Applications of Fuzzy Parameterized Relative Soft Sets in Decision-Making Problems

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Abstract—**This paper, aims to define fuzzy parameterized relative soft sets along with the presentation of their related properties. Following, the proposal of a decision-making method based on fuzzy parameterized relative soft set is made. In addition to that, a practical example is being demonstrated showing that the method can be successfully applied to the decision-making problems.**

Index Terms—**fuzzy parameterized relative soft sets, decision-making problems.**

I. Introduction

DAILY many fields have to deal with uncertain data, with the main issue being that it may not be possible to model using classical mathematics approach successfully. There have been a plethora of useful methods using well-known mathematical tools when it comes to describing uncertainty, such as the fuzzy set theory [1] , the probability theory, the soft set theory [2], [3], the fuzzy soft set theory [4]. Since then, the applications and properties of the soft set theory have been studied by many authors the applications and properties of the soft set theory have been studied by many authors ([5], [6], [7], [8], [9], [10], [11], [12]). In 2011, Çağman *et al.* [13] were the first to have worked on a detailed theoretical study of the fuzzy parameterized soft set (FP-soft sets). They discussed the model for solving decision-making problems based on the FP-soft sets. The soft set theory has been expanded further by embedding the ideas of the relative soft set (e.g.[14]). In 2013, Balami and Musa defined relative soft sets and their basic properties, which are the generalization of soft sets. Moreover, they also discussed their operations, such as union and intersection. The theory of the fuzzy set is a valuable mathematical tool when it comes to dealing with uncertainty. However, it is also a new notion when it comes to applying it to abstract algebraic structures. In 2017, Julath and Siripitukdet [15] examined some characterizations of fuzzy bi-ideals and fuzzy quasi-ideals of semigroups. Later in 2020, Yairayong [16] discussed the idea of combining the theories of hesitant fuzzy sets on semigroups and establishing a new framework for hesitant fuzzy sets on semigroups.

This paper aims to purpose the concept of fuzzy parameterized relative soft sets as a hybrid model of the FP-soft set and the relative soft set, which is a generalization of the FP-soft set and the relative soft set. The discussion of the score value in the model of the fuzzy parameterized relative soft sets in the decision-making problem is also taking place. The results show that the model of fuzzy parameterized relative soft sets is practical when it comes to solving decision-making problems.

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II. Preliminaries

This section concentrates on presenting the basic definitions and results of the soft set theory that will be used.

Definition 1 [9] Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. Then, a **soft set** F_A over U is a set defined by a function f_A representing a mapping

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A.$$

Here, f_A is called an approximate function of the soft set F_A, and the value $f_A(x)$ is a set called $x-$element of the soft set for all $x \in E$. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Example 1. [9] Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universe set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_2, x_3, x_4\}$ and $f_A(x_2) = \{u_2, u_4\}, f_A(x_3) = \emptyset, f_A(x_4) = \{u_1, u_2, u_3, u_4, u_5\}$, then the soft set F_A is written by

$$F_A = \{(x_2, \{u_2, u_4\}), (x_3, \emptyset), (x_4, \{u_1, u_2, u_3, u_4, u_5\})\}.$$

We can represent this soft set F_A in a tabular form as shown below. This style of representation will be useful for storing a relative soft set in a computer memory. If $h_i \in f_A(x)$ for all $a \in A$ then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table I.

TABLE I
THE TABULAR FORM OF THE SOFT SET F_A

$A\backslash U$	u_1	u_2	u_3	u_4	u_5
x_2	0	1	0	1	0
x_3	0	0	0	0	0
x_4	1	1	1	1	1

Definition 2 [9] Let U be an initial universe. A **fuzzy set** X over U is a set defined by a function μ_X representing a mapping $\mu_X : E \to [0, 1]$. Here, μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the

degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows,

$$X = \{(u/\mu_X(u)) : u \in U, \mu_X(u) \in [0,1]\}.$$

Note that the set of all fuzzy sets over U will be denoted by $F(U)$.

Definition 3 [9] Let U be an initial universe, E be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over U for all $x \in E$. Then, a **fuzzy soft set** (fs-set) Γ_A over U is a set defined by a function γ_A representing a mapping

$$\gamma_A : E \to F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } x \notin A.$$

Here, γ_A is called a fuzzy approximate function of the fs-set Γ_A, and the value $\gamma_A(x)$ is a fuzzy set called $x-$element of the fs-set for all $x \in E$. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U).$$

Note that the sets of all fs-set over U will be denoted by $FS(U)$.

Example 2. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universe set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_2, x_3, x_4\}$ and
$\gamma_A(x_2) = \{u_1/1, u_2/0.9, u_3/1, u_4/0.8, u_5/0.7\}$,
$\gamma_A(x_3) = \{u_1/0.1, u_2/1, u_3/0.4, u_4/1, u_5/0.3\}$,
$\gamma_A(x_4) = \{u_1/0.4, u_2/0.3, u_3/1, u_4/0.8, u_5/1\}$,
then the soft set Γ_A is written by
$\Gamma_A = \{(x_2, \{u_1/1, u_2/0.9, u_3/1, u_4/0.8, u_5/0.7\})$,
$(x_3, \{u_1/0.1, u_2/1, u_3/0.4, u_4/1, u_5/0.3\})$,
$(x_4, \{u_1/0.4, u_2/0.3, u_3/1, u_4/0.8, u_5/1\})\}$.
We can represent this fuzzy soft set Γ_A in a tabular form as Table II.

