Multi-Valued Neutrosophic Sets Based on Improved PROMETHEE Method and Its Application in Multi-Attribute Decision-Making

Dongsheng Xu, Xiaolan Wei, Yanran Hong, Li Liu, and Bo Wang

***Abstract*—Multi-attribute decision-making problem refers to how to rank and select the best scheme when the decision problem contains multiple attributes. PROMETHEE is decision-making method based on the comparison of schemes and outranking relation. The principle is to introduce a priority function to describe the priority degree between schemes with objective criteria. In this paper, PROMETHEE method is applied to multi-attribute decision-making problem, and the decision information is given in the form of multi-valued neutrosophic numbers. Through the introduction of the multi-valued neutrosophic sets and the classical PROMETHEE method, an improved PROMETHEE method is proposed to redefine the parameters of the preference function. Finally, an example is given to verify the effectiveness of the method.**

***Index Terms*—multi-valued neutrosophic set, PROMETHEE, outranking relation, multi-attribute decision-making**

I. INTRODUCTION

MULTI-attribute decision-making (MADM) problems exist in various fields, such as economics, management, and engineering. However, the information in MADM problems is often uncertain, incomplete and inconsistent. Therefore, in order to solve the fuzzy MADM problem, Zadeh [1] proposed the concept of fuzzy set and obtained in-depth research. On this basis, Atanassov [2] proposed the non-membership function, forming the intuitionistic fuzzy set theory. Smarandache [3] proposed the concept of neutrosophic sets, which is an extension of the intuitionistic fuzzy set, and it contains truth-membership function, indetermincy-membership function, falsity-membership function. After that, some scholars successively proposed interval neutrosophic sets (INSs), multi-valued neutrosophic sets (MVNSs) and so on [4], [5]. The MVNSs play an important role in MADM problem, which can help decision makers to make effective decisions [6]-[8]. For example, Juan et al. [9] provided an approach for solving MCGDM problems by applying the power aggregation operators. Liu et al. [10] proposed a decision-making method based on the MVNWGBM operator and the MVNWBM operator. Guan et al. [11] proposed a novel prediction model based on MVNSs to find the fluctuation law of time series. Among them, neutrosophic sets described the fluctuation pattern of time series. Peng [12] proposed a multi-valued neutrosophic distance based QUALIFLEX method. In fact, the development of MVNSs is not perfect, and further research and exploration are still needed.

Neutrosophic sets need not only theoretical research, but also application [13]-[19]. Based on the application of MVNSs in MADM problems, a series of methods are proposed to solve the problems, such as TODIM [4], [20], ELECTRE [21], VIKOR [22], PROMETHEE [23]-[25] and so on. Among them, PROMETHEE is a multi-criteria decision-making (MCDM) method based on the priority relationship [26], [27], which is also widely used in MADM problems. For example, Liu [28], [29] proposed a PROMETHEE method for MADM problems where attribute values are intuitive language numbers. Based on the theory of PROMETHEE method, Liu [30] proposed the PROMETHEE method to solve MADM problems. Tan et al. [31] proposed PROMETHEE method for hesitating fuzzy language based on possibility. Gen et al. [32] proposed a decision-making method to calculate index weight by thinking of hesitant fuzzy quality function expansion. Yuan and Zhang [33] proposed a cloud PROMETHEE method for MCDM. Wang and Liu [34] proposed a PROMETHEE optimization method based on INSs to address the shortcomings of the MCDM method. Yu et al. [35] used the maximizing deviation method to obtain the pollutant weight and applied PROMETHEE to the air quality evaluation. A. Aherwar et al. [36] used the entropy method to determine the index weight, and used the PROMETHEE method to determine the selection of biological materials. Zhao et al. [37] applied the improved PROMETHEE to the two-dimensional linguistic MADM problems. Haddad and Sanders [38] used PROMETHEE II to determine the direction of the powered wheelchair. Muhammet et al. [39] proposed PROMETHEE method based on trapezoidal fuzzy interval numbers for the application of automotive instrument panel materials. Among them, there are many methods to determine the index weight, and the difference in weight also affects the judgment of decision-making schemes [40]-[42]. This method has been applied in various fields [43]-[47], and it is constantly developing and improving.

