The Effect of the Alliance between Supply Members on Supply Chain Performance Based on Free Riding

Yongzhao Wang, Wenqiong Hou, Bingrui Zhao

Abstract—**In this paper, we investigate a dual channel supply chain in which a common retailer sells complementary products to customers in a market with two manufacturers (a dominant one and a weak one). The dominant manufacturer sells products through dual channels and provides promotional information services in retail channels, while the weak manufacture sells complementary products through retail channel. We consider three cooperation scenarios, including no-alliance (NA) , two-manufacturers alliance (MM), and retailer-weaker manufacturer alliance (RM) with the spillover effect of promotional information services. All of these alliance models are developed to determine the optimal pricing and retail service strategies. We use some numerical examples to demonstrate the results and analyse the sensitivity of main parameters to obtain some managerial insights. The results show that the alliance of supply chain members improves the operation efficiency of the supply chain, and the mechanism of increasing efficiency in the MM and RM scenarios is different. Interestingly, our results show that the dominant manufacturers will continue to increase the cost of service input with the enhancement of service spill-over effects.**

Index Terms—**Dual-channel supply chain, alliance selection, complementary product, spillover effects, supply chain efficiency**

I. Introduction

With the rapid development of internet technology, it is more and more convenient for consumers to use online access, which also providing a solid guarantee for the popularization of Internet channels [1]. With the formation of online shopping habits and the rapid growth of online sales, many manufacturers have entered the online market, trying to re-plan their channel sales models through online sales, in order to seize more shares in the fierce market competition [2]. In a multi-channel environment, consumers are faced with more choices of products and services, and they can easily switch between different channels to meet their purchase needs [3]. In response to the threat of manufacturers online channels, many promotional advertising services in physical channels will be increased to promote product sales. Therefore, which operation strategy the decision makers choose is a current issue facing the enterprise, and it is also a hot spot in the field of supply chain operation research [4].

In the purchase process, access to product information services is very important for consumers to choose the ideal product. In today's fierce competition among products, the main challenge faced by the company's operators is to find a way to make more people understand their products, thereby piercing greater consumer demand. Advertising services are one of the key tools to improve the product's perception of consumers in operational practice. Therefore, advertising promotion services have also attracted widespread attention from the academic community. The coordination of cooperative advertisement in a manufacturer-retailer supply chain is investigated when the manufacturer offers price deductions to customers in [5]. [6] explored the role of co-op advertising through brand name investments, local advertising expenditures, and sharing rules of advertising expenses in a two-echelon supply chain. While, [7] incorporated the reference price effect into the co-op advertising in a vertical supply chain. The diversification of the participants in the dual-channel supply chain determines the diversification of sales channels. Consumers will make full use of the diversified channels in the process of purchasing products, which determines the universality of consumers' free-riding behavior between different channels [8]. As product information services have the characteristics of public products, that is, information services have spill-over effects, free-riding behavior between channels based on information services is inevitable [9]. Promotional advertising services will attract consumers to buy products, for which companies need to

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pay high costs. Although companies that provide information services face the problem of free-riding consumers, they still provide information services in the traditional physical market. The results of [10] found that if companies that provide services (or sales efforts) do not obtain sales revenue due to consumers' free-riding behavior, this will inhibit their enthusiasm for providing services. [11] addresses a dual-channel supply chain and shows that online channels and traditional retail channels are in competition. The free-riding behavior of online sales channels will reduce the enthusiasm of traditional retailers to provide services. Free-riding behavior has an impact on the service decision of the dual-channel supply chain, and it also has an impact on the price decision of the supply chain. [12] separately investigated the impact of free-riding behavior on pricing under two service efforts strategies. In the pre-sale and after-sales effort strategies, the manufacturer's wholesale price showed different laws with the degree of free-riding. Zhou et.al discussed the dual-channel pricing and service issues in the case of online and offline sharing of service costs. The study found that the impact of free-riding on manufacturers pricing depends on the price sensitivity and service costs between channels [13]. In summary, it can be found that the impact of free-riding behavior on service effort decision-making and sales price is more complicated, which may increase, decrease or remain unchanged. Inspired by the above literatures, this paper will focus on how service spillover effects affect supply chain services and price decisions.

In the e-commerce environment, in addition to the competitive relationship between various industries and many brands, there are complementary relationships, and the strong product relevance makes the company's market decision will have an important impact on the performance of the other party [14]. [15] explores the effect of market structure on quality determination for complementary products in the airline industry. Xia et.al analyzed a problem of distribution channel strategies for an incumbent manufacturer who produces two complementary products and must determine whether or not to have another company to sell its products[16]. Considering that the nature of the conflict between direct and retail channels is due to the substitutability of products between channels, the spillover effect on the operation of the supply chain is worthy of further study when products between channels have complementary cooperation.