TABLE II
THE TABULAR FORM OF THE FUZZY SOFT SET Γ_A

$A\backslash U$	u_1	u_2	u_3	u_4	u_5
x_2	1.0	0.9	1.0	0.8	0.7
x_3	0.1	1.0	0.4	1.0	0.3
x_4	0.4	0.3	1.0	0.8	1.0

Next, the definition of fuzzy parameterized soft sets (FP-soft sets) over the universe set can be formulated.

Definition 4. [9] Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0,1]$. Then, a **fuzzy parameterized soft set** (FP-soft set) F_X over U is a set defined by a function f_X representing a mapping

$$f_X : E \to P(U) \text{ such that } f_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, f_X is called an approximate function of the FP-soft set F_X, and the value $f_X(x)$ is a set called $x-$element of the FP-soft set for all $x \in E$. Thus, a FP-soft set F_X over U can be represented by the set of ordered pairs
$F_X = \{(x/\mu_X(x), f_X(x)) : x \in E, f_X(x) \in P(U)$,
$\mu_X(x) \in [0,1]\}$.

Note that the sets of all FP-soft sets over U will be denoted by $FPS(U)$.

Example 3. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $X = \{x_2/0.2, x_3/0.5, x_4/1\}$ and $f_X(x_2) = \{u_2, u_4\}, f_X(x_3) = \{u_1, u_2\}$, $f_X(x_4) = \{u_1, u_2, u_3, u_4, u_5\}$, then the fps-set F_X is written by $F_X = \{(x_2/0.2, \{u_2, u_4\}), (x_3/0.5, \{u_1, u_2\}), (x_4/1, \{u_1, u_2, u_3, u_4, u_5\})\}$. We can represent this FP-soft sets F_X in a tabular form as Table III.

TABLE III
THE TABULAR FORM OF THE FP-SOFT SET F_X

$A\backslash U$	u_1	u_2	u_3	u_4	u_5
$x_2/0.2$	0	1	0	1	0
$x_3/0.5$	1	1	0	0	0
$x_4/1.0$	1	1	1	1	1

It is important to mention the concept of a relative soft set was introduced by Balami and Musa [14].

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{Ui} : i \in I\}$ be a collection of set of parameters $U = P(U_i)$ denotes the power set of $U_i, E = E_{Ui}$ and $A \subseteq E$.

Definition 5. [14] A pair (F, A) is called a **relative soft set** over U, where F is a mapping given by $F : A \to U$. In other words, a relative soft set over U is a parameterized family of subsets of the universe U. As for $e \in A, F(e)$, they may be considered as the set of $e-$approximate elements of the relative soft set (F, A). Based on the definition above, any change in the ordering of the universes will produce a different relative soft set.

Example 4. [14] Presuming that there are three universes U_1, U_2 and U_3. Let us consider the relative soft set (F, A), which describes the condition of some states in a country where Mr.X has enough capital and is considering various locations for his manufacturing industries.

Let $U_1 = \{S_1, S_2, S_3\}$ be a set of states with availability of land, $U_2 = \{S_4, S_5, S_6\}$ be a set of states with availability of labour, and $U_3 = \{S_7, S_8, S_9\}$ be a set of states with availability of raw materials. Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of the set of parameters related to the above universes, where $E_{U_1} = \{e_{U_1}, 1 =$ peaceful state, $e_{U_1}, 2 =$ commercial state, $e_{U_1}, 3 =$ armed robbery state, $e_{U_1}, 4 =$ state with good climate, $e_{U_1}, 5 =$ densely populated state$\}$. $E_{U_2} = \{e_{U_2}, 1 =$ power state, $e_{U_2}, 2 =$ harsh weather state, $e_{U_2}, 3 =$ violent state, $e_{U_2}, 4 =$ densely populated state$\}$. $E_{U_3} = \{e_{U_3}, 1 =$ accessible state, $e_{U_3}, 2 =$ state with good climate, $e_{U_3}, 3 =$ power state, $e_{U_3}, 4 =$ sparsely populated state$\}$.

Let $U = P(U_i), E_{U_i}$ and $A \subseteq E$ such that $i = 1, 2, 3$.
Let $A = \{a_1 = (e_{U_1}, 1, e_{U_2}, 1, e_{U_3}, 1)$,
$a_2 = (e_{U_1}, 2, e_{U_2}, 4, e_{U_3}, 2)$,
$a_3 = (e_{U_1}, 4, e_{U_2}, 3, e_{U_3}, 4)$,
$a_4 = (e_{U_1}, 3, e_{U_2}, 4, e_{U_3}, 4)$,
$a_5 = (e_{U_1}, 5, e_{U_2}, 1, e_{U_3}, 1)$,
$a_6 = (e_{U_1}, 1, e_{U_2}, 4, e_{U_3}, 4)\}$.
Suppose that $F(a_1) = (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})$,
$F(a_2) = (\{S_1, S_2\}, \{S_6\}, \{S_9\})$,
$F(a_3) = (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)$,
$F(a_4) = (\{S_2\}, \{S_6\}, \emptyset)$,
$F(a_5) = (\{S_3\}, \{S_5\}, \{S_8\})$,
$F(a_6) = (\{S_2, S_3\}, \{S_6\}, \emptyset)$.

