Augmented Lyapunov Function-based Exponential Stability Control of Fuzzy Systems with Stochastic Input Delay

Hejun Yao

Abstract—**The problem of exponential stability control of fuzzy systems is considered via augmented Lyapunov function approach. The T-S fuzzy system is established by introducing a new random variable on the basis of considering the stochastic input delay satisfying Bernoulli distribution. By designing the weighted Lyapunov function with exponential term, an exponential stability condition based on the bound of random input delay is obtained by using linear matrix inequality method, and the state feedback fuzzy controller based on random input time delay is given. The validity of the results is demonstrated by a numerical example.**

Index Terms—**Fuzzy systems, stochastic delay, exponential stability, augmented Lyapunov function**

I. Introduction

Time delay exists in most practical systems, which often leads to system performance deterioration and even system instability [1−2]. Therefore, the research on the stability of time-delay systems is not only of theoretical significance, but also of important practical application value. In the past decades, the stability analysis of time-delay systems has become one of the research hotspots in various fields [3−5]. Wirtinger's integral inequality is used to deal with the integral term generated in the derivation of L-K functional, which reduces the conservatism of the stability criteria to a certain extent [6]. The stability of T-S fuzzy systems is studied by using convex combination inequality [7]. A triple integral is introduced to study the robust stability of uncertain systems with time delay in the construction of L-K functional. A stability criterion with less conservatism is obtained [8]. However, delay information is not fully considered in the upper bound of the inner integral of the triple integral. Similarly, the methods adopted in [9-11] have obtained good results, However, the information of the upper and lower bounds of delay is not fully considered in the construction of L-K functional, which can further reduce the conservatism of the results. In [12], Qi et al. studied the problem of controller design for time-delay systems with stochastic disturbances and actuator saturation. By using the method of frequency domain analysis, Gherfi et al. proposed a control design method of proportional integral fractional order filter for first-order systems with time delay [13].

Since 1980s, fuzzy control technology and its theory have made great progress. In recent years, the stability analysis and system design of fuzzy control system have attracted much attention, and some beneficial results have been achieved, but the theoretical system has not yet been formed [14]. T-S fuzzy model was proposed by Takagi and Sugeno in 1985. T-S fuzzy model is to fit the same nonlinear system by multiple linear systems, use fuzzy algorithm to deconstruct the input variables, and then de fuzzify through fuzzy calculus reasoning to generate several equations representing the relationship between each group of input and output, and transform the complex nonlinear problem into the problem on different small line segments [15]. The stability analysis of T-S model is based on Lyapunov stability theory. It is necessary to find a common positive definite matrix P for each fuzzy subsystem to satisfy a family of linear matrix inequalities, so that the global system is asymptotically stable. The difficulty is that there is no good method to solve the matrix P and its application is limited. Because the application of T-S fuzzy control method in the system which is difficult to establish accurate mathematical model has been paid more and more attention, many achievements have been made in the study of stability of T-S fuzzy control system. Therefore, most of these results are based on the solution of common Lyapunov function proposed by Tanaka et al [16−17]. Xu et al. studied the stabilization of T-S fuzzy systems with mixed delays. A new Lyapunov function is designed by using the switching idea, which depends on both integral variables and membership functions. Therefore, the time derivative information of membership function can also be used. In addition, the requirement of positive definiteness of matching matrix for Lyapunov function is relaxed. Then the switch controller is designed to ensure the stability of the closed-loop system [18]. Ren studies the robust H_∞ stabilization of stochastic T-S fuzzy singular Markov jump systems with time delay by using integral sliding mode control. On the basis of fully considering the singular derivative matrix, a new fuzzy integral sliding surface function is designed. A delay dependent criterion is derived by Using LMI technique. In addition, in order to ensure the stochastic admissibility of the closed-loop system, a suitable sliding mode control law is designed to make the trajectory of the controlled system reach the specified sliding surface in finite time [19]. A sufficient

This work was supported in part by the National Natural Science Foundation of China under Grant 61073065, the Science and Technology Key Project Henan Province under Grant 202102210128, the Young Teacher Training Plan of Colleges and Universities of Henan Province under Grant 2019GGJS192, the Education Department of Henan Province Key Foundation under Grant 20A110009.

H. J. Yao is with the School of Mathematics and Statistics, Anyang Normal University, Anyang, China (+86-0372-3300967; e-mail: yaohejun@126.com).

condition for stochastic stability dependent on time-varying delay bounds is established. The L1-gain performance of standard linear programming is analyzed [20] . Ge et al. Studied the robust H_∞ stabilization of time-varying delay T-S fuzzy systems based on memory sampling control [21]. The α-exponential stability analysis and control problems of T-S fuzzy systems are discussed in [22]. A sufficient condition for α-exponential stability of controlled systems with L1-gain constraints is given.

However, most of the above results are concerned with the asymptotic stability analysis of fuzzy systems with state delay, while the research results are few on input delay, especially stochastic input delay system. Moreover, the results involving exponential stability are fewer. Because of this, the exponential stability control of T-S fuzzy systems with input delay is studied in this paper. The T-S fuzzy control method is used to describe the stochastic input delay by using the stochastic variable satisfying Bernoulli, and the mathematical model of the nonlinear system with input delay is established. By constructing an improved augmented Lyapunov function, the exponential stability condition is explored.

