Finite-time Control of Complex Networked Systems with Structural Uncertainty and Network Induced Delay

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Abstract—**By considering the uncertainties, a new mathematical model of the complex networked systems has been obtained in this paper. With the appearance of the network induced delay, the model has been transformed to a model with control input delay. A new Laypunov function has been constructed. Then, a finite-time stable sufficient condition has been obtained by using linear matrix inequalities. Finally, three simulation examples are given to verify the effectiveness of the proposed method.**

Index Terms—**Finite-time control, induced delay, state feedback, networked systems**

I. Introduction

As we know, the networked control systems have more advantages [1-3] . But the existence of network often induces time delay. And the time delay usually reduces the performance of the control systems, or makes the control systems instable. Therefore, more and more experts and scholars have studied the networked systems [4-6] . Dacic and Nesic designed the observer of the networked systems by using linear matrix inequalities. The state-feedback controller has been obtained in [7]. Gao and Chen gave a new delay systems approach to obtain a sufficient condition with less conservatism [8] . Xie et al. studied the design approach of the state feedback guaranteed cost controller for uncertain networked systems. The sufficient condition for the existence of the networked guaranteed quadratic cost controller was obtained by using the matrix inequality approach. And then, the controller design method was derived [9] . In [10], the packet loss and network induced delay in sensor to controller channel and network induced delay in controller to actuator channel were fully considered. A new model of the networked control systems was established. Based on this new model, a new controller design method was proposed.

The above research results are mainly concerned with the asymptotic stability of the networked control systems. In network engineering practice, the transient performance will be the focus of research. Therefore, it is particularly important to study the finite-time stability of the networked systems [11−15] . Chen et al. studied the finite-time control problem of the uncertain switched systems with state and control input delays. The sufficient condition of finite-time boundedness for the networked switched systems has been obtained by using linear matrix inequality. The average dwell time was used to design the state feedback controller of the systems [16] . In [17], the finite-time control problem of a class of networked control systems with short time varying delay and sampling jitter have been studied. The closed-loop systems ware described as a discrete-time linear systems model, and a robust control method was proposed to solve the finite-time stable problem. Elahi et al. has studied the finite-time control problem of the uncertain discrete-time networked control systems with random communication delay. By using the method of cone complementary linearization, an iterative algorithm was proposed to calculate the parameters of the controller [18] . In [19], the finite-time stable conditions and the design strategy of state-feedback controller for networked systems were obtained by using Lyapunov stability theory and the linear matrix inequality method. Elahi et al. proposed a new control algorithm for the discrete-time networked systems, and designed the finite-time H_∞ controller to overcome the adverse effects of the limited channel, delays and packet loss [20] .

However, most of the above studies aim at the traditional systems. The research result on finite-time control of the networked systems is extremely rare, which motivates this paper. In this paper, we will see the network induced delay as input delay to model a new mathematical model. The effects of the uncertainties and the network induced delay on the systems performance are considered. With the aid of the linear matrix inequalities, a sufficient condition can be obtained to make the networked systems finite-time stable.

II. Modelling and Preliminaries

Considering the following complex networked systems with uncertainty and external disturbance

$$\dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t) + B_1\omega(t) \tag{1}$$

the constant matrices $A \in R^{n\times n}, B_1 \in R^{n\times l}, B \in R^{n\times m}$ are systems matrices. The systems states are $x(t) \in R^n$, the systems input are $u(t) \in R^m$. The external disturbance $\omega(t) \in R^l$ satisfies

$$\omega^T(t)\omega(t) \le d$$

where $d \ge 0$ is a constant. And the structural uncertainty

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$\Delta A(t) \in R^{n\times n}$ satisfying

$$\Delta A(t) = DF(t)E \tag{2}$$

where $F(t)$ is an unknown matrix satisfying

$$F^T(t)F(t) \le I$$

and D, E are constant matrices.

Due to the introduction of the network, the network induced delay will inevitably appear in the control systems. We use τ represent the network induced delay. Moreover, in the networked systems, the induced delay will affect the control effect. Therefore, the induced delay should be considered, we can obtained the mathematical model of the networked systems such as

$$\dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t-\tau) + B_1\omega(t) \tag{3}$$

Remark1. In the systems (3), the influence of network induced delay is considered, especially the influence of the delay in the controller. At the same time, the influence of external disturbance on the system performance is also considered.