Then a relative soft set (F, A) can be viewed as consisting of the following approximation:

$(F, A) = \{(a_1, (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})),$
$(a_2, (\{S_1, S_2\}, \{S_6\}, \{S_9\})),$
$(a_3, (\{S_1, S_2, S_3\}, \{S_6\}, \emptyset)),$
$(a_4, (\{S_2\}, \{S_6\}, \emptyset)),$
$(a_5, (\{S_3\}, \{S_5\}, \{S_8\})),$
$(a_6, (\{S_2, S_3\}, \{S_6\}, \emptyset))\}.$

We can represent this relative soft set (F, A) in a tabular form as Table IV.

TABLE IV
THE TABULAR FORM OF THE RELATIVE SOFT SET (F, A)

$A\backslash U_1$	S_1	S_2	S_3
a_1	0	1	1
a_2	1	1	0
a_3	1	1	1
a_4	0	1	0
a_5	0	0	1
a_6	0	1	1
$A\backslash U_2$	S_4	S_5	S_6
a_1	0	1	0
a_2	0	0	1
a_3	0	0	1
a_4	0	0	1
a_5	0	1	0
a_6	0	0	1
$A\backslash U_3$	S_7	S_8	S_9
a_1	1	1	0
a_2	0	0	1
a_3	0	0	0
a_4	0	0	0
a_5	0	1	0
a_6	0	0	0

Each approximation has two parts : a predicate name and an approximate value set.

The example can be logically explained as follows: For $F(a_1) = (\{S_2, S_3\}, \{S_5\}, \{S_7, S_8\})$. If $\{S_2, S_3\}$ is the set of peaceful states for Mr.X then the states he can obtain regular electric power supply from is $\{S_5\}$ and $\{S_2, S_3\}$ is the set of peaceful states for Mr.X and $\{S_5\}$ is the set of state he can obtain regular electric power supply then the set of relatively accessible state to him is $\{S_7, S_8\}$. It is obvious that the relative soft set is a conditional relation.

III. Fuzzy Parameterized Relative Soft Sets

The purpose of this section , is to define the fuzzy parameterized relative soft sets as a hybrid model of the fuzzy parameterized soft sets and the relative soft sets.

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_i : i \in I\}$ be a collection of set of parameters. Let $\{X_i : i \in I\}$ be a collection of set of fuzzy sets over E_{U_i}. $U = P(U_i)$ denotes the power sets over U_i, $E = E_{U_i}$, $X = X_{U_i}$ and $A \subseteq E$.

Definition 6. A **fuzzy parameterized relative soft set** (fpr-soft set) $(F, A)_X$ over U is define by $F_X : A \to U$. Thus, a fpr-soft set $(F, A)_X$ over U can be represented as follows, $(F, A)_X = \{(\varepsilon/(\mu_X(\varepsilon_1), \mu_X(\varepsilon_2), ..., \mu_X(\varepsilon_n)), F_X(\varepsilon)) : \varepsilon \in A, \mu_X(\varepsilon_i : i \in I) \in [0, 1], (\varepsilon_i : i \in I) \in \varepsilon\}$,
where μ_X is the membership function of X. Note that the sets of all fpr-soft set over U will be denoted by $FPR(U)$.

Example 5. Let us deem that there are three universes U_1, U_2 and U_3. Making a consideration that the fpr-soft set $(F, A)_X$ which describes the condition of some states in a country where Mr.X which already has enough capital, is considering the building location for his hotel. Let $U_1 = \{S_1, S_2, S_3\}$ be a set of states with the availability of land, $U_2 = \{S_4, S_5, S_6\}$ would represent a set of states with the availability of labour required for the project, and $U_3 = \{S_7, S_8, S_9\}$ be a set of states with the necessary level of state security. Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of the set of parameter related to the above universes, where $E_{U_1} = \{e_{11}\{$the location near famous tourist attractions$\}$, $e_{12}\{$a suitable climate$\}$, $e_{13}\{$dense population$\}$, $e_{14}\{$harsh climate$\}$, $E_{U_2} = \{e_{21}\{$the population to speak English$\}$, $e_{22}\{$passion for service$\}$, $e_{23}\{$high level of education$\}$, e_{24} $\{$low criminal record$\}$, $E_{U_3} = \{e_{31}\{$availability of security features like alarms$\}$, $e_{32}\{$has CCTV$\}$, $e_{33}\{$availability of security guard$\}$, $e_{34}\{$location relatively near an entertainment venue$\}$. Let $U = P(U_i), E = E_{U_i}, i = 1, 2, 3$ and $A \subseteq E$. Let $X = \{X_{U_1}, X_{U_2}, X_{U_3}\}$ be a collection of the set of fuzzy set over E, where
$X_{U_1} = \{e_{U_{11}}/0.8, e_{U_{12}}/0.5, e_{U_{13}}/0.3, e_{U_{14}}/0.1\}$,
$X_{U_2} = \{e_{U_{21}}/0.6, e_{U_{22}}/0.7, e_{U_{23}}/0.2, e_{U_{24}}/0.1\}$,
$X_{U_3} = \{e_{U_{31}}/0.6, e_{U_{32}}/0.5, e_{U_{33}}/0.7, e_{U_{34}}/0.2\}$.
Let $A = \{a_1 = (e_{U_{11}}/0.8, e_{U_{21}}/0.6, e_{U_{34}}/0.2),$
$a_2 = (e_{U_{12}}/0.5, e_{U_{22}}/0.2, e_{U_{31}}/0.6),$
$a_3 = (e_{U_{13}}/0.3, e_{U_{24}}/0.1, e_{U_{32}}/0.5),$
$a_4 = (e_{U_{14}}/0.1, e_{U_{22}}/0.7, e_{U_{33}}/0.7),$
$a_5 = (e_{U_{11}}/0.8, e_{U_{22}}/0.7, e_{U_{32}}/0.5),$
$a_6 = (e_{U_{12}}/0.8, e_{U_{21}}/0.6, e_{U_{33}}/0.7)\}.$
Now, suppose that
$F_X(a_1) = (\{S_1, S_2, S_3\}, \{S_4, S_6\}, \emptyset),$
$F_X(a_2) = (\{S_2, S_3\}, \{S_4, S_5\}, \{S_8, S_9\}),$
$F_X(a_3) = (\{S_1, S_3\}, \{S_5\}, \{S_7, S_8, S_9\}),$
$F_X(a_4) = (\{S_2\}, \{S_4, S_5, S_6\}, \{S_8, S_9\}),$
$F_X(a_5) = (\{S_1, S_2, S_3\}, \{S_4, S_5, S_6\}, \{S_7, S_8, S_9\}),$
$F_X(a_6) = (\{S_2, S_3\}, \{S_4, S_6\}, \{S_8, S_9\}).$
Then the fpr-soft set $(F, A)_X$ over U is written by
$(F, A)_X = \{(a_1/(0.8, 0.6, 0.2), (\{S_1, S_2, S_3\}, \{S_4, S_6\}, \emptyset)),$
$(a_2/(0.5, 0.2, 0.6), (\{S_2, S_3\}, \{S_4, S_5\}, \{S_8, S_9\})),$
$(a_3/(0.3, 0.1, 0.5), (\{S_1, S_3\}, \{S_5\}, \{S_7, S_8, S_9\})),$
$(a_4/(0.1, 0.7, 0.7), (\{S_2\}, \{S_4, S_5, S_6\}, \{S_8, S_9\})),$
$(a_5/(0.8, 0.7, 0.5), (\{S_1, S_2, S_3\}, \{S_4, S_5, S_6\}, \{S_7, S_8, S_9\})),$
$(a_6/(0.8, 0.7, 0.5), (\{S_2, S_3\}, \{S_4, S_6\}, \{S_8, S_9\}))\}.$