In general, a supply chain in which all of its constituent entities operates in an un-coordinated manner, gives the least profit to the entities [17]. It is becoming a common practice that the alliance between supply chain members is an important way to increase their profits. In operations management, a group of papers discussed cooperation among members from a supply chain perspective [18-20]. These studies have found that whether to form an alliance is an important decision for supply chain members, which will affect their profit performance. Therefore, the impact of different cooperation structures on supply chain operations will be a research focus of this paper.

Inspired by the above discussions, the objective of this paper is concerned with the issue of supply members on supply chain performance based on free riding. the main contributions of this paper are summarized as follows: (i) pricing and service decisions model of complementary products in a dual-channel supply chain is established. (ii) How the different alliance forms of supply chain members affect the performance of the supply chain is studied in detail. (iii) The management inspiration of the influence of service spillover effect intensity on the decision-making of supply chain members is obtained.

The paper is organized as follows. In the next section, we formulate the mathematical model of the game problem. In Section 3, three cooperation structures models are established. Section 4 investigates the models with numerical studies and Section 5 concludes the paper.

II. Model

In this study, we consider pricing and service decisions of complementary products in a dual-channel supply chain with two manufacturers, labeled M_1 and M_2, and one common retailer, labeled R. The manufacturer M_1 sells the product 1 through the outlet of traditional retailer at wholesale price w_1 as well as through its own direct E-shop at direct retail price p_1. Due to the different channel costs, the production costs of product 1 in online and off-line channel are c_0 and c_1, respectively. In order to increase sales, the manufacturer M_1 also provides services including advertising and promotion in its off-line retail channels. The manufacturer M_2 produces product 2 at a cost c_2, and wholesales it only through the retail channel at wholesale price w_2. The retailer makes a decision about the sales price of product 2 p_2. The two products are complementary for each other. To reduce channel conflict, this paper adopts consistent pricing strategy in the two channels, which means that the product 1's retail price in the direct channel is equal to that in the retail channel [21,22]; that is both p_1. Considering that the service of product 1 in the retail channel has a double spill-over effect on its direct channel and its complementary products, the model structure is shown in Figure 1.

Similar to [23], to avoid trivial cases, it is assumed that $0 < c_i < w_i < p_i, (i = 1, 2)$, which makes sure that each channel member is willing to enter the market, and $w_1 < p_1$, which keeps the retailers away from buying the products from the direct channel.

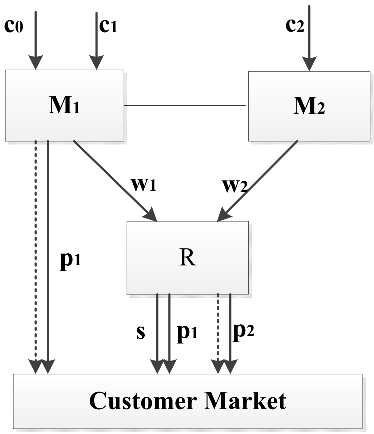

Fig. 1: Model structure.

Similar to [24] and [25], we set the demand functions to be linear with the prices. At the time the impact of services on demand is also considered. The demand function of product 1 and 2 in the traditional retail channel is expressed as

$$D_1 = \alpha_1 - p_1 - \beta_1 p_2 + s, \tag{1}$$

$$D_2 = \alpha_2 - p_2 - \beta_2 p_1 + \gamma_2 s, \tag{2}$$

and the demand function of direct channel can be expressed as

$$D_0 = \alpha_0 - p_1 - \beta_0 p_2 + \gamma_0 s. \tag{3}$$

where $\alpha_i (i = 0, 1, 2)$ represents the market basis of the product in channel i, $\beta_i (i = 0, 1, 2)$ denotes sensitivity of product demand to the price of its complementary product. The impact of complementary products on demand is less than the impact of the product's own price on demand, thus $0 < \beta_i < 1 (i = 0, 1, 2)$. $0 < \gamma_0 < 1$ is the spillover coefficient of services from retail channels to direct channels, and $0 < \gamma_2 < 1$ represents the spill-over coefficient of services from product 1 to complementary products. The larger the values of γ_0 and γ_2 suggest the greater the spillover brought to the direct channels and complementary products, which in turn leads to a stronger incentive for them to "free riding".

M_1 should take some measures to promote advertising in retail channels to increase the demand of product 1. The service cost is assumed to be a quadratic function of the promotional service effort [26, 27]. As a result, the related service cost of M_1 is $f(s) = \frac{1}{2}\tau s^2$, where $\tau > 0$ is the cost effectiveness of the M_1 service.

