Performance Evaluation of Credibilistic Mean Semi-absolute Deviation Portfolio with Psychology of Investors

Yechun Yu, Wen Fang and Cuirong Huang

Abstract—**By comprehensively thinking over the fuzzy nondeterminacy of the financial market and the psychological factors of investors, this paper studies the performance evaluation problems with different risk attitudes under the credibilistic environment based on DEA method. Firstly, the risk attitude parameter *k* is introduced into the trapezoidal fuzzy number. Based on credibility theory, the credibilistic mean and semi-absolute deviation with risk attitude are deduced through rigorous mathematical proof. Furthermore, the credibilistic mean semi-absolute deviation portfolio and corresponding portfolio efficiency evaluation models with risk attitude are constructed, wherein the real constraints such as transaction costs and transaction volume are also considered. Finally, an example shows that no matter what risk attitude an investor holds, the DEA frontiers generated by adequate sample size can effectively approximate real frontiers of the credibility mean-semi-absolute deviation models. Through correlation analysis, the feasibility and effectiveness of the proposed portfolio performance evaluation models with investor psychological factors are further verified.**

Index Terms—**portfolio performance evaluation, data envelopment analysis (DEA), risk attitude, credibilistic mean semi-absolute deviation, real efficient frontier.**

I. Introduction

Data envelopment analysis (DEA), as a non-parametric linear evaluation technique to deal with multiple input and multiple output problems, is favored by many researchers in various fields. Many scholars have launched out scientific research about DEA. For example, Liu and Wang [1] proposed a principal component analysis method to aggregate the DEA cross efficiency. In order to evaluate company performance, Leyer and Hüttel [2] put forward a novel method based on DEA. Karadayi and Karsak [3] proposed vague DEA model to evaluate the performance of public hospitals, in which quantitative and qualitative data are expressed as linguistic variables.

In the financial market, many scholars construct diversified portfolio performance evaluation models with the help of DEA technology. Murthi [4] applied DEA method to the field of portfolio performance efficiency for the first time, and pointed out that this method is consistent with Sharpe index and Jensen index method. Morey et al. [5] proposed a quadratic constrained non-linear DEA model to evaluate the portfolio performance. Joro and Na [6] evaluated the mean-variance-skewness portfolio performance by utilizing the non-linear DEA model. Liu [7] researched the theoretical basis of applying DEA model to portfolio performance evaluation. Banihashem et al. [8] used DEA method to study portfolio efficiency under the framework of mean conditional value at risk (CVaR). Zhou [9] put forward the improvement strategy of DEA frontier, and further extended the DEA method to evaluate the portfolio performance under the general income-risk framework.

The above literature is all about portfolio performance evaluation in stochastic environment. In fact, many vague factors exist in the financial market, such as domestic and foreign financial conditions, national economic policies, culture, market rules, etc. Zadeh [10] put forward the theory of fuzzy sets. Then, based on the possibility theory proposed by Zadeh [11], some scholars [12]-[13] studied the possibility fuzzy portfolio optimization problem. Liu [14] proposed the credibility theory. What's more, various credibility portfolio optimization models have been developed [15]-[17]. So far only a few scholars have discussed the portfolio performance evaluation in fuzzy environment. Chen [18] adopted three different risk measures to study the performance evaluation of possible fuzzy portfolio based on DEA method. Gupta et al. [19] constructed the mean-var portfolio performance evaluation model under the credibility fuzzy environment.

With the development of behavioral finance, it is found that investors are not information-efficient and rational at every time point. Subjective psychological factors have a significant impact on investment decisions. Tsaur [20] considered three different risk attitudes of investors: risk aversion, neutrality and preference, and discussed the possibility fuzzy portfolio selection problem considering investors' psychology. Based on this, Jin [21] studied the fuzzy portfolio model with investors' subjective psychological factors by using the possibility semi-variance as the risk measure. Note that the current portfolio performance evaluation methods are ignoring the impact of investors' psychological factors on the evaluation results.

In view of the above analysis, this paper comprehensively

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considers the fuzzy uncertainty and the psychological factors of investors, and studies the portfolio performance evaluation problem considering the psychological characteristics of investors under the credibility environment based on DEA method. This paper will include investors' risk attitude factors in the DEA efficiency evaluation model. The related content of portfolio efficiency evaluation theory will also be expanded. The model can also guide different types of investors, and help them make the best personal investment plan in the actual decision-making process.

In Section II, we will introduce and prove the preliminaries. In Section III, we first introduce the credibility mean-semi-absolute deviation model considering investors' psychology, and then propose the corresponding DEA portfolio model. In Section IV, the paper demonstrates the feasibility of the method through examples and we draw a conclusion in Section V.

II. Preliminaries

A. Credibility Theory and Related Definitions

Definition 1.1 [14]: ξ is a fuzzy variable with membership function μ, the credibility of $\xi \in B$ is defined as

$$Cr\{\xi \in B\} = \frac{1}{2}\left\{\sup_{x\in B}\mu(x) + 1 - \sup_{x\in B^C}\mu(x)\right\}. \tag{1}$$

Definition 1.2 [14]: ξ is a fuzzy variable, its expectation is defined as

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \geq x\}dx - \int_{-\infty}^{0} Cr\{\xi \leq x\}dx. \tag{2}$$

Theorem 1.1 [14]: ξ and η are independent fuzzy variable with limited expectation, for $\forall a, b \in R$, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \tag{3}$$

Definition 1.3 [16]: ξ is a fuzzy variable, expectation is $E[\xi]$, its semi-absolute deviation is defined as

$$SAD[\xi] = E(\xi - E\xi)^{-}, \tag{4}$$

where $(\xi - E\xi)^{-} = \max(0, E\xi - \xi)$.

Definition 1.4 [10]: LR fuzzy number $\tilde{A} = (a, b, \alpha, \beta)_{LR}$, whose membership function is defined as

$$\mu_{\tilde{A}} = \begin{cases} L_{\tilde{A}}\left(\dfrac{a-x}{\alpha}\right), a-\alpha \leq x < a, \\ 1, \quad a \leq x < b, \\ R_{\tilde{A}}\left(\dfrac{x-b}{\beta}\right), b \leq x < b+\beta, \\ 0, \quad \text{others.} \end{cases} \tag{5}$$

In addition, $L_{\tilde{A}}, R_{\tilde{A}} : [0,1] \to [0,1]$ is continuous monotonous decreasing function and $L_{\tilde{A}}(0) = R_{\tilde{A}}(0) = 1$, $L_{\tilde{A}}(1) = R_{\tilde{A}}(1) = 0$.