The fpr-soft set $(F, A)_X$ can be written as Table V.

Now, we present operations of the fpr-soft sets.

Definition 7. Let $(F, A)_X, (G, B)_Y \in FPR(U)$. Then, a **union** of $(F, A)_X$ and $(G, B)_Y$, denoted by $(F, A)_X \tilde{\cup} (G, B)_Y$ is the fpr-soft set $(H, C)_Z$ such that $C = A \cup B$ and $Z = X \cup Y$ where

$$\mu_{X\tilde{\cup}Y}(x) = \begin{cases} \mu_X(x) & \text{if } x \in X - Y, \\ \mu_Y(x) & \text{if } x \in Y - X, \\ \max\{\mu_X(x), \mu_Y(x)\} & \text{if } x \in X \cap Y, \end{cases}$$

and for all $\varepsilon \in C$,

$$H_Z(\varepsilon) = \begin{cases} F_X(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G_Y(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F_X(\varepsilon) \cup G_Y(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases}$$

Example 6. According to Example 5, let
$A = \{a_1 = (e_{U_{11}}/0.8, e_{U_{21}}/0.6, e_{U_{34}}/0.2),$

TABLE V
THE TABULAR FORM OF THE fpr-SOFT SET $(F,A)_X$

$A\backslash U_1$	S_1	S_2	S_3
$a_1/0.8$	1	1	1
$a_2/0.5$	0	1	1
$a_3/0.3$	1	0	1
$a_4/0.1$	0	1	0
$a_5/0.8$	1	1	1
$a_6/0.8$	0	1	1
$A\backslash U_2$	S_4	S_5	S_6
$a_1/0.6$	1	0	1
$a_2/0.2$	1	1	0
$a_3/0.1$	0	1	0
$a_4/0.7$	1	1	1
$a_5/0.7$	1	1	1
$a_6/0.7$	1	0	1
$A\backslash U_3$	S_7	S_8	S_9
$a_1/0.2$	0	0	0
$a_2/0.6$	0	1	1
$a_3/0.5$	1	1	1
$a_4/0.7$	0	1	1
$a_5/0.5$	1	1	1
$a_6/0.5$	0	1	1

$a_2 = (e_{U_{12}}/0.5, e_{U_{22}}/0.2, e_{U_{31}}/0.6),$
$a_3 = (e_{U_{13}}/0.3, e_{U_{24}}/0.1, e_{U_{32}}/0.5),$
$a_4 = (e_{U_{14}}/0.1, e_{U_{22}}/0.7, e_{U_{33}}/0.7),$
$a_5 = (e_{U_{11}}/0.8, e_{U_{22}}/0.7, e_{U_{32}}/0.5),$
$a_6 = (e_{U_{12}}/0.8, e_{U_{21}}/0.6, e_{U_{33}}/0.7)\}$, and
$B = \{b_1 = (e_{U_{11}}/0.3, e_{U_{21}}/0.5, e_{U_{34}}/0.7),$
$b_2 = (e_{U_{12}}/0.3, e_{U_{22}}/0.6, e_{U_{31}}/0.3),$
$b_3 = (e_{U_{13}}/0.6, e_{U_{24}}/0.3, e_{U_{32}}/0.2),$
$b_4 = (e_{U_{14}}/0.7, e_{U_{22}}/0.4, e_{U_{33}}/0.9).$

