Solving Inventory Models by the Intuitive Algebraic Method

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***Abstract*—Many researchers have sought to solve inventory models without using calculus so that practitioners who have no knowledge of calculus can still understand and apply inventory models in their studies. Algebraic methods, which include completing the square for the constant term, completing the square for the middle term, determining the arithmetic–geometric mean inequality, and cost difference comparison are critical algebraic approaches with distinct features. However, all lack the elegance of the more challenging computations of calculus. This paper provides an intuitive algebraic solution for inventory models.**

***Index Terms*—Economic ordering quantity model, Economic production quantity model, Algebraic method**

I. Introduction

Approximately one hundred papers have been written on strategies for solving inventory models without the use of calculus, which indicates the degree of concerted efforts made by researchers to help practitioners who lack analytic skills to establish inventory systems. One series of papers considers inventory models that use algebraic methods to deal with linear backorder costs. These include papers by Grubbström and Erdem [9], Cárdenas-Barrón [2], Ronald et al. [15], Chang et al. [3], and Luo and Chou [12]. Here, we briefly review these five papers and identify their questionable results, then suggest specific improvements. Our approach realizes a genuine algebraic spirit by solving inventory models without the need for lengthy computations.

In this paper, we discuss the following two inventory models: the economic order quantity (EOQ) and economic production quantity (EPQ) models. The EOQ model used by Grubbström and Erdem [9], and Ronald et al. [15] is as follows:

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}B^2+\frac{h}{2D}Q^2+K\right), \qquad (1.1)$$

where Q is the maximum inventory level and B is the maximum backorder level.

The EPQ model used by Cárdenas-Barrón [2], Chang et al. [3], and Luo and Chou [12] is as follows:

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}B^2+\frac{h}{2D}Q^2+K\rho\right)+cD, \qquad (1.2)$$

where Q is the maximum inventory level and B is the maximum backorder level.

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If we compare Equations (1.1) and (1.2), we find that the EOQ model of Equation (1.1) and EPQ model of Equation (1.2) have almost identical expressions, such that a solution approach used for the EOQ model of Equation (1.1) can be directly applied to the EPQ model of Equation (1.2).

II. Notation and Assumptions

Achieving compatibility between Grubbström and Erdem [9], Cárdenas-Barrón [2], Ronald et al. [15], Chang et al. [3], and Luo and Chou [12] is difficult because although the notations they use are the same, they have different meanings. For example, "Q" denotes the maximum inventory level of the EOQ models of Grubbström and Erdem [9] and Ronald et al [15]. In contrast, "Q" represents the economic production quantity of the EPQ models used by Cárdenas-Barrón [2], Chang et al. [3], and Luo and Chou [12]. The paper by Grubbström and Erdem [9] was the first to apply an algebraic method for solving inventory models. Therefore, we will follow the notation as Grubbström and Erdem [9] as possible.

B maximum backorder level,

b backorder cost per unit per unit of time,

c cost of production per unit

D demand rate,

$f(k)=(h+bk^2)/(1+k)^2$, an auxiliary function proposed by Ronald et al. [15],

h holding cost per unit per unit of time,

K setup cost,

P production rate, with $P>D$, for EPQ models,

Q maximum inventory level,

ρ $=(P-D)/P$,

$C(Q,B)$ the average cost per unit time.

The goal is to solve minimum problems in the EOQ model, i.e., $C(Q,B)$ in Equation (1.1), or in the EPQ model, i.e., $C(Q,B)$ in Equation (1.2), under the restrictions $Q>0$ and $B>0$, from a purely algebraic perspective without reference to the analytic approach and calculus.

III. Review of Grubbström and Erdem [9]

Grubbström and Erdem [9] use

$$K=\frac{b}{2D}\left(\frac{2DKh}{b(b+h)}\right)+\frac{h}{2D}\left(\frac{2DKb}{h(b+h)}\right), \qquad (3.1)$$

to represent

$$K=\frac{b}{2D}\alpha^2+\frac{h}{2D}\beta^2, \qquad (3.2)$$

with $\alpha=\sqrt{2DKh/b(b+h)}$ and $\beta=\sqrt{2DKb/h(b+h)}$.

