On Formulation of the Vehicle Routing Problems Objective Function with Focus on Time Windows, Quantities and Split Delivery Priorities

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***Abstract*—More firms and companies around the globe are working assiduously to organize better and much more timely deliveries of goods and services employing various of today's invented technologies in order to satisfy their teeming customers. The advent of multivariant priorities that real-life situations have brought into Vehicle Routing Problems (VRP) necessitates the inflow of researches in order to solve VRPs as they occur on daily basis. The paper discusses the formulation of a comprehensive VRP objective function that incorporates three priorities: Time Windows, Quantities and Split Deliveries which arise in Vehicle Routing Problems (VRP). The paper classifies Split Deliveries in relation to VRP and generates dynamics for each class so identified. Also, the interconnectivities between customers' priorities: Time windows and Quantity Priorities and the vehicular conditioning: Split Deliveries are explained. The formulated VRP objective function is aimed at calculating the distance carrying cost, computing the fixed cost as well as evaluating the priorities so involved.**

***Index Terms*—Keywords: Vehicle Routing Problems, Split Deliveries, Time Windows, Quantities Priorities, Priorities Interplay.**

I. Introduction

THE Vehicle Routing Problems (VRP) by [1] is a generic name given to a whole class of problems involving the visiting of 'Customers' by 'Vehicles'. The VRP appears frequently in practical situations which are not directly related to the physical delivery of goods alone but includes the collection of mails from mail boxes, the picking up and returning of children by schools' buses, house-call tours by doctors, preventive maintenance inspection tours, the delivery of laundry and a host of others. These are all referred to as VRPs in which the delivery operation may be collections, collections and deliveries or exclusively deliveries in which the goods and vehicles can take a variety of forms.

In view of the enormous number of real-life situations which gave rise to VRPs, it is not surprising to find out that a large number of constraints and/or objectives appear in such problems. Here, we consider problems in which a set of geographically dispersed customers with known requirements must be served with a fleet of vehicles stationed at a central facility or depot with the aim to minimize some distribution objectives. The basis assumption is that, all vehicle routes must start and finish at the depot, all the vehicles are of the same make, type and all the routes are open to all the vehicles at all hours of the day. This calls for a basic VRP that can be characterized in what follows. The main objective is to minimize the total cost of delivery or to maximize the profit while taking into consideration constraints which vary from one case to another.

The most general version of the VRP is the Capacitated Vehicle Routing Problem (CVRP). It is a kind of problem in which all the customers must be satisfied, all demands are known, and all vehicles have identical, limited capacity and are based at a central depot. The objectives are to minimize the vehicle fleet and the sum of travel time while the total demand of commodities for each route may not exceed the capacity of the vehicle which serves that route [2]. One of the most important extensions of the CVRP is the Vehicle Routing Problem with Time Window (VRPTW) in which each customer must be served within a specific time window. The objective is to minimize the vehicle fleet with the sum of travel time and waiting time needed to supply all customers in their required hour. The CVRP has been described by many as a variant of VRP in which all transportation requests are made up of the distribution of goods from the depot to the various customers. Other types of transportation can be classified based on requests in the node routing context as delivery and collection.

The complement of distribution deliveries to customers is mainly collection from customers, where all tasks involve the movement of goods and collection of empty containers or waste from the customers back to the depot. On the other hand, collections are often referred to as pickups. An associated routing problem to collection often occur either at the start of a supply chain, e.g., as in raw-milk collection (see [3]), or at the very end of the supply, e.g., in reverse logistics where returned empties have to be collected or waste has to be disposed (see [4] and [5]).

The equivalence of pure distribution and pure collection VRP turns out to be obvious when the routes are reversed so that collection becomes distribution and vice versa. When

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both the collection and the distribution as in pickups and deliveries of materials occur together in a route, it definitely leads to variant VRP. It is acclaimed that while distribution begins at the depot, collection ends at the depot. Therefore, problems with collection are known as many-to-one VRP and problems with distribution are seen as one-to-many VRP.