Theorem 1.2 [12]: Supposed $\tilde{A} = (a_1, b_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{B} = (a_2, b_2, \alpha_2, \beta_2)_{LR}$ are LR fuzzy number, $\lambda \in R$, we have

$$\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}, \tag{6}$$

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda\alpha_1, \lambda\beta_1)_{LR} & \lambda \geq 0, \\ (\lambda b_1, \lambda a_1, |\lambda|\beta_1, |\lambda|\alpha_1)_{LR} & \lambda < 0. \end{cases} \tag{7}$$

B. Portfolio performance Definition

Reference [5] proposed the definition of portfolio efficiency based on real efficient frontier. As shown in Fig. 1, $A_1(r_1, \sigma_1)$ and $A_2(r_2, \sigma_2)$ are the optimal portfolio on the real efficient frontier of portfolio, and $A(r, \sigma)$ is any portfolio to be evaluated. According to two different projection paths, the relative distance from the real effective frontier to $A(r, \sigma)$ is calculated. The profit-oriented efficiency and risk-oriented efficiency are obtained. The mathematical expression is as follows:

$$\text{PE}_r = \frac{r}{r_2}; \quad \text{PE}_\sigma = \frac{\sigma_1}{\sigma}. \tag{8}$$

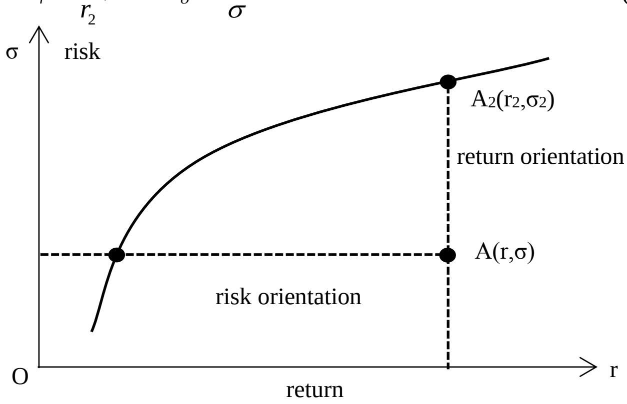

Fig. 1. Portfolio performance

III. Model Building

A. Credibility Mean-Semi-Absolute Deviation Model Considering Investors' Psychology

Based on the possibility theory, reference [23] derived the possibility mean and standard deviation with risk attitude parameter k to characterize the return and risk objectives of portfolio. This paper considers the more complex case, which introduce the risk attitude parameter k into the trapezoidal fuzzy numbers. Under the credibility environment, we derive the credibility mean and semi-absolute deviation of psychological factors with risk attitude.

$\xi = (a, b, \alpha, \beta)_k$ is trapezoidal fuzzy number considering investors' risk attitude, its membership function is

$$u_\xi(x) = \begin{cases} 1 - \left(\dfrac{a-x}{\alpha}\right)^k, a-\alpha \leq x < a, \\ 1, \quad a \leq x < b, \\ 1 - \left(\dfrac{x-b}{\beta}\right)^k, b \leq x < b+\beta, \\ 0, \quad \text{others.} \end{cases} \tag{9}$$

k is a risk attitude parameter. If $0 < k < 1$, it means investors have risk aversion attitude; if $k = 1$, it means that they have risk neutral attitude; if $k > 1$, it means that they have risk preference attitude.

Definition 2.1: Fuzzy number $\xi = (a, b, \alpha, \beta)_k$ credibility mean is

$$E(\xi) = \frac{b+a}{2} + \frac{k}{k+1}\frac{\beta - \alpha}{2}. \tag{10}$$

Prove: According to Definition 1.1, $\xi \geq y$ credibility mean

$$Cr(\xi \ge y)=\begin{cases}1, & y \le a-\alpha,\\ \frac{1}{2}\left[1+\left(\frac{a-y}{\alpha}\right)^k\right], & a-\alpha < y \le a,\\ \frac{1}{2}, & a < y \le b,\\ \frac{1}{2}\left[1-\left(\frac{y-b}{\beta}\right)^k\right], & b < y \le b+\beta,\\ 0, & b+\beta < y.\end{cases} \tag{11}$$

According to Definition 1.2, we have

$$\begin{aligned}E(\xi) &= \int_0^{+\infty} Cr\{\xi \ge y\}dy - \int_{-\infty}^0 Cr\{\xi \le y\}dy\\ &= \int_0^{a-\alpha} 1dy + \frac{1}{2}\int_{a-\alpha}^{a}\left[1+\left(\frac{a-y}{\alpha}\right)^k\right]dy\\ &\quad+\int_a^b \frac{1}{2}dy + \frac{1}{2}\int_b^{b+\beta}\left[1-\left(\frac{y-b}{\beta}\right)^k\right]dy\\ &= \frac{b+a}{2}+\frac{k}{k+1}\frac{\beta-\alpha}{2}.\end{aligned} \tag{12}$$

Theorem 2.2: Credibility semi-absolute deviation is

$$ASD(\xi)=\begin{cases}\frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}+\frac{\alpha^{-k}}{2(k+1)}\left(\frac{a-b}{2}+\frac{k}{k+1}\frac{\alpha-\beta}{2}\right)^{k+1}, & a-\alpha < E(\xi) \le a,\\ \frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}, & a < E(\xi) \le b,\\ \frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}+\frac{\beta^{-k}}{2(k+1)}\left(\frac{a-b}{2}+\frac{k}{k+1}\frac{\beta-\alpha}{2}\right)^{k+1}, & b < E(\xi) \le b+\beta,\\ 0, & \text{otherwise.}\end{cases} \tag{13}$$

Prove: If $\eta=\xi-E(\xi)$, then $Cr\{\eta \le y\}=Cr\{\xi \le y+E(\xi)\}$,

$$Cr\{\eta \le y\}=\begin{cases}0, & y \le a-\alpha-E(\xi),\\ \frac{1}{2}\left[1-\left(\frac{a-y-E(\xi)}{\alpha}\right)^k\right], & a-\alpha-E(\xi) < y \le a-E(\xi),\\ \frac{1}{2}, & a-E(\xi) < y \le b-E(\xi),\\ \frac{1}{2}\left[1+\left(\frac{y+E(\xi)-b}{\beta}\right)^k\right], & b-E(\xi) < y \le b+\beta-E(\xi),\\ 1, & b+\beta-E(\xi) < y.\end{cases} \tag{14}$$

