Prescribed Performance Adaptive Neural Network Tracking Control of Strict-Feedback Nonlinear Systems with Nonsymmetric Dead-zone

Yi-Qin Zhou, Xin-Yu Ouyang, Nan-Nan Zhao, Hai-Bo Xu and Hui Li

***Abstract*—For a class of strict-feedback nonlinear systems with dead-zone input, the problem of adaptive neural tracking control is studied. Considering that the system contains unknown functions and asymmetric dead-zone. Firstly, the radial basis function (RBF) neural network (NN) is introduced to approach the unknown nonlinear functions in the system model. Then, a new error transformation method is proposed and applied to the design of performance controller. Based on the differential Lyapunov function method, the stability of closed-loop control system is analyzed, and the tracking error of the closed-loop system can converge to the preset boundary. Finally, the simulation results are used to further verify the control effect of the proposed controller.**

***Index Terms*—Adaptive neural tracking control, strict-feedback nonlinear systems, dead-zone input, prescribed performance control (PPC), backstepping**

I. Introduction

In practical engineering applications, most of the systems show nonlinear characteristics [1]–[4]. In the past ten years, the combination of backstepping design with neural network and other general function approximators to realize adaptive control of nonlinear systems has attracted extensive attention. [5]–[13]. There exist several significant advantages, such as simple structure, fast learning algorithm and strong approximation ability [14]–[16]. For example, In [17], an adaptive backstepping controller is designed based on NN control for a class of disturbed nonlinear systems. In [18], the adaptive switched neural controller and the corresponding robust compensation control law are designed, and the unknown function of the robot is approached by the neural network.

In recent years, with the increasingly complex structure of the control object, the control accuracy is required to be higher, and the real-time performance is also stronger. In 2008, the Greek scholar Bechlioulis first proposed a new control method called prescription performance control (PPC) [19]. The problem of prescribe performance control for SISO nonlinear systems is considered, and the error can be converge to the predetermined region through adaptive control and prescribed performance function in [20]. For a class of unknown pure-feedback nonlinear systems, [21] proposed a prescribed backsteping control scheme, which has strong robustness. In [22], aiming at a class of master-slave asymmetric remote control system, the stability and position synchronization control of the system is studied by combining with the prescribed performance control technology.

In practice, dead-zones exist in various equipment and actual systems, such as DC servo drives, gear set, and processes of industrial production [23]–[26]. As a result, the existence of this nonlinearity can lead to serious deterioration of system performance and even system instability without proper suppression. It is generally known that the nonlinearity of dead-zone input is a non-smooth function in real systems, which shows some insensitivity to small control inputs. According to the SISO and MIMO nonlinear strict-feedback systems with dead-zone, an adaptive backstepping controller is proposed by using fuzzy or neural network technology, which further solves the nonlinear parameterized nonlinear systems with dead-zone and uncertainties in [27], [28]. The effect of nonsymmetric dead-zone input can be eliminated by using unknown dead-zone parameters to set an adaptation auxiliary signal. in [29]. The fuzzy approximation method is used to design adaptive controller with given constraints, which can guarantee the transient and steady-state performance of system tracking error. [30]. Moreover, in order to better illustrate the uncertainty of the dead-zone in the physical system, a fuzzy dead-zone model is shown in [31]. In [32], an adaptive output feedback control method is presented for a kind of SIMO nonlinear systems. Chen et al. introduced an auxiliary design system to analyze the influence of input constraints, and proposed an adaptive tracking control scheme for those uncertain nonlinear systems with asymmetric input in [33].

Inspired by the works of [33] and [30], a new adaptive neural network tracking controller is proposed for a class of nonlinear systems with strict-feedback nonlinear asymmetric unknown dead-zone input. The main contributions of this paper are as follows:

(1) Combining adaptive neural network control with backstepping control technology, an effective adaptive control scheme for nonlinear systems is proposed. It not only guarantees that all signals in closed-loop system are bounded, but also the tracking error is limited to a given error range;

Manuscript received November 6, 2020; revised April 24, 2021. This work is supported in part by the Scientific Research Fundation of Liaoning Provincial Education Department of China (Grant Nos. 2019LNJC11 and 2019LNJC13).

Yi-Qin Zhou is Postgraduate of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (e-mail: 1179820057@qq.com).

Xin-yu Ouyang is Professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (Corresponding author: e-mail: 13392862@qq.com).

Nan-nan Zhao is Associate Professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (Corresponding author: 723306003@qq.com).

Hai-Bo Xu is Postgraduate of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (e-mail: 502679740@qq.com).

Hui Li is Postgraduate of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (e-mail: 1778169103@qq.com).

(2) A new error transfer function in the form of logarithmic function is used to design the controller, because of dead-zone disturbance, the original error transfer function has to be reconstructed;

(3) Overcome the nonlinear design difficulties of non-smooth input asymmetric dead-zone.

II. System descriptions and preparatory knowledge

A. System descriptions

The mathematical description of SISO strick-feedback nonlinear system with saturation constraints is as follows:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \lambda_i(t) \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)D(v) + \lambda_n(t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$ $(i = 1, 2, \cdots, n)$ and $y \in \mathbb{R}$ are the system state vector and the system output, respectively, $\lambda_i(\cdot)$ is a bound disturbance which converges to $\overline{\lambda_i}$, $g_i(\cdot)$ and $f_i(\cdot) : \mathbb{R}^i \rightarrow \mathbb{R}$ are all the unknown smooth functions, and $D(v)$ is the plant input constrained by nonsymmetric dead-zone nonlinearity. The dead-zone function with input v and output u is described by

$$u = D(v) = \begin{cases} \kappa_r(v-m), v \geq m \\ 0, p < v < m \\ \kappa_l(v-p), v \leq p \end{cases} \tag{2}$$

where $\kappa_r, \kappa_l, m > 0$ and $p < 0$ are unknown constants of input dead-zone function.

Then, the dead-zone can be defined as the follows:

$$u = \kappa v(t) + s(t) \tag{3}$$

and

$$s(t) = \begin{cases} -\kappa_r m, v \geq m \\ -\kappa v, p < v < m \\ -\kappa_l p, v \leq p \end{cases} \tag{4}$$

$$\kappa = \begin{cases} \kappa_r, v \geq 0 \\ \kappa_l, v \leq 0 \end{cases} \tag{5}$$

From (4),we have

$$\begin{aligned} |s(t)| &= |D(v) - \kappa v| \\ &\leq \max\{\kappa_r m, -\kappa_l p\} \\ &= \bar{s} \end{aligned} \tag{6}$$

define $\underline{\kappa} = \min\{\kappa_r, \kappa_l\}$, $\bar{\kappa} = \max\{\kappa_r, \kappa_l\}$, $\underline{\kappa} \leq \kappa \leq \bar{\kappa}$.

The control purpose of this thesis: An adaptive neural network controller is designed under the prescribed performance constraint, so that the closed-loop system can track the reference input signal y_r stably. The tracking error z_i satisfies the prescribed performance function and all signals can be ensured semi-globally consistently bounded in the closed-loop system.

