{T}}{\partial \bar{y}}\bigg)_{\bar{y}=0}$$

The coefficient of rate of heat transfer from the plate to the fluid in terms of Nusselt Number is given by

$$Nu = \frac{q}{\rho U_0 Cp(\bar{T}_w - \bar{T}_\infty)}$$

$$= \frac{\theta_c}{\theta - \theta_c} \frac{\partial \theta}{\partial y}\bigg)_{y=0} = \frac{\theta_c}{t - \theta_c}\left(\frac{\theta_{i,j,2} - t}{\Delta y}\right)$$

(c) Sherwood number:

The mass flux q_m from the plate to the fluid is given by the Fick's law in the form

$$q_m = -D \frac{\partial \bar{C}}{\partial \bar{y}}\bigg)_{\bar{y}=0} = -D \frac{U_0(\bar{C}_w - \bar{C}_\infty)}{\vartheta_\infty} \frac{\partial \phi}{\partial y}\bigg)_{y=0}$$

The Sherwood number measures the rate of mass transfer at the plate and is given by $Sh = \dfrac{q_m}{U_0(\bar{C}_w - \bar{C}_\infty)}$

$$= -\frac{1}{Sc} \frac{\partial \phi}{\partial y}\bigg)_{y=0} = -\frac{1}{Sc}\left(\frac{\phi_{i,j,2} - t}{\Delta y}\right)$$

III. RESULT AND DISCUSSION

The non dimensional discretized governing equations together with the non-dimensional boundary conditions have been solved with the help of AB and CF fractional derivative method by developing suitable programming code in MATLAB using the method described in section C. This analysis has been done to study the effects of various parameters such as $\theta_r, \theta_c, Kr, Sc, Ec, \Pr, M, \alpha$ and β, ξ, Sc etc. on velocity(u), temperature (θ) and species concentration(ϕ) profiles in presence of time. The numerical results have been presented graphically in figures (1) to (14). In the following discussion, the initial values of the parameters are considered as $\theta_r = -7$, $\theta_c = -7$, $\alpha = 0.3$, $\beta = 0.3$, $M = 0.5$, $Gr_m = 0.1$, $Gr = 0.1$, $Kr = 0.05$, $Ec = 0.1$, $\Pr = 0.71$, $Sc = 0.22$ and $\xi = 0.01$ unless otherwise stated.