For the above networked system, we will design the following state- feedback controller to make the systems finite-time stable

$$u(t) = Kx(t) \tag{4}$$

where K is a constant matrix to be determined. The closed-loop systems can be obtained as follows

$$\dot{x}(t) = \overline{A}x(t) + BKx(t-\tau) + B_1\omega(t) \tag{5}$$

where

$$\overline{A} = A + \Delta A(t)$$

Lemma1 [3]. For a given n-order symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where S_{11} is r-order matrix, then the following three conditions are equivalent

(1) $S < 0$

(2) $S_{11} < 0,\ \ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$

(3) $S_{22} < 0,\ \ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.$

Lemma2 [14]. For the given constant matrix Y, D and E with appropriate dimension, where Y is symmetric matrix, then $Y + DFE + E^T F^T D^T < 0$ for matrix F satisfying $F^T F \le I$, if and only if there is a constant $\varepsilon > 0$, such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

III. Results

In this section, we aim to explore a sufficient condition for the finite-time stability of the systems (5). And then, the controller gain matrix K will be determined. Furthermore, the matrix inequalities are equivalent to the linear matrix inequalities by using the matrix similarity transformation.

3.1 The nonlinear sufficient condition of the finite-time stability

Theorem1. The time delay networked systems (5) can be finite-time stabilized with respect to (c_1, c_2, T, R, d) with $c_1 < c_2$, if there exist positive definite matrices $P, T \in R^{n\times n}$, $S \in R^{l\times l}$, R, scalar $\alpha \ge 0$ and matrix $K \in R^{m\times n}$ satisfying the matrix inequality

$$\begin{bmatrix} \Xi & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} < 0 \tag{6a}$$

and

$$\frac{c_1(\lambda_{\max}(\tilde{P}) + d\lambda_{\max}(S)(1-e^{-\alpha T}) + \tau\lambda_{\max}(\tilde{T}))}{\lambda_{\min}(\tilde{P})} < c_2 e^{-\alpha T} \tag{6b}$$

where

$$\Xi = P\overline{A} + \overline{A}^T P + T - \alpha P$$

$$\tilde{T} = R^{-1/2} T R^{-1/2}$$

$$\tilde{P} = R^{-1/2} P R^{-1/2}$$

Proof. Using the positive definite matrix P, T to construct the following Lyapunov function

$$V(t) = \int_{t-\tau}^{t} x^T(\theta)Tx(\theta)d\theta + x^T(t)Px(t) \tag{7}$$

With the solution of the equation (5), it is easy to obtain

$$\begin{aligned}
\dot{V}(t) &= x^T(t)Tx(t) - x^T(t-\tau)Tx(t-\tau) + 2x^T(t)P\dot{x}(t) \\
&= x^T(t)Tx(t) - x^T(t-\tau)Tx(t-\tau) \\
&\quad + 2x^T(t)P[\overline{A}x(t) + BKx(t-\tau) + B_1\omega(t)] \\
&= x^T(t)(T + P\overline{A}^T + \overline{A}^T P)x(t) - x^T(t-\tau)Tx(t-\tau) \\
&\quad + 2x^T(t)PBKx(t-\tau) + 2x^T(t)PB_1\omega(t) \\
&= \xi^T(t)\Theta\xi(t)
\end{aligned}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}$$

$$\Theta = \begin{bmatrix} P\overline{A} + \overline{A}^T P + T & PBK & PB_1 \\ * & -T & 0 \\ * & * & 0 \end{bmatrix}$$

Inserting the condition (6a) into the above inequality, we obtain

$$\dot{V}(t) < \alpha V(t) + \alpha\omega^T(t)S\omega(t) \tag{8}$$

Multiplying the above inequality with $e^{-\alpha t}$, we have

$$\frac{d}{dt}(e^{-\alpha t}V(t)) < \alpha e^{-\alpha t}\omega^T(t)S\omega(t)$$

Integrating from 0 to t on the both sides of the above inequality, it follows that

$$e^{-\alpha t}V(t) - V(0) < \int_0^t \alpha e^{-\alpha\theta}\omega^T(\theta)S\omega(\theta)d\theta \tag{9}$$

Giving some transforms such as $\tilde{P} = R^{-1/2}PR^{-1/2}$, $\tilde{T} = R^{-1/2}TR^{-1/2}$, it is obviously

$$\begin{aligned}
&x^T(t)Px(t) \le V(x(t)) \\
&\quad < e^{\alpha T}[c_1(\lambda_{\max}(\tilde{P}) + d\lambda_{\max}(S)(1-e^{-\alpha t}) + \tau\lambda_{\max}(\tilde{T}))]
\end{aligned} \tag{10}$$

therefore

$$\begin{aligned}
\lambda_{\min}(\tilde{P})x^T(t)Rx(t) &\le x^T(t)Px(t) \\
&= x^T(t)R^{1/2}\tilde{P}R^{1/2}x(t)
\end{aligned} \tag{11}$$