Since $Cr\{\eta \ge y\}=1-Cr\{\eta \le y\}$. For $\eta^-=\max\{0,-\eta\}$,

$$\begin{aligned}Cr\{\eta^- \le y\} &= Cr\{\max\{0,-\eta\} \le y\}\\ &= \begin{cases}Cr\{\eta \ge -y\}, & y \ge 0,\\ 0, & y < 0,\end{cases}\end{aligned} \tag{15}$$

$$Cr\{\eta^- \ge y\}=\begin{cases}Cr\{\eta \le -y\}, & y \ge 0,\\ 1, & y < 0.\end{cases} \tag{16}$$

According to Definition 1.2,

$$\begin{aligned}E(\xi-E\xi)^- &= E(\eta^-)\\ &= \int_0^{+\infty} Cr\{\eta^- \ge y\}dy - \int_{-\infty}^0 Cr\{\eta^- \le y\}dy\\ &= \int_0^{+\infty} Cr\{\eta \le -y\} = \int_{-\infty}^0 Cr\{\eta \le y\}dy.\end{aligned} \tag{17}$$

According to Definition 1.1, and $E(\eta)=E(\xi-E\xi)=0$, so

$$\int_0^{+\infty} Cr\{\eta \ge y\}dy = \int_{-\infty}^0 Cr\{\eta \le y\}dy. \tag{18}$$

We know

$$ASD(\xi)=E(\xi-E\xi)^-=\int_{-\infty}^0 Cr\{\eta \le y\}dy=\int_0^{+\infty} Cr\{\eta \ge y\}dy. \tag{19}$$

Combine Formulas (14) and (19), we will divide it into the following situations to discuss:

Case 1: If $E\xi \le a-\alpha$,

$$ASD(\xi)=\int_{-\infty}^0 Cr\{\eta \le y\}dy=0. \tag{20}$$

Case 2: If $a-\alpha < E\xi \le a$,

$$\begin{aligned}ASD(\xi) &= \int_{-\infty}^0 Cr\{\eta \le y\}dy\\ &= \frac{1}{2}\int_{a-\alpha-E\xi}^0\left[1-\left(\frac{a-y-E(\xi)}{\alpha}\right)^k\right]dy\\ &= \frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}\\ &\quad+\frac{1}{2(k+1)\alpha^k}\left(\frac{a-b}{2}+\frac{k}{k+1}\frac{\alpha-\beta}{2}\right)^{k+1}.\end{aligned} \tag{21}$$

Case 3: If $a < E\xi \le b$,

$$\begin{aligned}ASD(\xi) &= \int_{-\infty}^0 Cr\{\eta \le y\}dy\\ &= \frac{1}{2}\int_{a-\alpha-E\xi}^{a-E\xi}\left[1-\left(\frac{a-y-E(\xi)}{\alpha}\right)^k\right]dy+\int_{a-E\xi}^0\frac{1}{2}dy\\ &= \frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}.\end{aligned} \tag{22}$$

Case 4: If $b < E\xi \le b+\beta$,

$$\begin{aligned}ASD(\xi) &= \int_0^{+\infty} Cr\{\eta \ge y\}dy\\ &= \frac{1}{2}\int_0^{b+\beta-E\xi}\left[1-\left(\frac{y+E(\xi)-b}{\beta}\right)^k\right]dy\\ &= \frac{b-a}{4}+\frac{k}{k+1}\frac{\beta+\alpha}{4}\\ &\quad+\frac{1}{2(k+1)\beta^k}\left(\frac{a-b}{2}+\frac{k}{k+1}\frac{\beta-\alpha}{2}\right)^{k+1}.\end{aligned} \tag{23}$$

Case 5: If $E\xi > b+\beta$,

$$ASD(\xi)=\int_0^{+\infty} Cr\{\eta \ge y\}dy=0. \tag{24}$$

The return of asset i is $r_i=(a_i,b_i,\alpha_i,\beta_i)_k$. The investment proportion of asset i is $x_i, i=1,...,n$, and the unit transaction cost vector is $(c_1,\cdots,c_n)^T$. The return of the portfolio is expressed as

$$RE\left(\sum_{i=1}^n r_i x_i\right)=E\left(\sum_{i=1}^n r_i x_i\right)-\sum_{i=1}^n c_i x_i. \tag{25}$$

According to Definition1.2, we have

$$\sum_{j=1}^n x_i r_i=\left(\sum_{i=1}^n a_i x_i, \sum_{i=1}^n b_i x_i, \sum_{i=1}^n \alpha_i x_i, \sum_{i=1}^n \beta_i x_i\right)_k. \tag{26}$$

According to Definition 2.1, the return of the portfolio is

$$RE\left(\sum_{i=1}^n r_i x_i\right)=\sum_{i=1}^n\left(\frac{b_i+a_i}{2}+\frac{k}{k+1}\frac{\beta_i-\alpha_i}{2}\right)x_i-\sum_{i=1}^n c_i x_i. \tag{27}$$

According to Definition 2.2, the semi-absolute deviation of portfolio credibility is

$$ASD\left(\sum_{i=1}^{n} r_i x_i\right)=\begin{cases} p+0.5(k+1)^{-1}\left(\sum_{i=1}^{n}\alpha_i x_i\right)^{-k} q_1^{k+1}, & \sum_{i=1}^{n}(a_i-\alpha_i)x_i \le E\left(\sum_{i=1}^{n} r_i x_i\right)\le \sum_{i=1}^{n} a_i x_i, \\ p, & \sum_{i=1}^{n} a_i x_i \le E\left(\sum_{i=1}^{n} r_i x_i\right)\le \sum_{i=1}^{n} b_i x_i, \\ p+0.5(k+1)^{-1}\left(\sum_{i=1}^{n}\beta_i x_i\right)^{-k} q_2^{k+1}, & \sum_{i=1}^{n} b_i x_i \le E\left(\sum_{i=1}^{n} r_i x_i\right)\le \sum_{i=1}^{n}(b_i+\beta_i)x_i, \\ 0, & \text{otherwise.}\end{cases} \quad (28)$$