The foremost and probably the simplest variant of VRP is the VRP with Backhauls (VRPB). For example, whenever bulky materials are to be transported, all deliveries to the so-called linehaul customers must be performed first. This makes the vehicle lighter and empty as at the time of arrival at the first collection point usually referred to as the backhaul customer. Since movement from a backhaul customer to a linehaul customer is forbidden then, the model remains applicable if the corresponding arcs are removed from the arc set. Alternatively, one can set the costs of these arcs to a sufficiently large number M .

Backhauling constraints result from the difficulty of rearranging the loaded items inside the vehicle. If the loading space allows rearrangements, e.g., where the vehicle can be loaded from rear and front or all sides, the resulting problem is a VRP with mixed deliveries and collections, or simply Mixed VRPB (MVRPB) [6]. Here, the vehicle capacity must be checked at each edge (or arc) traversed; i.e., the load already collected from backhaul customers plus the load to be delivered to linehaul customers can never exceed the carrying capacity of the vehicle.

On like cases where each customer in the VRPB and MVRPB requires either a delivery or a collection and not both, the VRP with Simultaneous Pickup and Delivery (VRPSPD) (see [7]) comprises two transportation requests for each customer, namely a delivery from the depot to the customer and a pickup from the customer to the depot. Both transportation requests must be performed by the same vehicle in a single visit. This practice is common in several real-world applications such as; the delivery of beverages and the simultaneous collection of empty bottles and as in common practice to tour operators in many holiday regions, the same bus that convenes newly arriving guest also transports the departing hotel guests between a local airport and several hotels. Likewise, the capacity constraint ensures that no vehicle is overloaded at any point.

In the past few decades, the Vehicle Routing Problems (VRP) and its variants have grown more popular in the academic literature [8]. Yet, the problem's characteristics and assumptions vary widely from one place to another and from person to person. However, some authors have made efforts to classify existing articles accordingly. Based on an adapted version of the comprehensive taxonomy suggested in [9], we formulate a VRP objective function, generate relations for the priorities and show the interplay among Time Windows, Quantities and Split Deliveries Priorities in this paper.

Section II and subsequent sections will be devoted to discussing the fundamental requirements of VRP, the priorities one after the other and formulation of a comprehensive objective function that aggregates the priorities.

II. Fundamentals of VRP Requirements

In order to spell out the fundamentals of VRP requirements clearly, there is need to consider the VRP over a given time, T. Let $C = \{c_i \mid i = 0, 1, 2, 3, \ldots, N\}$ be the set of N customers with c_0 the depot. Let $V = \{v_k \mid k = 1, 2, 3, \ldots, M\}$ be the set of M homogenous vehicles stationed at the depot, c_0. By [10], every pair of locations, (i, j), between two successive nodes declared as customers, where $i, j \leq N$ and $i \neq j$, associates with the travel time, t_{ij}, from customer c_i to c_j and the metric distance traveled, $d(i, j) = d_{ij} = d(c_j)$ is symmetrical, i.e. $d_{ij} = d_{ji}$.

Every customer, c_i, is bound by the following fundamental requirements:

(a) there should be specified quantity, $q(c_i)$, of the products or services required to be delivered by the vehicles v_k to the customer, c_i. Where the vehicle v_k alone cannot deliver the entire quantity required by the customer, c_i, then, the quantity is split between the vehicles v_k and v_{k+1} or split among the vehicles v_k, v_{k+1}, v_{k+2} and ... and v_M;

(b) there should be specified time, t_{ij}, required by the vehicle, v_k, to move from the depot, c_0, or from a customer, c_i, to visit the next customer, c_j, in order to unload the quantity, $q(c_i)$, and leave the customer, c_j, for the next customer, c_{j+1}, or return to the depot, should all the customers on that route have been serviced or all the quantity carried by the vehicle, $Q(v_k)$, have been exhausted. Where two or more vehicles, v_k and v_{k+1} or $v_1, v_2, \ldots v_k$ respectively, have to service a customer, the time, t_{ij}, is required by the vehicles, v_1, v_2, ..., v_k, to move either from the depot, c_0, or from a customer, c_i, to visit the customer, c_j, to unload the quantity, $q(c_i)$, and leave the customer, c_j, for the next customer, c_{j+1}, or return to the depot, should the customer with split delivery on that route have been serviced or all the quantities, $Q(v_1) + Q(v_2) + \ldots + Q(v_k)$, have been exhausted.