From (10) and (11), the following inequality can be obtained

$$\begin{aligned}
&x^T(t)Rx(t) \\
&< \frac{e^{\alpha T}}{\lambda_{\min}(\tilde{P})}[c_1(\lambda_{\max}(\tilde{P}) + d\lambda_{\max}(S)(1-e^{-\alpha T}) + \tau\lambda_{\max}(\tilde{T}))]
\end{aligned} \tag{12}$$

Combining (6b) and (12), it implies,

$$x^T(t)Rx(t) \le c_2, \forall\, t \in [0,T]$$

The networked systems (5) can be finite-time stabilized.

Remark2. In theorem1, the Lyapunov function (7) with integral term is designed to effectively compensate the network induced delay. The delay-independence sufficient conditions (6a) and (6b) are conservative, but they are easy to be solved.

Remark3. The matrix inequality (6a) in theorem1 is nonlinear to the variables P and K, and the condition (6b) is an inequality about the eigenvalue of the matrix, which cannot be directly solved. Next, with some appropriate matrix transformations, we will transform the condition (6a) and (6b) into linear matrix inequalities.

3.2 The linearization of the finite time stable condition

Theorem2. The controller $u(t) = \bar{K}X^{-1}x(t)$ can make the networked systems (5) finite-time stable with respect to (c_1, c_2, T, R, d) ($c_1 < c_2$), if there exist positive definite matrices $X \in R^{n\times n}$, $\bar{T} \in R^{n\times n}$, $S \in R^{l\times l}$, matrix $\bar{K} \in R^{m\times n}$, and scalars $\alpha \ge 0$, $\varepsilon>0, \lambda_1, \lambda_2, \lambda_3 > 0$ satisfying the linear matrix inequalities

$$\begin{bmatrix} AX + XA^T + \bar{T} - \alpha X + \varepsilon DD^T & B\bar{K} & B_1 & XE^T \\ * & -\bar{T} & 0 & 0 \\ * & * & -\alpha S & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{13a}$$

$$\lambda_1 R^{-1} < X < R^{-1} \tag{13b}$$

$$\lambda_2 \bar{T} < \lambda_1 X \tag{13c}$$

$$0 < S < \lambda_3 I \tag{13d}$$

$$\begin{bmatrix} d\lambda_3(1-e^{-\alpha T}) - c_2 e^{-\alpha T} & \sqrt{c_1} & \sqrt{\tau} \\ * & -\lambda_1 & 0 \\ * & * & -\lambda_2 \end{bmatrix} < 0 \tag{13e}$$

Proof. Inserting (2) into (6a), we obtain

$$\begin{bmatrix} P(A+\Delta A(t)) + (A+\Delta A(t))^T P + T - \alpha P & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} < 0$$

i.e.

$$\Pi = \begin{bmatrix} \Pi_{11} & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} < 0$$

where

$$\Pi_{11} = P(A+DF(t)E) + (A+DF(t)E)^T P + T - \alpha P$$

The above inequality can be rewritten as

$$\begin{bmatrix} PA + A^T P + T - \alpha P & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} + \begin{bmatrix} PDF(t)E + (DF(t)E)^T P & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0$$

i.e.

$$\begin{bmatrix} PA + A^T P + T - \alpha P & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} + \begin{bmatrix} PD \\ 0 \\ 0 \end{bmatrix} F(t)\begin{bmatrix} E & 0 & 0 \end{bmatrix} + \begin{bmatrix} E & 0 & 0 \end{bmatrix}^T F^T(t) \begin{bmatrix} PD \\ 0 \\ 0 \end{bmatrix}^T < 0$$

With lemma2, there exists a constant $\varepsilon > 0$, such that the above inequality is equivalent to

$$\begin{bmatrix} PA + A^T P + T - \alpha P & PBK & PB_1 \\ * & -T & 0 \\ * & * & -\alpha S \end{bmatrix} + \varepsilon \begin{bmatrix} PD \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PD \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon^{-1}\begin{bmatrix} E & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E & 0 & 0 \end{bmatrix} < 0$$