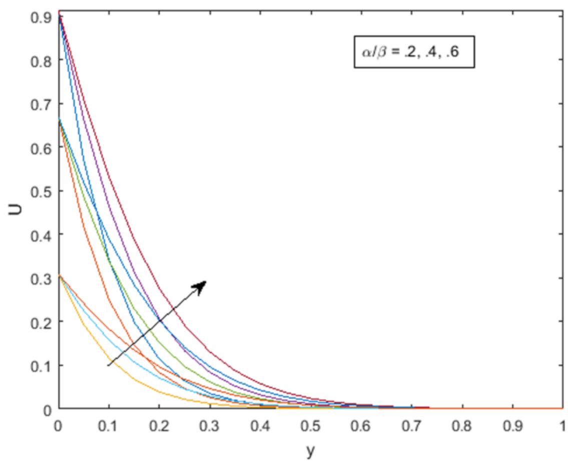

Fig 1: Effects of α and β on velocity profile

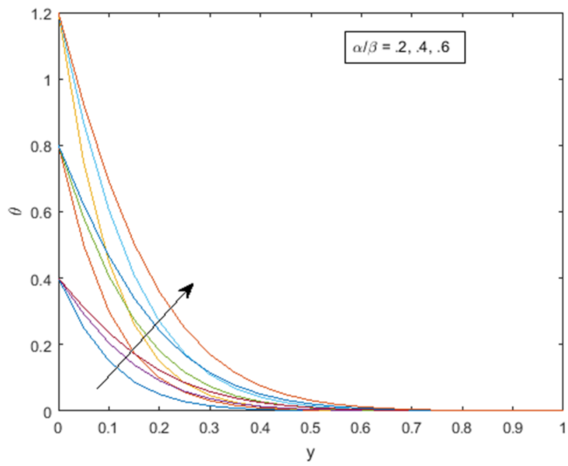

Fig 2: Effects of α and β on temperature profile

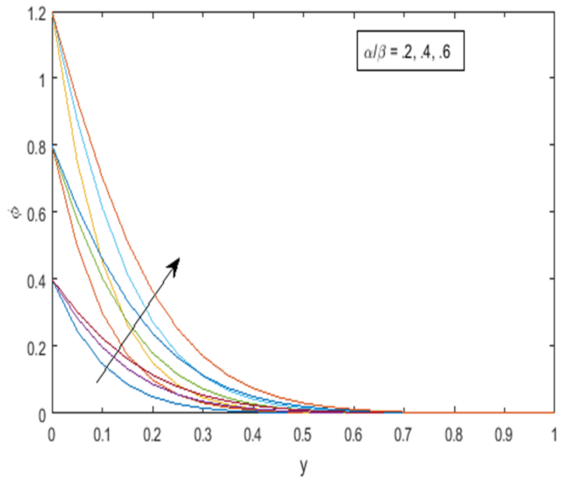

Fig 3: Effects of α and β on concentration profile

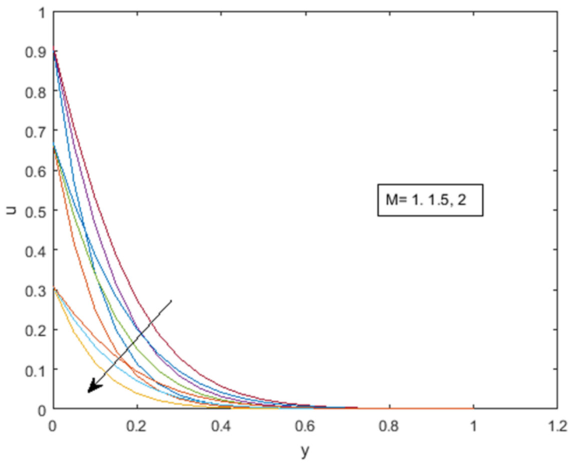

Fig 4: Effects of M on velocity Profile

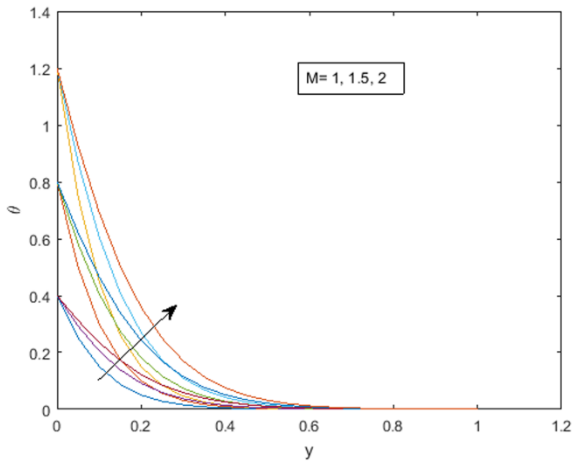

Fig 5: Effects of M on temperature Profile

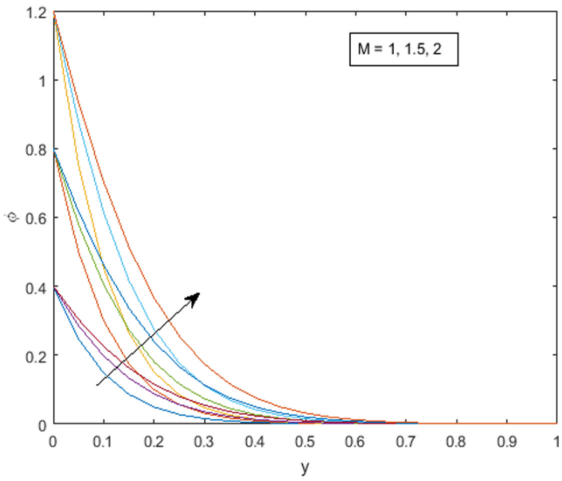

Fig 6: Effects of M on concentration Profile

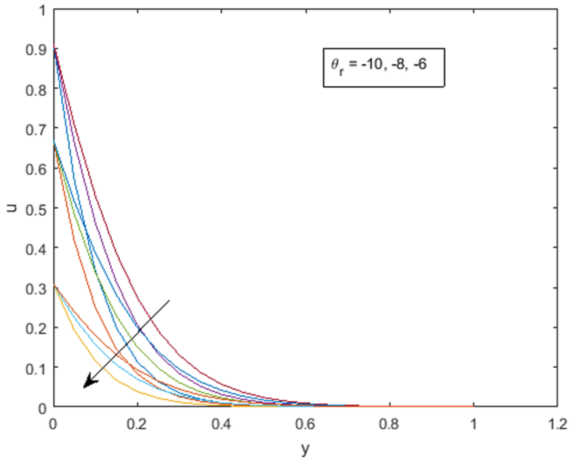

Fig 7: Effects of θ_r on velocity profile

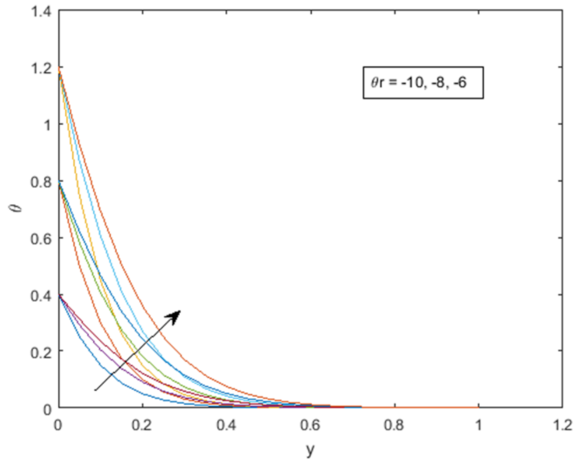

Fig 8: Effects of θ_r on temperature profile

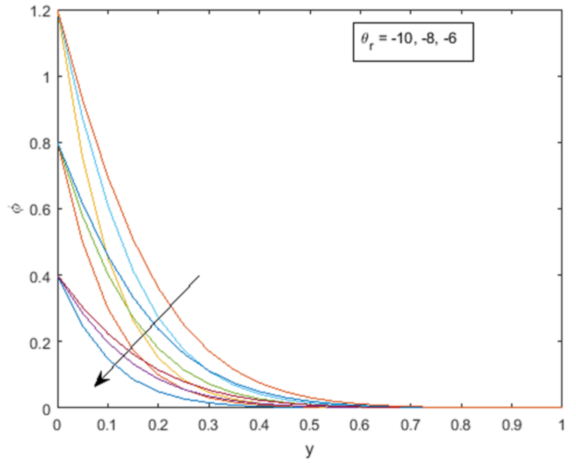

Fig 9: Effects of θ_r on concentration profile

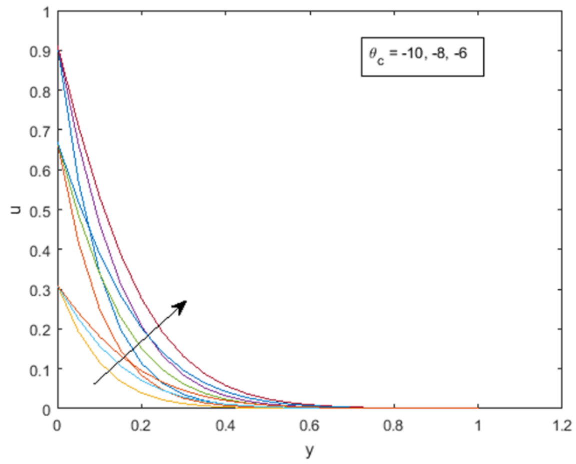

Fig 10: Effects of θ_c on velocity profile

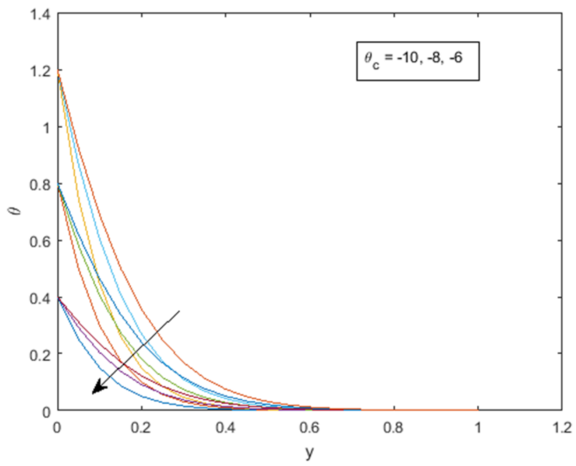