Among them,

$$p=\sum_{i=1}^{n}\left(\frac{b_i-a_i}{4}+\frac{k}{k+1}\frac{\beta_i+\alpha_i}{4}\right)x_i,$$
$$q_1=\sum_{i=1}^{n}\left(\frac{a_i-b_i}{2}+\frac{k}{k+1}\frac{\alpha_i-\beta_i}{2}\right)x_i, \quad (29)$$
$$q_2=\sum_{i=1}^{n}\left(\frac{a_i-b_i}{2}+\frac{k}{k+1}\frac{\beta_i-\alpha_i}{2}\right)x_i.$$

In this paper, the mean of portfolio considering investors' risk attitude is used to represent the return objective function of portfolio, and the semi- absolute deviation is used to represent the risk objective function of portfolio. Therefore, when minimizing the risk objective function, the credibility mean semi-absolute deviation model considering investors' risk attitude is constructed as follows.

$$\begin{cases} \min \ ASD\left(\sum_{i=1}^{n} r_i x_i\right) \\ \text{s.t.} \quad RE\left(\sum_{i=1}^{n} r_i x_i\right)=\sum_{i=1}^{n}\left(\frac{b_i+a_i}{2}+\frac{k}{k+1}\frac{\beta_i-\alpha_i}{2}\right)x_i-\sum_{i=1}^{n} c_i x_i \ge \mu_0, \\ \qquad \sum_{i=1}^{n} x_i = 1, \\ \qquad 0\le l_i \le x_i \le u_i, \quad i=1,2,\cdots,n. \end{cases} \quad (30)$$

when maximizing the return objective function, the credibility mean semi-absolute deviation model considering investors' risk attitude is constructed as follows.

$$\begin{cases} \max \ RE\left(\sum_{i=1}^{n} r_i x_i\right)=\sum_{i=1}^{n}\left(\frac{b_i+a_i}{2}+\frac{k}{k+1}\frac{\beta_i-\alpha_i}{2}\right)x_i-\sum_{i=1}^{n} c_i x_i \\ \text{s.t.} \quad ASD\left(\sum_{i=1}^{n} r_i x_i\right)\ge \sigma_0, \\ \qquad \sum_{i=1}^{n} x_i = 1, \\ \qquad 0\le l_i \le x_i \le u_i, \quad i=1,2,\cdots,n. \end{cases} \quad (31)$$

In models (28) and (29), k is a parameter of risk attitude, when $0<k<1$, it means risk aversion; when $k=1$, it means risk neutrality; when $k>1$, it means risk preference. μ_0, σ_0 are the given return and risk, and l_i, u_i are the lower and upper limits.

B. Credibility Mean Semi-Absolute Deviation DEA Model Considering Investor Psychology

Randomly generate N portfolios, the investment proportion vector of the j portfolio is $x^j=(x_1^{\,j},...,x_n^{\,j})^T$, $j=1,...,N$, and satisfies $\sum_{i=1}^{n} x_i^{\,j}=1, 0\le l_i \le x_i^{\,j}\le u_i$, $i=1,...,n.$ For DMU_j, the output index is $RE\left(\sum_{i=1}^{n} r_i x_i^j\right)$ and the input index is $ASD\left(\sum_{i=1}^{n} r_i x_i^j\right)$. In order to evaluate the efficiency of DMU_{j_0}, the following presents two different oriented efficiency evaluation models considering investor psychology. The mean semi-absolute deviation BCC evaluation model considering investor psychology under risk orientation is as follows.

$$\begin{cases} \min \ \theta_{j_0} \\ \text{s.t} \ \sum_{j=1}^{N}\lambda_j ASD\left(\sum_{i=1}^{n} r_i x_i^j\right)\le \theta_0 ASD\left(\sum_{i=1}^{n} r_i x_i^{j_0}\right), \\ \qquad \sum_{j=1}^{N}\lambda_j RE\left(\sum_{i=1}^{n} r_i x_i^j\right)\ge RE\left(\sum_{i=1}^{n} r_i x_i^{j_0}\right), \\ \qquad \sum_{j=1}^{N}\lambda_j = 1, \quad \lambda_j\ge 0, \quad j=1,2,\cdots,N. \end{cases} \quad (32)$$

Among them, the optimal solution of model (32) is $0\le\theta_{j_0}^*\le 1$, and the efficiency of DMU_{j_0} under risk orientation is $\text{DE}_\sigma=\theta_{j_0}^*$.

The mean semi-absolute deviation BCC evaluation model considering investors' psychology is as follows.

$$\begin{cases} \max \ \varphi_{j_0} \\ \text{s.t} \ \sum_{j=1}^{N}\lambda_j ASD\left(\sum_{i=1}^{n} r_i x_i^j\right)\le ASD\left(\sum_{i=1}^{n} r_i x_i^{j_0}\right), \\ \qquad \sum_{j=1}^{N}\lambda_j RE\left(\sum_{i=1}^{n} r_i x_i^j\right)\ge \varphi_{j_0} RE\left(\sum_{i=1}^{n} r_i x_i^{j_0}\right), \\ \qquad \sum_{j=1}^{N}\lambda_j = 1, \quad \lambda_j\ge 0, \quad j=1,2,\cdots,N. \end{cases} \quad (33)$$

The optimal solution of (33) is $\varphi_{j_0}^*\ge 1$, and the efficiency of DMU_{j_0} under revenue guidance is $\text{DE}_r=1/\varphi_{j_0}^*$.

IV. Empirical Research

We use an example to verify the validity of the credibility mean semi-absolute deviation BCC efficiency evaluation model considering investors' psychology. We select five kinds of risk assets in Shanghai stock market and follow the trapezoidal fuzzy return data of literature [19], as shown in Table I.

TABLE I
FUZZY RETURNS OF FIVE KINDS OF RISK ASSETS

Number	a_i	b_i	α_i	β_i
1	0.0220	0.0260	0.0200	0.0300
2	0.0530	0.0600	0.0500	0.0540
3	0.0780	0.0850	0.0720	0.0940
4	0.1200	0.1640	0.0080	0.1760
5	0.0060	0.0860	0.0720	0.0900

The lower and upper limits are 0, 0.6; the unit transaction cost vector is $c=(0.003,0.003,0.003,0.003,0.003)^T$; the risk attitude parameters k are 0.5, 1, 2, respectively. $k=0.5$ represents the attitude of risk aversion; $k=1$ represents the risk neutrality; $k=2$ represents the attitude of risk preference. The specific steps are as follows.

Step 1: Firstly, four groups of investment proportion samples with different sample sizes $N=20$, 100, 500, 2000 are randomly generated to meet the investment proportion constraints of the model. Then calculate the credibility mean and semi- absolute deviation of these samples under three different risk attitudes.