II. Problem Formulation

Consider the following fuzzy system with stochastic input delay

Rule i:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i , ⋯, and $z_n(t)$ is M_n^i

THEN

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + B_i u(t-\tau(t)) \tag{1}$$

where $z(t) = [z_1(t) \ \ z_2(t) \ \ \cdots \ \ z_n(t)]^T$ is the premise variable, $x(t) \in R^n$ is the system state, $u(t-\tau(t)) \in R^m$ is the controlled input with stochastic delay, $M_k^i (i=1, \ 2, \ \cdots, \ r \ ; \ k=1, \ 2, \ \cdots, \ n)$ are fuzzy sets. $A_i \in R^{n\times n}$ are constant matrices, $B_i \in R^{n\times m}$ are input matrix, r is the number of IF-THEN rules. $\tau(t)$ is a stochastic input delay satisfying $\tau(t) \in [0, \ \tau]$. $\Delta A_i(t) \in R^{n\times n}$ satisfies:

$$\Delta A_i(t) = DF(t)E_i$$

where D, E_i are constant matrices , $F(t)$ satisfies $F^T(t)F(t) \le I$. T-S fuzzy system is described by

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))[(A_i + \Delta A_i(t))x(t) \\ + B_i u(t-\tau(t))]\end{aligned} \tag{2}$$

$$x(t) = \psi(t) \qquad t \in [-\tau, \ 0]$$

where

$$\omega_i(z(t)) = \prod_{k=1}^{n} M_k^i(z_k(t))$$

$$\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r} \omega_i(z(t))}$$

and $\omega_i(z(t))$ satisfying

$$\omega_i(z(t)) \ge 0, \qquad \sum_{i=1}^{r} \omega_i(z(t)) > 0$$

We will design the controller as

$$u(t) = \sum_{i=1}^{r} \mu_i(z(t))K_i x(t) \tag{3}$$

With (3) and (2), we obtain

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))[(A_i + \Delta A_i(t))x(t) \\ + B_i K_j x(t-\tau(t))]\end{aligned} \tag{4}$$

$$x(t) = \psi(t) \qquad t \in [-\tau, \ 0]$$

where $\psi(t)$ is the initial condition of the state, and satisfies

$$\| \dot{\psi}(t) \| \le \bar{\psi} \qquad t \in [-\tau, \ 0]$$

where $\bar{\psi}$ is a positive constant

In order to deal with the stochastic input delay, we make $\tau_1 \in [0, \ \tau]$,

$$\Omega_1 = \{t : \tau(t) \in [0, \ \tau_1)\}$$

$$\Omega_2 = \{t : \tau(t) \in [\tau_1, \ \tau]\}$$

Obviously

$$\Omega_1 \cap \Omega_2 = \Phi$$

Then we define two functions as

$$h_1(t) = \begin{cases} \tau(t) & t \in \Omega_1 \\ 0 & t \notin \Omega_1 \end{cases}$$

$$h_2(t) = \begin{cases} \tau(t) & t \in \Omega_2 \\ \tau_1 & t \notin \Omega_2 \end{cases} \tag{5}$$

$\beta(t)$ is defined as

$$\beta(t) = \begin{cases} 1 & t \in \Omega_1 \\ 0 & t \in \Omega_2 \end{cases} \tag{6}$$

where we suppose that $\beta(t)$ is a Bernoulli distributed sequence satisfying

$$\Pr ob\{\beta(t) = 1\} = E\{\beta(t)\} = \beta$$

where $\beta \in [0,1]$ is a constant.

With the functions $h_1(t), h_2(t)$ and the stochastic variable $\beta(t)$, the closed system (4) can be equivalently written as

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))[\overline{A}_i x(t) \\ &\quad + \beta(t)B_iK_j x(t-h_1(t)) \\ &\quad + (1-\beta(t))B_iK_j x(t-h_2(t))] \qquad (7) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))\overline{A}_{ij}\xi(t) \\ x(t) &= \psi(t) \qquad t\in[-\tau,\ 0]\end{aligned}$$

where

$$\overline{A}_{ij} = [\overline{A}_i \quad \beta(t)B_iK_j \quad (1-\beta(t))B_iK_j]$$

$$\xi^T(t) = [x^T(t),\ x^T(t-h_1(t)),\ x^T(t-h_2(t))]$$

$$\overline{A}_i = A_i + \Delta A_i(t)$$

Remark1. The fuzzy system is modeled as a stochastic system satisfying Bernoulli distribution according to the probability $\beta(t)$ of random delay in different intervals. And the influence of external disturbance on the system was considered.

III. Main Results

Definition1 [10] For the system (7), if there exist constants $\alpha > 0$ and $\gamma \geq 1$ such that

$$E\{\| x(t)\|\} \leq \gamma \sup_{-\bar{d}\leq s\leq 0} E\{\|\psi(s)\|\}\mathrm{e}^{-\alpha t} \qquad t\geq 0$$

then the system (7) is mean-square exponentially stable.