Remark 1. *For the systems shown in (1), the adaptive state feedback control problem with symmetric [34] and asymmetric dead-zone nonlinear [35] is studied respectively. In practical applications, due to the physical characteristics of the driver and the influence of the environment, the dead-zone parameters are often uncertain and inconsistent. Because RBF neural network is highly adaptive and can approximate any nonlinear function, this paper will use RBF neural network to deal with the uncertain features.*

Assumption 1. *The sign of $g_i(\bar{x}_i)$ are known, and there are unknown constants d_m and d_M, such that $0 < d_m \leq |g_i(\bar{x}_i)| \leq d_M < \infty$. Without affecting the conclusion, it can be assumed that $d_m \leq g_i(\bar{x}_i) \leq d_M < \infty$*

Substituting (3) into (1) results in

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \lambda_i(t) \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)\kappa v + g_n(\bar{x}_n)s(t) + \lambda_n(t) \\ y = x_1 \end{cases} \tag{7}$$

For convenience, $f_i(\cdot)$ and $g_i(\cdot)$ will be replaced by f_i and g_i, respectively. In addition, the time t in the functions will be omitted, for example, λ_i for $\lambda_i(t)$, v for $v(t)$, u for $u(t)$ and so on.

Lemma 1. *For $\forall(x, y) \in R^2$, the inequality as follows:*

$$xy \leq \frac{\eta^j}{j}|x|^j + \frac{1}{q\eta^q}|y|^q \tag{8}$$

where $\eta > 0, j > 1, q > 1$, and $(j-1)(q-1) = 1$.

B. Prescribed performance control

Next, the performance function is introduced to preset the performance index of tracking error z_i.

The tracking error is defined as z_1, and y_r is the desired trajectory signal, α_i is the virtual control signal. Based on backstepping technique, the derivation process needs n steps. We first assume that z_i satisfies coordinate transformation as follows:

$$\begin{aligned} z_1 &= x_1 - y_r, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, 3, \cdots, n \end{aligned} \tag{9}$$

A possible boundary function τ_1 is formulated as:

$$\tau_1 = (\tau_0 - \tau_\infty)e^{-\delta t} + \tau_\infty, \tag{10}$$

Lemma 2. *Continuous function is called performance function, if:*

(1) τ_1 is the boundary function;

(2) $\lim_{t\to\infty} \tau_t = \tau_\infty \geq 0$.

Where $\tau_0 > \tau_\infty$, τ_0, τ_∞ and δ_i are prescribed positive constants, τ_0 denotes the initial value, τ_∞ is the upper bound of steady-state error, and δ_i represents the convergence speed of exponential function.

For backstepping design of adaptive neural controller, a novel error variable is defined as follows:

$$\varsigma_1 = \ln(\frac{\tau_1 + z_1}{\tau_1 - z_1}) \tag{11}$$

The derivative of time of ς_1 is calculated by:

$$\dot{\varsigma}_1 = 2\Lambda_1\left(f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 + \lambda_1 - \dot{y}_r - \frac{z_1\dot{\tau}_1}{\tau_1}\right) \tag{12}$$

where $\Lambda_1 = \frac{\tau_1}{\tau_1^2 - z_1^2}$.

Assumption 2. *For the function y_r and its time derivatives are known and bounded.*

C. NN basics

Artificial neural network is the use of mathematical model to simulate the human brain information processing function. In 1985, Powell proposed the RBF of multivariate interpolation. In 1988, Broomhead and Lowe applied RBF to neural network design and constructed RBF neural network. Because neural network has the advantages of nonlinear function approximation, learning and fault tolerance, this paper introduces the following RBF neural network [36] to approach continuous function:

$$f_{nn} = W^T\psi(Z) \tag{13}$$

where $Z \in \Omega_Z \subset R^n$, $W = [w_1, w_2, ..., w_l]^T \in R^l$ and $l > 1$ are input vector, weight vector and NN node number respectively; and $\psi(Z) = [\psi_1(Z), \psi_2(Z), ..., \psi_l(Z)]^T$, where $\psi_i(Z)$ is Gaussian functions as follows:

$$\psi_i(Z) = \exp[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{N^2}], i = 1, 2, ..., l \tag{14}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{in}]$ is the centre of the receptive field and N denotes the width of the Gaussian function. It has been proven in [37] that the RBF neural network (13) can approach any continuous function on a compact set $\Omega_Z \subset R^n$ with arbitrary accuracy in the form of

$$f(Z) = W^{*T}\psi(Z) + \beta(Z), \forall Z \in \Omega_Z \subset R^n \tag{15}$$

where W^* is the ideal constant weight vector, and $\beta(Z)$ is the approximation error, it has $|\beta(Z)| \le \bar{\varepsilon}, Z \in \Omega_Z$, $\bar{\varepsilon} > 0$ is an unknown constant.

In general, W^* is selected as the value of W that minimises $\beta(Z)$ over Ω_Z, that is

$$W^* := \arg \min_{W\in R^l} \sup_{Z\in\Omega_Z} |f(Z) - W^T\psi(Z)| \tag{16}$$

The stability results acquired in the neural network control literature are semi-global, only the input variables Z of the NN still within some pre-fixed compact set, $\Omega_Z \subset R^n$, where the compact set Ω_Z can be made as large as desired, there exists controller with a large number of NN nodes guarantee all the signals in the closed-loop keep bounded.

D. Controller design

Next, a backstepping method based on adaptive neural control design procedure will be presented. For the ith subsystem, the virtual control signal α_1 and actual control input v are constructed as follows:

$$\alpha_1 = -\frac{\varsigma_1}{\Lambda_1}(a_1 + \frac{1}{2} + \frac{1}{2c_1^2}\hat{\theta}\psi_1^T\psi_1) \tag{17}$$

where ψ_1 is the function of$[x_1, y_r, \dot{y}_r, \tau_1, \dot{\tau}_1]$ and α_1 is the function of $[x_1, \hat{\theta}, y_r, \dot{y}_d, \tau_1, \dot{\tau}_1]$,

$$\alpha_i = -(a_i + \frac{1}{2})z_i - \frac{1}{2c_i^2}z_i\hat{\theta}\psi_i^T\psi_i \tag{18}$$

$$v = -(a_n + \frac{1}{2\eta^2})z_n - \frac{1}{2c_n^2}z_n\hat{\theta}\psi_n^T\psi_n \tag{19}$$

where ψ_i is the function of$[\bar{x}_i, y_r, \dot{y}_r \cdots y_r^{(i)}, \tau_1, \dot{\tau}_1 \cdots \tau_1^{(i)}]$, α_i is the function of $[\bar{x}_n, \hat{\theta}, y_d, \dot{y}_r \cdots y_r^{(i)}, \tau_1, \dot{\tau}_1 \cdots \tau_1^{(i)}]$, $\hat{\theta}$ is the estimate value of θ, which are unknown constants can be written as:

$$\theta = \max_{1\le i\le n}\left\{\frac{1}{d_m}\|W_i\|^{*2}\right\} \tag{20}$$

The adaptive law is defined as follow:

$$\dot{\hat{\theta}} = \frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \sum_{i=2}^{n}\frac{r}{2c_i^2}z_i^2\psi_i^T\psi_i - \sigma\hat{\theta} \tag{21}$$

where r and σ are designed to be a positive constant and $\tilde{\theta} = \theta - \hat{\theta}$ denotes parameter error.