Based on the strong position of manufacturer M_1, the supply chain system conducts a manufacturer-led Stackelberg game. Following the Stackelberg game framework, the manufacturer M_1 decides wholesale price w_1 and promotional service effort s, whereas the manufacturer M_2 and the retailer determine the wholesale price of complementary products w_1 and the retail price of the two products p_1 and p_2 to maximize their profit, respectively. Let Π_{M_1}, Π_{M_2} and Π_R denote the profit of M_1, M_2 and R, respectively. Therefore, the profit for each member is

$$\Pi_{M_1}(w_1, s) = (p_1 - c_0)D_0 + (w_1 - c_1)D_1 - \frac{1}{2}\tau s^2, \tag{4}$$

$$\Pi_{M_2}(w_2) = (w_2 - c_2)D_2, \tag{5}$$

$$\Pi_R(p_1, p_2) = (p_1 - w_1)D_1 + (p_2 - w_2)D_2. \tag{6}$$

In order to analyze the impact of spillover effect of promotional information services on the decision-making of supply chain members, we will consider different cooperation scenarios in the following section.

III. Theoretical analysis and results

In the following, we assume that all of members have complete information. Three cooperation scenarios are discussed with the spillover effect, which including a no-alliance (NA) scenario, a two-manufacturers alliance (MM) scenario, and a retailer and weaker manufacturer alliance (RM) scenario.

A. The completely no-alliance scenario (NA)

In this section, we first consider the scenario where supply chain members make independent decisions with the objective of maximizing their own profits. The two manufacturers as Stackelberg leaders make decisions first, and then the retailer decides the retail price. The game model is established as follows.

$$\begin{cases} \max\limits_{w_1,s} \Pi_{M_1}^{NA} = (p_1 - c_0)D_0 + (w_1 - c_1)D_1 - \frac{1}{2}\tau s^2, \\ \max\limits_{w_2} \Pi_{M_2}^{NA} = (w_2 - c_2)D_2, \\ \quad s.t. \max\limits_{p_1,p_2} \Pi_R^{NA} = (p_1 - w_1)D_1 + (p_2 - w_2)D_2. \end{cases} \tag{7}$$

The equilibrium solutions can be obtained by backward induction. In order to obtain the optimal solution, taking first-order partial derivatives of Π_R^{NA} with with respect to the retail price p_1 and p_2, we could get the optimal retail price reaction functions are:

$$p_1^{NA^*} = A_{11}w_1 + A_{12}s + A_{13}w_2 + A_{14}, \tag{8}$$

$$p_2^{NA^*} = A_{21}w_1 + A_{22}s + A_{23}w_2 + A_{24}, \tag{9}$$

where $A_{11} = \frac{2-\beta_1^2-\beta_1\beta_2}{4-(\beta_1+\beta_2)^2}$, $A_{12} = \frac{2-\beta_1\gamma_2-\beta_2\gamma_2}{4-(\beta_1+\beta_2)^2}$, $A_{13} = -A_{21} = \frac{\beta_2-\beta_1}{4-(\beta_1+\beta_2)^2}$, $A_{22} = \frac{2\gamma_2-\beta_1-\beta_2}{4-(\beta_1+\beta_2)^2}$, $A_{23} = \frac{2-\beta_2^2-\beta_1\beta_2}{4-(\beta_1+\beta_2)^2}$, $A_{i4} = \frac{2\alpha_i-\beta_1\alpha_{3-i}-\beta_2\alpha_{3-i}}{4-(\beta_1+\beta_2)^2} (i = 1, 2)$.

From Eqs. (4), (5) and the retailer's price response functions, we can obtain the objective functions of the retailer and manufacturer. Considering the wholesale prices of products w_1, w_2 and the service level s as decision variables, the necessary conditions for the manufacturers make maximum profits are $\frac{\partial \Pi_{M_1}}{\partial w_1}=0$, $\frac{\partial \Pi_{M_1}}{\partial s}=0$ and $\frac{\partial \Pi_{M_2}}{\partial w_2}=0$. According to the first-order condition, we can calculate the following result.

$$\begin{bmatrix} w_1^{NA^*} \\ s^{NA^*} \\ w_2^{NA^*} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}^{-1} \begin{bmatrix} B_{14} \\ B_{24} \\ B_{34} \end{bmatrix}. \quad (10)$$