By using the lemma1, it is obvious that the inequality (6a) is equivalent to

$$\begin{bmatrix} PA + A^T P + T - \alpha P + \varepsilon PDD^T P & PBK & PB_1 & E^T \\ * & -T & 0 & 0 \\ * & * & -\alpha S & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0$$

The matrix $diag\{P^{-1}, P^{-1}, I, I\}$ is multiplied at both sides of the above inequality, we have

$$\Sigma = \begin{bmatrix} \Sigma_{11} & BKP^{-1} & B_1 & P^{-1}E^T \\ * & -P^{-1}TP^{-1} & 0 & 0 \\ * & * & -\alpha S & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0$$

where

$$\Sigma_{11} = AP^{-1} + P^{-1}A^T + P^{-1}TP^{-1} - \alpha P^{-1} + \varepsilon DD^T$$

By making some substitutions such as

$$\bar{K} = KP^{-1}, \bar{T} = P^{-1}TP^{-1}, X = P^{-1}$$

the above inequality is equivalent to (13a).

In addition, the replacement is as follows $\tilde{X} = R^{-1/2}XR^{-1/2}, \tilde{P} = R^{-1/2}PR^{-1/2}$ and $\tilde{T} = R^{-1/2}TR^{-1/2}$. From the inequalities (13b-13d), we can obtain

$$\lambda_{\max}(\tilde{P}) < \frac{1}{\lambda_1}, \lambda_{\min}(\tilde{P}) > 1$$

and

$$\lambda_{\max}(\tilde{T}) < \frac{\lambda_1}{\lambda_2}\lambda_{\max}(\tilde{P}), \lambda_{\max}(S) < \lambda_3 \tag{14}$$

Reusing the lemma1, the inequality (13e) and the following inequality are equivalent.

$$d\lambda_3(1-e^{-\alpha T}) - c_2 e^{-\alpha T} + \frac{c_1}{\lambda_1} + \frac{\tau}{\lambda_2} < 0 \tag{15}$$

With (15), the condition (6b) follows that

$$\begin{aligned} &\frac{1}{\lambda_{\min}(\tilde{P})}[c_1(\lambda_{\max}(\tilde{P}) + d\lambda_{\max}(S)(1-e^{-\alpha T}) + \tau\lambda_{\max}(\tilde{T}))] \\ &< \frac{\tau}{\lambda_2} + \frac{c_1}{\lambda_1} + d\lambda_3(1-e^{-\alpha T}) \end{aligned} \tag{16}$$

Inserting (15) into (16), the inequality (6b) is established. The proof is completed.

Remark4. The nonlinear conditions in theorem1 have been transformed successfully into linear conditions in theorem2 by some appropriate matrix transformations. Although the number of the inequalities increases, but the obtained conditions (13a-13e) are linear for the variables $X,\bar{T},S,\bar{K},\alpha,\varepsilon,\lambda_1,\lambda_2,\lambda_3$, which can be easily solved by using the control tool box in MATLAB.

3.3 The design method of finite-time controller in the networked systems with state delay

Considering the following complex networked systems with state delay and external disturbance

$$\dot{x}(t)=Ax(t)+A_h x(t-h)+Bu(t)+B_1\omega(t) \tag{17}$$

where A_h is a constant matrix, h is the state time delay.

With the influence of network induced delay, the networked systems (17) can be changed as

$$\dot{x}(t)=Ax(t)+A_h x(t-h)+Bu(t-\tau)+B_1\omega(t) \tag{18}$$

With the controller (4), the closed-loop systems can be obtained

$$\dot{x}(t)=Ax(t)+A_h x(t-h)+BKx(t-\tau)+B_1\omega(t) \tag{19}$$

Theorem3. The time delay networked systems (17) can be finite-time stabilized with respect to (c_1,c_2,T,R,d) with $c_1<c_2$, if there exist positive definite matrices $P,Q\in R^{n\times n}$, $T\in R^{n\times n}$, $S\in R^{l\times l}$, and matrix $K\in R^{m\times n}$ satisfying the matrix inequalities

$$\begin{bmatrix} \Xi & PA_h & PBK & PB_1 \\ * & -Q & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -\alpha S \end{bmatrix}<0 \tag{20a}$$

$$\frac{c_1(\lambda_{\max}(\tilde{P})+h\lambda_{\max}(\tilde{Q})+\tau\lambda_{\max}(\tilde{T}))+d\lambda_{\max}(S)(1-e^{-\alpha T})}{\lambda_{\min}(\tilde{P})}<c_2e^{-\alpha T} \tag{20b}$$

where

$$\Xi=PA+A^TP+Q+T-\alpha P$$

Proof. Using the positive definite matrix P,Q,T to construct the Lyapunov function

$$V(x(t))=x^T(t)Px(t)+\int_{t-h}^{t}x^T(\theta)Qx(\theta)d\theta+\int_{t-\tau}^{t}x^T(\theta)Tx(\theta)d\theta \tag{21}$$