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Dongsheng Xu is an associate professor in the School of Science of Southwestern Petroleum University (e-mail: xudongsheng1976@163. com).

Xiaolan Wei is a master student in the School of Science of Southwest Petroleum University, Chengdu 610500, China (phone: +86-1848-216-7219; e-mail: weixiaolan9767@163.com).

Yanran Hong is a doctoral student in the School of Economics and Management of Southwest Jiaotong University, Chengdu 611756, China (e-mail: UranusHYR@163.com).

Li Liu is a teacher at the No.1 Middle School in Ziyang (e-mail: 1096153814@qq.com).

Bo Wang is a student in the School of Science of Southwest Petroleum University, Chengdu 610500, China (e-mail: 2366236026@qq.com).

In PROMETHEE method, the selection and establishment of the preference function are important to help the decision maker make better judgment. Among them, Liao et al. [48] improved the preference function, allowing decision makers to choose parameters based on their strict preference over the scheme, and it be used in the evaluation and decision-making of Sichuan wine brands. For priority criteria, a lot of preference information is no longer static. Qi et al. [49] designed a dynamic weighting method for the preference expectation. Deng and Mei [50] combined the multi-criterion preference order index of the PROMETHEE method with the TOPSIS principle to establish a solution model for the MCDM problem with incomplete criterion weight coefficient. Zhang [51] combined absolute value with relative value, coexists linearly with nonlinearly, making the preferred function form of expert evaluation more reasonable and reliable. Li and Yue [52] extended the concept of the priority function in PROMETHEE II method, and proposed a class of priority functions and parameter determination methods. Lazim [53] proposed a preference of green suppliers using the PROMETHEE under the usual criterion preference functions. Nassereddine [54] proposed a new preference function, which expanded the chosen range of decision makers. Zhang et al. [55] improved the PROMETHEE method, replaced the priority function with the utility function, and proposed a method to determine the attribute weight. Ren et al [56] proposed the PROMETHEE II method and introduced the threshold calculation every two schemes on the criterion of preference function. Zhou et al. [57] introduced the priority function and the lattice-valued degree difference of the linguistic-valued lattice implication algebra in the PROMETHEE method. Tian et al. [58] put forward the ranking and different degree of performance values on each attribute determine the preference function of alternatives.

PROMETHEE is a method to determine the full ranking of schemes based on the priority relationship. When solving the MADM problem, the selection of preference function is very important to the decision result. Therefore, the influence of preference function on the result should be considered when making decision. This paper first introduces the MVNSs and the classical PROMETHEE method, then proposes improved PROMETHEE method by redefining the preference function. Finally, an example is given to verify the feasibility of the method.

II. PRELIMINARIES

A. Neutrosophic Set

Definition 1: Let X be a space of points (objects) with a generic element in X denoted by x, a neutrosophic set A in X is characterized by a truth-membership function $T_{A(x)}$, a indeterminacy-membership function $I_{A(x)}$ and a falsity-membership function $F_{A(x)}$, where $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ are the function of finite discrete subset of $]0^-,1^+[$, that is $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$: $X \to]0^-,1^+[$. So, A can be expressed by

$$A=\{\langle x,T_{A(x)},I_{A(x)},F_{A(x)}\rangle \mid x\in X\}$$

with the condition of

$$0^{-1}\le \sup T_{A(x)}+\sup I_{A(x)}+\sup F_{A(x)}\le 3^+.$$

Definition 2: The complement of a neutrosophic set A is denoted by A^c and is defined as

$$T_{A^c}=\{1^+\}\odot T_{A(x)},\ I_{A^c}=\{1^+\}\odot I_{A(x)},\ F_{A^c}=\{1^+\}\odot F_{A(x)},$$

for every x in X.