III. Main result

The design process consists of n steps.

Step 1: A positive definite Lyapunov is defined as follows:

$$V_1 = \frac{1}{4}\varsigma_1^2 + \frac{d_m}{2r}\tilde{\theta}^2 \tag{22}$$

By using (12) and (22), The derivative of time of $V_1(t)$ can be obtained

$$\begin{aligned} \dot{V}_1 &= \frac{1}{2}\varsigma_1\dot{\varsigma}_1 - \frac{d_m}{r}\tilde{\theta}\dot{\hat{\theta}} \\ &= \varsigma_1\Lambda_1(f_1 + g_1x_2 + \lambda_1 - \dot{y}_r - \frac{\dot{\tau}_1 z_1}{\tau_1}) \\ &\quad - \frac{d_m}{r}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{23}$$

Based on Lemma 2, it is not hard to obtain that $\varsigma_1\Lambda_1\lambda_1 \le \frac{\varsigma_1^2\Lambda_1^2}{2} + \frac{\bar{\lambda}_1^2}{2}$. Using this inequality into (23) gives

$$\begin{aligned} \dot{V}_1 &\le \varsigma_1\Lambda_1(f_1 + g_1x_2 + \frac{\varsigma_1\Lambda_1}{2} - \dot{y}_r - \frac{\dot{\tau}_1 z_1}{\tau_1}) \\ &\quad + \frac{\bar{\lambda}_1^2}{2} - \frac{d_m}{r}\tilde{\theta}\dot{\hat{\theta}} \\ &\le \varsigma_1\Lambda_1 g_1 x_2 + \varsigma_1\bar{f}_1 + \frac{\bar{\lambda}_1^2}{2} - \frac{d_m}{r}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{24}$$

where $\bar{f}_1 = \Lambda_1(f_1 + \frac{\varsigma_1\Lambda_1}{2} - \dot{y}_r - \frac{\dot{\tau}_1 z_1}{\tau_1})$. The RBF neural network $W_1^T\psi_1(Z_1)$ can approach the unknown function $\bar{f}_1$, where $Z_1 = [x_1, y_r, \dot{y}_r, \tau_1, \dot{\tau}_1]^T \in \Omega_{Z_1} \subset R^5$.

$$\bar{f}_1 = W_1^{*T}\psi_1(Z_1) + \beta_1(Z_1), |\beta_1(Z_1)| \le \bar{\varepsilon}_1 \tag{25}$$

where $\beta_1(Z_1)$ is the approximation error and $\bar{\varepsilon}_1 > 0$. By using Young's inequality, the results are as follows:

$$\begin{aligned} \varsigma_1\bar{f}_1 &\le \frac{1}{2c_1^2}\varsigma_1^2\|W_1^*\|^2\psi_1^T\psi_1 + \frac{c_1^2}{2} + \frac{\varsigma_1^2}{2}d_m + \frac{1}{2}\frac{\bar{\varepsilon}_1^2}{d_m} \\ &\le \frac{d_m}{2c_1^2}\varsigma_1^2\theta\psi_1^T\psi_1 + \frac{c_1^2}{2} + \frac{\varsigma_1^2}{2}d_m + \frac{1}{2}\frac{\bar{\varepsilon}_1^2}{d_m} \end{aligned} \tag{26}$$

By choosing (17), from $z_2 = x_2 - \alpha_1$ the following equality can be obtained:

$$\varsigma_1\Lambda_1 g_1\alpha_1 \le -d_m\varsigma_1^2(a_1 + \frac{1}{2} + \frac{1}{2c_1^2}\hat{\theta}\psi_1^T\psi_1) \tag{27}$$

It yields

$$\begin{aligned} \dot{V}_1 &\le -a_1 d_m\varsigma_1^2 + \varsigma_1\Lambda_1 g_1 z_2 \\ &\quad + \frac{d_m}{r}\tilde{\theta}(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 - \dot{\hat{\theta}}) + \Delta_1 \end{aligned} \tag{28}$$

where $\Delta_1 = \frac{c_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2d_m}$.

Step 2:To stabilize the subsystem in (7), here let's select the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{29}$$

The derivative of time of V_2 is calculated by:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 \\ &= \dot{V}_1 + z_2\left(f_2 + g_2x_3 + \lambda_2 - \dot{\alpha}_1\right)\end{aligned} \tag{30}$$

Substituting (28) into (30) yields

$$\begin{aligned}\dot{V}_2 \leq & -a_1d_m\varsigma_1^2 + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 - \dot{\hat{\theta}}\right) + \mathit{\Delta}_1 \\ & + z_2(f_2 + g_1\varsigma_1\mathit{\Lambda}_1 + g_2x_3 + \lambda_2 - \dot{\alpha}_1)\end{aligned} \tag{31}$$

According to the (8) and (31), we have

$$\begin{cases} z_2\lambda_2 \leq \frac{1}{2}z_2^2 + \frac{\bar{\lambda}_2^2}{2} \\ -z_2\frac{\partial\alpha_1}{\partial x_1}\lambda_1 \leq \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \frac{\bar{\lambda}_1^2}{4}\end{cases} \tag{32}$$

It yields

$$\begin{aligned}\dot{V}_2 \ & \leq -a_1d_m\varsigma_1^2 + \frac{d_m}{r}\tilde{\theta}(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 - \dot{\hat{\theta}}) + \Delta_1 \\ & + z_2[g_1\varsigma_1\Lambda_1 + f_2 \\ & + g_2x_3 + \frac{z_2}{2} - \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1x_2) \\ & + z_2(\frac{\partial\alpha_1}{\partial x_1})^2 - \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} \\ & - \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(k)}}y_r^{(k+1)} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 \\ & - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_2^2}z_2^2\psi_2^T\psi_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta}] \\ & - \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{r}{2c_l^2}z_l^2\psi_l^T\psi_l + \frac{\bar{\lambda}_2^2}{2} + \frac{\bar{\lambda}_1^2}{4}\end{aligned} \tag{33}$$

where

$$\begin{aligned}\dot{\alpha}_1 = & \frac{\partial\alpha_1}{\partial x_1}\left(f_1 + g_1x_2\right) + \frac{\partial\alpha_1}{\partial x_1}\lambda_1 \\ & + \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} + \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(k)}}y_r^{(k+1)} \\ & + \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_2^2}z_2^2\psi_2^T\psi_2 \\ & + \frac{\partial\alpha_1}{\partial\hat{\theta}}\sum_{k=3}^{n}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k - \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta}\end{aligned} \tag{34}$$