Step 2: Take the above credibility mean and semi-absolute deviation as the output and input data of BCC model, and use BCC evaluation models (32) and (33) to calculate the DEA efficiency (denoted as DE_σ and DE_r) and ranking of portfolio samples under the guidance of risk and return.

Step 3: By solving the optimal solutions of the portfolio optimization models (30) and (31), and combining with the portfolio efficiency Formula (8), the return-oriented efficiency PE_r and risk-oriented efficiency PE_σ of the above portfolio samples based on the real effective frontier method are calculated.

Step 4: Quantitative correlation analysis is used to further illustrate the validity of the credibility mean semi-absolute deviation BCC evaluation model considering investors' psychology.

Table II shows the samples ($N = 20$) under three different risk attitudes. It can be found that for each portfolio sample (DMU), the credibility mean-semi absolute deviation under risk aversion attitude are the smallest, while under risk preference attitude are the largest. This shows that for risk averse people, they have the characteristics of high return and high risk, while for risk averse people, the opposite is true. According to Table II, we have drawn Fig. 2. In the case of different k values, they show a monotonic increasing trend basically.

Based on three different risk attitudes, Tables III (A) - (C) show the efficiency and ranking of 20 DMUs. It can be found that the DEA efficiency of each DMU in Table III is higher than that based on the real frontier method. This is because the real effective frontier of the portfolio is always at the top left of the DEA frontier. When the return level is the same, the risk value of the real effective frontier is smaller than that of the DEA effective frontier. When the risk level is the same, the return value of the real effective frontier is larger than that of the DEA effective frontier.

In order to fully characterize the correlation between BCC frontier efficiency (DE) and portfolio real frontier efficiency (PE), Pearson correlation coefficient and Spearman rank correlation coefficient [22] are introduced for quantitative correlation analysis. Table IV shows the correlation analysis results based on the credibility mean semi-absolute deviation framework, considering the efficiency of different risk attitudes and sample sizes. The results are plotted as shown in Fig. 3-6. It can be found that in the case of three different risk attitudes, the correlation coefficient of the two efficiency is more than 0.94 while the sample size reaches 2000. This further shows that the mean semi-absolute deviation BCC evaluation method considering investor psychology is effective.

TABLE II
CREDIBILITY MEAN AND SEMI-ABSOLUTE DEVIATION WITH DIFFERENT RISK ATTITUDES

DMU	$k = 0.5$		$k = 1.0$		$k = 2.0$	
	Mean	Semi-Absolute Deviation	Mean	Semi-Absolute Deviation	Mean	Semi-Absolute Deviation
1	0.0472	0.0181	0.0482	0.0230	0.0492	0.0280
2	0.0630	0.0169	0.0663	0.0218	0.0696	0.0266
3	0.0680	0.0157	0.0705	0.0212	0.0731	0.0266
4	0.0791	0.0166	0.0837	0.0218	0.0882	0.0270
5	0.0665	0.0203	0.0699	0.0258	0.0733	0.0313
6	0.0787	0.0188	0.0829	0.0245	0.0872	0.0302
7	0.0480	0.0186	0.0494	0.0237	0.0507	0.0288
8	0.0858	0.0201	0.0903	0.0263	0.0947	0.0325
9	0.0552	0.0140	0.0566	0.0189	0.0580	0.0237
10	0.0662	0.0175	0.0684	0.0232	0.0706	0.0289

TABLE II (CONTINUE)
CREDIBILITY MEAN AND SEMI-ABSOLUTE DEVIATION WITH DIFFERENT RISK ATTITUDES

DMU	$k = 0.5$		$k = 1.0$		$k = 2.0$	
	Mean	Semi-Absolute Deviation	Mean	Semi-Absolute Deviation	Mean	Semi-Absolute Deviation
11	0.0624	0.0122	0.0652	0.0168	0.0680	0.0214
12	0.0861	0.0209	0.0907	0.0271	0.0953	0.0334
13	0.0687	0.0151	0.0717	0.0202	0.0746	0.0253
14	0.0930	0.0211	0.0985	0.0273	0.1040	0.0335
15	0.0746	0.0177	0.0788	0.0229	0.0831	0.0282
16	0.0745	0.0174	0.0777	0.0230	0.0809	0.0286
17	0.0813	0.0157	0.0867	0.0205	0.0921	0.0253
18	0.0628	0.0166	0.0650	0.0219	0.0673	0.0272
19	0.0581	0.0125	0.0603	0.0172	0.0624	0.0218
20	0.0828	0.0159	0.0874	0.0213	0.0921	0.0267

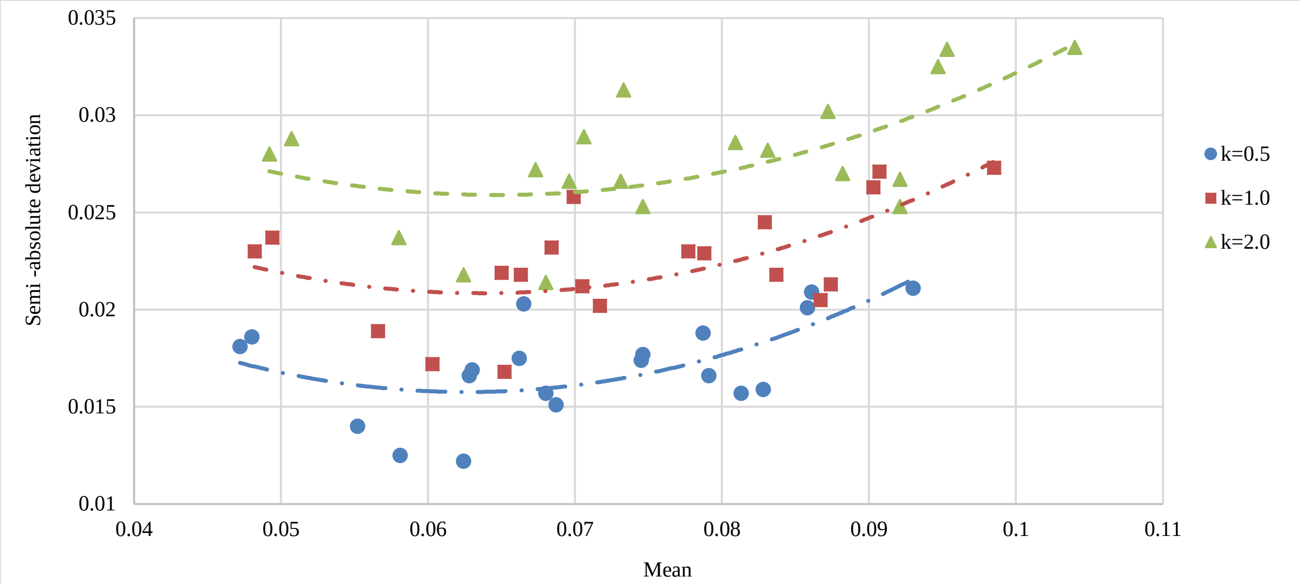

Fig. 2. Credibility mean and semi-absolute deviation under different risk attitudes