With the solution of the systems (19), it is easy to obtain

$$\begin{aligned}\dot{V}(x(t))&=x^T(t)(PA+A^TP)x(t)+2x^T(t)PA_hx(t-h)\\&\quad+2x^T(t)PBKx(t-\tau)+2x^T(t)PB_1\omega(t)\\&\quad+x^T(t)Qx(t)+x^T(t-h)Qx(t-h)\\&\quad+x^T(t)Tx(t)+x^T(t-\tau)Tx(t-\tau)\\&=x^T(t)(PA+A^TP+Q+T)x(t)\\&\quad+2x^T(t)PA_hx(t-h)+2x^T(t)PBKx(t-\tau)\\&\quad+2x^T(t)PB_1\omega(t)+x^T(t-h)Qx(t-h)\\&\quad+x^T(t-\tau)Tx(t-\tau)\\&=\begin{bmatrix}x(t)\\x(t-h)\\x(t-\tau)\\\omega(t)\end{bmatrix}^T\bar{\Pi}\begin{bmatrix}x(t)\\x(t-h)\\x(t-\tau)\\\omega(t)\end{bmatrix}\end{aligned}$$

where

$$\bar{\Pi}=\begin{bmatrix} PA+A^TP+Q+T & PA_h & PBK & PB_1 \\ * & -Q & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & 0 \end{bmatrix}$$

Inserting the condition (20a) into the above inequality, we obtain

$$\begin{aligned}\dot{V}(x(t))&<\alpha x^T(t)Px(t)+\alpha\omega^T(t)S\omega(t)\\&<\alpha V(x(t))+\alpha\omega^T(t)S\omega(t)\end{aligned} \tag{22}$$

Multiplying the upper formula with $e^{-\alpha t}$, we have

$$e^{-\alpha t}\dot{V}(x(t))-e^{-\alpha t}\alpha V(x(t))<\alpha e^{-\alpha t}\omega^T(t)S\omega(t)$$

therefore

$$\frac{d}{dt}(e^{-\alpha t}V(x(t)))<\alpha e^{-\alpha t}\omega^T(t)S\omega(t)$$

Integrating from 0 to t on both sides of the above inequality, it follows that

$$e^{-\alpha t}V(x(t))-V(x(0))<\int_0^t\alpha e^{-\alpha\theta}\omega^T(\theta)S\omega(\theta)d\theta \tag{23}$$

Giving some transforms such as $\tilde{P}=R^{-1/2}PR^{-1/2}$, $\tilde{T}=R^{-1/2}TR^{-1/2}$, it is obviously

$$\begin{aligned}&x^T(t)Px(t)\le V(x(t))\\&<e^{\alpha t}V(x(0))+\alpha d\lambda_{\max}(S)e^{\alpha t}\int_0^t e^{-\alpha\theta}d\theta\\&<e^{\alpha t}[x^T(0)Px(0)+\int_{-h}^0x^T(\theta)Qx(\theta)d\theta\\&\quad+\int_{-\tau}^0x^T(\theta)Tx(\theta)d\theta+d\lambda_{\max}(S)(1-e^{-\alpha t})]\\&<e^{\alpha t}[x^T(0)R^{1/2}\tilde{P}R^{1/2}x(0)\\&\quad+\int_{-h}^0x^T(\theta)R^{1/2}\tilde{Q}R^{1/2}x(\theta)d\theta\\&\quad+\int_{-\tau}^0x^T(\theta)R^{1/2}\tilde{T}R^{1/2}x(\theta)d\theta+d\lambda_{\max}(S)(1-e^{-\alpha t})]\\&<e^{\alpha t}[\lambda_{\max}(\tilde{P})x^T(0)Rx(0)\\&\quad+\lambda_{\max}(\tilde{Q})\int_{-h}^0x^T(\theta)Rx(\theta)d\theta\\&\quad+\lambda_{\max}(\tilde{T})\int_{-\tau}^0x^T(\theta)Rx(\theta)d\theta+d\lambda_{\max}(S)(1-e^{-\alpha t})]\\&<e^{\alpha T}[c_1(\lambda_{\max}(\tilde{P})+h\lambda_{\max}(\tilde{Q})\\&\quad+\tau\lambda_{\max}(\tilde{T}))+d\lambda_{\max}(S)(1-e^{-\alpha t})]\end{aligned} \tag{24}$$