Applying $z_3 = x_3 - \alpha_2$, we have:

$$\begin{aligned}\dot{V}_2 \leq & -a_1d_m\varsigma_1^2 + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 - \dot{\hat{\theta}}\right) \\ & + \mathit{\Delta}_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{r}{2c_l^2}z_l^2\psi_l^T\psi_l \\ & + z_2\left(g_2z_3 + g_2\alpha_2 + \bar{f}_2\right) + \frac{\bar{\lambda}_1^2}{4} + \frac{\bar{\lambda}_2^2}{2}\end{aligned} \tag{35}$$

where

$$\begin{aligned}\bar{f}_2 = & g_1\varsigma_1\Lambda_1 + f_2 + \frac{z_2}{2} - \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1x_2) \\ & + z_2(\frac{\partial\alpha_1}{\partial x_1})^2 - \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} \\ & - \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(k)}}y_r^{(k+1)} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 \\ & - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{r}{2c_2^2}z_2^2\psi_2^T\psi_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta}\end{aligned} \tag{36}$$

A similar method was used as described above, we can apply RBF neural network $W_2^T\psi_2(Z_2)$ to approach $\bar{f}_2$.

$$\bar{f}_2 = W_2^{*T}\psi_2(Z_2) + \beta_2(Z_2), |\beta_2(Z_2)| \leq \bar{\varepsilon}_2 \tag{37}$$

where $\beta_2(Z_2)$ is the approximation error and $\bar{\varepsilon}_2 > 0$. According to the Young's inequality, we have

$$z_2\bar{f}_2 \leq \frac{d_m}{2c_2^2}z_2^2\theta\psi_2^T\psi_2 + \frac{1}{2}c_2^2 + \frac{1}{2}d_mz_2^2 + \frac{1}{2}\frac{\bar{\varepsilon}_2^2}{d_m} \tag{38}$$

By choosing (18), the following equality can be obtained:

$$z_2g_2\alpha_2 \leq -d_mz_2^2\left(a_2 + \frac{1}{2}\right) - \frac{d_m}{2c_2^2}z_2^2\hat{\theta}\psi_2^T\psi_2 \tag{39}$$

Substituting (38) and (39) to (35) yields

$$\begin{aligned}\dot{V}_2 \leq & -a_1d_m\varsigma_1^2 - a_2d_mz_2^2 + z_2g_2z_3 \\ & + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \frac{r}{2c_2^2}z_2^2\psi_2^T\psi_2 - \dot{\hat{\theta}}\right) \\ & + \mathit{\Delta}_1 + \mathit{\Delta}_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{r}{2c_l^2}z_l^2\psi_l^T\psi_l\end{aligned} \tag{40}$$

where $\Delta_2 = \frac{c_2^2}{2} + \frac{\bar{\lambda}_2^2}{2} + \frac{\bar{\lambda}_1^2}{4} + \frac{\bar{\varepsilon}_2^2}{2d_m}$

Step i ($3 \leq i \leq n-1$): The virtual control signal $\dot{\alpha}_i$ will construct control the z_{i+1} system, we have

$$\begin{aligned}\dot{\alpha}_{i-1} = & \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left(f_k + g_kx_{k+1}\right) \\ & + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\lambda_k + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 \\ & + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sum_{k=2}^{i}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sigma\hat{\theta} \\ & + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k)}}y_r^{(k+1)} \\ & + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sum_{k=i+1}^{n}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k\end{aligned} \tag{41}$$

Select the Lyapunov function as follows:

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{42}$$

The derivative of time of V_i can be obtained by

$$\dot{V}_i = \dot{V}_{i-1} + z_i\dot{z}_i \tag{43}$$

where V_i in (43) can be acquired by duplicating the similar procedures as those in Step 1:

$$\begin{aligned}\dot{V}_{i-1} \leq & -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{i-1} a_k d_m z_k^2 + z_{i-1} g_{i-1} z_i \\ & + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2 \psi_1^T \psi_1 + \sum_{k=2}^{i-1} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k - \dot{\hat{\theta}}\right) \\ & + \sum_{k=1}^{i-1} \Delta_k - \sum_{k=1}^{i-2} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \sum_{l=i}^{n} \frac{r}{2c_l^2} z_l^2 \psi_l^T \psi_l \end{aligned} \tag{44}$$

where $\Delta_k = \frac{c_k^2}{2} + \frac{\bar{\lambda}_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2d_m}$.
Substituting (44) and $\dot{z}_i$ into (43), it follows that:

$$\begin{aligned}\dot{V}_i \leq & -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{i-1} a_k d_m z_k^2 \\ & + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2 \psi_1^T \psi_1 + \sum_{k=2}^{i-1} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k - \dot{\hat{\theta}}\right) \\ & + \sum_{k=1}^{i-1} \Delta_k - \sum_{k=1}^{i-2} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \sum_{l=i+1}^{n} \frac{r}{2c_l^2} z_l^2 \psi_l^T \psi_l \\ & + z_i[g_{i-1} z_{i-1} - \frac{r}{2c_i^2} z_i^2 \psi_i^T \psi_i \sum_{k=1}^{i-2} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \\ & + f_i + g_i x_{i+1} + \frac{z_n}{2} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(f_k + g_k x_{k+1}) \\ & + z_i \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1}}{\partial x_k})^2 - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tau_1^{(k)}} \tau_1^{(k+1)} \\ & - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(k)}} y_r^{(k+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \frac{r}{2c_1^2} \varsigma_1^2 \psi_1^T \psi_1 \\ & - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sum_{k=2}^{i} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k \\ & - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sum_{k=i+1}^{n} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sigma \hat{\theta}] \\ & + \frac{\bar{\lambda}_i^2}{2} + \sum_{k=1}^{i-1} \frac{\bar{\lambda}_k^2}{4} \end{aligned} \tag{45}$$

From $x_{i+1} = z_{i+1} + \alpha_i$, it follows that:

$$\begin{aligned}\dot{V}_i \leq & -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{i-1} a_k d_m z_k^2 \\ & + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2 \psi_1^T \psi_1 + \sum_{k=2}^{i-1} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k - \dot{\hat{\theta}}\right) \\ & + \sum_{k=1}^{i-1} \Delta_k - \sum_{k=1}^{i-1} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \sum_{l=i+1}^{n} \frac{r}{2c_l^2} z_l^2 \psi_l^T \psi_l \\ & + z_i[g_i z_{i+1} + g_i \alpha_i + \bar{f}_i] \\ & + \frac{\bar{\lambda}_i^2}{2} + \sum_{k=1}^{i-1} \frac{\bar{\lambda}_k^2}{4} \end{aligned} \tag{46}$$

where

$$\begin{aligned}\bar{f}_i = & g_{i-1} z_{i-1} - \frac{r}{2c_i^2} z_i^2 \psi_i^T \psi_i \sum_{k=1}^{i-2} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \\ & + f_i + \frac{z_n}{2} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sum_{k=2}^{i} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k \\ & - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(f_k + g_k x_{k+1}) \\ & + z_i \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1}}{\partial x_k})^2 - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tau_1^{(k)}} \tau_1^{(k+1)} \\ & - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(k)}} y_r^{(k+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \frac{r}{2c_1^2} \varsigma_1^2 \psi_1^T \psi_1 \\ & - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sum_{k=i+1}^{n} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sigma \hat{\theta} \end{aligned} \tag{47}$$