Fig 11: Effects of θ_c on temperature profile

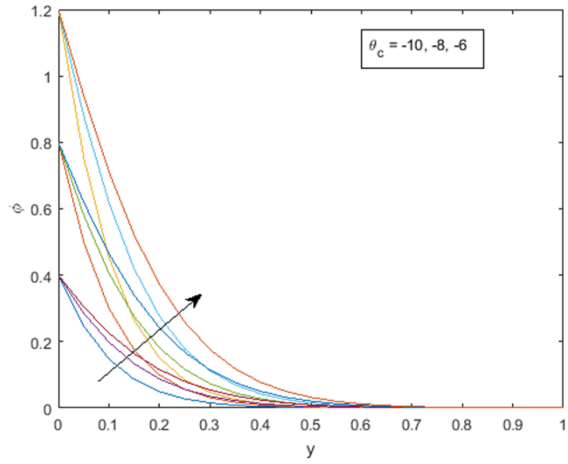

Fig 12: Effects of θ_c on concentration profile

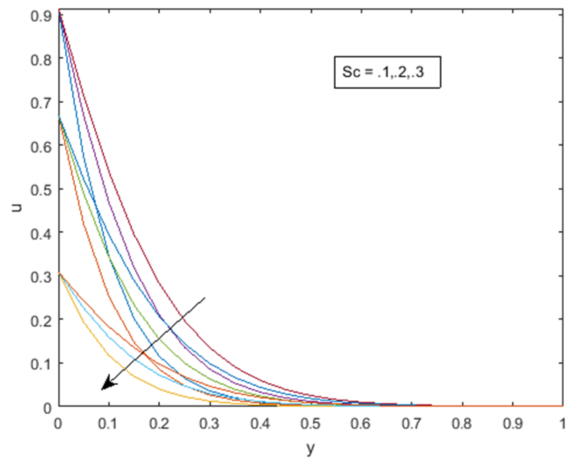

Fig 13: Effects of Sc on velocity profile

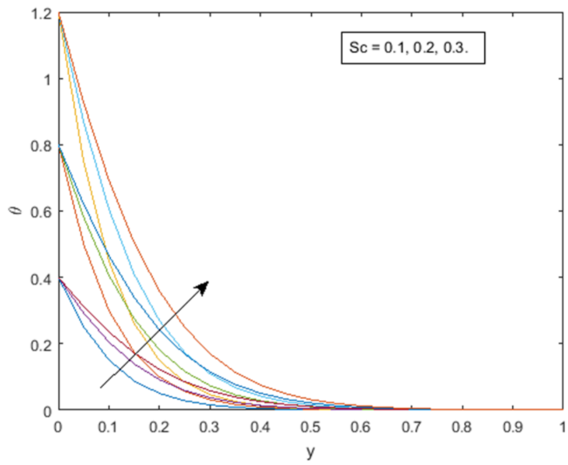

Fig 14: Effects of Sc temperature profile

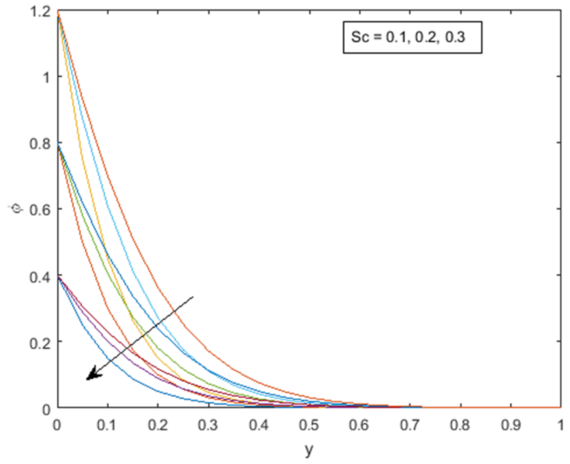

Fig 15: Effects of Sc concentration profile

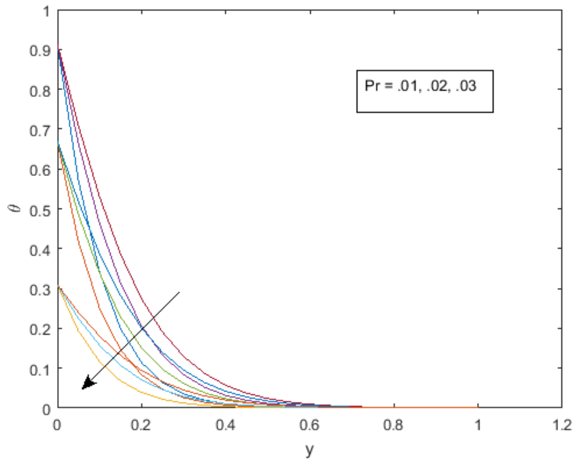

Fig 16: Effects of $\Pr$ temperature profile

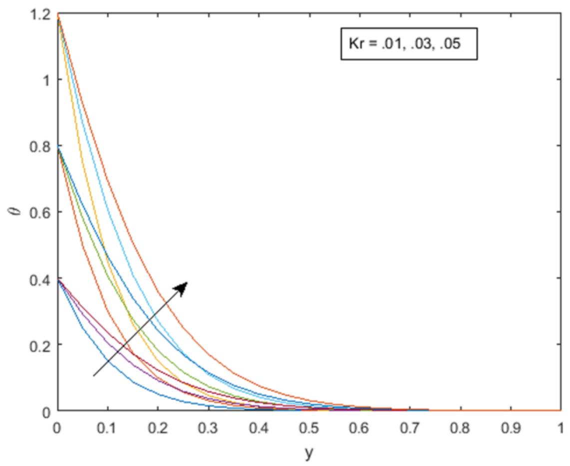

Fig 17: Effects of Kr temperature profile

The solutions have been found for different values of Physical parameters such as the AB fractional operator (α) and CF fractional operator (β), magnetic parameter (M), viscosity variation parameter (θ_r), thermal conductivity parameter (θ_c), Schmidt number (Sc), Prandlt $(\Pr)$, Radiation parameter (Kr) on velocity, temperature and species concentration of fluid and are expressed graphically in Fig1 to Fig17.