Suppose $(F,A)_X$ and $(G,B)_Y$ are two fpr-soft sets over the same U such that
$(F,A)_X = \{(a_1/(0.8,0.6,0.2), (\{S_1,S_2,S_3\},\{S_4,S_6\},\emptyset)),$
$(a_2/(0.5,0.2,0.6), (\{S_2,S_3\},\{S_4,S_5\},\{S_8,S_9\})),$
$(a_3/(0.3,0.1,0.5), (\{S_1,S_3\},\{S_5\},\{S_7,S_8,S_9\})),$
$(a_4/(0.1,0.7,0.7), (\{S_2\},\{S_4,S_5,S_6\},\{S_8,S_9\})),$
$(a_5/(0.8,0.7,0.5), (\{S_1,S_2,S_3\},\{S_4,S_5,S_6\},\{S_7,S_8,S_9\})),$
$(a_6/(0.8,0.7,0.5), (\{S_2,S_3\},\{S_4,S_6\},\{S_8,S_9\}))\}.$
$(G,B)_Y = \{(b_1/(0.3,0.6,0.7), (\{S_2,S_3\},\{S_4\},\emptyset)),$
$(b_2/(0.3,0.6,0.3), (\{S_1,S_3\},\{S_5\},\{S_8\})),$
$(b_3/(0.6,0.3,0.2), (\{S_3\},\{S_4\},\{S_7,S_8\})),$
$(b_4/(0.7,0.4,0.9), (\{S_1\},\{S_5,S_6\},\{S_9\}))\}.$

Therefore $(F,A)_X \widetilde{\cup} (G,B)_Y = (H,C)_Z$, where
$(H,C)_Z = \{(c_1/(0.8,0.6,0.7), (\{S_1,S_2,S_3\},\{S_4,S_6\},\emptyset)),$
$(c_2/(0.5,0.6,0.6), (\{S_1,S_2,S_3\},\{S_4,S_5\},\{S_8,S_9\})),$
$(c_3/(0.6,0.3,0.5), (\{S_1,S_3\},\{S_4,S_5\},\{S_7,S_8,S_9\})),$
$(c_4/(0.7,0.7,0.9), (\{S_1,S_2\},\{S_4,S_5,S_6\},\{S_8,S_9\})),$
$(c_5/(0.8,0.7,0.5), (\{S_1,S_2,S_3\},\{S_4,S_5,S_6\},\{S_7,S_8,S_9\})),$
$(c_6/(0.8,0.7,0.5), (\{S_2,S_3\},\{S_4,S_6\},\{S_8,S_9\}))\}.$

Definition 8. Let $(F,A)_X, (G,B)_Y \in FPR(U)$. Then, an **intersection** of $(F,A)_X$ and $(G,B)_Y$, denoted by $(F,A)_X \widetilde{\cap} (G,B)_Y$ is the fpr-soft set $(H,C)_Z$ such that $C = A \cup B$ and $Z = X \cap Y$ where

$$\mu_{X\widetilde{\cap}Y}(x) = \begin{cases} \mu_X(x) & \text{if } x \in X - Y, \\ \mu_Y(x) & \text{if } x \in Y - X, \\ \min\{\mu_X(x), \mu_Y(x)\} & \text{if } x \in X \cap Y, \end{cases}$$

and for all $\varepsilon \in C$,

$$H_Z(\varepsilon) = \begin{cases} F_X(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G_Y(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F_X(\varepsilon) \cap G_Y(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases}$$

Example 7. According to Example 6, then
$(F,A)_X \widetilde{\cap} (G,B)_Y = (H,C)_Z$, where
$(H,C)_Z = \{(c_1/(0.3,0.5,0.2), (\{S_2,S_3\},\{S_4\},\emptyset)),$
$(c_2/(0.3,0.2,0.3), (\{S_3\},\{S_5\},\{S_9\})),$
$(c_3/(0.3,0.1,0.2), (\{S_3\},\emptyset,\{S_7,S_8\})),$
$(c_4/(0.1,0.4,0.7), (\emptyset,\{S_5,S_6\},\{S_9\})),$
$(c_5/(0.8,0.7,0.5), (\{S_1,S_2,S_3\},\{S_4,S_5,S_6\},\{S_7,S_8,S_9\})),$
$(c_6/(0.8,0.7,0.5), (\{S_2,S_3\},\{S_4,S_6\},\{S_8,S_9\}))\}.$

Proposition 1. Let $(F,A)_X, (G,B)_Y$ and $(I,D)_W$ be three fpr-soft sets over U. If $(F,A)_X, (G,B)_Y$ and $(I,D)_W$ are conformable for the union and intersection then
i.) $(F,A)_X \widetilde{\cup} (F,A)_X = (F,A)_X$,
ii.) $(F,A)_X \widetilde{\cup} (G,B)_Y = (G,B)_Y \widetilde{\cup} (F,A)_X$,
iii.) $(F,A)_X \widetilde{\cup} ((G,B)_Y \widetilde{\cup} (I,D)_W)$
$= ((F,A)_X \widetilde{\cup} (G,B)_Y) \widetilde{\cup} (I,D)_W$,
iv.) $(F,A)_X \widetilde{\cap} (F,A)_X = (F,A)_X$,
v.) $(F,A)_X \widetilde{\cap} (G,B)_Y = (G,B)_Y \widetilde{\cap} (F,A)_X$,
vi.) $(F,A)_X \widetilde{\cap} ((G,B)_Y \widetilde{\cap} (I,D)_W)$
$= ((F,A)_X \widetilde{\cap} (G,B)_Y) \widetilde{\cap} (I,D)_W$,
vii.) $(F,A)_X \widetilde{\cup} ((G,B)_Y \widetilde{\cap} (I,D)_W)$
$= ((F,A)_X \widetilde{\cup} (G,B)_Y) \widetilde{\cap} ((F,A)_X \widetilde{\cup} (I,D)_W)$,
viii.) $(F,A)_X \widetilde{\cap} ((G,B)_Y \widetilde{\cup} (I,D)_W)$
$= ((F,A)_X \widetilde{\cap} (G,B)_Y) \widetilde{\cup} ((F,A)_X \widetilde{\cap} (I,D)_W)$.