Obviously

$$x^T(t)Px(t)=x^T(t)R^{1/2}\tilde{P}R^{1/2}x(t)\ge\lambda_{\min}(\tilde{P})x^T(t)Rx(t) \tag{25}$$

therefore

$$\begin{aligned}&x^T(t)Rx(t)\\&<\frac{e^{\alpha T}[c_1(\lambda_{\max}(\tilde{P})+h\lambda_{\max}(\tilde{Q})+\tau\lambda_{\max}(\tilde{T}))+d\lambda_{\max}(S)(1-e^{-\alpha T})]}{\lambda_{\min}(\tilde{P})}\end{aligned} \tag{26}$$

From (20b) and (26), the following inequality can be obtained

$$x^T(t)Rx(t)\le c_2,\ \forall\, t\in[0,T].$$

The time delay networked systems (17) can be finite-time stabilized.

By using the proof method in theorem 2, it is easy to get the following conclusion.

Theorem4. The controller $u(t)=\bar{K}X^{-1}x(t)$ can make the delay networked systems (17) finite-time stable with respect to (c_1,c_2,T,R,d) ($c_1<c_2$), if there exist positive definite

matrices $X, \bar{Q} \in R^{n\times n}$, $\bar{T} \in R^{n\times n}$, $S \in R^{l\times l}$, matrix $\bar{K} \in R^{m\times n}$, and scalars $\alpha \ge 0$, $\lambda_1, \lambda_2, \lambda_3 > 0$ satisfying the linear matrix inequalities

$$\begin{bmatrix} \bar{\Theta} & A_h X & B\bar{K} & B_1 \\ * & -\bar{Q} & 0 & 0 \\ * & * & -\bar{T} & 0 \\ * & * & * & -\alpha S \end{bmatrix} < 0 \quad (27a)$$

$$\lambda_1 R^{-1} < X < R^{-1} \quad (27b)$$

$$\lambda_2 \bar{Q} < \lambda_1 X \quad (27c)$$

$$\lambda_3 \bar{T} < \lambda_1 X \quad (27d)$$

$$0 < S < \lambda_4 I \quad (27e)$$

$$\begin{bmatrix} d\lambda_4(1-e^{-\alpha T}) - c_2 e^{-\alpha T} & \sqrt{c_1} & \sqrt{h} & \sqrt{\tau} \\ * & -\lambda_1 & 0 & 0 \\ * & * & -\lambda_2 & 0 \\ * & * & * & -\lambda_3 \end{bmatrix} < 0 \quad (27f)$$

where

$$\bar{\Theta} = AX + XA^T + \bar{Q} + \bar{T} - \alpha X$$

(**The proof is omitted.**)

IV. Simulation

Example 1

In order to illustrate the convenience of calculation of the proposed method, the networked systems in the form of (3) will be considered, where

$$A = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 \\ 0.7 \end{bmatrix}, F(t) = 0.5\cos t, E = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}$$

$\tau = 1$.

By solving the linear matrix inequality (13), we obtain

$$X = \begin{bmatrix} 1.4650 & 0.6548 \\ 0.6548 & 0.4670 \end{bmatrix}, T = \begin{bmatrix} 0.6937 & 4.4542 \\ 4.4542 & 3.6730 \end{bmatrix},$$

$$S = \begin{bmatrix} 2.6735 & 0.3435 \\ 0.3435 & 1.6358 \end{bmatrix}, \bar{K} = \begin{bmatrix} -1.5408 & 2.4068 \end{bmatrix},$$

$\alpha = 0.4359, \varepsilon = 1.3176, \lambda_1 = 0.4531, \lambda_2 = 1.3551,$

$\lambda_3 = 0.1392.$

The state-feedback controller can be obtained

$$u(t) = \bar{K}X^{-1}x(t) = \begin{bmatrix} -8.9882 & 17.7565 \end{bmatrix}$$

Example 2

In order to verify the effectiveness of the control design method proposed in this paper, the networked systems (3) is considered, where

$$A = \begin{bmatrix} -6 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0 \\ 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ -1 \\ -0.2 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.1 & -0.2 & 0.3 \end{bmatrix}^T, E = \begin{bmatrix} -0.1 & -0.03 & 0.1 \end{bmatrix}$$