Similar to (38), it can be written as

$$z_i \bar{f}_i \leq \frac{d_m}{2c_i^2} z_i^2 \theta \psi_i^T \psi_i + \frac{1}{2} c_i^2 + \frac{1}{2} d_m z_i^2 + \frac{1}{2}\frac{\bar{\varepsilon}_i^2}{d_m} \tag{48}$$

By choosing (18), the following equality can be obtained:

$$z_i g_i(\bar{x}_i)\alpha_i \leq -d_m z_i^2 \left(a_i + \frac{1}{2}\right) - \frac{d_m}{2c_i^2} z_i^2 \hat{\theta} \psi_i^T \psi_i \tag{49}$$

Combining (48) and (49) to (46) yields

$$\begin{aligned}\dot{V}_i \leq & -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{i} a_k d_m z_k^2 + z_i g_i z_{i+1} \\ & + \frac{d_m}{r}\tilde{\theta}\left(\frac{r}{2c_1^2}\varsigma_1^2 \psi_1^T \psi_1 + \sum_{k=2}^{i} \frac{r}{2c_k^2} z_k^2 \psi_k^T \psi_k - \dot{\hat{\theta}}\right) \\ & + \sum_{k=1}^{i} \Delta_k - \sum_{k=1}^{i-1} \frac{\partial \alpha_k}{\partial \hat{\theta}} z_{k+1} \sum_{l=i+1}^{n} \frac{r}{2c_l^2} z_l^2 \psi_l^T \psi_l \end{aligned} \tag{50}$$

where $\Delta_i = \frac{c_i^2}{2} + \frac{\bar{\lambda}_i^2}{2} + \sum_{k=1}^{i-1}\sum_{l=1}^{k} \frac{\bar{\lambda}_l^2}{4} + \frac{\bar{\varepsilon}_i^2}{2d_m}$.

Step n: take the following Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 \tag{51}$$

From (7), we obtain:

$$\begin{aligned}\dot{V}_n =& \dot{V}_{n-1} + z_n\dot{z}_n \\ \leq& -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{n-1} a_k d_m z_k^2 + \sum_{k=1}^{n-1}\Delta_k \\ &+ \frac{d_m}{r}\tilde{\theta}(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \sum_{k=2}^{n-1}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k - \dot{\hat{\theta}}) \\ &+ z_n[g_{n-1}z_{n-1} + f_n \\ &- \frac{r}{2c_n^2}z_n\psi_n^T\psi_n\sum_{k=1}^{n-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ &+ g_n\kappa v + g_n s(t) + \frac{z_n}{2} \\ &- \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k + g_k x_{k+1}) \\ &+ z_n\sum_{k=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_k})^2 - \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} \\ &- \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k)}}y_r^{(k+1)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}] \\ &+ \frac{\bar{\lambda}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4}\end{aligned} \tag{52}$$

The following relation can be derived:

$$\begin{aligned}\dot{V}_n \leq& -a_1 d_m \varsigma_1^2 - \sum_{k=2}^{n-1} a_k d_m z_k^2 + \sum_{k=1}^{n-1}\Delta_k \\ &+ \frac{d_m}{r}\tilde{\theta}(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \sum_{k=2}^{n-1}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k - \dot{\hat{\theta}}) \\ &+ z_n(g_n\kappa v + g_n s(t) + \bar{f}_n) - \frac{1}{2}d_m z_n^2 \\ &+ \frac{\bar{\lambda}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4}\end{aligned} \tag{53}$$

The $\dot{\alpha}_{n-1}$ is expressed by (38), where $i = n$

$$\begin{aligned}\bar{f}_n =& g_{n-1}z_{n-1} - \frac{r}{2c_n^2}z_n\psi_n^T\psi_n\sum_{k=1}^{n-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ &+ f_n + \frac{z_n}{2} - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k + g_k x_{k+1}) \\ &+ z_n\sum_{k=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_k})^2 - \sum_{k=0}^{n-1}\frac{\partial\alpha_{i-1}}{\partial\tau_1^{(k)}}\tau_1^{(k+1)} \\ &- \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k)}}y_r^{(k+1)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}d_m z_n\end{aligned} \tag{54}$$

Similar to (48), it can be written as

$$z_n\bar{f}_n \leq \frac{d_m}{2c_n^2}z_n^2\theta\psi_n^T\psi_i + \frac{1}{2}c_n^2 + \frac{d_m}{2}z_n^2 + \frac{1}{2}\frac{\bar{\varepsilon}_n^2}{d_m} \tag{55}$$

where a_n is a design parameter. At this stage, using the actual control input v in (19) and (6), it produces

$$z_n g_n s(t) \leq \frac{1}{2\eta^2}g_n\underline{\kappa}z_n^2 + \frac{\eta^2}{2\underline{\kappa}}d_M\bar{s}^2 \tag{56}$$

$$\begin{aligned}z_n g_n\kappa v \leq& -a_n g_n\underline{\kappa}z_n^2 - \frac{1}{2\eta^2}g_n\underline{\kappa}z_n^2 \\ &- \frac{d_m}{2c_n^2}z_n^2\hat{\theta}\psi_n^T\psi_n\end{aligned} \tag{57}$$

From (56) and (57), (53) can be rewritten as

$$\begin{aligned}\dot{V}_n \leq& -a_1 d_m\varsigma_1^2 - \sum_{i=2}^{n-1}a_i d_m z_i^2 - a_n g_n\underline{\kappa}z_n^2 + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} \\ &+ \sum_{i=1}^{n-1}\Delta_i + \frac{1}{2}c_n^2 + \frac{1}{2}\bar{\lambda}_n^2 + \frac{1}{2}\frac{\bar{\varepsilon}_n^2}{d_m} + \frac{d_M}{2k}\eta^2\bar{s}^2 \\ &+ \frac{d_m}{r}\tilde{\theta}(\frac{r}{2c_1^2}\varsigma_1^2\psi_1^T\psi_1 + \sum_{k=2}^{n}\frac{r}{2c_k^2}z_k^2\psi_k^T\psi_k - \dot{\hat{\theta}})\end{aligned} \tag{58}$$

Now, by applying the adaptive law $\dot{\hat{\theta}}$ in (21) into (58), it follows

$$\begin{aligned}\dot{V}_n \leq& -a_1 d_m\varsigma_1^2 - \sum_{i=2}^{n-1}a_i d_m z_i^2 - a_n g_n\underline{\kappa}z_n^2 \\ &+ \sum_{i=1}^{n-1}\Delta_i + \frac{1}{2}c_n^2 + \frac{1}{2}\bar{\lambda}_n^2 + \frac{1}{2}\frac{\bar{\varepsilon}_n^2}{d_m} + \frac{d_M}{2k}\eta^2\bar{s}^2 \\ &+ \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + \frac{d_m\sigma}{r}\tilde{\theta}\hat{\theta}\end{aligned} \tag{59}$$

Theorem 1. *Consider the system of (1) and unknown input dead-zone nonlinearities (2), the controller (19), and the adaptive law (21). Under the premise of Assumption $1-2$, and the package functions $\bar{f}_i$ can be approached by RBF NN with a bounded approximation error z_i. If the initial error $e(0) < \tau_1(0)$. Then, there exist design parameters σ, τ_1 and d_m such that for all signals in the closed loop system are consistently, semi-globally and ultimately bounded, and the output tracking error $e(t) = y(t) - y_r(t)$ converges to prescribed boundness and satisfies the prescribed performance.*