Definition 9. Let $(F,A)_X \in FPR(U)$. The **complement** of a fpr-soft set $(F,A)_X$ is denoted by $(F,A)^{\tilde{c}}_X$ such that

$\mu_{X^{\tilde{c}}}(\varepsilon) = 1 - \mu_X(\varepsilon)$ and $F_{X^{\tilde{c}}}(\varepsilon) = U \setminus F_X(\varepsilon)$ for all $\varepsilon \in A$.

Example 8. According to Example 6, then
$(F,A)^{\tilde{c}}_X = \{(a_1/(0.2,0.4,0.8), (\emptyset,\{S_5\},\{S_7,S_8,S_9\})),$
$(a_2/(0.5,0.8,0.4), (\{S_1\},\{S_6\},\{S_7\})),$
$(a_3/(0.7,0.9,0.5), (\{S_2\},\{S_4,S_6\},\emptyset)),$
$(a_4/(0.9,0.3,0.3), (\{S_1,S_3\},\emptyset,\{S_7\})),$
$(a_5(0.2,0.3,0.5), (\emptyset,\emptyset,\emptyset)),$
$(a_6/(0.2,0.4,0.3), (\{S_1\},\{S_5\},\{S_7\}))\}.$

Definition 10. Let $(F,A)_X, (G,B)_Y \in FPR(U)$. Define $(F,A)_X \subseteq (G,B)_Y$ is given by $A \subseteq B$ and $\mu_X(\varepsilon) \le \mu_Y(\varepsilon)$ and $F_X(\varepsilon) \subseteq G_Y(\varepsilon)$ for all $\varepsilon \in A$. Then $(F,A)_X$ is said to be **fuzzy parameterized relative soft subset** of $(G,B)_Y$.

Definition 11. Let $(F,A)_X, (G,B)_Y \in FPR(U)$. Then $(F,A)_X, (G,B)_Y$ are **fuzzy parameterized relative soft equal** written as $(F,A)_X = (G,B)_Y$ if $A = B$ and $\mu_X(\varepsilon) = \mu_Y(\varepsilon)$ and $F_X(\varepsilon) = G_Y(\varepsilon)$ for all $\varepsilon \in A$.

Proposition 2. Let $(F,A)_X, (G,B)_Y$ and $(I,D)_W$ be three fpr-soft sets over U. Then
i.) $(F,A)_X \subseteq (F,A)_X$,
ii.) If $(F,A)_X \subseteq (G,B)_Y$ and $(G,B)_Y \subseteq (I,D)_W$, then $(F,A)_X \subseteq (I,D)_W$,
iii.) If $(F,A)_X = (G,B)_Y$ and $(G,B)_Y = (I,D)_W$, then $(F,A)_X = (I,D)_W$,
iv.) If $(F,A)_X \subseteq (G,B)_Y$ and $(G,B)_Y \subseteq (F,A)_X$, then $(F,A)_X = (G,B)_Y$,
v.) $((F,A)^{\tilde{c}}_X)^{\tilde{c}} = (F,A)_X$.

IV. APPLICATIONS

This section is used to define a score value of $(F,A)_X$ and construct a model for solving decision-making problems based on $fpr-$soft sets.

Definition 11. Let $(F,A)_X \in FPR(U)$. Then a **score value** of $(F,A)_X$, denoted by $(F,A)^d_X$, is defined by

$$(F,A)^d_X = \{u/\mu_{F^d_X}(u) : u \in U\},$$

which is a fuzzy set over U, its membership function $\mu_{F^d_X}$ is define by

$$\mu_{F^d_X} : U \to [0,1], \mu_{F^d_X}(u) = \frac{1}{|A|}\sum_{x\in|A|}\mu_X(x)\chi_{F_X(x)}(u),$$

where $|A|$ is the number of element of A, $F_X(x)$ is the subset determined by the parameter x and

$$\chi_{F_X(x)}(u) = \begin{cases} 1, & u \in F_X(x). \\ 0, & u \notin F_X(x). \end{cases}$$

Therefore, we can make a decision by the following algorithm.

Algorithm

Step 1. Construction of the $fpr-$soft sets $(F,A)_X$ and $(G,B)_Y$ over U.

Step 2. Computation of the $fpr-$soft set $(H,C)_Z$ by using a convenient operation of the $fpr-$soft sets $(F,A)_X$ and $(G,B)_Y$.

Step 3. Computation of the score value $(H,C)^d_Z$.

Step 4. Selection of the largest membership grade $\mu_{H^d_Z}(u)$.

The following example is provided for the usage of the new algorithm for the $fpr-$soft set.