They convert the objective function, $C(Q,B)$, as follows

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}\left(B^2+\alpha^2\right)+\frac{h}{2D}\left(Q^2+\beta^2\right)\right), \qquad (3.3)$$

and then they complete the square to derive that

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}(B-\alpha)^2+\frac{h}{2D}(Q-\beta)^2\right)+\frac{D}{B+Q}\left(\frac{b}{2D}2\alpha B+\frac{h}{2D}2\beta Q\right). \tag{3.4}$$

They mention that $b\alpha B+h\beta Q$ contains a factor of $B+Q$ such that

$$b\alpha B+h\beta Q=(B+Q)\delta \tag{3.5}$$

with

$$\delta=\sqrt{\frac{2DKbh}{b+h}} \tag{3.6}$$

to imply

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}(B-\alpha)^2+\frac{h}{2D}(Q-\beta)^2\right)+\delta. \tag{3.7}$$

Based on Equation (3.7), Grubbström and Erdem [9] claim that $B^*=\alpha$, $Q^*=\beta$ and $C(Q^*,B^*)=\delta$.

IV. Review of Cárdenas-Barrón [2]

Cárdenas-Barrón [1] examined an Economic Production Quantity (EPQ) model. He followed Grubbström and Erdem [4] to use

$$K\rho=\frac{b}{2D}\left(\frac{2DKh\rho}{b(b+h)}\right)+\frac{h}{2D}\left(\frac{2DKb\rho}{h(b+h)}\right), \tag{4.1}$$

to derive his optimal solution.

Hence, we can claim that Cárdenas-Barrón [2] applied the same approach proposed by Grubbström and Erdem [9] to solve his EPQ model.

V. Review of Ronald et al. [15]

Ronald et al. [15] claimed that Grubbström and Erdem [9] and Cárdenas-Barrón [2] to use Equations (3.1) and (3.2), with $\alpha=\sqrt{2DKh/b(b+h)}$ and $\beta=\sqrt{2DKb/h(b+h)}$, to derive that

$$b\alpha B+h\beta Q=b\sqrt{\frac{2DKh}{b(b+h)}}B+h\sqrt{\frac{2DKb}{h(b+h)}}Q$$
$$=\sqrt{\frac{2DKbh}{(b+h)}}(B+Q), \tag{5.1}$$

such that $b\alpha B+h\beta Q$ contain the desired factor, $B+Q$.

However, the decomposition of Equation (3.1) is beyond the ability of ordinary practitioners. Hence, Ronald et al. [15] propose their next two-step solution method. The original domain is $Q>0$ and $B>0$ to minimize $C(Q,B)$.

Ronald et al. [15] considered on each ray, $\{(Q,kQ):Q>0\}$ with $0<k<\infty$, to find the local minimum on each ray as

$$C(Q,kQ)=\frac{a}{Q}\left(Q-\sqrt{\frac{d}{a}}\right)^2+2\sqrt{ad}, \tag{5.2}$$

with $a=(h+bk^2)/2(1+k)$ and $d=DK/(1+k)$.

Hence, they derived that on each ray $\{(Q,kQ):Q>0\}$, the optimal ordering quantity $Q^*=\sqrt{2DK/(h+bk^2)}$ and the minimum value

$$C(Q^*,kQ^*)=2\sqrt{ad}=\frac{\sqrt{2(h+bk^2)DK}}{1+k}, \tag{5.3}$$

for $0<k<\infty$. Motivated by Equation (5.3), they assumed an auxiliary function, say $f(k)$, with

$$f(k)=\frac{h+bk^2}{(1+k)^2} \tag{5.4}$$

satisfying $C(Q^*,kQ^*)=\sqrt{2DKf(k)}$.