Proof 1. *On account of the stability analysis of the strict-feedback system, the form of Lyapunov function is reconstructed $V = V_n$*

$$\begin{aligned}\dot{V}_n \leq& -a_1 d_m\varsigma_1^2 - \sum_{i=2}^{n}a_i d_m z_i^2 + \sum_{i=1}^{n-1}\Delta_i \\ &+ \frac{1}{2}c_n^2 + \frac{1}{2}\bar{\lambda}_n^2 + \frac{1}{2}\bar{\varepsilon}_n^2 + \frac{d_M}{2k}\eta^2\bar{s}^2 \\ &+ \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + \frac{d_m\sigma}{r}\tilde{\theta}\hat{\theta}\end{aligned} \tag{60}$$

Due to

$$\frac{d\sigma}{r}\tilde{\theta}\hat{\theta} \leq -\frac{d_m\sigma}{2r}\tilde{\theta}^2 + \frac{d_m\sigma}{2r}\theta^2 \tag{61}$$

Substituting (58) into (57) produces

$$\dot{V}_n \leq -a_1 d_m\varsigma_1^2 - \sum_{i=2}^{n}a_i d_m z_i^2 - \frac{d_m\sigma}{2r}\tilde{\theta}^2 + \sum_{i=1}^{n}\Delta_i \tag{62}$$

where

$$\begin{aligned}\Delta_n =& \frac{d_m\sigma}{2r}\theta^2 + \frac{1}{2}c_n^2 + \frac{1}{2}z_n^2 + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} \\ &+ \frac{1}{2}\frac{\bar{\varepsilon}_n^2}{d_m} + \frac{d_M}{2k}\eta^2\bar{s}^2\end{aligned} \tag{63}$$

Select the control gains $k_1 = \frac{1}{4a_1}\chi_1, k_j = \frac{1}{2a_j}\chi_j, j = 2,3,...,n$, $\sigma = \chi$, *Let* $\chi = \min[\chi_1, \chi_2, ..., \chi_n]$, *and* $\Upsilon = \sum_{i=1}^{n} \Delta_i$ *are positive constants. Then we have*

$$\dot{V} \leq -\chi V + \Upsilon \tag{64}$$

Equation (64) can become $\dot{V} + \chi V \leq \Upsilon$ *and* $e^{\chi t}(\dot{V} + \chi V) \leq e^{\chi t}\Upsilon$, *then we have*

$$\frac{de^{\chi t}V}{dt} \leq e^{\chi t}\Upsilon \tag{65}$$

Let $C = \frac{\Upsilon}{\chi}$ *and integrating (50) over* $[0, t]$, *we have*

$$e^{\chi t}V(t) - V(0) \leq Ce^{\chi t} - C \tag{66}$$

From (66), it easily to obtain

$$0 \leq V(t) \leq C + (V(0) - C)e^{-\chi t} \tag{67}$$

Therefore, $z_i (i = 1, 2, ..., n)$ *and* $\tilde{\theta}$ *are bounded.* θ *defined as a constant, and* $\hat{\theta}$ *is also bounded. We have* $z_1 = x_1 - y_d$, *and* y_d *is a reference signal, it can deduce the* x_1 *is bounded. Besides,* α_1 *is the function of* z_1 *and* $\hat{\theta}$ *which are bounded variables; thus,* $x_2 = z_2 + \alpha_1$ *is also bounded. Similarly, it can be easily verified that* $x_j (j = 3, ..., n)$ *is bounded. From (19), it is follows that* v *is also bounded. Therefore, all the signals in the closed-loop systems remain bounded. From (62) and (67), it can be inferred that* $|z_i| \leq \sqrt{2C + 2(V(0) - C)\exp(-\chi t)}$. *If* $V(0) = C$, z_i *can converge to* $\sqrt{2C}$, *i.e,* $\lim_{t\to\infty} |z_i(t)| = \sqrt{2C}$. *Therefore ,this implies the tracking errors can converge to the prescribed bounds*

IV. SIMULATION EXAMPLE

In order to verify the effectiveness of our result via example. Take the following third-order strict-feedback nonlinear system with input dead-zone constraints:

$$\begin{cases} \dot{x}_1 = x_2 + 0.1 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_2 + (2 + sin(x_2^2))u + 0.1cos(u) \\ y = x_1 \end{cases} \tag{68}$$

where x_1, x_2, x_3 represent the state variables, y is the system output, u denotes the output of the saturation limits are choosen as $m = 1$ and $p = -1$, respectively. Based on Theorem 1, the adaptive neural network controller is designed for the system (68). Let the system output y follows desired reference signal $y_r = 0.5sin(t) + 0.5sin(0.5t)$.

The initial values of the prescribed performance function for tracking error are as follows: $\tau_0 = 3$, $\tau_\infty = 0.07$, and $\delta = -1$. Thus, the boundary function is $\rho = (3-0.07)e^{-t}+0.07$. The virtual control signal α_i and the actual control input v are as follows:

$$\alpha_i = -(a_i + \frac{1}{2})z_i - \frac{1}{2c_i^2} z_i \hat{\theta} \psi_i^T \psi_i \tag{69}$$

$$v = -(a_3 + \frac{1}{2\eta^2})z_3 - \frac{1}{2c_3^2} z_3 \hat{\theta} \psi_i^T \psi_i \tag{70}$$

The adaptive law is defined as follow:

$$\dot{\hat{\theta}} = \frac{r}{2c_1^2} \varsigma_1^2 \psi_1^T \psi_1 + \sum_{i=2}^{3} \frac{r}{2c_i^2} z_i^2 \psi_i^T \psi_i - \sigma \hat{\theta} \tag{71}$$

where a_3, c_3, σ and r take appropriate positive parameters, from formula (9), it has $z_3 = x_3 - \alpha_2$. From the simulation, the system parameters are taken as follows: $a_1 = 10$, $a_2 = 10$, $a_3 = 11$, $c_1 = 11$, $c_2 = 11$, $c_3 = 11$, $r = 3$, $\sigma = 2$, and $\eta = 1$. The initial state is $[x_1(0), x_2(0), x_3(0)]^T = [-0.2, 0.1, 0.1]^T$, and $\hat{\theta}(0) = 0$.

The simulation time is set to 40 seconds, and the simulation results of the proposed control scheme are shown in Figs. 1-5. The output error z_1 and the performance prescribed function τ_1 are shown in Fig. 1. The system output y can track the reference signal y_r well in Fig. 2 despite the existence of external disturbances. And then, Figs.3 shows the actual control signal $v(t)$ of the system and the function $u(t)$ constrained by the dead-zone function $D(\cdot)$. The adaptive parameter $\hat{\theta}$ is shown in Fig. 4. Finally, other states x_2 and x_3 of the system are shown in Fig. 5. It is no hard to find from these simulation figures that PPC is implemented in the case of input dead-zone nonlinearity, unknown nonlinearity and unknown external disturbance.