where

$$\begin{aligned}
B_{11} &= A_{11}(-1-2A_{11}-\beta_0 A_{21}) + A_{21}(-\beta_1-\beta_0 A_{11}) \\
&\quad -(A_{11}+\beta_1 A_{21}), \\
B_{12} &= A_{12}(-1-2A_{11}-\beta_0 A_{21}) + A_{22}(-\beta_1-\beta_0 A_{11}) \\
&\quad +(1+\gamma_0 A_{11}), \\
B_{13} &= A_{13}(-1-2A_{11}-\beta_0 A_{21}) + A_{23}(-\beta_1-\beta_0 A_{11}), \\
B_{14} &= A_{14}(1+2A_{11}+\beta_0 A_{21}) + A_{24}(\beta_1+\beta_0 A_{11}) - \alpha_1 \\
&\quad -\alpha_0 A_{11} - c_0(A_{11}+\beta_0 A_{21}) - c_1(A_{11}+\beta_1 A_{21}), \\
B_{21} &= A_{11}(\gamma_0-2A_{12}-\beta_0 A_{22}) - \beta_0 A_{21}A_{12} + 1 - A_{12} \\
&\quad -\beta_1 A_{22}, \\
B_{22} &= A_{12}(\gamma_0-2A_{12}-\beta_0 A_{22}) - \beta_0 A_{22}A_{12} + \gamma_0 A_{12} - \tau, \\
B_{23} &= A_{13}(\gamma_0-2A_{12}-\beta_0 A_{22}) - \beta_0 A_{23}A_{12}, \\
B_{24} &= A_{14}(2A_{12}+\beta_0 A_{22}-\gamma_0) + \beta_0 A_{24}A_{12} - \alpha_0 A_{12} \\
&\quad -c_0(A_{12}+\beta_0 A_{22}-\gamma_0) - c_1(A_{12}+\beta_1 A_{22}-1), \\
B_{31} &= -A_{21}-\beta_2 A_{11}, \\
B_{32} &= \gamma_2 A_{11} + A_{21}, \\
B_{33} &= -2A_{23} - 2\beta_2 A_{13}, \\
B_{34} &= -\alpha_1 A_{21} - \alpha_2 A_{11} - c_2(\beta_2 A_{13}+A_{23}).
\end{aligned}$$

Substituting $w_1^{NA^*}, s^{NA^*}$, and $w_2^{NA^*}$ into the retailer's price response functions, we can get the optimal retail prices as

$$p_1^{NA^*} = A_{11}w_1^{NA^*} + A_{12}s^{NA^*} + A_{13}w_2^{NA^*} + A_{14}, \quad (11)$$
$$p_2^{NA^*} = A_{21}w_1^{NA^*} + A_{22}s^{NA^*} + A_{23}w_2^{NA^*} + A_{24}. \quad (12)$$

The following propositions can be derived by the analysis of the optimal solution under the completely no-alliance scenario.

Proposition 1. *In the completely no-alliance scenario, with the rise of the service level, the consistent product 1's retail price of dual channel will increase, which the upward trend will slow down with the increase of service spillover effect. Moreover, the retail price of complementary products is affected by both the cross-price elasticity coefficient and the spillover coefficient, but in general, product 2's retail price will strongly change with the rise of the spillover effect.*

Proof. As before, using the values of $p_1^{NA^*}$ and $p_2^{NA^*}$ in Eqs.(11) and (12), we get

$$\begin{aligned}
\frac{\partial p_1^{NA^*}}{\partial s} &= \frac{2-\beta_1\gamma_2}{4-(\gamma_1+\gamma_2)^2} > 0, \\
\frac{\partial^2 p_1^{NA^*}}{\partial s \partial \gamma_2} &= \frac{-\beta_1-\beta_2}{4-(\gamma_1+\gamma_2)^2} < 0, \\
\frac{\partial p_2^{NA^*}}{\partial s} &= \frac{2\gamma_2-\beta_1-\beta_2}{4-(\gamma_1+\gamma_2)^2}.
\end{aligned}$$

When $2\gamma_2 > \beta_1+\beta_2$, it is easy find that $\frac{\partial p_2^{NA^*}}{\partial s} > 0$; When $2\gamma_2 \le \beta_1+\beta_2$, thus $\frac{\partial p_2^{NA^*}}{\partial s} \le 0$, $\frac{\partial^2 p_2^{NA^*}}{\partial s \partial \gamma_2} > 0$. This proves the Proposition 1.

Proposition 1 shows that the cost of product advertising and promotion will increase with the improvement of service. Manufacturers would set a high wholesale price which can fulfill the loss of profit caused by increased costs. At the same time, the retailer will also take the opportunity to increase sales prices in order to expand margins. Since the "free riding" customers increase as γ_2 increases, the decision makers have to slow down the increase rate of prices both in direct channel and retail channel in order to reduce customer defection. When the spillover coefficient of services is large, while the price sensitivity of complementary products on each other's demand are small (i.e., $2r_2 > b_1 + b_2$), the retail price will increase with the increase of service, and the increasing trend will be more significant with r_2. On the contrary, the retail price p_2 will decrease with the increase of s, and the downward trend will become more significant with γ_2 increase.

Proposition 2. *In the completely no-alliance scenario, the impact of service on service providers is always greater than the complementary products.*

Proof. From Eqs.(11) and (12), we get

$$\frac{\partial p_1^{NA^*}}{\partial s} - \frac{\partial p_2^{NA^*}}{\partial s} = \frac{(2+\beta_1+\beta_2)(1-\gamma_2)}{4-(\gamma_1+\gamma_2)^2} > 0.$$

This proves the Proposition 2.