Lemma1 [2] For any vectors a, b and matrices N, X, Y, Z with appropriate dimensions, if the following matrix inequality holds

$$\begin{bmatrix} X & Y \\ Y^T & Z\end{bmatrix} \geq 0$$

then we have

$$-2a^TNb \leq \inf_{X,Y,Z}\begin{bmatrix} a \\ b\end{bmatrix}^T\begin{bmatrix} X & Y-N \\ Y^T-N^T & Z\end{bmatrix}\begin{bmatrix} a \\ b\end{bmatrix}$$

Lemma2 [8] For the matrices $X_i, Y_i (1\leq i\leq r)$ and positive-definite matrix $S > 0$ with appropriate dimensions, the following inequality is hold

$$\begin{aligned}&2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{p=1}^{r}\sum_{l=1}^{r}\mu_i\mu_j\mu_p\mu_l X_{ij}^T S Y_{pl} \\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(X_{ij}^TSX_{ij} + Y_{ij}^TSY_{ij})\end{aligned}$$

where $\mu_i(1\leq i\leq r)$ denotes $\mu_i(z(t))\geq 0$, and

$$\sum_{i=1}^{r}\mu_i(z(t)) = 1$$

Lemma3 [4] The linear matrix inequality

$$\begin{bmatrix} Y(x) & W(x) \\ * & R(x)\end{bmatrix} > 0$$

is equivalent to

$$R(x) > 0,\ Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where the matrices $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Lemma4 [9] For constant $\varepsilon > 0$ and matrices D, E, F, satisfying $F^TF\leq I$, then the following inequality holds

$$DEF + E^TF^TD^T \leq \varepsilon DD^T + \varepsilon^{-1}E^TE$$

Theorem1 If there exist positive-definite matrices $P, R\in R^{n\times n}$ and matrices $K_j\in R^{m\times n}$ $(j=1,2,\cdots,r)$ and X_{ij}, Y_i $(i,j=1,2,3)$ with appropriate dimensions, and constants $\alpha > 0, 1\geq\beta\geq 0$, such that the following matrix inequality holds

$$\Theta = \begin{bmatrix}\Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33}\end{bmatrix} < 0 \qquad (8)$$

where

$$\Theta_{11} = P\overline{A}_i + \overline{A}_i^TP + 2\alpha P + \tau X_{11} + \tau\overline{A}_i^TR\overline{A}_i$$

$$\Theta_{12} = P\beta B_iK_j + Y_1 + \tau X_{12} + \tau\overline{A}_i^TR\beta B_iK_j$$

$$\Theta_{13} = P(1-\beta)B_iK_j + \tau X_{13} - Y_1 + \tau\overline{A}_i^TR(1-\beta)B_iK_j$$

$$\Theta_{22} = \tau X_{22} + Y_2 + Y_2^T + \tau K_j^TB_i^TR\beta B_iK_j$$

$$\Theta_{23} = -Y_2 + Y_3^T + \tau X_{23}$$

$$\Theta_{33} = \tau X_{33} - Y_3 - Y_3^T + \tau K_j^TB_i^TR(1-\beta)B_iK_j$$

the closed system (7) is mean-square exponentially stable.

Proof The Lyapunov function is selected as

$$V(t) = x^T(t)Px(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta$$

where P, R are positive-definite matrices. And, we have

$$\begin{aligned}&\dot{V}(t) + 2\alpha V(t) \\ &= 2x^T(t)P\dot{x}(t) + \tau\dot{x}^T(t)R\dot{x}(t) \\ &\quad - \int_{-\tau}^{0}\dot{x}^T(t+\theta)Re^{2\alpha\theta}\dot{x}(t+\theta)d\theta \\ &\quad - 2\alpha\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta + 2\alpha V(t) \\ &= 2x^T(t)P\dot{x}(t) + \tau\dot{x}^T(t)R\dot{x}(t) \\ &\quad - \int_{t-\tau}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds \\ &\quad - 2\alpha\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta \\ &\quad + 2\alpha x^T(t)Px(t) + 2\alpha\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta \\ &= 2x^T(t)P\dot{x}(t) + \tau\dot{x}^T(t)R\dot{x}(t) + 2\alpha x^T(t)Px(t) \\ &\quad - \int_{t-\tau}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds\end{aligned} \qquad (9)$$

With

$$x(t-h_1(t)) - x(t-h_2(t)) - \int_{t-h_2(t)}^{t-h_1(t)}\dot{x}(s)ds = 0$$

for any $4n \times n$ matrix $N = \begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T$, we know

$$0 = \xi^T(t)N[x(t-h_1(t)) - x(t-h_2(t)) - \int_{t-h_2(t)}^{t-h_1(t)} \dot{x}(s)ds] \tag{10}$$

With lemma1 and the formula (10), we obtain

$$\begin{aligned}
0 &\le 2\xi^T(t)N[x(t-h_1(t)) - x(t-h_2(t))] \\
&\quad + \int_{t-h_2(t)}^{t-h_1(t)} \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Re^{2\alpha(s-t)} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds \\
&= 2\xi^T(t)Y[x(t-h_1(t)) - x(t-h_2(t))] \\
&\quad + (h_2(t) - h_1(t))\xi^T(t)X\xi(t) \\
&\quad + \int_{t-h_2(t)}^{t-h_1(t)} \dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds \\
&\le 2\xi^T(t)Y[x(t-h_1(t)) - x(t-h_2(t))] \\
&\quad + \tau\xi^T(t)X\xi(t) + \int_{t-\tau}^{t} \dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds
\end{aligned} \tag{11}$$