Fig. 1- Fig.3 demonstrate the velocity, temperature and concentration profile for various values of the AB fractional operator (α) and CF fractional operator (β). It is observed in all the three figures that the velocity, temperature and concentration u, θ and ϕ respectively increases for increasing values of α and β.

In Fig.4, it is seen that with the increasing value of the Hartmann number M , velocity decreases. The presence of magnetic field in the normal direction of the flow in an electrically conducting fluid produces Lorentz force which opposes the flow. To overcome this opposing force, some extra work should be done which is transformed to heat energy. Hence temperature increases (Fig.5). With the increase of M species concentration also increases (Fig.6).

The effects of viscosity parameter θ_r on velocity, temperature and species concentration distribution are plotted in Fig.7, Fig.8 and Fig.9. In Fig.7 displays that dimensionless velocity u decreases with the increase of θ_r . It happens with the increase of the viscosity parameter the thickness of the velocity boundary layer decreases. Physically, this is due to a larger θ_r , implies higher temperature difference between the fluid and the surface. Fig.8 shows that temperature profile θ rises with the increase of θ_r . The species concentration ϕ decreases for increasing value of θ_r (Fig.9). Fig.10 depicts the distribution of velocity with the variation of the thermal conductivity parameter θ_c . Velocity increases and temperature decreases with the increasing value of θ_c (Fig.11) which implies decreasing of viscosity and so velocity increases. With the increase of θ_c concentration of species increases (Fig.12).