(c) The priority, δ, of the customer, $\delta(c_i)$, to be serviced by the vehicle, v_k, must be placed.

All the customers are serviced from only one depot by a homogeneous and limited fleet. The vehicles leave and ultimately return to the depot after the last customer has been serviced or the vehicle has exhausted its carrying capacity. There is a set, V, of vehicles with identical capacities. The capacity of each vehicle is represented by $Q(v_k)$. Much as the customer are having some requirements also, the vehicles, v_k, have the following characteristics to be met:

(a) there is a limited working period, T, of the vehicle, $T(v_k)$, from the starting time, $T^s(v_k)$, to the finishing time, $T^f(v_k)$.

(b) there is a fixed cost, $FC(v_k)$, of wages to the driver and the loaders attached to each vehicle, v_k.

(c) the carrying capacity, Q, of the vehicle, $Q(v_k)$ must be spelt from the onset.

Hinging on the customers' requirements as well as the vehicles' characteristics, the following general assumptions are made:

(a) the variable cost, VC_{ij}, is given as the cost of the least path from customer c_i to the next customer c_j.
(b) the time, t_{ij}, is the corresponding travel time from customer c_i to customer c_j which is assumed to be symmetrical, $t_{ij} = t_{ji}$, provided there is no traffic congestion.
(c) let $R_i = \{r_i(1), \ldots, r_i(x)\}$ represent the set of routes for vehicle v_k, where $r_i(x)$ indicates the ith customer visited and x is the number of customers on the route. We also assume that every route terminates at the depot with $r_i(x+1) = 0$.
(d) the distance from where the vehicle can pack to unload to the warehouse or store of each customers is equal hence, the time to unload per item is fixed.

There is a need to discuss the priorities in what follows to usher us into the VRP objectives and problem formulations.

III. The VRP Priorities

In recent times, the priorities that a customer can place on the demand that are to be met by the delivery vehicle have begun to increase e.g. Time of delivery, Quantity to be supplied, Dynamical situations, Split deliveries, Heterogenous vehicles, Periodic e. t. c. A customer can require that one or more of these priorities be met at a particular time. This work admits only three of such priorities which are: Priority based on Time, Quantity and on Split Deliveries of the products to be delivered.

A. Priority Based on Time

A well-known priority placed on VRPs is the Time Windows, where every customer is associated with a time horizon, (see [11]), within which, the customer must be visited or serviced. Thus, VRP with Time Priority (VRPTP) is one of the most important extensions of the VRP. In the VRPTP, each customer specifies a time window within which the service must start and probably finish. The VRPTP can be used to model various real-life applications, such as itemized in [12].

An extension of the VRP with Time Priority is given by [13], which considers multiple periods and assumes that each customer is required to be visited based on a given frequency and given feasible combinations of visiting periods. Recent applications include: routing and scheduling of service teams for preventive maintenance of elevators at customer locations [14], Pick-up Scheduling of Two-dimensional Loading [15], Cash Distribution using Skip Concept [16], Minimizing Electrical Energy Consumption [17] and periodic delivery of blood products to hospitals by the Austrian Red Cross [18].

In addition to time-window constraints are some practically relevant constraints in quite a number of VRP variants relating to scheduling, i.e., such requiring attention for the travel time to the customer, waiting time at the customers place and service time. In the VRP with Time Windows (VRPTW) credited to [19], the traversal time, t_{ij}, for each arc $(i, j) \in C$ and a time window (t^e, t^l), (See [10]), which corresponds, respectively, to the earliest time, t^e, and the latest time, t^l, within which the vehicle should service the customer, c_i. A schedule, which is the entire time, T_{ik} for the service at a customer, c_i, when visited by vehicle v_k, is considered feasible if

$$t^e \leq T_{ik} \leq t^l \quad \forall i \in C, k \in V, \tag{1}$$

(if vehicle v_k does not visit customer c_i, the time T_{ik} is irrelevant) and $\xi_{ijk} = 1$ implies that:

$$T_{ik} + t_{ij} \leq T_{jk} \quad \forall (i, j) \in C, k \in V, \tag{2}$$

holds. The latter constraint, (2), merges the routing decisions with the time schedule. This can be linearized by means of MTZ-like [20] to obtain the constraints of the form:

$$T_{ik} - T_{jk} + M\xi_{ijk} \leq M - t_{ij} \quad \forall (i, j) \in C, k \in V \tag{3}$$