$F(t) = \cos t, \tau = 0.5.$

By solving the linear matrix inequality (13), we obtain

$$u(t) = \bar{K}X^{-1}x(t) = \begin{bmatrix} 2.5783 & -3.7851 & 1.5546 \end{bmatrix} x(t)$$

If we select the initial states

$$x(0) = \begin{bmatrix} 3 \\ -1 \\ -6 \end{bmatrix}$$

the response curves of the systems states $x_1(t), x_2(t), x_3(t)$ are obtained in Fig.1-3.

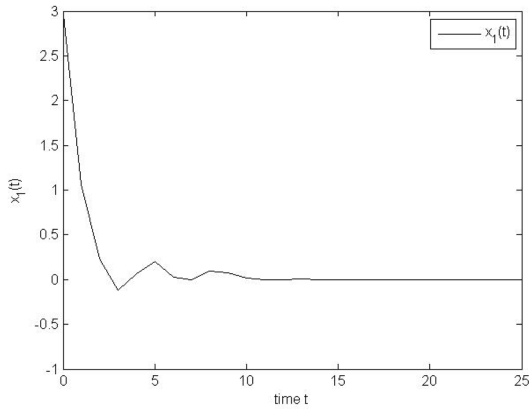

Fig.1 The response curve of the systems state $x_1(t)$

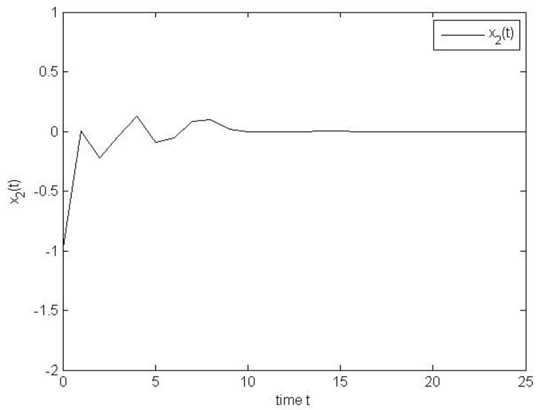

Fig.2 The response curve of the systems state $x_2(t)$

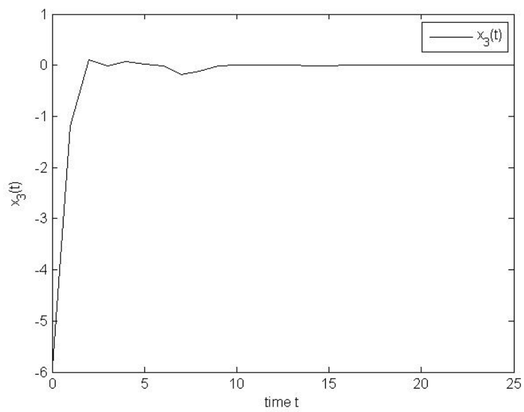

Fig.3 The response curve of the systems state $x_3(t)$

It can be seen from Fig.1-3 that the three systems states can converge to 0 in 10 seconds. Therefore, the control design method proposed in this paper is feasible and effective.

Example 3

To examine our proposed controller and compare with reference [14], for simplicity, we consider the uncertain networked control systems (3), where

$$A=\begin{bmatrix}-5 & 1\\ 2 & -2\end{bmatrix}, B=\begin{bmatrix}0.1\\ 0.5\end{bmatrix}, B_1=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}, D=\begin{bmatrix}0.01\\ 0.2\end{bmatrix},$$

$$F(t)=\sin t, E=\begin{bmatrix}1 & 1\end{bmatrix}, \tau=0.1.$$

By using the algorithm in [14], the state feedback controller can be obtained as

$$u(t)=[1.3445 \quad -2.4457]x(t)$$

On the other, we solve the linear matrix inequality (13), the state feedback controller can be obtained by using the proposed approach in this paper

$$u(t)=Kx(t)=[0.2478 \quad -0.1458]x(t)$$

(1) Comparison of the simulation results of the systems states

By selecting the initial states as

$$x(0)=\begin{bmatrix}-3\\ 2\end{bmatrix}$$

the response curves of the systems states $x_1(t), x_2(t)$ are obtained in Fig.4 and Fig.5.