Proposition 2 shows that manufacturers will provide effective services to attract customer groups and increase their profits. In this process, the retailer will benefit from the presence of service spillover effect and increase the number of "free-rider" groups, but the change in service level still has a greater impact on service providers, which is consistent with the facts.

Proposition 3. *In the completely no-alliance scenario, there is a threshold. When the service is less than this value, the retailer will increase the selling price by a larger extent than the manufacturer's cost of service. Conversely, when the service is greater than this threshold, the retailer will increase the price by a smaller amount than the increase in the cost of the service by the manufacturer.*

Proof. Take the following derivatives:

$$\frac{\partial p_1^{NA^*}}{\partial s} = \frac{2-\beta_1\gamma_2-\beta_2\gamma_2}{4-(\gamma_1+\gamma_2)^2}, \qquad \frac{\partial f(s)}{\partial s} = \tau s.$$

Now we give a threshold $\delta_0 = \frac{2-\beta_1\gamma_2-\beta_2\gamma_2}{\tau(4-(\gamma_1+\gamma_2)^2)}$. It is easy to find that i) $\frac{\partial p_1^{NC^*}}{\partial s} > \frac{\partial f(s)}{\partial s}$, when $s < \delta_0$; ii) $\frac{\partial p_1^{NC^*}}{\partial s} \leq \frac{\partial f(s)}{\partial s}$, when $s \leq \delta_0$. This proves the Proposition 3.

Proposition 3 shows that the magnitude of the retail price p_1 increase for retailers is higher than the service cost paid by the manufacturer when the service is less than the threshold δ_0. From this perspective, the retailer benefits more than the manufacturer; On the contrary, when the services provided in the market are sufficiently abundant and comprehensive, further service improvement by manufacturers cannot bring greater promotion to the sales of goods in the retail channel, and it cannot bring more profit margins.

B. The two-manufacturers alliance scenario (MM)

In this subsection, we present the results when the two manufacturers cooperate in offering service and setting wholesale prices. Two manufacturers acting as a whole system to make their joint pricing and service decision by maximizing the total profit $\Pi_M^{MM} = \Pi_{M_1} + \Pi_{M_2}$. Similar to the calculation process in NA scenario, the MM model is established as follows.

$$\begin{cases} \max\limits_{w_1,w_2,s} \Pi_M^{MM} = (p_1-c_0)D_0 + (w_1-c_1)D_1 \\ \qquad\qquad +(w_2-c_2)D_2 - \frac{1}{2}\tau s^2, \\ s.t. \ \max\limits_{p_1,p_2} \Pi_R^{MM} = (p_1-w_1)D_1 + (p_2-w_2)D_2. \end{cases} \tag{13}$$

The backward induction method is used to solve the model equilibrium solution. In the case of a given wholesale price and service, the retailer's optimal retail price response function is consistent with the NA scenario and will not be described again.

Substituting the retailer's price response function into Π_M^{MM}, and considering the wholesale prices of products w_1 and the service level s as decision variables, the necessary conditions for the manufacturers make maximum profit are $\frac{\partial \Pi_M^{MM}}{\partial w_1} = 0$, $\frac{\partial \Pi_M^{MM}}{\partial w_2} = 0$ and $\frac{\partial \Pi_M^{MM}}{\partial s} = 0$. Thus, we obtain

$$\begin{bmatrix} w_1^{MM^*} \\ s^{MM^*} \\ w_2^{MM^*} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13}+B_{31} \\ B_{21} & B_{22} & B_{23}+B_{32} \\ B_{13}+B_{31} & B_{23}+B_{32} & F_1 \end{bmatrix}^{-1} * \begin{bmatrix} B_{31}c_2 - B_{14} \\ B_{32}c_2 - B_{24} \\ F_2 \end{bmatrix}, \tag{14}$$

where $F_1 = B_{33} - 2A_{13}(A_{13}+\beta_0 A_{23})$, $F_2 = 2A_{13} - B_{34} - \beta_0 A_{13}A_{24} - \alpha_0 A_{13} - c_0(A_{13}+\beta_0 A_{23})$.