Fig.13 and Fig.14 displays the variation of velocity and concentration profile for various values of Schmidt number Sc . Velocity decreases with the increasing value of Sc (Fig.13). The species concentration decreases with the increasing value of Sc . With the increase of *Sc*, the concentration boundary layer becomes thinner due to which the concentration gradient increases. As a result, the species concentration decreases (Fig.14). In Fig.16 it is seen that temperature is decreases with the increasing value of Prandlt number $(\Pr)$. In Fig.17 shows that temperature is increases with the increasing value of radiation parameter (Kr) .

A. Comparision of AB and CF Fractional Derivative Methods for Various Values of the Parameters

Here we compare between AB and CF fractional derivative for various values of the parameters taking $y = 0.35$. From the following tables it is observed that the values of the velocity, temperature and concentration profiles for various parameters are almost the same for both the the methods- AB and CF fractional derivative but the values of CF is slidly greater than the values of AB. From Table I, II, III, IV,V and VI it is observed that the values of CF is slightly greather than the values of AB for velicity (u)

and temperature (θ) but for concentarion (ϕ) the values of CF is slightly less than the values of AB.

TABLE I : EFFECT OF α or β ON u, θ AND ϕ

α/β	t	u		θ		ϕ	
		AB	CF	AB	CF	AB	CF
0.2	0.4	0.019534	0.019535	0.008676	0.008679	0.008187	0.008187
	0.8	0.014359	0.014379	0.017176	0.017187	0.016858	0.016858
	1.2	0.006799	0.006814	0.025699	0.025725	0.025567	0.025567
0.4	0.4	0.052086	0.052137	0.023211	0.023209	0.021670	0.021661
	0.8	0.038286	0.038396	0.045909	0.045924	0.045092	0.045058
	1.2	0.018185	0.018308	0.068283	0.068496	0.068904	0.068898
0.6	0.4	0.087558	0.087666	0.039193	0.039159	0.035939	0.035912
	0.8	0.064684	0.064981	0.077228	0.077186	0.075289	0.075170
	1.2	0.031495	0.031982	0.114996	0.115771	0.113639	0.113596

TABLE II: EFFECT OF M ON u, θ AND ϕ

M	t	u		θ		ϕ	
		AB	CF	AB	CF	AB	CF
1	0.4	0.086655	0.086714	0.008627	0.008630	0.008188	0.008188
	0.8	0.063689	0.063825	0.017174	0.017186	0.016861	0.016861
	1.2	0.030350	0.030513	0.025694	0.025724	0.025575	0.025575
1.5	0.4	0.051863	0.051903	0.023206	0.023205	0.021677	0.021670
	0.8	0.038132	0.038212	0.045898	0.045908	0.045108	0.045084
	1.2	0.018112	0.018194	0.068306	0.068449	0.068943	0.068941
2	0.4	0.0194990	0.019513	0.039151	0.039138	0.035998	0.035986
	0.8	0.014356	0.014378	0.077106	0.077092	0.075552	0.075515
	1.2	0.006797	0.006814	0.114287	0.114567	0.116222	0.116218

TABLE III: EFFECT OF θ_c ON u, θ AND ϕ

θ_c	t	u		θ		ϕ	
		AB	CF	AB	CF	AB	CF
-10	0.4	0.019499	0.019513	0.039072	0.039059	0.008192	0.008192
	0.8	0.014356	0.014378	0.076795	0.076782	0.016877	0.016877
	1.2	0.006797	0.006814	0.113611	0.113892	0.025611	0.025611
-8	0.4	0.052089	0.052129	0.023185	0.023183	0.021731	0.021724
	0.8	0.038299	0.038380	0.045814	0.045824	0.045321	0.045297
	1.2	0.018191	0.018274	0.068122	0.068265	0.069412	0.069410
-6	0.4	0.087573	0.087632	0.008625	0.008628	0.036197	0.036185
	0.8	0.064367	0.064504	0.017167	0.017180	0.076339	0.076302
	1.2	0.030674	0.030839	0.025679	0.025709	0.117941	0.117939