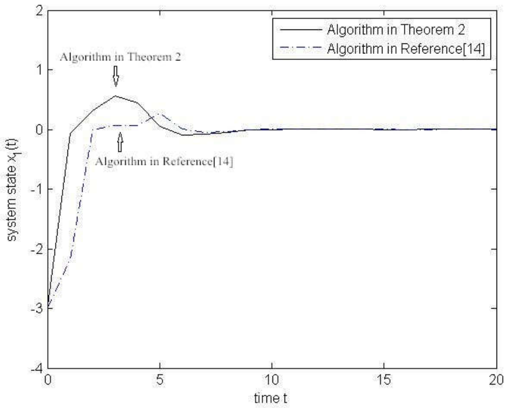

Fig.4 The response curves of the systems state $x_1(t)$

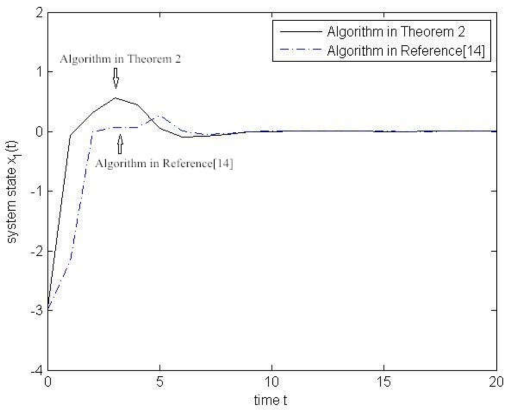

Fig.5 The response curves of the systems state $x_2(t)$

In the Fig.4, the solid line is the response curve of the systems states with algorithm in theorem2. The dot-dashed line presents the response with algorithm in reference [14]. The convergence speed of the solid line is obviously faster than that of the dot-dashed line. The smoothness of the solid line is slightly better than that of the dot-dashed line. But the overshoot of the solid line is slightly larger than that of the dot-dashed line.

In the Fig.5, the solid line is the response curve of the systems states with algorithm in theorem2. The dot-dashed line presents the response with algorithm in reference [14]. The convergence speed and smoothness of solid line are especially good. The convergence speed of the dot-dashed line is slow, and there is a large frequency oscillation. The solid line has no overshoot, while the dot-dashed line has larger overshoot. The control effect of theorem 2 is obviously better than the algorithm in reference [14].

(2) Verification of the systems performance

In order to compare the systems performance, the dispersed IAE function will be used as performance indicators to evaluate the system performance

$$IAE=\int_0^\infty |e(t)|\,dt$$

The curves of IAE function that use algorithm in theorem2 and the algorithm in reference [14] are shown in the Fig.6.

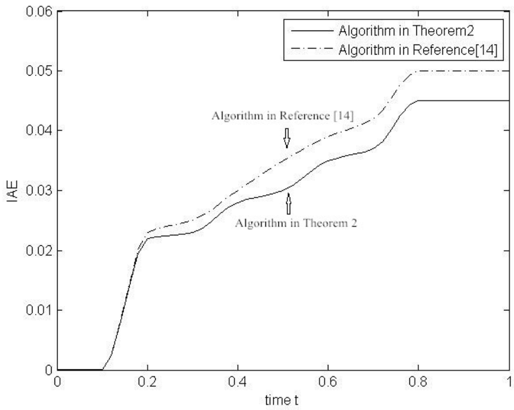

Fig.6 The curves of IAE of the two algorithms

The curve of IAE function obtained by the algorithm in theorem2 is represented by solid line, and the curve of IAE function obtained by the algorithm in reference [14] is represented by dot-dashed line. It can be seen from the figure that over time, the change of algorithm used in theorem 2 is significantly smaller than that used in reference [14]. Therefore, the algorithm in theorem2 can improve the systems performance more effectively than that in reference [14].

V. Conclusion

In order to study the finite-time stabilization problem of the networked systems, we see the network induced delay as the input time delay to model a new mathematical model. With the network induced delay, a new Laypunov function has been explored to reduce conservatism of the sufficient condition. Therefore, the results of this paper are more practical.

On the other hand, it should be pointed out that the results based on linear matrix inequality are still conservative. With the development of research, it will be more interesting to explore some more effective conditions by using some recently developed technologies dealing with delay and uncertainty. In addition, it is worth mentioning that the results of this paper can be extended to other networked control systems, such as distributed delay, fast varying delay, random delay, etc., which will be our next research work. For the nonlinear time-delay network system, the corresponding finite-time control problem can be solved by using T-S fuzzy method.

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