Definition 3: A neutrosophic set A is contained in another neutrosophic set B, $A\subseteq B$ if and only if

$\inf T_{A(x)}\le \inf T_{B(x)}$, $\sup T_{A(x)}\le \sup T_{B(x)}$,

$\inf I_{A(x)}\ge \inf I_{B(x)}$, $\sup I_{A(x)}\ge \sup I_{B(x)}$,

$\inf F_{A(x)}\ge \inf F_{B(x)}$ and $\sup F_{A(x)}\ge \sup F_{B(x)}$, $x\in X$.

B. Multi-Valued Neutrosophic Set

Definition 4: Let X be a space of points (objects), a MVNS A in X is characterized by

$$A=\{\langle x,T_{A(x)},I_{A(x)},F_{A(x)}\rangle \mid x\in X\} \tag{1}$$

where the truth-membership function $T_{A(x)}$, indeterminacy-membership function $I_{A(x)}$, falsity-membership function $F_{A(x)}$, with the condition of

(i) $0\le \gamma_A$, η_A, $\xi_A\le 1$, $0\le \gamma_A^+ +\eta_A^+ +\xi_A^+\le 3$;

(ii) $\gamma_A\in T_{A(x)}$, $\eta_A\in I_{A(x)}$, $\xi_A\in F_{A(x)}$;

(iii) $\gamma_A^+=\sup T_{A(x)}$, $\eta_A^+=\sup I_{A(x)}$, $\xi_A^+=\sup F_{A(x)}$

$\gamma_A^-=\inf T_{A(x)}$, $\eta_A^-=\inf I_{A(x)}$,

Especially,

(i) If $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ all have only one value, then the MVNSs are reduced to single-valued neutrosophic sets (SVNSs);

(ii) If $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ all are interval value, then the MVNSs are reduced to INSs;

(iii) If $T_{A(x)}=\varnothing$, then the MVNSs are reduced to double hesitant fuzzy sets (DHFSs);

(iv) If $T_{A(x)}=F_{A(x)}=\varnothing$, then the MVNSs are reduced to hesitant fuzzy sets (HFSs).

Definition 5. The complement of a MVNS A is denoted by A^c and is defined as

$$A^c=\langle \bigcup_{\gamma_A\in T_A}\{1-\gamma_A\}, \bigcup_{\eta_A\in I_A}\{1-\eta_A\}, \bigcup_{\xi_A\in F_A}\{1-\xi_A\}\rangle \tag{2}$$

Definition 6: Let $A=\langle T_A,I_A,F_A\rangle$ and $B=\langle T_B,I_B,F_B\rangle$ are two MVNNs, $A\prec B$ if and only if $\forall\ T_A^a\in T_A$, $T_B^b\in T_B$, $I_A^a\in I_A$, $I_B^b\in I_B$, $F_A^a\in F_A$, $F_B^b\in F_B$ and $T_A^a<T_B^b$, $I_A^a<I_B^b$, $F_A^a<F_B^b$.

Definition 7: Let $A=\langle T_A,I_A,F_A\rangle$ and $B=\langle T_B,I_B,F_B\rangle$ are two MVNSs, the likelihood of preference relation between A and B is defined as

$$P(A,B)=\frac{1}{3}\{\tilde{P}(T_A,T_B)+\tilde{P}(I_A,I_B)+\tilde{P}(F_A,F_B)\} \tag{3}$$

where

$$\tilde{P}(T_A,T_B)=\begin{cases}\dfrac{\sum_{\gamma_A\in T_A}\sum_{\gamma_B\in T_B}\dfrac{\gamma_A}{\gamma_A+\gamma_B}}{|T_A|.|T_B|}, T_A\ne\{0\}/T_B\ne\{0\}\\ 0.5, T_A=T_B=\{0\}\end{cases} \tag{4}$$

$$\tilde{P}(I_A, I_B) = \begin{cases} 1 - \dfrac{\sum_{\eta_A \in I_A} \sum_{\eta_B \in I_B} \dfrac{\eta_A}{\eta_A + \eta_B}}{|I_A| \cdot |I_B|}, I_A \neq \{0\} / I_B \neq \{0\} \\ 0.5, I_A = I_B = \{0\} \end{cases} \quad (5)$$

$$\tilde{P}(F_A, F_B) = \begin{cases} 1 - \dfrac{\sum_{\xi_A \in F_A} \sum_{\xi_B \in F_B} \dfrac{\xi_A}{\xi_A + \xi_B}}{|F_A| \cdot |F_B|}, F_A \neq \{0\} / F_B \neq \{0\} \\ 0.5, F_A = F_B = \{0\} \end{cases} \quad (6)$$

For example, let $A = \langle 0.4, 0.3, 0.2 \rangle$, $B = \langle 0.4, 0.3, 0.1 \rangle$, $C = \langle 0.4, 0.2, 0.2 \rangle$, the likelihood of preference relation between them can be obtained as Table I.