TABLE IV:EFFECT OF θ_r ON u, θ AND ϕ

θ_r	t	u		θ		ϕ	
		AB	CF	AB	CF	AB	CF
-10	0.4	0.086886	0.086946	0.008627	0.008630	0.035797	0.035785
	0.8	0.063406	0.063542	0.017173	0.017186	0.074782	0.074745
	1.2	0.030044	0.030207	0.025693	0.025723	0.114595	0.114592
-8	0.4	0.051837	0.051877	0.023203	0.023201	0.021612	0.021604
	0.8	0.037944	0.038024	0.045895	0.045905	0.044853	0.044830
	1.2	0.017955	0.018037	0.068305	0.068448	0.068399	0.068397
-6	0.4	0.019465	0.019479	0.039142	0.039128	0.008180	0.008180
	0.8	0.014307	0.014329	0.077095	0.077081	0.016832	0.016832
	1.2	0.006764	0.006781	0.114282	0.114563	0.025513	0.025512

TABLE V: EFFECT OF Sc ON u, θ AND ϕ

Sc	t	u		θ		ϕ	
		AB	CF	AB	CF	AB	CF
0.1	0.4	0.087576	0.087636	0.008627	0.008630	0.008260	0.036788
	0.8	0.064363	0.064500	0.017174	0.017186	0.016851	0.075551
	1.2	0.030659	0.030823	0.025694	0.025724	0.025439	0.114781
0.2	0.4	0.052090	0.052131	0.023206	0.023205	0.022081	0.022078
	0.8	0.038298	0.038378	0.045898	0.045908	0.045090	0.045081
	1.2	0.018186	0.018268	0.068306	0.068449	0.068186	0.068186
0.3	0.4	0.019499	0.019513	0.039153	0.039139	0.036795	0.008260
	0.8	0.014356	0.014378	0.077107	0.077094	0.075573	0.016851
	1.2	0.006796	0.006813	0.114287	0.114568	0.114783	0.025439

TABLE VI: EFFECT OF Kr AND Pr ON θ

Kr	t	θ		Pr	t	θ	
		AB	CF			AB	CF
0.01	0.4	0.008627	0.008630	0.01	0.4	0.025650	0.025694
	0.8	0.017174	0.017186		0.8	0.017167	0.017185
	1.2	0.025694	0.025724		1.2	0.008643	0.008647
0.03	0.4	0.023201	0.023200	0.02	0.4	0.068050	0.068258
	0.8	0.045898	0.045908		0.8	0.045898	0.045913
	1.2	0.068456	0.068917		1.2	0.023326	0.023329
0.05	0.4	0.039133	0.039140	0.03	0.4	0.113733	0.114133
	0.8	0.077102	0.077190		0.8	0.077152	0.077163
	1.2	0.114323	0.114592		1.2	0.039446	0.039487

B. Comparison of AB and CF method for Coefficient of skin friction, Nusselt number and Sherwood number

We are considering the values of the AB operator (α) and the CF operator (β) as 0.25 and 0.5. For these two values C_f, Nu and Sh are calculated by both the AB and CF method for various values of the involved parameters. The variation of Coefficient of skin- friction (C_f), Nusselt number (Nu) and Sherwood number (Sh) at the plate $y=0$ and time $t=0.35$ against θ_c, M and θ_r are demonstrated from Table VII to Table IX. In table VII, it is seen that coefficient of Skin- friction (C_f) increases but Nu and Sh are decreases for the increasing the values of θ_r. In table VIII, it is seen that C_f increases but Nu and Sh are decreases for the increasing the values of θ_c. In table IX, it is seen that C_f increases but Nu and Sh are decreases for the increasing the values of M.