It is worthy of note that, with the definitions above, time windows are asymmetric in the sense that, arriving at a customer, c_i, before time t^e is allowed. In which case the vehicle has to wait until time t^e, while arriving later than time t^l is prohibited. Some authors also add service times, s_i, at vertices to their models. This is only a minor extension and can be included by properly redefining the travel times and time windows.

Owing to the afore, the time windows can be sub-divided into four frames. Each customer, $c_i \in N$, has a time windows, $t^e < t_{ij} < t^l$, i.e. an interval (t^e, t^l). According to [12], the following scenarios arise as time window sub-divisions:

PT$_1$= $\left(t^e, t^l\right)$: This time window indicates that; the vehicle can arrive any time after the earliest, t^e, and must leave before the latest time, t^l. This implies that, the vehicle can arrive at the customer's place at any time of the day and depart at any time as long as the delivery is done before the latest departure time required by the customer. Of all the time windows, $\left(t^e, t^l\right)$ is one that gives room for the vehicle to service the customer at any convenient time within the working period, T_k.
PT$_2$= $\left(t^e, t^l\right]$: Here, the vehicle may arrive at the customer's location any time within the working period but must depart on or before the allotted latest departure time set by the customer.
PT$_3$= $\left[t^e, t^l\right)$: Here, the vehicle arrives on or after the earliest time and leaves at any time before the latest departure time. It is closed at the earliest time but open at the latest time interval. In this case, should the vehicle arrive ahead of the arrival time, it cannot be allowed to discharge, hence, has to wait till the earliest arrival time.
PT$_4$= $\left[t^e, t^l\right]$: A vehicle that arrives earlier than t^e, has to wait until t^ebefore it can start serving the customer. Arriving later than $t^l$is not allowed rather the vehicle must leave at most by t^l. This case places restriction at both the arrival and departure time. It gives no room for the vehicle to come at just any time earlier than the earliest time and must depart on or before the latest departure time. Should the vehicle not have finished discharging, it must leave at the latest departure time to give room for other things as the case may be. This case calls for the unloading time not to be elongated unnecessarily as the customer might have

other things to attend to.

The time priority may be represented using the tree diagram in figure 1.

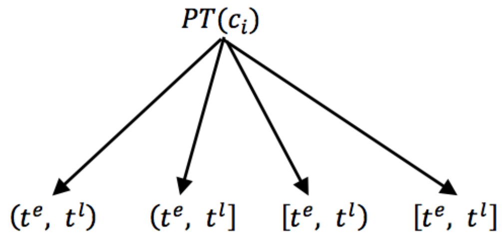

Fig. 1. Time Priority Tree

From figure 1, $PT(c_i)$ represents the time window priority of a customer, c_i, that can only be linked to a time window priority, $(t^e, t^l), (t^e, t^l], [t^e, t^l)$ or $[t^e, t^l]$ at a time. Such that:

$$PT(c_i\ ) = PT_1\ or\ PT_2\ or\ PT_3\ or\ PT_4 \tag{4}$$

B. Priority Based on Quantity

Two important cases of priority based on quantity will be considered here as opined by [12], namely:
(i) cases where a customer requires a quantity that is within the carrying capacity of the vehicle. Servicing such customer on that route leads to specifying the quantities without any tradeoff. Each customer, c_i, has a quantity demanded, i.e. an interval $[q^{min}, q^{max}]$, which corresponds, respectively, to the minimum and maximum quantities demanded by the customer, c_i, (see [11]).
(ii) cases where the request of a customer may be fulfilled by more than a vehicle. This occurs in situations where the customer's demand exceeds the carrying capacity of the vehicle. This turns out to be cost effective on the side of supplier in that, the vehicle goes directly to the customer. Variations in priority based on quantity are thus:
PQ$_1$= $\left(q^{min}, q^{max}\right)$: The expression indicates that, the minimum and maximum quantities that the customer requires are open at both ends.This implies that, the customer has got no fixed quantity to be delivered. Hence, the vehicle can deliver any amount of the product to the customer as long as the carrying capacity of the vehicle is not exceeded. The flexibility of this case makes planning on quantities to be delivered to each customer difficult for the vehicle from on set.