TABLE I

	$\tilde{L}_T$	$\tilde{L}_I$	$\tilde{L}_F$	
(A,B)	1/2	1/2	1/3	8/18
(A,C)	1/2	2/5	1/2	14/30
(B,A)	1/2	1/2	2/3	10/18
(C,A)	1/2	3/5	1/2	16/30

C. PROMETHEE

In the classical PROMETHEE method, six preference functions are provided. At the same time, the preference parameters are indifference threshold and strict preference threshold. The decision maker can judge each index function according to his own judgment, and construct a preference function of general criterion in advance. For the convenience of the study, six preference functions have been proposed by some scholars, as shown in Table II:

TABLE II
THE PREFERENCE FUNCTIONS

Usual Function	$P(d) = \begin{cases} 1 & ,d > 0 \\ 0 & ,d = 0 \end{cases}$
V-shape Function	$P(d) = \begin{cases} \dfrac{d}{q} & ,d > 0 \\ 1 & ,d = 0 \end{cases}$
Level Function	$P(d) = \begin{cases} 0 & ,d \leq q \\ 0.5 & ,q < d \leq p \\ 1 & ,d > p \end{cases}$
Liner Function	$P(d) = \begin{cases} 0 & ,d \leq q \\ \dfrac{d-p}{p-q} & ,q < d \leq p \\ 1 & ,d > p \end{cases}$
Gauss Function	$P(d) = \begin{cases} 1 - e^{\frac{d^2}{2s^2}} & ,d > 0 \\ 0 & ,d \leq 0 \end{cases}$
U-shape Function	$P(d) = \begin{cases} 1 & ,d \geq q \\ 0 & ,0 \leq d < q \end{cases}$

III. PROMETHEE METHOD BASED ON MVNS

Let the alternatives be $A = (A_1, ..., A_m)$, and the attributes be $C = (C_1, C_2, ..., C_n)$. Let the weights of the attributes be $W = (w_1, w_2, ..., w_n)$, where $0 \leq w_j \leq 1$, $\sum_{j=1}^{n} w_j = 1$. Let a_{ij}, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, be the attribute value of the alternative A_i with attribute C_j, so $A = (a_{ij})_{m \times n}$ is a decision matric. The following is the calculation procedure of extended PROMETHEE method.

Step 1: Establish decision matrix.

Step2: Standardize decision information matrix. If the decision is a cost factor, the decision information should be changed by its complementary set, while an efficient factor, it should not be changed.

Step 3: Construct preference function $P_j(a_i, a_r)$ of scheme A_i relative to A_r under the attribute C_j by

$$P_j(a_i, a_r) = \begin{cases} 0 & ,L_j(a_i, a_r) \leq \mathrm{p} \\ \dfrac{L_j(a_i, a_r) - p}{q - p} & ,p < L_j(a_i, a_r) < q \\ 1 & ,L_j(a_i, a_r) \geq q \end{cases} \quad (7)$$

the $L_j(a_i, a_r)$ is obtained by definition 7.