TABLES FOR NUMERICAL VALUES OF C_f , Nu AND Sh

TABLE VII: VARIATION OF PHYSICAL QUANTITIES WITH THERMAL CONDUCTIVITY PARAMETER FOR $\theta_r = -10, M = .5,\ S_0 = 5\ , K_r = .01$

α/β	θ_c	C_f		Nu		Sh	
		AB	CF	AB	CF	AB	CF
.25	-12	-14.380461	-14.379962	2.876189	2.879594	5.922515	5.905266
	-10	-10.431785	-9.719499	2.057259	1.893972	4.455403	4.877412
	-8	-8.52968	-8.010561	1.665563	1.54772	3.748706	4.056809
	-6	-7.347156	-6.921358	1.416665	1.321974	3.309653	3.564089
0.5	-12	-6.516398	-6.148493	1.297239	1.214924	3.031169	3.241514
	-10	-5.888037	-5.561046	1.168106	1.095565	2.816664	3.003347
	-8	-5.389216	-5.093224	1.063685	0.998631	2.646667	2.815835
	-6	-4.979286	-4.707939	0.972888	0.913938	2.507186	2.66312

TABLE VIII: VARIATION OF PHYSICAL QUANTITIES WITH MAGNETIC FIELD PARAMETER FOR $\theta_c = -10, \theta_r = -10,\ S_0 = 5\ , K_r = .01$

α/β	M	C_f		Nu		Sh	
		AB	CF	AB	CF	AB	CF
.25	0.3	-14.371963	-14.371464	2.876188	2.879593	5.922519	5.905269
	0.4	-10.422982	-9.711021	2.069326	1.905571	4.461536	4.881495
	0.5	-8.525322	-8.006635	1.689689	1.571011	3.762675	4.06727
	0.6	-7.350862	-6.925328	1.457594	1.360612	3.334035	3.583069
0.5	0.3	-6.504223	-6.136033	1.294805	1.21245	3.043622	3.254078
	0.4	-5.874657	-5.54823	1.173643	1.101037	2.826903	3.012431
	0.5	-5.381041	-5.085716	1.078538	1.013185	2.658466	2.825455
	0.6	-4.980536	-4.709631	1.001124	0.941463	2.523318	2.675773

TABLE IX: VARIATION OF PHYSICAL QUANTITIES WITH VISCOSITY PARAMETER FOR $\theta_c = -10, M = .5,\ S_0 = 5\ , K_r = .01$

α/β	θ_r	C_f		Nu		Sh	
		AB	CF	AB	CF	AB	CF
.25	-12	-14.482981	-14.482481	2.876101	2.879506	5.957634	5.94033
	-10	-10.464855	-9.747952	2.069283	1.904258	4.461086	4.884307
	-8	-8.50729	-7.987997	1.689836	1.570678	3.731955	4.037108
	-6	-7.250114	-6.827735	1.458046	1.361271	3.26419	3.511475
.5	-12	-6.637243	-6.273966	1.294816	1.213616	3.067395	3.275105
	-10	-5.971478	-5.638657	1.173634	1.099497	2.830117	3.019577
	-8	-5.431367	-5.131772	1.078626	1.012163	2.638866	2.808491
	-6	-4.961446	-4.689	1.001423	0.941184	2.471469	2.624914

IV. CONCLUSION

This investigation presents a numerical solution to study the effects of variable viscosity and thermal conductivity on MHD flow over a hot stretching sheet. Based on above study we may conclude that:

Velocity, temperature and species concentration are increases with the increasing value of AB fractional parameter and CF fractional parameter. Increasing value of Magnetic field parameter decreases the value of velocity but increases the values of temperature and species concentration. When the viscosity parameter increases, the velocity and the species concentration decreases whereas temperature increases. With the increasing thermal conductivity parameter, the velocity and the species concentration increases but the temperature decreases. Velocity and species concentration decrease with the increasing value of the Schmidt number.The Coefficient of skin friction increases due to viscosity, thermal conductivity and magnetic field.The rate of heat and mass transfer decreases due to viscosity, thermal conductivity and magnetic field.

The values of the velocity, temperature and concentration profiles for various parameters are almost the same for both the the methods- AB and CF fractional derivative. As gamma function is present inside the exponential function in AB fractional derivative method, so the result obtained by it is more accurate over the CF fractional derivative method.

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