Inserting (11) into (9)

$$\begin{aligned}
\dot{V}(t) + 2\alpha V(t) &\le 2x^T(t)P\dot{x}(t) + \tau\dot{x}^T(t)R\dot{x}(t) \\
&\quad + 2\alpha x^T(t)Px(t) \\
&\quad + 2\xi^T(t)Y[x(t-h_1(t)) \\
&\quad - x(t-h_2(t))] + \tau\xi^T(t)X\xi(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\{x^T(t)[P\overline{A}_i + \overline{A}_i^T P \\
&\quad + 2\alpha P]x(t) + 2x^T(t)P\overline{A}_{di}x(t-d) \\
&\quad + 2x^T(t)P\beta(t)B_iK_jx(t-h_1(t)) \\
&\quad + 2x^T(t)P(1-\beta(t))B_iK_jx(t-h_2(t)) \\
&\quad + 2\xi^T(t)Y\begin{bmatrix} 0 & 0 & I & -I \end{bmatrix}\xi(t) \\
&\quad + \tau\xi^T(t)X\xi(t) + \tau\dot{x}^T(t)R\dot{x}(t)
\end{aligned} \tag{12}$$

With lemma2, we have

$$\begin{aligned}
&\tau\dot{x}^T(t)R\dot{x}(t) \\
&= \tau\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{p=1}^{r}\sum_{l=1}^{r} \mu_i(z(t))\mu_j(z(t))\mu_p(z(t))\mu_l(z(t)) \\
&\quad \cdot (\overline{A}_{ij}\xi(t))^T R(\overline{A}_{pl}\xi(t)) \\
&\le \tau\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\xi^T(t)\overline{A}_{ij}^T R\overline{A}_{ij}\xi(t) \\
&= \tau\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\xi^T(t)\Pi\xi(t)
\end{aligned} \tag{13}$$

where

$$\Pi = \begin{bmatrix} \overline{A}_i^T R\overline{A}_i & \overline{A}_i^T R\beta(t)B_iK_j & \overline{A}_i^T R(1-\beta(t))B_iK_j \\ * & \beta^2(t)K_j^T B_i^T RB_iK_j & \beta(t)(1-\beta(t))K_j^T B_i^T RB_iK_j \\ * & * & (1-\beta(t))^2 K_j^T B_i^T RB_iK_j \end{bmatrix}$$

Obviously

$$\begin{aligned}
&2\xi^T(t)\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}\begin{bmatrix} 0 & I & -I \end{bmatrix}\xi(t) \\
&= \xi^T(t)\begin{bmatrix} 0 & Y_1 & -Y_1 \\ * & Y_2 + Y_2^T & -Y_2 + Y_3^T \\ * & * & -Y_3 - Y_3^T \end{bmatrix}\xi(t)
\end{aligned} \tag{14}$$

Inserting the formulas (13), (14) into the inequality (12), and with inequality (5), we obtain

$$\begin{aligned}
E\{V\} &< E\{V(0)\}e^{-2\alpha t} \\
&\le [\lambda_{\max}(P) + \tau\lambda_{\max}(R)\overline{\psi}^2]E\{\|\psi(t)\|^2\}e^{-2\alpha t}
\end{aligned} \tag{15}$$

Obviously

$$E\{V(t)\} \ge \lambda_{\min}(P)E\{\|x(t)\|^2\} \tag{16}$$

From the inequalities (15) and (16), we obtain

$$E\{\|x(t)\|\} < \sqrt{\frac{\lambda_{\max}(P) + \tau\lambda_{\max}(R)\overline{\psi}^2}{\lambda_{\min}(P)}}E\{\|\psi(t)\|\}e^{-\alpha t}$$

With the definition1, the closed system (7) is exponentially stable.

Remark2. In theorem 1, the sufficient condition (8) is not a linear matrix inequality, which cannot be solved by the tool of the LMI toolbox in MATLAB.

Theorem 2 If there exist positive-definite matrices $\overline{P}, \overline{R} \in R^{n\times n}$ and matrices $\overline{K}_j \in R^{m\times n}$ $(j = 1, 2, \cdots, r)$, $\overline{X}_{ij}, \overline{Y}_i$ $(i, j = 1, 2, 3)$ and constants $\alpha > 0, 1 \ge \beta \ge 0$, such that the following linear matrix inequalities hold