TABLE III (A)
EFFICIENCY AND RANKING OF 20 PORTFOLIO SAMPLES UNDER RISK AVERSION ATTITUDE

DMU	$k=0.5$							
	DE_{σ}	Ranking	PE_{σ}	Ranking	DE_{r}	Ranking	PE_{r}	Ranking
1	0.6773	18	0.4949	19	0.5414	20	0.4273	19
2	0.7305	17	0.6616	17	0.7433	17	0.6145	17
3	0.8401	11	0.7547	7	0.8276	13	0.7179	8
4	0.9145	6	0.8097	5	0.9388	6	0.7857	5
5	0.6381	20	0.5745	18	0.7269	18	0.5375	18
6	0.8059	14	0.7138	13	0.8893	9	0.6864	12
7	0.6581	19	0.4874	20	0.5448	19	0.4229	20
8	0.8661	9	0.7180	12	0.9420	5	0.6999	10
9	0.8745	8	0.7206	11	0.7643	15	0.6674	14
10	0.7357	16	0.6628	16	0.7688	14	0.6189	16
11	1.0000	1	0.9067	1	1.0000	1	0.8855	1
12	0.8421	10	0.6945	14	0.9302	7	0.6800	13
13	0.8867	7	0.7957	6	0.8774	10	0.7629	6
14	1.0000	1	0.7351	9	1.0000	1	0.7275	7
15	0.8146	13	0.7251	10	0.8637	12	0.6916	11
16	0.8274	12	0.7366	8	0.8683	11	0.7037	9
17	0.9960	4	0.8798	3	0.9957	4	0.8640	3
18	0.7394	15	0.6700	15	0.7448	16	0.6230	15
19	0.9773	5	0.8377	4	0.9079	8	0.8018	4
20	1.0000	3	0.8824	2	1.0000	1	0.8672	1

TABLE III (B)
EFFICIENCY AND RANKING OF 20 PORTFOLIO SAMPLES WITH RISK NEUTRAL ATTITUDE

DMU	$k=1.0$							
	DE_{σ}	Ranking	PE_{σ}	Ranking	DE_{r}	Ranking	PE_{r}	Ranking
1	0.7314	18	0.5268	19	0.5292	20	0.4149	19
2	0.7824	15	0.6840	15	0.7459	15	0.6094	15
3	0.8383	11	0.7337	7	0.8026	13	0.6698	12
4	0.9149	6	0.8030	5	0.9397	5	0.7659	4
5	0.6830	20	0.5977	18	0.7287	18	0.5447	18
6	0.8113	14	0.7120	11	0.8861	9	0.6757	11
7	0.7095	19	0.5186	20	0.5349	19	0.4127	20
8	0.8577	9	0.7078	13	0.9330	6	0.6929	8
9	0.8919	7	0.7105	12	0.7339	16	0.6167	14
10	0.7475	17	0.6539	17	0.7474	14	0.5817	17

TABLE III (B) (CONTINUE)
EFFICIENCY AND RANKING OF 20 PORTFOLIO SAMPLES WITH RISK NEUTRAL ATTITUDE

DMU	$k = 1.0$							
	DE_σ	Ranking	PE_σ	Ranking	DE_r	Ranking	PE_r	Ranking
11	1.0000	1	0.8740	2	1.0000	1	0.8241	2
12	0.8406	10	0.6887	14	0.9237	7	0.6796	10
13	0.8876	8	0.7770	6	0.8429	12	0.7190	7
14	1.0000	1	0.7327	8	1.0000	1	0.7343	6
15	0.8348	12	0.7320	9	0.8665	10	0.6809	9
16	0.8246	13	0.7228	10	0.8533	11	0.6694	13
17	1.0000	1	0.8784	1	1.0000	1	0.8554	1
18	0.7684	16	0.6705	16	0.7293	17	0.5930	16
19	0.9805	5	0.8132	4	0.8974	8	0.7413	5
20	0.9815	4	0.8508	3	0.9923	4	0.8239	3

TABLE III (C)
EFFICIENCY AND RANKING OF 20 PORTFOLIO SAMPLES UNDER RISK PREFERENCE ATTITUDE

DMU	$k = 2.0$							
	DE_σ	Ranking	PE_σ	Ranking	DE_r	Ranking	PE_r	Ranking
1	0.7664	17	0.5467	19	0.5132	20	0.4018	19
2	0.8149	14	0.6976	14	0.7403	14	0.6027	14
3	0.8372	10	0.7197	9	0.7779	13	0.6340	13
4	0.9134	6	0.7979	5	0.9324	5	0.7496	4
5	0.7117	20	0.6119	18	0.7271	15	0.5439	18
6	0.8133	15	0.7098	11	0.8795	9	0.6649	11
7	0.7427	19	0.5382	20	0.5219	19	0.3995	20
8	0.8353	11	0.6987	13	0.9243	6	0.6846	8
9	0.9021	7	0.7032	12	0.7046	18	0.5782	15
10	0.7550	18	0.6470	17	0.7247	16	0.5543	17
11	1.0000	1	0.8542	2	1.0000	1	0.7782	3
12	0.8243	12	0.6826	15	0.9180	7	0.6752	9
13	0.8878	8	0.7645	6	0.8097	12	0.6860	7
14	1.0000	1	0.7291	8	1.0000	1	0.7348	5
15	0.8462	9	0.7354	7	0.8626	10	0.6708	10
16	0.8222	13	0.7130	10	0.8352	11	0.6425	12
17	1.0000	1	0.8768	1	1.0000	1	0.8478	1
18	0.7887	16	0.6696	16	0.7096	17	0.5683	16
19	0.9823	4	0.7980	4	0.8871	8	0.6956	6
20	0.9468	5	0.8301	3	0.9780	4	0.7931	2