Example 9. Assume that a family X, Mr. X and Mrs.X, which enough capital is considering for the location of their hotel. Suppose that there are three universes U_1, U_2 and U_3. Let us consider the fpr-soft set $(F,A)_X$ and $(G,B)_Y$ which describes the condition of some states in a country which Mr.X and Mrs X going to choose, respectively. Let $U_1 = \{S_1,S_2,S_3\}$ be a set of states with the availability of land, $U_2 = \{S_4,S_5,S_6\}$ be a set of states with the availability of labour, $U_3 = \{S_7,S_8,S_9\}$ be a set of states with the availability of safety, Let $E_{U_i} = \{E_{U_1},E_{U_2},E_{U_3}\}$ be a collection of the set of parameter related to the above universes, where $E_{U_1} = \{e_{11}\{$the location near famous tourist attractions$\}$, $e_{12}\{$a suitable climate$\}$, $e_{13}\{$dense population$\}$, $e_{14}\{$harsh climate$\}$, $E_{U_2} = \{e_{21}\{$the population to speak English$\}$, $e_{22}\{$passion for service$\}$, $e_{23}\{$high level of education$\}$, e_{24} $\{$low criminal record$\}$, $E_{U_3} = \{e_{31}\{$availability of security features like alarms$\}$, $e_{32}\{$has CCTV$\}$, $e_{33}\{$availability of security guard$\}$, $e_{34}\{$location relatively near an entertainment venue$\}$. Let $U = P(U_i), E = E_{U_i}, i = 1,2,3$ and $A \subseteq E$. Let $Y = X = \{X_{U_1}, X_{U_2}, X_{U_3}\}$ be a collection of the set of fuzzy set over E, where

$X_{U_1} = \{e_{U_{11}}/0.8, e_{U_{12}}/0.5, e_{U_{13}}/0.3, e_{U_{14}}/0.1\}$,
$X_{U_2} = \{e_{U_{21}}/0.6, e_{U_{22}}/0.7, e_{U_{23}}/0.2, e_{U_{24}}/0.1\}$,
$X_{U_3} = \{e_{U_{31}}/0.6, e_{U_{32}}/0.5, e_{U_{33}}/0.7, e_{U_{34}}/0.2\}$.

Step 1: Construction of the $fpr-$soft sets $(F,A)_X$ and $(G,B)_Y$ over U. Let

$A = \{a_1 = (e_{U_{11}}/0.8, e_{U_{21}}/0.6, e_{U_{34}}/0.2)$,
$a_2 = (e_{U_{12}}/0.5, e_{U_{22}}/0.2, e_{U_{31}}/0.6)$,
$a_3 = (e_{U_{13}}/0.3, e_{U_{24}}/0.1, e_{U_{32}}/0.5)$,
$a_4 = (e_{U_{14}}/0.1, e_{U_{22}}/0.7, e_{U_{33}}/0.7)$,
$a_5 = (e_{U_{11}}/0.8, e_{U_{22}}/0.7, e_{U_{32}}/0.5)$,
$a_6 = (e_{U_{12}}/0.8, e_{U_{21}}/0.6, e_{U_{33}}/0.7)\}$ and
$B = \{b_1 = (e_{U_{11}}/0.3, e_{U_{21}}/0.5, e_{U_{34}}/0.7)$,
$b_2 = (e_{U_{12}}/0.3, e_{U_{22}}/0.6, e_{U_{31}}/0.3)$,
$b_3 = (e_{U_{13}}/0.6, e_{U_{24}}/0.3, e_{U_{32}}/0.2)$,
$b_4 = (e_{U_{14}}/0.7, e_{U_{22}}/0.4, e_{U_{33}}/0.9)$.

Suppose $(F,A)_X$ and $(G,B)_Y$ are two fpr-soft sets over the same U such that

$(F,A)_X = \{(a_1/(0.8,0.6,0.2), (\{S_1,S_2,S_3\},\{S_4,S_6\},\emptyset))$,
$(a_2/(0.5,0.2,0.6), (\{S_2,S_3\},\{S_4,S_5\},\{S_8,S_9\}))$,
$(a_3/(0.3,0.1,0.5), (\{S_1,S_3\},\{S_5\},\{S_7,S_8,S_9\}))$,
$(a_4/(0.1,0.7,0.7), (\{S_2\},\{S_4,S_5,S_6\},\{S_8,S_9\}))$,
$(a_5/(0.8,0.7,0.5), (\{S_1,S_2,S_3\},\{S_4,S_5,S_6\},\{S_7,S_8,S_9\}))$,
$(a_6/(0.8,0.7,0.5), (\{S_2,S_3\},\{S_4,S_6\},\{S_8,S_9\}))\}$ and
$(G,B)_Y = \{(b_1/(0.3,0.6,0.7), (\{S_2,S_3\},\{S_4\},\emptyset))$,
$(b_2/(0.3,0.6,0.3), (\{S_1,S_3\},\{S_5\},\{S_8\}))$,
$(b_3/(0.6,0.3,0.2), (\{S_3\},\{S_4\},\{S_7,S_8\}))$,
$(b_4/(0.7,0.4,0.9), (\{S_1\},\{S_5,S_6\},\{S_9\}))\}$.