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & \Xi_{35} & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \tag{17}$$

where

$$\begin{aligned}
\Xi_{11} &= A_i\overline{P} + \overline{P}A_i^T + 2\alpha\overline{P} + \tau\overline{X}_{11} + \varepsilon_1 DD^T \\
\Xi_{12} &= \beta B_i\overline{K}_j + \tau\overline{X}_{12} + \overline{Y}_1 \\
\Xi_{13} &= (1-\beta)B_i\overline{K}_j - \overline{Y}_1 + \tau\overline{X}_{13} \\
\Xi_{14} &= \tau\beta\overline{P}A_i^T \\
\Xi_{15} &= \tau(1-\beta)\overline{P}A_i^T \\
\Xi_{16} &= \overline{P}E_i^T \\
\Xi_{17} &= \tau\beta\overline{P}E_i^T \\
\Xi_{18} &= \tau(1-\beta)\overline{P}E_i^T
\end{aligned}$$

$$\Xi_{22} = \tau \bar{X}_{22} + \bar{Y}_2 + \bar{Y}_2^T$$
$$\Xi_{23} = \tau \bar{X}_{23} + \bar{Y}_3^T - \bar{Y}_2$$
$$\Xi_{24} = \tau\beta \bar{K}_j^T B_i^T$$
$$\Xi_{33} = \tau \bar{X}_{33} - \bar{Y}_3 - \bar{Y}_3^T$$
$$\Xi_{35} = \tau(1-\beta)\bar{K}_j^T B_i^T$$
$$\Xi_{44} = -\tau_1 \beta \bar{R} + \varepsilon_2 DD^T$$
$$\Xi_{55} = -\tau_1(1-\beta)\bar{R} + \varepsilon_3 DD^T$$

with the controller $u(t) = \sum_i^r \mu_i(z(t))\bar{K}_i \bar{P}^{-1} x(t)$, the closed system (7) is mean-square exponentially stable.

Proof.

$$\Theta = \Theta_0 + \tau \begin{bmatrix} \bar{A}_i^T R \bar{A}_i & \bar{A}_i^T R\beta B_i K_j & \bar{A}_i^T R(1-\beta)B_i K_j \\ * & K_j^T B_i^T R\beta B_i K_j & 0 \\ * & * & (1-\beta)K_j^T B_i^T R B_i K_j \end{bmatrix}$$
$$= \Theta_0 + \tau\beta \begin{bmatrix} \bar{A}_i^T R \bar{A}_i & \bar{A}_i^T R B_i K_j & 0 \\ * & K_j^T B_i^T R B_i K_j & 0 \\ * & * & 0 \end{bmatrix}$$
$$+ \tau(1-\beta) \begin{bmatrix} \bar{A}_i^T R \bar{A}_i & 0 & \bar{A}_i^T R B_i K_j \\ * & 0 & 0 \\ * & * & K_j^T B_i^T R B_i K_j \end{bmatrix}$$
$$= \Theta_0 + \alpha_1^T \frac{1}{\tau\beta} R^{-1} \alpha_1 + \alpha_2^T \frac{1}{\tau(1-\beta)} R^{-1} \alpha_2$$

where

$$\Theta_0 = \begin{bmatrix} \Theta_{011} & \Theta_{012} & P(1-\beta)B_i K_j + \tau X_{13} - Y_1 \\ * & \Theta_{022} & -Y_2 + Y_3^T + \tau X_{23} \\ * & * & \tau X_{33} - Y_3 - Y_3^T \end{bmatrix}$$
$$\Theta_{011} = P\bar{A}_i + \bar{A}_i^T P + 2\alpha P + \tau X_{11}$$
$$\Theta_{012} = P\beta B_i K_j + Y_1 + \tau X_{12}$$
$$\Theta_{022} = \tau X_{22} + Y_2 + Y_2^T$$
$$\alpha_1 = \begin{bmatrix} \tau\beta R\bar{A}_i & \tau\beta R B_i K_j & 0 \end{bmatrix}$$
$$\alpha_2 = \begin{bmatrix} \tau(1-\beta) R\bar{A}_i & 0 & \tau(1-\beta) R B_i K_j \end{bmatrix}$$

With lemma3, $\Theta < 0$ is equivalent to

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \tau\beta \bar{A}_i^T R & \tau(1-\beta)\bar{A}_i^T R \\ * & \Sigma_{22} & \Sigma_{23} & \tau\beta K_j^T B_i^T R & 0 \\ * & * & \Sigma_{33} & 0 & \tau(1-\beta)K_j^T B_i^T R \\ * & * & * & -\tau\beta R & 0 \\ * & * & * & * & -\tau(1-\beta)R \end{bmatrix} < 0$$

$$\Sigma_{11} = P\bar{A}_i + \bar{A}_i^T P + 2\alpha P + \tau X_{11}$$
$$\Sigma_{12} = P\beta B_i K_j + Y_1 + \tau X_{12}$$
$$\Sigma_{13} = P(1-\beta)B_i K_j + \tau X_{13} - Y$$
$$\Sigma_{22} = \tau X_{22} + Y_2 + Y_2^T$$
$$\Sigma_{23} = \tau X_{23} + Y_3^T - Y_2$$
$$\Sigma_{33} = \tau X_{33} - Y_3 - Y_3^T$$