TABLE IV
EFFICIENCY CORRELATION ANALYSIS CONSIDERING DIFFERENT RISK ATTITUDES IN CREDIBILITY MEAN SEMI-ABSOLUTE DEVIATION FRAMEWORK

Correlation Coefficient	Sample Size N	Risk Orientation			Return Orientation		
		$k = 0.5$	$k = 1.0$	$k = 2.0$	$k = 0.5$	$k = 1.0$	$k = 2.0$
r_p	20	0.8977	0.8872	0.9068	0.8737	0.9308	0.9398
	100	0.9842	0.9817	0.9743	0.9952	0.9936	0.9800
	500	0.9946	0.9943	0.9926	0.9894	0.9963	0.9975
	2000	**0.9969**	**0.9962**	**0.9959**	**0.9969**	**0.9990**	**0.9968**
r_s	20	0.9101	0.8908	0.8649	0.9259	0.9561	0.9657
	100	0.9860	0.9819	0.9745	0.9959	0.9924	0.9794
	500	0.9944	0.9941	0.9926	0.9907	0.9966	0.9975
	2000	**0.9962**	**0.9955**	**0.9949**	**0.9967**	**0.9982**	**0.9959**

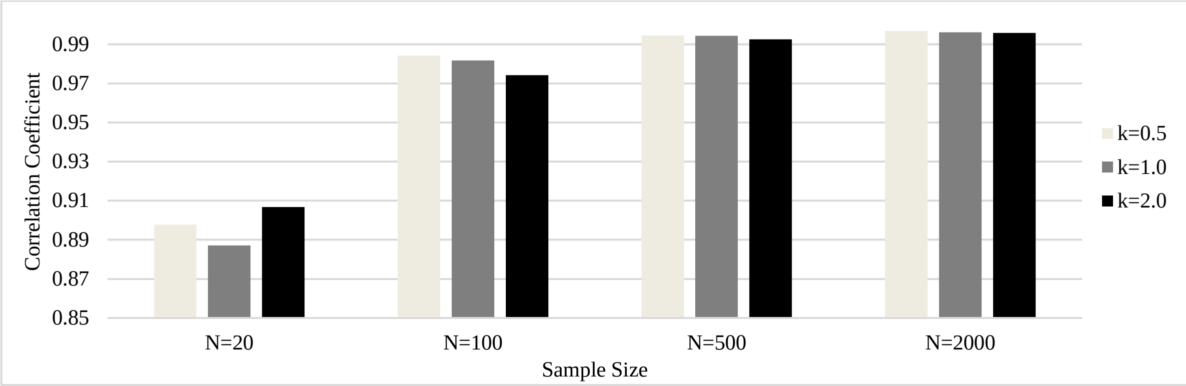

Fig. 3. Analysis of the correlation coefficient r_p in the case of risk orientation

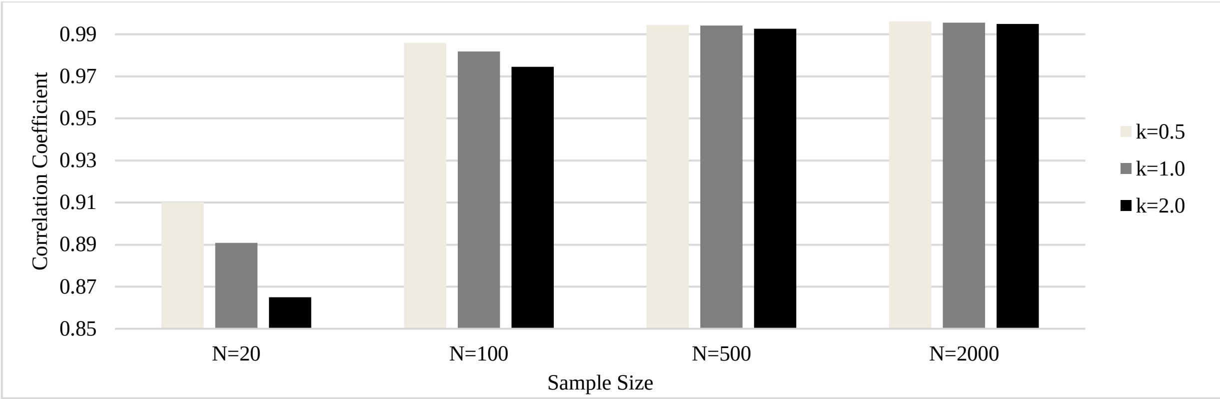

Fig. 4. Analysis of the correlation coefficient r_s in the case of return orientation

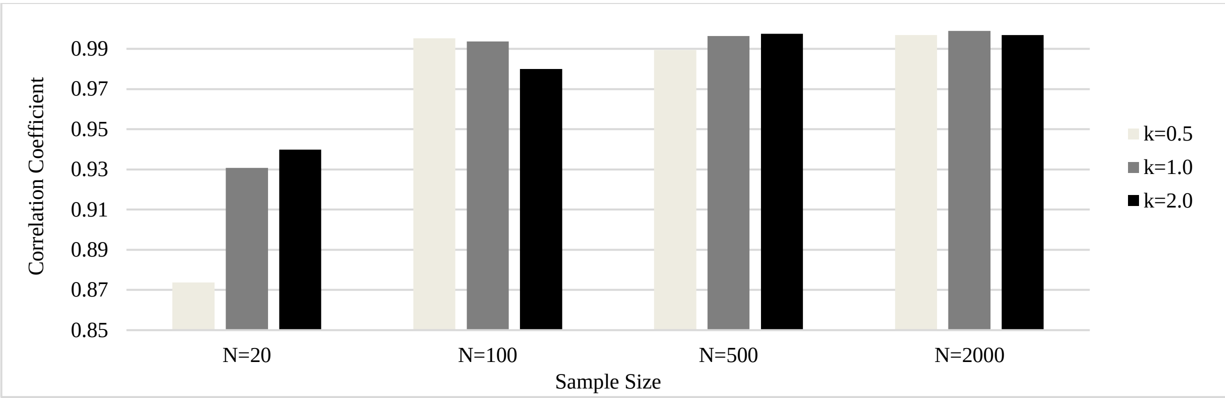

Fig. 5. Analysis of the correlation coefficient r_s in the case of risk orientation