Step4: We define the priority index $\pi(a_i, a_r)$ of the scheme B_i relative to B_r by

$$\pi(a_i, a_r) = \frac{\sum_{j=1}^{n} w_j P_j(a_i, a_r)}{\sum_{j=1}^{n} w_j} = \sum_{j=1}^{n} w_j P_j(a_i, a_r) \quad (8)$$

Step 5: Calculate the inflow $\phi^+(a_i)$, outflow $\phi^-(a_i)$ and net flow $\phi(a_i)$ of the object, as following

$$\phi^+(a_i) = \sum_{r=1}^{n} \pi(a_i, a_r) = \sum_{r=1}^{n} \sum_{j=1}^{m} w_j P_j(d_{ir}) \quad (9)$$

$$\phi^-(a_i) = \sum_{r=1}^{n} \pi(a_r, a_i) = \sum_{r=1}^{n} \sum_{j=1}^{m} w_j P_j(d_{ri}) \quad (10)$$

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i) \quad (11)$$

Step6: Rank all alternatives according to the value of ϕ, the greater the value of ϕ, the better the alternative is.

IV. PROBLEM DESCRIPTION

In real life, most decision-making problems involve multiple schemes and multiple attributes, which are called MADM problems. In traditional MADM methods, alternatives are evaluated with crisp value generally. However, due to the complexity of objective things and the subjectivity of human, MADM problems are usually accompanied by uncertainty, so the decision information often given is fuzzy. The PROMETHE is a MADM method based on outranking relation, this method compares the advantages and disadvantages of two schemes one by one through the priority function, and finally determines the priority relation ranking of all schemes, avoiding the influence of the decision-making compensation on the evaluation results . In addition, there is no need to standardize the evaluation information in decision-making, which ensures the integrity of the evaluation information.

This paper mainly takes investment engineering project as an example. The features of project investment roughly meet the following conditions:

(i) There are clear construction goals;
(ii) It includes construction period and operation period;
(iii) Large investment amount and long cycle;
(iv) Consider capital turnaround time and add value.

A few important attributes are listed here:

(i) Income: Regardless of any investment, the main purpose is for business. Therefore, income should be used as a factor of evaluation;
(ii) Social benefits: In business competition, investments with significant social benefits can not only improve business competition, but also obtain social recognition more easily. Therefore, social benefits should be regarded as an evaluation indicator of investment;
(iii) Market effect: In the process of social development, the market effect is very significant. The first is the speed of market preemption, the second is the cost. Technology, project experience and social benefits will reduce development costs, so it can be carried out in the form of minor profits or losses;
(iv) Technical difficulty: In order to improve the safety and practicability of the project, higher requirements will be put forward for high technology;
(v) Risk: Market risk, financing risk, social risk, resource risk.

V. Application and Analysis

We give an example to illustrate feasibility and rationality of the extended method. Suppose there are four enterprises $A=(A_1,A_2,A_3,A_4)$ and three attributes $C=(C_1, C_2,C_3)$. Suppose a_{ij} is given for the alternative A_i under the attribute C_i, $i=1,2,3,4$ and $j=1,2,3$. The weight of each attribute is given by $w=\{0.2,0.25,0.55\}$. Now we need to select the best one from the four enterprises.

Here, the decision information is given the form of multi-valued neutrosophic number $a_{ij}=(T,I,F)$, where T is acceptable degree, I is hesitant degree, F is unacceptable degree ($T,I,F\in]0^-,1^+[$). The decision matrix show in the following.

$$A=\begin{bmatrix} \langle\{0.4,0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.4\},\{0.2,0.3\},\{0.3\}\rangle & \langle\{0.2\},\{0.2\},\{0.5\}\rangle \\ \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.5\},\{0.2\},\{0.1,0.2\}\rangle \\ \langle\{0.3,0.4\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.2,0.3\},\{0.2\}\rangle \\ \langle\{0.7\},\{0.1,0.2\},\{0.1\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.4\},\{0.3\},\{0.2\}\rangle \end{bmatrix}$$

Attribute C_1, C_2 are the efficient factor, the C_3 is the cost factor. Therefore, we can get a standardized information decision matrix.

$$\tilde{A}=\begin{bmatrix} \langle\{0.4,0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.4\},\{0.2,0.3\},\{0.3\}\rangle & \langle\{0.8\},\{0.8\},\{0.5\}\rangle \\ \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.5\},\{0.8\},\{0.9,0.8\}\rangle \\ \langle\{0.3,0.4\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.8,0.7\},\{0.8\}\rangle \\ \langle\{0.7\},\{0.1,0.2\},\{0.1\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.6\},\{0.7\},\{0.8\}\rangle \end{bmatrix}$$