Obviously

$$\Sigma = \Sigma_0 + \gamma_1^T F(t)\bar{\gamma}_1 + \bar{\gamma}_1^T F^T(t)\gamma_1 + \gamma_2^T F^T(t)\bar{\gamma}_2$$
$$+ \bar{\gamma}_2^T F(t)\gamma_2 + \gamma_3^T F^T(t)\bar{\gamma}_3 + \bar{\gamma}_3^T F(t)\gamma_3$$
$$\le \Sigma_0 + \varepsilon_1 \gamma_1^T \gamma_1 + \frac{1}{\varepsilon_1}\bar{\gamma}_1^T \bar{\gamma}_1 + \varepsilon_2 \bar{\gamma}_2^T \bar{\gamma}_2$$
$$+ \frac{1}{\varepsilon_2}\gamma_2^T \gamma_2 + \varepsilon_3 \bar{\gamma}_3^T \bar{\gamma}_3 + \frac{1}{\varepsilon_3}\gamma_3^T \gamma_3$$

where

$$\Sigma_0 = \begin{bmatrix} \Sigma_{011} & \Sigma_{012} & \Sigma_{013} & \tau\beta A_i^T R & \tau(1-\beta)A_i^T R \\ * & \Sigma_{022} & \Sigma_{023} & \tau\beta K_j^T B_i^T R & 0 \\ * & * & \Sigma_{033} & 0 & \tau(1-\beta)K_j^T B_i^T R \\ * & * & * & -\tau\beta R & 0 \\ * & * & * & * & -\tau(1-\beta)R \end{bmatrix}$$
$$\Sigma_{011} = PA_i + A_i^T P + 2\alpha P + \tau X_{11}$$
$$\Sigma_{012} = P\beta B_i K_j + Y_1 + \tau X_{12}$$
$$\Sigma_{013} = P(1-\beta)B_i K_j + \tau X_{13} - Y_1$$
$$\Sigma_{022} = \tau X_{22} + Y_2 + Y_2^T$$
$$\Sigma_{023} = \tau X_{23} + Y_3^T - Y_2$$
$$\Sigma_{033} = \tau X_{33} - Y_3 - Y_3^T$$
$$\gamma_1 = \begin{bmatrix} D^T P & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\gamma_2 = \begin{bmatrix} \tau\beta E_i & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\gamma_3 = \begin{bmatrix} \tau(1-\beta) E_i & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\bar{\gamma}_1 = \begin{bmatrix} E_i & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\bar{\gamma}_2 = \begin{bmatrix} 0 & 0 & 0 & D^T R & 0 \end{bmatrix}$$
$$\bar{\gamma}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & D^T R \end{bmatrix}$$

With lemma4, $\Sigma < 0$ is equivalent to

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} & \Delta_{16} & \Delta_{17} & \Delta_{18} \\ * & \Delta_{22} & \Delta_{23} & \Delta_{24} & 0 & 0 & 0 & 0 \\ * & * & \Delta_{33} & 0 & \Delta_{35} & 0 & 0 & 0 \\ * & * & * & \Delta_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Delta_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (18)$$

where

$\Delta_{11} = PA_i + A_i^T P + 2\alpha P + \tau X_{11} + \varepsilon_1 PDD^T P$
$\Delta_{12} = P\beta B_i K_j + \tau X_{12} + Y_1$
$\Delta_{13} = \tau X_{22} + Y_2 + Y_2^T P(1-\beta)B_i K_j - Y_1 + \tau X_{13}$
$\Delta_{14} = \tau\beta A_i^T R$
$\Delta_{15} = \tau(1-\beta)A_i^T R$
$\Delta_{16} = E_i^T$
$\Delta_{17} = \tau\beta E_i^T$
$\Delta_{18} = \tau(1-\beta)E_i^T$
$\Delta_{22} = \tau X_{22} + Y_2 + Y_2^T$
$\Delta_{23} = \tau X_{23} + Y_3^T - Y_2$
$\Delta_{24} = \tau\beta K_j^T B_i^T R$
$\Delta_{33} = \tau X_{33} - Y_3 - Y_3^T$
$\Delta_{35} = \tau(1-\beta)K_j^T B_i^T R$
$\Delta_{44} = \varepsilon_2 RDD^T R - \tau\beta R$
$\Delta_{55} = \varepsilon_3 RDD^T R - \tau(1-\beta)R$

Multiply by $diag\{P^{-1}\ P^{-1}\ P^{-1}\ R^{-1}\ R^{-1}\ I\ I\ I\}$ on both sides of equation (18), and assuming $\overline{P} = P^{-1}\ \overline{K}_j = K_j P^{-1}\ \overline{X}_{ij} = P^{-1}X_{ij}P^{-1}\ \overline{Y}_i = P^{-1}Y_i P^{-1}\ \overline{R} = R^{-1}$ t the inequality $\Delta < 0$ is equivalent to (17). Therefore, (17) is equivalent to (8).

Remark3. With the lemma1-4, a sufficient condition (17) is obtained in terms of linear matrix inequality in theorem2.

Remark4. In theorem 2, when the value of exponential stability α is given, condition (17) is a linear matrix inequality. If we want to optimize the exponential stability α, we can select different values of α and solve it many times, so as to get the optimization result.