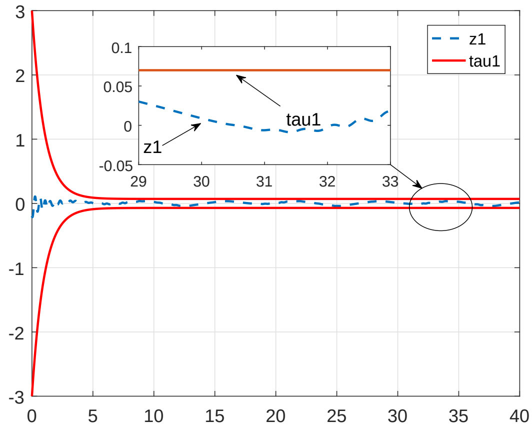

Fig. 1: Performance bound τ_1 and tracking error z_1

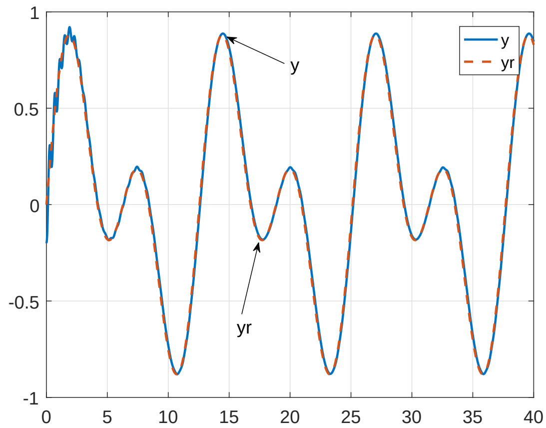

Fig. 2: Output y and the reference signal y_r

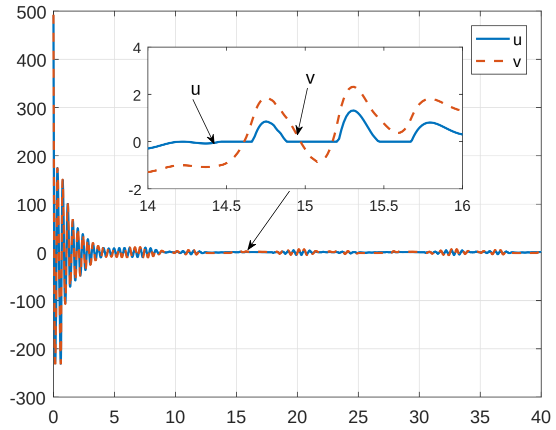

Fig. 3: Control input signal v and dead-zone output signal u

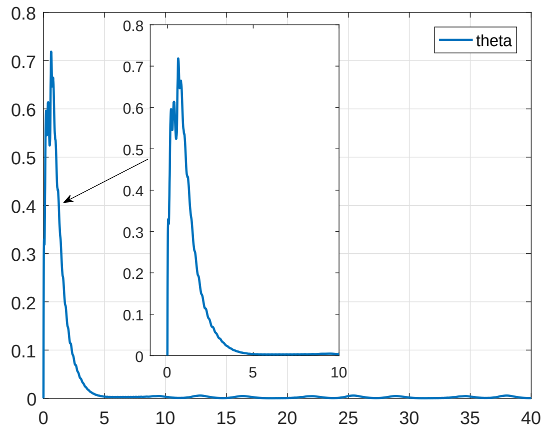

Fig. 4: Adaptive parameter $\hat{\theta}$

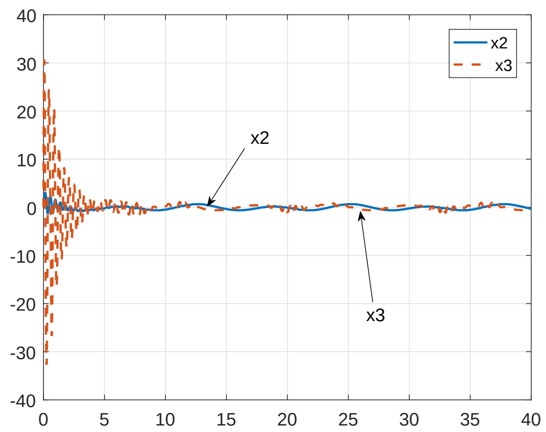

Fig. 5: State variables x_2 and x_3

V. CONCLUSION

The paper presented an adaptive neural control method for a class of strict-feedback nonlinear systems with nonsymmetric dead-zone input and external disturbances. Then, a new error transformation method and a prescribed performance control method are proposed to realize the prescribed performance constraint of tracking error. The unknown function in the system is approximated by RBF neural network, which is combined with backstepping technology and adaptive neural network control technology to complete the design of the controller. Moreover, the stability of the closed-loop system and the tracking performance are guaranteed. By simulating, it indicates that the scheme is effective. It can be further studied and applied to switching systems, nonlinear multi-agent systems and nonlinear interconnected systems.

REFERENCES

[1] X. Sun, D. Wu, G. Liu, and W. Wang, “Input-to-state Stability for Networked Predictive Control with Random Delays in Both Feedback and Forward Channels,” *IEEE Transactions on Industrial Electronics*, vol. 61, no. 7, pp. 3519–3526, 2014.

[2] Z. Wang, J. Xu, X. Song, and H. Zhang, “Consensus Conditions for Multi-Agent Systems Under Delayed Information,” *IEEE Transactions on Circuits and Systems II-express Briefs*, vol. 65, no. 11, pp. 1773–1777, 2018.

[3] X. Li and S. Song, “Stabilization of Delay Systems: Delay-dependent Impulsive Control,” *IEEE Transactions on Automatic Control*, vol. 62, no. 1, pp. 406–411, 2017.

[4] X. Liu, S. Li, and K. Zhang, “Optimal Control of Switching Time in Switched Stochastic Systems with Multi-switching Times and Different Costs,” *International Journal of Control*, vol. 90, no. 8, pp. 1604–1611, 2017.

[5] Y. Li, S. Qiang, X. Zhuang, and O. Kaynak, “Robust and Adaptive Backstepping Control for Nonlinear Systems Using RBF Neural Networks,” *IEEE Transactions on Neural Networks*, vol. 15, no. 3, pp. 693–701, 2004.

[6] C. Kwan and F. L. Lewis, “Robust Backstepping Control of Nonlinear Systems Using Neural Networks,” *IEEE Transactions on Systems Man and Cybernetics, Part A Systems and Humans*, vol. 30, no. 6, pp. 753–766, 2000.

[7] S. S. Ge and C. Wang, “Direct Adaptive NN Control of a Class of Nonlinear Systems,” *IEEE Transactions on Neural Networks*, vol. 13, no. 1, pp. 214–221, 2002.

[8] Y. Liu, G. Wen, and S. Tong, “Direct Adaptive NN Control for a Class of Discrete-time Nonlinear Strict-feedback Systems,” *Neurocomputing*, vol. 73, no. 13, pp. 2498–2505, 2010.