Ronald et al. [15] found that

$$f(k)=(b+h)\left(\frac{1}{1+k}-\frac{b}{b+h}\right)^2+\frac{bh}{b+h}, \tag{5.5}$$

to imply that $k^*=h/b$ and $f(k^*)=bh/(b+h)$, and then

$$Q^*=\sqrt{\frac{2bDK}{h(b+h)}}, \tag{5.6}$$

$$B^*=k^*Q^*=\sqrt{\frac{2hDK}{b(b+h)}}, \tag{5.7}$$

and

$$C(Q^*,B^*)=\sqrt{2DKf(k^*)}=\sqrt{\frac{2bhDK}{b+h}}. \tag{5.8}$$

VI. Review of Chang et al. [3]

Chang et al. [3] criticized that the two-step solution method of Ronald et al. [15] is too complicated for ordinary readers, such that Chang et al. [3] applied the following solution approach for the EPQ model of Equation (1.2). We do not recall the original derivations of Chang et al. [3]. Instead, we repeat the derivation of Chang et al. [3] in the expression of Cárdenas-Barrón [2]. To solve the minimum problem of Equation (1.2), by the solution procedure of Chang et al. [3], the objective function is rewritten as

$$C(Q,B)=\frac{b+h}{2(Q+B)}\left(B-\frac{h(Q+B)}{b+h}\right)^2+\frac{bh(Q+B)}{2(b+h)}+\frac{DK\rho}{Q+B}+cD, \tag{6.1}$$

to attain the minimum, then they found the relation

$$B^*(Q)=\frac{h(Q+B)}{b+h}. \tag{6.2}$$

They converted $C(Q,B)$ to $C(Q+B,B^*(Q))$ as

$$C(Q+B,B^*(Q))=\frac{bh(Q+B)}{2(b+h)}+\frac{DK\rho}{Q+B}+cD, \tag{6.3}$$

to derive

$$(Q+B)^*=\sqrt{\frac{2(b+h)DK\rho}{bh}}. \tag{6.4}$$

Consequently, they obtained the optimal backlog level for the EPQ model,

$$B^*=\sqrt{\frac{2h\rho DK}{b(b+h)}}, \tag{6.5}$$

and the minimum cost for the EPQ model,

$$C(Q^*,B^*)=\sqrt{\frac{2bh\rho DK}{b+h}}+cD. \tag{6.6}$$

Chang et al. [3] further claimed a direction for future research to rewrite Equation (1.2) as

$$C(Q,B)=\left(\frac{b+h}{2}B^2+DK\rho\right)\frac{1}{Q+B}+\frac{h}{2}(Q+B)-hB+cD$$
$$=\left(\sqrt{\frac{(b+h)B^2+2DK\rho}{2(Q+B)}}-\sqrt{\frac{h(Q+B)}{2}}\right)^2+\sqrt{h(b+h)B^2+2h\rho DK}-hB+cD. \tag{6.7}$$

Hence, Chang et al. [3] derived a relation

$$(Q+B)^*(B)=\sqrt{\frac{2DK\rho}{h}+\frac{b+h}{h}B^2}, \tag{6.8}$$

and

$$C\left((Q+B)^*(B),B\right)=\sqrt{h(b+h)B^2+2h\rho DK}-hB+cD. \quad (6.9)$$

They abstractly expressed Equation (6.9) to solve the following auxiliary function

$$f(B)=\sqrt{(1+\alpha_1)B^2+\beta_1}-B, \quad (6.10)$$

with $\alpha_1=b/h$ and $\beta_1=2\rho DK/h$.

They predicted that to solve Equation (6.10) by an algebraic method will be an interesting research topic in the future.

VII. Review of Luo and Chou [12]

Luo and Chou [12] not only solved the open question proposed by Chang et al. [3], but also handled a generalized problem proposed by Lau et al. [11] and Chiu et al. [5] as follows,

$$f(x)=\sqrt{ax^2+bx+c}-x, \quad (7.1)$$

with $f(x)>0$ for $x>0$.

We quote theorem 1 of Luo and Chou [12] in the following.