IV. Simulation

Consider the fuzzy system (7), where

$$A_1 = \begin{bmatrix} -2 & -2 \\ 5 & -7 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, E_{11} = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}, E_{21} = \begin{bmatrix} 1 & 2 \end{bmatrix},$$
$$E_{31} = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix},\ F(t) = 0.3\cos t, \tau = 1, \alpha = 0.2,,$$
$$\beta = 0.3$$

Solving the sufficient condition (17), the fuzzy controller gain matrix as

$$K_1 = \overline{K}_1 \overline{P}^{-1} = [-2.2445 \quad -1.5867]$$
$$K_2 = \overline{K}_2 \overline{P}^{-1} = [0.5672 \quad -1.6778]$$

Theoretically, the system (7) is exponentially stable. If we choose the following initial conditions

$$\begin{bmatrix} x_{11}(0) \\ x_{12}(0) \\ x_{21}(0) \\ x_{22}(0) \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \\ 3 \\ -5 \end{bmatrix}$$

the simulation results as

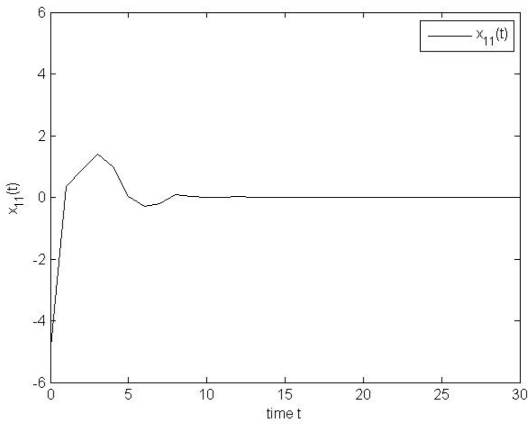

Fig.1 The response curve of the system state $x_{11}(t)$

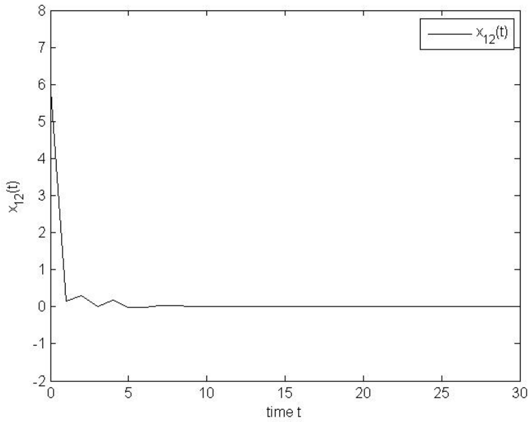

Fig.2 The response curve of the system state $x_{12}(t)$

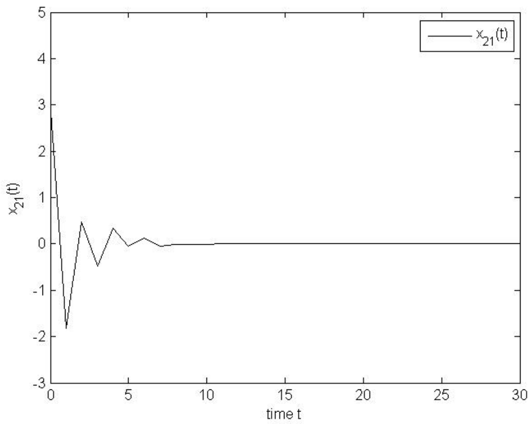

Fig.3 The response curve of the system state $x_{21}(t)$

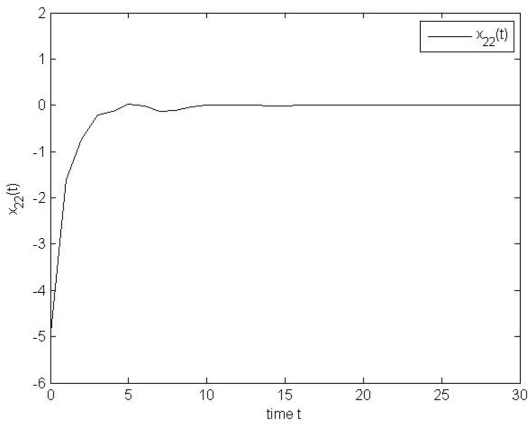

Fig.4 The response curve of the system state $x_{22}(t)$

From the response curves of the four system state, the state convergence speed of the system is fast and the state is exponential convergence. The overshoot of state convergence is small and the stability is good, so the control design method given in this paper is feasible.

V. CONCLUSION

T-S fuzzy approach is used to model a fuzzy system with a stochastic input delay in this paper. A stochastic variable is introduced to model the controlled system as a new stochastic system. An exponential stability condition is obtained by constructing a weighted Lyapunov function. Then, the state feedback fuzzy controller is designed.

REFERENCES

[1] G. Song, J. Lam and S. Xu, "Quantized feedback stabilization of continuous time-delay systems subject to actuator saturation," *Nonlinear Analysis: Hybrid Systems*, vol.30, pp.1-13, 2018.

[2] Y.L. Shi and C. Li, "Almost Periodic Solution, Local Asymptotical Stability and Uniform Persistence for A Harvesting Model of Plankton Allelopathy with Time Delays," *IAENG International Journal of Applied Mathematics*, vol. 49, no.3, pp.307-312, 2019.