Then, the preference degree $P_j(a_i,a_r)$ with respect to C_j can be calculated by liner function in Table II. The preference degree of each scheme relative to other schemes is shown as following matrix.

$$P_1(A_i,A_r)=\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0.000 & 0.415 & 0.521 & 0.352 \\ A_2 & 0.585 & 0.000 & 0.606 & 0.432 \\ A_3 & 0.497 & 0.394 & 0.000 & 0.333 \\ A_4 & 0.648 & 0.568 & 0.667 & 0.000 \end{array}$$

$$P_2(A_i,A_r)=\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0.000 & 0.364 & 0.465 & 0.364 \\ A_2 & 0.636 & 0.000 & 0.604 & 0.500 \\ A_3 & 0.535 & 0.396 & 0.000 & 0.396 \\ A_4 & 0.636 & 0.500 & 0.604 & 0.000 \end{array}$$

$$P_3(A_i,A_r)=\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0.000 & 0.582 & 0.571 & 0.551 \\ A_2 & 0.418 & 0.000 & 0.484 & 0.469 \\ A_3 & 0.429 & 0.510 & 0.000 & 0.479 \\ A_4 & 0.449 & 0.531 & 0.521 & 0.000 \end{array}$$

Next, the comprehensive priority index between the schemes can be obtained .

$$\pi=\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0.000 & 0.494 & 0.535 & 0.465 \\ A_2 & 0.506 & 0.000 & 0.538 & 0.469 \\ A_3 & 0.465 & 0.459 & 0.000 & 0.429 \\ A_4 & 0.535 & 0.531 & 0.571 & 0.000 \end{array}$$

Finally, the inflow, outflow and net flow of each scheme are calculated as Table III.

TABLE III
THE INFLOW, OUTFLOW AND NET FLOW OF A_i

Alternative	ϕ^+	ϕ^-	ϕ
A_1	1.493	1.507	-0.014
A_2	1.514	1.483	0.031
A_3	1.353	1.644	-0.291
A_4	1.637	1.363	2.274

According to the inflow ϕ^+, outflow ϕ^- and net flow ϕ of each scheme, it can be seen that $\phi_4 > \phi_2 > \phi_1 > \phi_3$, so the ranking order is $A_4 > A_2 > A_1 > A_3$ and the best choice is A_4.

VI. Comparison and Conclusion

In order to verify the effectiveness of the improved method, the result of the improved PROMETHEE method and the classical PROMETHEE method are compared, as shown in Table IV.

TABLE IV
COMPARISION

	Ranking	Optimal choice
Classical PROMETHEE method	$A_4 > A_2 > A_1 > A_3$	A_4
Improved PROMETHEE method	$A_4 > A_2 > A_1 > A_3$	A_4

The result shows that the improved method is basically consistent with the results of the classical method, which indicates that the improved method is feasible.

In MADM problem, due to the fuzziness of decision information, neutrosophic set is introduced to describe such information. This paper uses the improved PROMETHEE method to solve MADM problem. In PROMETHEE method, the selection of preference function is the key to solve the MADM problem. The improved PROMETHEE method redefines the parameters of preference function, and the decision information is given in the form of MVNSs. Finally, according to the inflow, outflow and net flow of each scheme, the full ranking of the schemes is determined based on the priority relationship. This case demonstrates the effectiveness of the improved method. In the future, PROMETHEE method can be continuously improved and applied in MADM problems, especially in fuzzy environment.

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Dongsheng Xu, born in Nanchong country, Sichuan province, P. R. China, on December 20, 1976. He received MSc degree in Science from Southwestern Petroleum University, P.R. China. He is an associate professor in the school of Science of Southwestern Petroleum University and doctoral candidate in school of economics and mathematics of Southwestern University of Finance and Economics. His research direction is financial asset pricing. He has published 20 papers in journals, books and conference proceedings including journals such as Journal of Computational and Applied Mathematics, Chaos, Solitons and Fractals, Journal of Southwest Petroleum University (natural science edition), and 1 book.