Theorem 1 of Luo and Chou [12]

For the existence and uniqueness of an interior minimum, we obtain the necessary conditions as

(i) When $b\ge 0$, we find that $a>1$ and $4c>b^2$, and

(ii) When $b<0$, we derive that $4c>b^2$ and $4(a-1)c>b^2$, with the minimum point, say x^*,

$$x^*=\frac{1}{2a}\left(\sqrt{\frac{4ac-b^2}{a-1}}-b\right), \quad (7.2)$$

and minimum value, say $f(x^*)$,

$$f(x^*)=\frac{b+\sqrt{(4ac-b^2)(a-1)}}{2a}. \quad (7.3)$$

To apply Theorem 1 of Luo and Chou [12], with $a=1+(b/h)$, $b=0$ and $c=2DK\rho/h$, we derive

$$B^*=x^*=\sqrt{\frac{2h\rho DK}{b(b+h)}}, \quad (7.4)$$

and

$$f(B^*)=f(x^*)=\sqrt{\frac{2b\rho DK}{h(b+h)}}. \quad (7.5)$$

Hence, we find

$$C(Q^*,B^*)=hf(B^*)+cD$$
$$=\sqrt{\frac{2bh\rho DK}{b+h}}+cD. \quad (7.6)$$

VIII. Our intuitive approach

Until now, the methods described in the five papers authored by Grubbström and Erdem [9], Cárdenas-Barrón [2], Ronald et al. [15], Chang et al. [3], and Luo and Chou [12] have been used to solve the minimum problems of the EOQ model or EPQ model using Equations (1.1) or (1.2), respectively. Although the algebraic methods described in these papers are exciting, they lack the elegance of other well-known algebraic methods.

We observe Equation (1.1) to find out that if we can add an extra condition as

$$a_1B=a_2Q \quad (8.1)$$

then we can rewrite

$$B+Q=\left((a_1+a_2)/a_2\right)B \quad (8.2)$$

and

$$bB^2+hQ^2=\left((ha_1^2+ba_2^2)/a_2^2\right)B^2. \quad (8.3)$$

If we compare Equations (8.2) and (8.3) to find out that if we try to simplify the expression of Equation (8.3), such that ha_1^2 and ba_2^2 have a common factor, if we select $a_1=b$ and $a_2=h$, then $a_1+a_2=b+h$ and $ha_1^2+ba_2^2=bh(b+h)$ to have a common factor $b+h$. Therefore, we assume $a_1=b$ and $a_2=h$, then we can rewrite Equations (8.1~8.3) as

$$bB=hQ, \quad (8.4)$$

$$B+Q=\left((b+h)/h\right)B, \quad (8.5)$$

and

$$bB^2+hQ^2=\left((h+b)b/h\right)B^2. \quad (8.6)$$

Consequently, we plug Equations (8.4~8.5) into Equation (1.1) to convert $C(Q,B)$ as $C(bB/h,B)$, then

$$C(bB/h,B)=\frac{b}{2}B+\frac{hDK}{(b+h)B}. \quad (8.7)$$

From Equation (7.13), we obtain

$$B^*=\sqrt{\frac{2hDK}{b(b+h)}}, \quad (8.8)$$

and then we derive

$$Q^*=\frac{b}{h}\sqrt{\frac{2hDK}{b(b+h)}}=\sqrt{\frac{2bDK}{h(b+h)}}, \quad (8.9)$$

and

$$C(Q^*,B^*)=bB^*=\sqrt{\frac{2bhDK}{b+h}}, \quad (8.10)$$

which is identical to the result of Equation (3.6) proposed by Grubbström and Erdem [4].

IX. Numerical example

As no numerical examples are provided in the papers by Grubbström and Erdem [9], Cárdenas-Barrón [2], Ronald et al. [15], Chang et al. [3], and Luo and Chou [12], we provide the following numerical example to illustrate our derivations. We assume that $b=\$4$ (\$/unit), $c=\$2$ (\$/unit), $D=800$ (unit/year), $h=\$3$ (\$/unit/year), $K=\$400$ (\$/setup), and $P=1200$ (unit/year). We obtain $B^*=261.861$. In addition, we find that $Q^*=349.149$ and $C(Q^*,B^*)=1047.446$.