[9] G. Wen, Y. Liu, and C. L. P. Chen, “Direct Adaptive Robust NN Control for a Class of Discrete-time Nonlinear Strict-feedback Siso Systems,” *Neural Computing and Applications*, vol. 21, no. 6, pp. 1423–1431, 2012.

[10] T. Zhang, S. S. Ge, and C. C. Hang, “Adaptive Neural Network Control for Strict-feedback Nonlinear Systems Using Backstepping Design,” *Automatica*, vol. 36, no. 12, pp. 1835–1846, 2000.

[11] C. Pukdeboon, “Adaptive Backstepping Finite-time Sliding Mode Control of Spacecraft Attitude Tracking,” *Journal of Systems Engineering and Electronics*, vol. 26, no. 4, p. 826, 2010.

[12] X. Zheng, P. Li, H. Li, and D. Ding, “Adaptive Backstepping-based NTSM Control for Unmatched Uncertain Nonlinear Systems,” *Journal of Systems Engineering and Electronics*, vol. 26, no. 3, pp. 557–564, 2015.

[13] R. F. Rachmadi, I. K. E. Purnama, M. H. Purnomo, and M. Hariadi, “A Systematic Evaluation of Shallow Convolutional Neural Network on CIFAR Dataset,” *Iaeng Internaitonal Journal of Computer Science*, vol. 46, no. 2, pp. 365–376, 2019.

[14] J. Yu, B. Chen, H. Yu, C. Lin, and L. Zhao, “Neural Networks-based Command Filtering Control of Nonlinear Systems with Uncertain Disturbance,” *Information Sciences*, vol. 426, pp. 50–60, 2018.

[15] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, “Adaptive Neural State-feedback Tracking Control of Stochastic Nonlinear Switched Systems: An Average Dwell-time Method,” *IEEE transactions on Neural Networks and Learning Systems*, vol. 30, no. 4, pp. 1076–1087, 2018.

[16] W. Hong and G. Tonghui, “Training Algorithm of Adaptive Neural Fuzzy Inference System Based on Improved SRUKF,” *IAENG International Journal of Computer Science*, vol. 44, no. 4, pp. 396–403, 2017.

[17] S. S. Ge and J. Wang, “Robust Adaptive Neural Control for a Class of Perturbed Strict Feedback Nonlinear Systems,” *IEEE Transactions on Neural Networks*, vol. 13, no. 6, pp. 1409–1419, 2002.

[18] L. Yu, S. Fei, J. Huang, and Y. Gao, “Trajectory Switching Control of Robotic Manipulators based on RBF Neural Networks,” *Circuits Systems Signal Processing*, vol. 33, no. 4, pp. 1119–1133, 2014.

[19] Z. Jia, Z. Hu, and W. Zhang, “Adaptive Output-feedback Control with Prescribed Performance for Trajectory Tracking of Underactuated Surface Vessels,” *Isa Transactions*, vol. 95, pp. 18–26, 2019.

[20] C. P. Bechlioulis and G. A. Rovithakis, "Approximation-free Prescribed Performance Control for Unknown SISO Pure Feedback Systems," *Control Conference*, 2013.
[21] C. P. Bechlioulis and G. A. Rovithakis, "A Low-complexity Global Approximation-free Control Scheme with Prescribed Performance for Unknown Pure Feedback Systems," *Automatica*, vol. 50, no. 4, pp. 1217–1226, 2014.
[22] Y. Yang, C. Ge, H. Wang, X. Li, and C. Hua, "Adaptive Neural Network based Prescribed Performance Control for Teleoperation System under Input Saturation," *Journal of the Franklin Institute*, vol. 352, no. 5, pp. 1850–1866, 2015.
[23] C. Shen, Y. Shi, and B. Buckham, "Trajectory Tracking Control of an Autonomous Underwater Vehicle Using Lyapunov-based Model Predictive Control," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 7, pp. 5796–5805, 2017.
[24] G.-Y. Gu, C.-Y. Su, and L.-M. Zhu, "Robust Inverse Compensation and Control of a Class of Non-linear Systems with Unknown Asymmetric Backlash non-linearity," *IET Control Theory and Applications*, vol. 9, no. 12, pp. 1869–1877, 2015.
[25] C. Siyu, T. Jinyuan, L. Caiwang, and W. Qibo, "Nonlinear Dynamic Characteristics of Geared Rotor Bearing Systems with Dynamic Backlash and Friction," *Mechanism and Machine Theory*, vol. 46, no. 4, pp. 466–478, 2011.
[26] X. Tian, Z. Yang, and Z. Yang, "Adaptive Stabilization of Fractional-order Energy Supply-demand System with Dead-zone Nonlinear Inputs," *IAENG International Journal of Applied Mathematics*, vol. 49, no. 4, pp. 500–504, 2019.
[27] S. Tong and Y. Li, "Adaptive Fuzzy Output Feedback Control of MIMO Nonlinear Systems with Unknown Dead-zone Inputs," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 134–146, 2013.
[28] Y. L. S. Tong, "Adaptive Fuzzy Decentralized Output Feedback Control for Nonlinear Large-scale Systems with Unknown Dead-zone Inputs," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 5, pp. 913–925, 2013.
[29] Y. Liu, Y. Gao, S. Tong, and Y. Li, "Fuzzy Approximation-based Adaptive Backstepping Optimal Control for a Class of Nonlinear Discrete-time Systems with Dead-zone," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 1, pp. 16–28, 2016.
[30] X. Liu, H. Wang, C. Gao, and M. Chen, "Adaptive Fuzzy Funnel Control for a Class of Strict Feedback Nonlinear Systems," *Neurocomputing*, vol. 241, pp. 71–80, 2017.
[31] X.-S. Wang, C.-Y. Su, and H. Hong, "Robust Adaptive Control of a Class of Nonlinear Systems with Unknown Dead-zone," *Automatica*, vol. 40, pp. 407–413, 2004.
[32] G. Tao and P. V. Kokotovic, "Adaptive Control of Plants with Unknown Dead-zones," *IEEE Transactions on Automatic Control*, vol. 39, no. 1, pp. 59–68, 1994.
[33] M. Chen, S. S. Ge, and B. Ren, "Adaptive Tracking Control of Uncertain MIMO Nonlinear Systems with Input Constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, 2011.
[34] X. S. Wang, C. Y. Su, and H. Hong, "Robust Adaptive Control of a Class of Nonlinear Systems with Unknown dead-zone," *Automatica*, vol. 40, no. 3, pp. 407–413, 2004.
[35] S. Ibrir, W. F. Xie, and C. Y. Su, "Adaptive Tracking of Nonlinear Systems with Non-symmetric Dead-zone Input," *Automatica*, vol. 43, no. 3, pp. 522–530, 2007.
[36] J. B. Bradley, "Neural Networks: A Comprehensive Foundation," *Information Processing Management*, vol. 31, no. 5, p. 786, 1995.
[37] E. B. Kosmatopoulos, M. M. Polycarpou, M. A. Christodoulou, and P. A. Ioannou, "High-order Neural Network Structures for Identification of Dynamical Systems," *IEEE transactions on Neural Networks*, vol. 6(2), pp. 422–431, 1995.