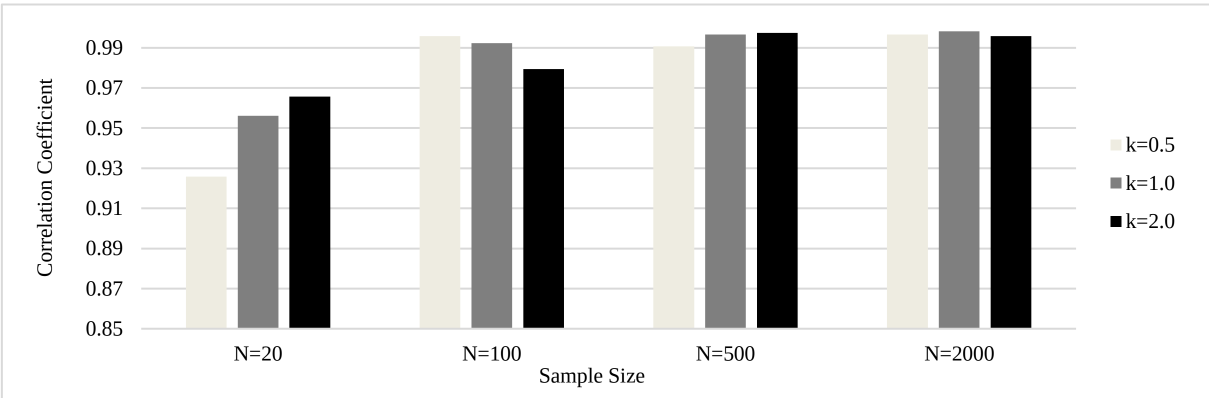

Fig. 6. Analysis of the correlation coefficient r_s in the case of return orientation

V. Conclusion

This paper focuses on the study of portfolio performance evaluation considering investors' psychology under the credibility environment, which extends the work of fuzzy portfolio performance evaluation. Firstly, based on the credibility theory, the credibility mean and semi-absolute deviation considering investors' risk attitude are derived. Secondly, combined with the DEA method, this paper applies the semi-absolute deviation of credibility risk measure to the efficiency evaluation of credibility portfolio for the first time, and constructs the portfolio optimization model and BCC efficiency evaluation model of credibility mean semi-absolute deviation considering investor psychology. Finally, the correlation analysis show that no matter what kind of risk attitude investors take, the credibility mean semi-absolute deviation efficiency evaluation model based on DEA method considering investor psychology can effectively evaluate it.

References

[1] P. Liu, L. F. Wang, "The use of principal component analysis for cross-efficiency aggregation," Engineering Letters, vol. 27, no.3, pp541-548, 2019.

[2] M. Leyer, S. Hüttel, "Performance Analysis with DEA, Process Mining and Business Process Simulation on a Livestock Process," Lecture Notes in Engineering and Computer Science, vol. 2228, pp938-943, 2017.

[3] M.A. Karadayi, E.A. Karsak, "Imprecise DEA framework for evaluating the efficiency of state hospitals in Istanbul," Lecture Notes in Engineering and Computer Science, vol. 2, pp1018-1023, 2014.

[4] B.P.S. Murthi, Y.K. Choi, P. Desai, "Efficiency of mutual funds and portfolio performance measurement: A non-parametric approach," European Journal of Operational Research, vol. 98, pp 408-418, 1997.

[5] M.R. Morey, R.C. Morey, "Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking," Omega, vol. 27, pp 241–58, 1999.

[6] T. Joro, P. Na, "Portfolio performance evaluation in mean-variance-skewness framework," European Journal of Operational Research, vol. 175, pp 446-461, 2006.

[7] W.B. Liu, Z.B. Zhou, D.B. Liu et al, "Estimation of portfolio efficiency via DEA," Omega, vol. 102, pp107-118, 2015.

[8] S. Banihashemi, S. Navidi, "Portfolio performance evaluation in Mean-CVaR framework: A comparison with non-parametric methods value at risk in Mean-VaR analysis," Operations Research Perspectives, vol. 4, pp21-28, 2017.

[9] Z.B. Zhou, H. Xiao, Q. Jin et al, "DEA frontier improvement and portfolio rebalancing: An application of China mutual funds on considering sustainability information disclosure," European Journal of Operational Research, vol. 269, pp111–131, 2018.

[10] L.A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, pp338-353, 1965.

[11] L.A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," Fuzzy Sets and Systems, vol. 1, pp3-28,1978.

[12] E. Vercher, J.D. Bermúdez, J.V. Segura, "Fuzzy portfolio optimization under downside risk measures," Fuzzy Sets and Systems, vol. 158, pp 769-782, 2007.

[13] W. Yue, Y.P. Wang, "A new fuzzy multi-objective higher order moment portfolio selection model for diversified portfolios," Physica A, vol. 465, pp124–140, 2017.

[14] B. Liu, Y.K. Liu, "Expected value of fuzzy variable and fuzzy expected value models," IEEE Transactions on Fuzzy Systems, vol. 10, pp445-450, 2002.

[15] X.X. Huang, "Mean-semivariance models for fuzzy portfolio selection," Journal of Computational and Applied Mathematics, vol. 217, pp1-8, 2008.

[16] E. Vercher, J.D. Bermudez, "Portfolio optimization using a credibility mean-absolute semi-deviation model," Expert Systems with Applications, vol. 42, pp7121-7131, 2015.

[17] M.K. Mehlawat, "Credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels," Information Sciences, vol. 345, pp9-26, 2016.

[18] W. Chen, Y. Gai, P. Gupta, "Efficiency evaluation of fuzzy portfolio in different risk measures via DEA," Annals of Operations Research, vol. 269, pp103–127, 2018.

[19] P. Gupta, M.K. Mehlawat, A. Kumar, et al., "Credibilistic fuzzy DEA approach for portfolio efficiency evaluation and rebalancing toward benchmark portfolios using positive and negative returns," International Journal of Fuzzy Systems, https://doi.org/10.1007/s40815-020-00801-4, 2020.

[20] R.C. Tsaur, "Fuzzy portfolio model with different investor risk attitudes," European Journal of Operational Research, vol. 227, pp385-390, 2013.

[21] X.Y. Zhou, J. Wang, et al, "Portfolio selection under different attitudes in fuzzy environment," Information Sciences, vol. 462, pp278-289, 2018.

[22] L. Jin, Y. Li, "Discrimination of several correlation coefficients and their realization in R language," Statistics & information forum, vol. 34, pp3-11 (in Chinese), 2019.