For completeness, we conduct a sensitivity analysis in which we change the values of the constant terms one at a time to examine their influence on the maximum backorder level B, the maximum inventory level Q, and the average cost per unit time $C(Q,B)$. In Table 1, we list the results with respect to the backorder cost *b*.

Table 1. Variation of *b*.

b	B^*	Q^*	$C(Q^*, B^*)$
-20%	311.086	331.825	995.473
-10%	284.268	341.121	1023.363
-5%	272.587	345.276	1.35.829
base	261.861	349.149	1047.446
5%	251.976	352.767	1058.301
10%	242.833	356.156	1068.467
20%	226.455	362.329	1086.986

We find that Q^* and $C(Q^*, B^*)$ have a positive relation with the backorder cost b, and B^* has a negative relation with b. In Table 2, we list the results with respect to the demand D.

Table 2. Variation of D.

D	B^*	Q^*	$C(Q^*, B^*)$
-20%	234.216	312.288	936.864
-10%	248.424	331.231	993.694
-5%	255.231	340.308	1020.924
base	261.861	349.149	1047.446
5%	268.328	357.771	1073.313
10%	274.643	366.190	1098.570
20%	286.855	382.473	1147.419

We find that B^*, Q^* and $C(Q^*, B^*)$ all have a positive relation with the demand D. In Table 3, we list the results with respect to the holding cost h.

Table 3. Variation of h.

D	B^*	Q^*	$C(Q^*, B^*)$
-20%	244.949	408.248	979.796
-10%	253.924	376.184	1015.698
-5%	258.010	362.120	1032.041
base	261.861	349.149	1047.446
5%	265.499	337.141	1061.994
10%	268.940	325.988	1075.760
20%	275.299	305.888	1101.196

We find that B^* and $C(Q^*, B^*)$ have a positive relation with the holding cost h, and Q^* has a negative relation with h. In Table 4, we list the results with respect to the setup cost K.

Table 4. Variation of K.

K	B^*	Q^*	$C(Q^*, B^*)$
-20%	234.216	312.288	936.864
-10%	348.424	331.231	993.694
-5%	255.231	340.308	1020.924
base	261.861	349.149	1047.446
5%	268.328	357.771	1073.313
10%	274.643	366.190	1098.570
20%	286.855	382.473	1147.419

We find that B^*, Q^* and $C(Q^*, B^*)$ all have a positive relation with the setup cost K.

X. An application of our approach

We call Chang and Schonfeld [20], to know the cost function of their first transit model,

$$C(r,h)=\frac{d_1}{rh}+d_2(r+d)+d_3h+d_4, \tag{10.1}$$

where d_1=BDTW, d_2=qxLTW/4g, d_3=qwzLTW, and d_4=qvLMTW are four abbreviations to simplify the expression of Equation (10.1).

Chang and Schonfeld [20] derived that $\frac{\partial}{\partial r}C(r,h)$ and $\frac{\partial}{\partial h}C(r,h)$, as

$$\frac{\partial}{\partial r}C(r,h)=\frac{-d_1}{r^2h}+d_2, \tag{10.2}$$

and

$$\frac{\partial}{\partial h}C(r,h)=\frac{-d_1}{rh^2}+d_3, \tag{10.3}$$

and then Chang and Schonfeld [20] solved the system of $\frac{\partial}{\partial r}C(r,h)=0$ and $\frac{\partial}{\partial h}C(r,h)=0$ to imply that

$$\frac{d_1}{d_2}=r^2h, \tag{10.4}$$

and

$$\frac{d_1}{d_3}=h^2r. \tag{10.5}$$

Based on Equations (10.4) and (10.5), Chang and Schonfeld [20] obtained that minimum solution

$$r^*=\left(\frac{d_1d_3}{d_2^{\,2}}\right)^{1/3}, \tag{10.6}$$

and

$$h^*=\left(\frac{d_1d_2}{d_3^{\,2}}\right)^{1/3}. \tag{10.7}$$

The above discussion is the analytical method by calculus.