PQ$_2$= $\left(q^{min}, q^{max}\right]$: Here, the customer has an upper limit of quantity required to be supplied. The quantity is open at the lower end and closed at the upper end. There is a definite quantity the customer looks forward to receiving hence, gives rooms for less supply.

PQ$_3$= $\left[q^{min}, q^{max}\right)$: Here, the quantity supplied to the customer cannot be less than a particular amount but, can be more should the vehicle be able to deliver it. In practice, this allows the customers to review order upward at the point of delivery. Though, this could impact the vehicles ability to service all the earlier marked customers.

PQ$_4$= $\left[q^{min}, q^{max}\right]$: It gives no room for the customer to receive more than earlier been requested. It is strictly bounded at both ends. In practical terms, there are situations in which a customer might be in need of more goods or services envisaged due to patronage. Such cases do not permit variation on the part of the customer even when it is at a disadvantage. It must be noted that, situations in which the intervals are locked at both ends, are not flexible enough. It's not an ideal priority for it gives no room for future business expansion.

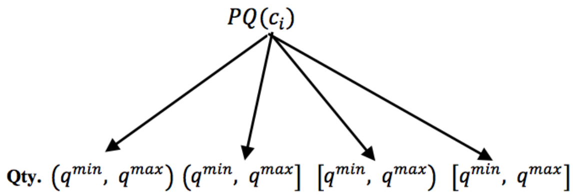

Fig. 2. Quantity Priority Tree

Figure 2 depicts that a customer, c_i, can only be linked to a quantity priority once such that:

$$PQ(c_i) = PQ_1\ or\ PQ_2\ or\ PQ_3\ or\ PQ_4 \tag{5}$$

where $PQ(c_i)$ denotes the priority of a customer. It must be noted that, every customer is bound to have priorities based on time and on quantity. If the priority based on quantity of a customer exceeds the carrying capacity of the vehicle then, such a customer is bound to have priority based on split deliveries else, split deliveries are not required. Also, each customer must fulfil only one line of priority condition.

C. Priority Based on Split Deliveries

In real-life settings where the vehicles used are homogenous, split deliveries occur when the demand of a customer cannot be met by just one vehicle as in [21], [22] and more, which [23] coined as split and non-split Services.

Over a long time, we have assumed that all tasks of servicing a particular customer is being performed by a single vehicle in one service operation, i.e., services are not split. However, with passing of time, there have been reasons for splitting some services: On one hand, if the demand of a customer exceeds the vehicle carrying capacity, more than one visit is unavoidable. On the other hand, splitting of a service into several smaller services request can cost savings in the long run. The Split Delivery VRP (SDVRP) opined by [24] and [25], allow, in principle that, each demand be split into arbitrarily many smaller deliveries served by more than a vehicle. Obviously, the reasons for split deliveries could be broadly and explicitly classified under the followings:

(i) the vehicle, v_k, has serviced some customer(s), $c_1, c_2, \ldots, c_{N-n}$, along the route, $r_i\,(x_i)$, with the quantities, $q(c_1), q(c_2), \ldots, q(c_{N-n})$, where n is the number of customers that have been serviced on the route, thereby causing the quantity, q, to be delivered to customers, c_{N-n+1}, along the route not sufficient. Hence, called for a split delivery by another vehicle, v_{k+1}, rather than v_k to make up for the remaining. If the serviced customers, $c_1, c_2, \ldots, c_{N-n}$, by the vehicle, v_k, get the quantities:

$$q\left[c_1(v_k)\right] + q\left[c_2(v_k)\right] + \ldots + q[c_{N-n}(v_k)]$$
$$= \sum_{i=1}^{N-n} q[c_i(v_k)] \qquad (6)$$

then, from (6) it implies that, the quantity, q, demanded by the customer, $q(c_{N-n+1})$, can be expressed as:

$$\sum_{i=1}^{N-n} q[c_i(v_k)] + q[c_{N-n+1}(v_k)] > Q(v_k) \qquad (7)$$

If (7) holds then, it leads to

$$q[c_{N-n+1}(v_k)] > Q(v_k) - \sum_{i=1}^{N-n} q\left[c_i(v_k)\right] \qquad (8)$$