Substituting $w_1^{MM^*}, s^{MM^*}$ and $w_2^{MM^*}$ into the retailer's price response function, we can get the optimal retail prices are

$$p_1^{MM^*} = A_{11}w_1^{MM^*} + A_{12}s^{MM^*} + A_{13}w_2^{MM^*} + A_{14}, \tag{15}$$

$$p_2^{MM^*} = A_{21}w_1^{MM^*} + A_{22}s^{MM^*} + A_{23}w_2^{MM^*} + A_{24}. \tag{16}$$

C. The retailer and weak manufacturer alliance scenario (RM)

In this subsection, we consider the alliance scenario in which the retailer and the weak manufacturer cooperate with each other, acting as a team, to maximize their joint profits. After the alliance is formed, their revenue are shared. The wholesale price of complementary products will be used as an internal transfer price, which is no need to make decisions. The strong manufacturer acts as the leader of Stackelberg game and the alliance between the retailer and the weak manufacturer acts as the follower. The strong manufacturer first decides the wholesale price and service; then the alliance decides the retail price according to the strong manufacturers decisions. The RM model is given as follows:

$$\begin{cases} \max\limits_{w_1,s} \Pi_{M_1}^{RM} = (p_1-c_0)D_0 + (w_1-c_1)D_1 - \dfrac{1}{2}\tau s^2, \\ s.t. \ \max\limits_{p_1,p_2} \Pi_R^{RM} = \Pi_{M_2} + \Pi_R \\ \qquad\qquad = (p_1-w_1)D_1 + (p_2-c_2)D_2. \end{cases} \tag{17}$$

Considering the retail prices p_1 and p_2 as decision variables, the necessary conditions for the retailer alliance make maximum profit are $\frac{\partial \Pi_R^{RM}}{\partial p_1} = 0$ and $\frac{\partial \Pi_R^{RM}}{\partial p_2} = 0$. Thus, we obtain

$$p_1^{RM^*} = A_{11}w_1 + A_{12}s + A_{13}w_2 + A_{14}, \tag{18}$$

$$p_2^{RM^*} = A_{21}w_1 + A_{22}s + A_{23}w_2 + A_{24}. \tag{19}$$

After getting the optimal values of $p_1^{RM^*}$ and $p_2^{RM^*}$, the corresponding profits of the strong manufacturer can be obtained. From the first-order conditions for $\Pi_{M_1}^{RM}$, we have the optimal solution as follows:

$$\begin{bmatrix} w_1^{RM^*} \\ s^{RM^*} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^{-1} \begin{bmatrix} -B_{31}c_2 - B_{14} \\ B_{32}c_2 - B_{24} \end{bmatrix}. \tag{20}$$

Substituting $w_1^{RM^*}$ and s^{RM^*} into the retailer's price response function, we can get the optimal retail prices are

$$p_1^{MM^*} = A_{11}w_1^{RM^*} + A_{12}s^{RM^*} + A_{13}c_2 + A_{14}, \tag{21}$$

$$p_2^{MM^*} = A_{21}w_1^{RM^*} + A_{22}s^{RM^*} + A_{23}c_2 + A_{24}. \tag{22}$$

IV. NUMERICAL EXAMPLES

This paper considers the spillover effect of promotional information services and the different alliance scenario between supply chain members. The optimal equilibrium is more complicated, and the profit function of each member contains many parameters. Therefore, some numerical examples are set in order to illustrate the theoretical results. The focus of this paper is to compare the optimal decisions of three different alliance scenario, and discuss the effects of service spillover coefficients and cross-price elasticity coefficients on supply chain member's decisions. The following parameter values are set as a benchmark,

Market parameters: $\alpha_0 = 400, \alpha_1 = 500, \alpha_2 = 600$,
Cost parameters: $c_0 = 22, c_1 = 18, c_2 = 5$,
Demand parameters: $\beta_0 = 0.4, \beta_1 = 0.6, \beta_2 = 0.7$,
Service parameter: $\gamma_0 = 0.6, \gamma_2 = 0.4, \tau = 0.8$.

which allow for a comprehensive illustration in the previous studies of spillover effects.

A. The analysis of optimal decisions under different alliance scenario

There are three alliance scenario among supply chain members, namely no-alliance (NA) scenario, two-manufacturers alliance (MM) scenario and retailer-weaker manufacturer alliance (RM) scenario. With the given data, we can get the system equilibrium solutions in different alliance scenario, see Tables 1 and 2.

TABLE I: The optimal decisions in different alliance scenario.

Model	w_1^*	s^*	w_2^*	p_1^*	p_2^*	D_0^*	D_1^*	D_2^*
NA	159	160	271	293	324	72	171	133
MM	134	212	223	311	291	98	225	174
RM	249	235	–	378	178	91	250	251

TABLE II: Maximum profits in different alliance scenario.

Model	$\Pi_{M_1 0}^*$	$\Pi_{M_1 1}^*$	$\Pi_{M_1}^*$	$\Pi_{M_2}^*$	Π_R^*	Π_C^*
NA	$1.9 * 10^4$	$2.4 * 10^4$	$3.3 * 10^4$	$3.5 * 10^4$	$3.0 * 10^4$	$9.9 * 10^4$
MM	$2.8 * 10^4$	$2.6 * 10^4$	$7.5 * 10^4$		$5.2 * 10^4$	$12.7 * 10^4$
RM	$3.2 * 10^4$	$5.7 * 10^4$	$6.8 * 10^4$	$7.5 * 10^4$		$14.4 * 10^4$

where $\Pi_{M_1 0}^*$ and $\Pi_{M_1 1}^*$ represent profits from direct channel and traditional channel of the strong manufacturers, respectively.