Next, we consider how to apply our approach to this transit problem. Based on the derivations of Equations (10.6) and (10.7), with the minimum solution

$$r^*h^*=\left(\frac{d_1^{\,2}}{d_2d_3}\right)^{1/3}. \tag{10.8}$$

Following our intuitive approach, we assume that r^*h^* is a constant such that we set the following condition

$$rh=C_0. \tag{10.9}$$

Consequently, we convert Equation (10.1) to

$$C(\frac{C_0}{h},h)=\frac{d_1}{C_0}+d_2(\frac{C_0}{h}+d)+d_3h+d_4. \tag{10.10}$$

We consider the following problem as $\frac{A}{h}+Bh$ as

$$\frac{A}{h}+Bh=(\sqrt{A/h}-\sqrt{Bh})^2+2\sqrt{AB}\ , \tag{10.11}$$

to derive that the minimum is attained when

$$h=\sqrt{A/B}\,. \tag{10.12}$$

According to Equation (10.12), we know that when

$$h^*=\sqrt{d_2C_0/d_3}\,, \tag{10.13}$$

we convert our minimum problem as follows,

$$C(\frac{C_0}{h^*},h^*)=\frac{d_1}{C_0}+d_2d+2\sqrt{d_2d_3C_0}+d_4. \tag{10.14}$$

We are facing the following minimum problem, with $A_1>0$ and $B_1>0$,

$$f(x)=\frac{A_1}{x^2}+B_1x. \tag{10.15}$$

Up to now, we cannot solve the minimum problem of Equation (10.15) by a pure algebraic method, so we compute $\frac{df}{dx}=\frac{-2A_1}{x^3}+B_1$ and then the minimum solution is

$$x^* = \left(\frac{2A_1}{B_1}\right)^{1/3}. \tag{10.16}$$

Based on Equation (10.16), then the minimum problem of Equation (10.14) will occur when

$$C_0^* = \left(\left(\frac{2d_1}{2\sqrt{d_2 d_3}}\right)^{1/3}\right)^2$$
$$= \left(\frac{d_1^2}{d_2 d_3}\right)^{1/3}. \tag{10.17}$$

We plug Equation (10.17) into Equation (10.13), then

$$h^* = \sqrt{d_2 C_0 / d_3}$$
$$= \left(\frac{d_2\left(\frac{d_1^2}{d_2 d_3}\right)^{1/3}}{d_3}\right)^{1/2}$$
$$= \left(\frac{\left(d_1^2 d_2^2\right)^{1/3}}{d_3^4}\right)^{1/2}$$
$$= \left(\frac{d_1 d_2}{d_3^2}\right)^{1/3}, \tag{10.18}$$

that is the same result as Equation (10.7).

From the above discussion, we demonstrate that our approach can be applied to solve other optimization problems.

XI. Direction for Future Research

Several articles are related to this paper. We list them in the following for interested practitioners: Braide and Idoniboyeobu [1], Cheng et al. [4], Chung [6], Cui et al. [7], Eke et al. [8], Grzybowski [10], Malik et al. [13], Qiu et al. [14], Tan et al. [16], Tian et al. [17], Wang et al. [18], and Wen et al. [19]. In addition, how to solve the Equation (10.15) by pure algebraic method will also be an interesting research topic in the future.

XII. Conclusion

In this paper, we reviewed previously established algebraic methods for solving inventory models, including completing the square of the constant term, completing the square of the intermediate term, determining the arithmetic–geometric mean inequality, and cost difference comparison. These algebraic approaches for solving inventory problems all involve long calculation times and lack elegance. Our proposed genuine algebraic method converts a two-variable problem into a one-variable problem, thereby helping practitioners to directly obtain the optimal solution.

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