[3] W. Qi, Y. Kao, X. Gao and Y. Wei, "Controller design for time-delay system with stochastic disturbance and actuator saturation via a new criterion," *Applied Mathematics and Computation,* vol. 320, pp. 535-546, 2018.

[4] Y. Ma, X. Jia and Q. Zhang, "Robust observer-based finite-time H_∞ control for discrete-time singular Markovian jumping system with time delay and actuator saturation," *Nonlinear Analysis: Hybrid Systems*, vol.28, pp.1-22, 2018.

[5] W. B. Chen and F. Gao, "Improved Delay-dependent Stability Criteria for Systems with Two Additive Time-varying Delays," *IAENG International Journal of Applied Mathematics*, vol. 49, no.4, pp.427-433, 2019.

[6] O.M. Kwon, M. J. Park, J. H. Park, "Improved results on stability of linear systems with time-varying delays via wirtinger-based integral inequality," *Journal of the Franklin Institute*, vol. 351, no.12, pp. 5386- 5398, 2014.

[7] L.K.Wang, H.K.Lam, "New stability criterion for continuous-time Takagi-sugeno fuzzy systems with time varying delay," *IEEE Transactions on Cybernetics*,vol.49, no.4,pp.1551-1556, 2019.

[8] W. Qian, M. M. Yuan, L. Wang, "Robust stability criteria for uncertain systems with interval time-varying delay based on multi-integral functional approach," *Journal of the Franklin Institute*, vol.355, no.2, pp.849-861, 2018.

[9] C.K. Zhang, Y. He, L. Jiang, "Stability analysis of systems with time-varying delay via relaxed integral inequalities," *Systems & Control Letters*, vol.92, no.1, pp.52-61, 2016.

[10] J. H. Kim, "Further improvement of Jensen inequality and application to stability of time-delayed systems,"*Automatica*, vol.64, no.1, pp. 121-125, 2016.

[11] Z. C. Li, Y. Bai, C.Z. Huang, "Novel delay-partitioning stabilization approach for networked control system via wirtinger-based inequalities," *ISA Transactions*, vol.61, no.1, pp.75-86, 2016.

[12] W. Qi, Y. Kao, X.Gao, & Y.ei, "Controller design for time-delay system with stochastic disturbance and actuator saturation via a new criterion," *Applied Mathematics & Computation*, vol. 320, pp. 535-546, 2018.

[13] K. Gherfi，A. Charef, H. A. Abbassi, "Proportional integral- fractional filter controller design for first order lag plus time delay system," *Journal International Journal of Systems Science*, vol.50, no.5, pp. 885-904, 2019.

[14] H. X. Guan, B. Yang and H.R. Wang, "Multiple Faults Diagnosis of Distribution Network Lines Based on Convolution Neural Network with Fuzzy Optimization," *IAENG International Journal of Computer Science*, vol.47, no.3, pp. 567-571, 2020.

[15] C. Hua , S. Wu , X .Guan, "Stabilization of T-S Fuzzy System With Time Delay Under Sampled-Data Control Using a New Looped-Functional," *IEEE Transactions on Fuzzy Systems*,vol.326, pp.400-407, 2020.

[16] Y. Ma, M. Chen, Q. Zhang, "Non-fragile static output feedback control for singular T–S fuzzy delay-dependent systems subject to Markovian jump and actuator saturation," *Journal of the Franklin Institute*, vol. 353, no.11, pp.2373-2397,2016.

[17] Z. Yin, X. Jiang, L. Tang, "On stability and stabilization of T-S fuzzy systems with multiple random variables dependent time-varying delay," *Neurocomputing*, vol.412, pp.91-100, 2020.

[18] M. Xu, Z. Xu, "Stabilization of T-S fuzzy systems with mixed time delays," *International Journal of Systems Science*, vol.2, pp.1-11,2020.

[19] J. Ren，G. He，J. Fu, "Robust H_∞ sliding mode control for nonlinear stochastic T-S fuzzy singular Markovian jump systems with time-varying delays," *Information Sciences*, vol.535, pp.42-63,2020.

[20] W.H. Qi, J. H. Park, J. Cheng, X. M. Chen, "Stochastic stability and L1 -gain analysis for positive nonlinear semi-Markov jump systems with time-varying delay via T-S fuzzy model approach," *Fuzzy Sets and Systems*, vol. 371, pp.110-122, 2019.

[21] C. Ge, Y. Shi, J. H. Park, C. Hua, "Robust H_∞ stabilization for T-S fuzzy systems with time-varying delays and memory sampled-data control," *Applied Mathematics and Computation*, vol. 346, pp. 500-512, 2019.

[22] M. Meng, J. Lam, J. Feng, X. D. Zhao, X. M. Chen, "Exponential stability analysis and H_∞ synthesis of positive T-S fuzzy systems with time-varying delays," *Nonlinear Analysis: Hybrid Systems*, vol. 24, pp.186-197, 2017.

Hejun Yao is a professor in School of Mathematics and Statistics at Anyang Normal University, China. He received his Ph. D from Applied Mathematics in Henan Normal University, China in 2006. His research interests include networked control systems, finite-time stability, delay-time systems, information sharing and service technology, computer systems. Currently, He has published more than 40 papers in various journals.