By comparing the data in Tables 1 and 2, the following conclusions can be drawn.

i) Compared with NA scenario where supply chain members are completely no alliance, the cooperation between members (MM and RM scenario) will always reduce system friction, in which the total profit of the supply chain will be greater than NA scenario. In particular, when the two players in the game have the same status (RM scenario), the total profit of the system is the largest, that is $\Pi_C^{RM^*} > \Pi_C^{MM^*} > \Pi_C^{NC^*}$. At the same time, the profit of the manufacturer in MM and RM scenario is greater than NA scenario, that is $\Pi_R^{MM^*} > \Pi_R^{NC^*}, \Pi_{M_1}^{RM^*} > \Pi_{M_1}^{NC^*}$. In fact, cooperation among supply chain members reduces some of the marginal costs, which in turn increases market demand for products. The increase in profits under the cooperation situation also makes manufacturers willing to further provide more service to stimulate sales and increase revenue (i.e. $s^{MM^*} > s^{NC^*}, s^{RM^*} > s^{NC^*}$).

ii) Compared with NA scenario, The cooperation between manufacturers (MM) reduces the wholesale price of products and their complementary products, but the increase in product demand makes up for the loss caused by the drop in wholesale prices. At the same time, the profit margin of the two products for the retailer has increased significantly (i. e., $p_i^{MM^*} - w_i^{MM^*} > p_i^{NA^*} - w_i^{NA^*}, i = 1, 2$), and consumers will also benefit from the low prices of the products. Due to the increased investment in service, manufacturers increase their wholesale prices to expand their profit margins in RM scenario, leading retailers to further increase the retail prices of their products. On the contrary, the cooperation has reduced the intermediate links of complementary products, so the retail price has dropped significantly, and consumers have benefited significantly.

B. The relationship between service cost and spillover coefficient

In this section, we discuss the impact of spillover effects on the promotional service cost. With the given data, we can get the different time paths of the promotional service cost under different values of spillover coefficient parameter γ_0 and γ_2 in Figures 2 and 3, where they varies in the interval (0,1).

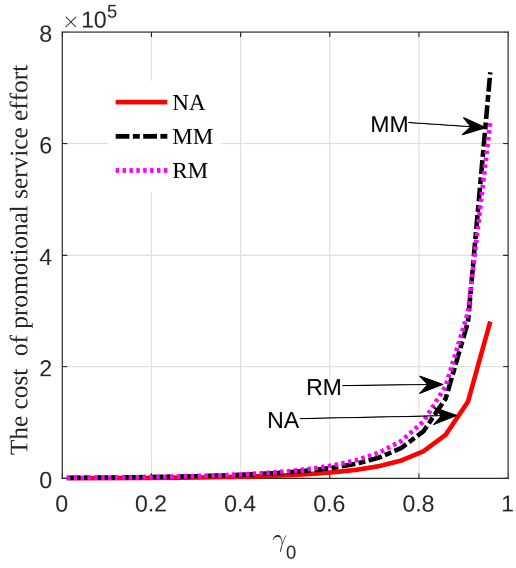

Fig. 2: The promotional service cost with different γ_0.

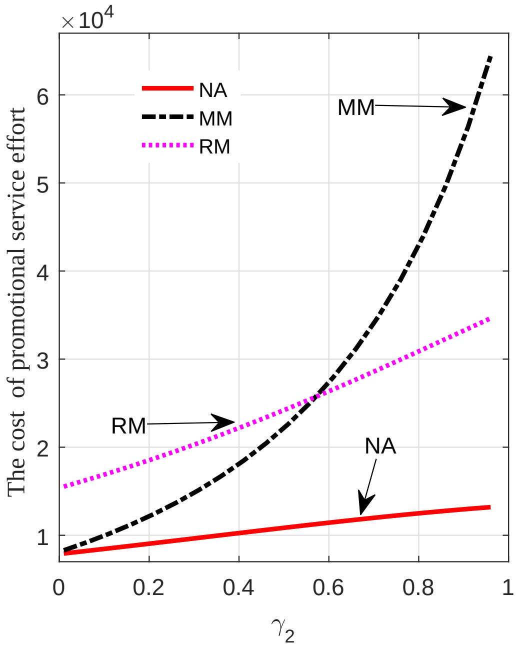

Fig. 3: The promotional service cost with different γ_2.