Step 2: Computation of the $fpr-$soft set $(H,C)_Z$ by the intersection operation. Then $(F,A)_X \widetilde{\cap} (G,B)_Y = (H,C)_Z$, where $(H,C)_Z = \{(c_1/(0.3,0.5,0.2), (\{S_2,S_3\},\{S_4\},\emptyset))$,
$(c_2/(0.3,0.2,0.3), (\{S_3\},\{S_5\},\{S_9\}))$,
$(c_3/(0.3,0.1,0.2), (\{S_3\},\emptyset,\{S_7,S_8\}))$,
$(c_4/(0.1,0.4,0.7), (\emptyset,\{S_5,S_6\},\{S_9\}))$,
$(c_5/(0.8,0.7,0.5), (\{S_1,S_2,S_3\},\{S_4,S_5,S_6\},\{S_7,S_8,S_9\}))$,
$(c_6/(0.8,0.7,0.5), (\{S_2,S_3\},\{S_4,S_6\},\{S_8,S_9\}))\}$.

Step 3: The score value of $(H,C)_Z$ can be written as Table VI.

TABLE VI
THE TABULAR FORM OF THE RELATIVE SOFT SET $(H,C)_Z$

$C\backslash U_1$	S_1	S_2	S_3
$c_1/0.3$	0	1	1
$c_2/0.3$	0	0	1
$c_3/0.3$	0	0	1
$c_4/0.1$	0	0	0
$c_5/0.8$	1	1	1
$c_6/0.8$	0	1	1
$\mu_{H^d_Z}(u)$	0.133	0.316	0.416
$C\backslash U_2$	S_4	S_5	S_6
$c_1/0.5$	1	0	0
$c_2/0.2$	0	1	0
$c_3/0.1$	0	0	0
$c_4/0.4$	0	1	1
$c_5/0.7$	1	1	1
$c_6/0.7$	1	0	1
$\mu_{H^d_Z}(u)$	0.316	0.216	0.300
$C\backslash U_3$	S_7	S_8	S_9
$c_1/0.2$	0	0	0
$c_2/0.3$	0	0	1
$c_3/0.2$	1	1	0
$c_4/0.7$	0	0	1
$c_5/0.5$	1	1	1
$c_6/0.5$	0	1	1
$\mu_{H^d_Z}(u)$	0.116	0.200	0.333

Step.3: Finally, the largest membership grade can be chosen by $\max \mu_{H^d_Z}(u) = \{S_3/0.416, S_4/0.316, S_8/0.333\}$. Hence the family X selected S_3 for the states with the availability of suitable land, while also selecting S_4 for the states with the availability of labour, and S_9 as states with suitable safety requirements.

V. Conclusion

In this paper, a detailed presentation of fuzzy parameterized relative soft sets and their applications were demonstrated. The new algorithm for multiple evaluations in decision-making problems based on fuzzy parameterized relative soft sets was constructed and presented along with examples.

References

[1] L. A. Zadeh, "Fuzzy Sets", *Information and Control,* vol. 8, no. 3, pp. 338-353, 1965.

[2] D. Molodtsov, "Soft Set Theory-First Results", *Computer and Mathematics with Applications,* vol. 37, pp. 19-31, 1999.

[3] P. K. Maji, R. Biswas and A. R. Roy, "Soft Set Theory", *Computer and Mathematics with Applications,* vol. 45, pp. 555-562, 2003.

[4] P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy Soft Set", *The Journal of Fuzzy Mathematics,* vol. 9, no. 3, pp. 589-602, 2001.

[5] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, "On Some New Operations in Soft Set Theory", *Computer and Mathematics with Applications,* vol. 57, pp. 1547-1553, 2009.

[6] N. Çağman and S. Enginoglu, "Soft Set Theory and Uni-int Decision Making", *European Journal of Operational Research,* vol 207, pp. 848-855, 2010.

[7] N. Çağman and S. Enginoglu, "Soft Matrix Theory and its Decision Making, *Computer and Mathematics with Applications,* vol. 59, pp. 3308-3314, 2010.

[8] F. Feng, X. Y. Liu, V. Leoreanu-Fotea and Y. B. Jun, "Soft Sets and Soft Rough Sets, *Information Sciences,* vol. 181, pp. 1125-1137, 2011.

[9] N. Çağman, F. Çitak and S. Enginoglu, "Fuzzy Parameterized Fuzzy Soft Set Theory and its Applications", *Turkish Journal of Fuzzy Systems*, vol. 1, no. 1, pp. 21-35, 2010.

[10] P. K. Maji, R. Biswas and A. R. Roy, "An Application of Soft Sets in a Decision Making Problem", *Computer and Mathematics with Applications,* vol. 44, pp. 1077-1083, 2002.

[11] A. Sezgin and A. O. Atagün, "On Operations of Soft Sets", *Computer and Mathematics with Applications,* vol. 61, pp. 1457-1467, 2011.

[12] Y. Zou and Z. Xiao, "Data Analysis approaches of Soft Sets under Incomplete Information, *Knowledge-Based Systems,* vol. 21, pp. 941-945, 2008.

[13] N. Çağman, F. Çitak and S. Enginoglu, "FP-soft Set Theory and its Application", *Annals of Fazzy Mathematics and Informatics,* vol. 2, no. 2, pp. 219-226, 2011.

[14] H. M. Balami and I. A. Musa, "On Relative Soft Set", *International Journal of Scientific Research and Management,* vol. 1, no.9, pp. 494-506, 2013.

[15] P. Julath and M. Siripitukdet, "Some Characterizations of Fuzzy Bi-ideals and Fuzzy Quasi-ideals of Semigroups", *Engineering Letters,* vol. 25, no. 2, pp. 160-166, 2017.

[16] P. Yairayong, "Applications of Hesitant Fuzzy Sets to Completely Regular Semigroups", *IAENG International Journal of Computer Science,* vol. 47, no. 1, pp. 98-109, 2020.