By observing the changes in service costs in the three alliance scenario in Figures 2 and 3, the following conclusions can be drawn.

i) Although the manufacturer as a service provider cannot internalize the revenue generated by the spillover effect in NA and RM scenario, more and more customers are more willing to purchase products through the direct channel after receiving the services from the retail channel with the increase of the direct channel spillover coefficient γ_0. The benefits of the "free-riding" of the direct channel are sufficient to offset the loss of cost input, so manufacturers will also continue to increase the cost of service input to stimulate sales in the direct channel. On the other hand, although the service spillover coefficient has become larger, manufacturers will still improve the service level of retail channels to ease conflicts between direct and traditional channels, and thus avoid fierce price competition among supply chain members. The alliance in MM scenario can help manufacturers internalize the benefits generated by spillover effects, so manufacturers will also appropriately increase investment to stimulate potential customer groups and increase sales and profits.

ii) In MM scenario the benefits can be internalized which generated by spillover effects. With the increase of the spillover coefficient γ_2 of complementary products, manufacturers need to continuously increase the cost of service input to achieve the promotion of their own products. In NA and RM scenario, manufacturers cannot internalize the revenue generated by spillover effects which cannot be made up for the cost loss caused by excessive service investment. Therefore, the growth rate of service input costs is slow. At this time, customers can be attracted to increase profits without excessively increasing losses. Considering that the "free-riding" behavior of complementary products will harm the manufacturers of service providers, deeper contractual cooperation or coordination methods are more beneficial to supply chain members to take full advantage of the spillover effects of retail channel services to obtain higher returns.

C. Sensitivity analysis of spillover effect coefficient

This section will discuss the influence of spillover effect coefficient and spillover ratio on the decision-making and profit of supply chain members. The differences of spillover effect coefficient in the three alliance scenario are shown in Figs 4 and 5.

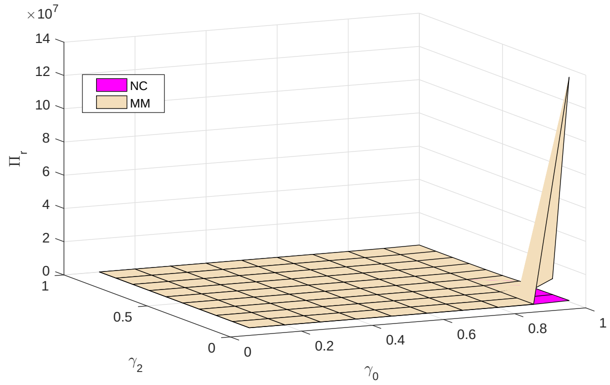

Fig. 4: The profits of manufacturer M_1 with different γ_0 and γ_2.

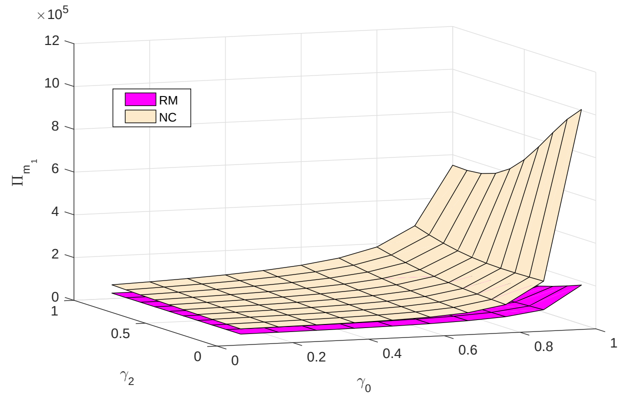

Fig. 5: The profits of retailer R with different γ_0 and γ_2.

Fig. 4 and 5 show the changes in the profits of manufacturer M_1 and retailer R with different combinations of spillover effects in different alliance scenario. Compared with NA scenario, cooperation between retailers and disadvantaged manufacturers can significantly increase the profits of manufacturer M_1, and cooperation between manufacturers can significantly increase the profits of the retailer. This also reflects that cooperation has reduced the system friction within the supply chain system and improved the operating efficiency of the supply chain.

V. Conclusions

In practice, it is common for supply chain members to cooperate with each other. In this article, we propose a complementary products supply chain model involving two complementary manufacturers and one retailer. This paper is concerned with a decision-making problem for supply chain members and the impact of service spillovers in different alliance scenario. Numerical examples are provided to illustrate the effectiveness of the main theoretical results and the solution procedure. Then, some managerial insights can be obtained according to the sensitive analysis of the key system parameters. Firstly, this paper provides a framework to research the effect of service spillovers in a complementary products supply chain. Secondly, the analytical solution of the optimal strategies for supply chain members under different cooperation modes. Thirdly, it is clear to show the impact of cooperation mode on supply chain efficiency based on free riding.

Some valuable extensions should be noted. For example, we test the findings considering cooperation among supply chain members. Future research can study the cooperative choice of supply chain members under fuzzy environment[28-30] based on the current model.

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