From (8), the split quantity required by $q[c_{N-n+1}\left(v_k\right)]$ is the remaining quantity:

$$q\left[c_{N-n+1}(v_k)\right] - \{Q(v_k) - \sum_{i=1}^{N-n} q\left[c_i(v_k)\right]\} = Q(v_{k+1}) \qquad (9)$$

Since the quantity that will be delivered by $Q(v_{k+1})$ cannot be determined a priori then:

$$q\left[c_{N-n+1}(v_k)\right] = Q(v_k) + Q(v_{k+1}) - \sum_{i=1}^{N-n} q\left[c_i(v_k)\right] \qquad (10)$$

(ii) the quantity, q, that is demanded by the customer, $q(c_i)$, exceeds the carrying capacity, Q, of the vehicle, $Q(v_k)$. This is given by the relation:

$$q(c_i) = q[c_i(v_k)] > Q(v_k) \qquad (11)$$

leading to

$$q(c_i) = Q(v_k) + Q(v_{k+1}) - q[c_i(v_k)] \qquad (12)$$

where $Q(v_{k+1})$ is the carrying capacity of vehicle v_{k+1} and $q[c_i\left(v_k\right)]$ implies the quantity delivered or to be supplied to customer c_i by v_k. The fractional part or whole of $Q(v_{k+1})$ that is added to $Q(v_k)$ in order to make up for the quantity required by the customer, $q(c_i)$ is the split quantity.

(iii) a rider to (12) is when the quantity, q, that is demanded by the customer, $q(c_i)$, exceeds the carrying capacity, Q, of the vehicle, $Q(v_k)$ and such customer's demand has to be met by more than two vehicles, $v_k, v_{k+1}, v_{k+2}, \ldots$ thus:

$$q(c_i) = Q(v_k) + Q(v_{k+1}) + Q(v_{k+2}) + \ldots - q[c_i(v_k)] \qquad (13)$$

Here, all the vehicles, $v_k, v_{k+1}, v_{k+2}, \ldots$ have to leave the depot for the customer directly. In order not to violate the time window priority, the vehicles might need to time their departure from the depot for the customer in order not to cluster at the customer's warehouse.

From figure 3, a customer, c_i, can only be linked to a split delivery priority only once such that:

$$PSD(c_i) = Q(v_k) + Q(v_{k+1}) - \sum_{i=1}^{N-n} q\left[c_i(v_k)\right] \qquad (14)$$

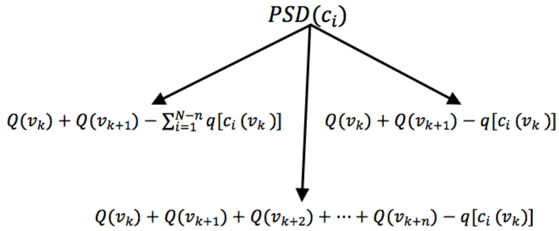

Fig. 3. Split Deliveries Priority Tree

or

$$PSD(c_i) = Q(v_k) + Q(v_{k+1}) - q[c_i(v_k)] \qquad (15)$$

or

$$PSD(c_i) = Q(v_k) + Q(v_{k+1}) + \ldots + Q(v_{k+n}) - q[c_i(v_k)] \qquad (16)$$

where $PSD(c_i)$ represents the priority of a customer. It must be put on record that, not every customer is bound to have priorities based on split deliveries.

D. Priorities Interplay Tree

Since only one set of time and quantity priorities conditions can be fulfilled by a customer, we discuss the interplay various priorities.

From the priority interplay tree Figure 4, if a customer, c_i, is not serviced, the Time Priority of the customer, $PT\left(c_i\right) = 0$ and the Quantity priority of the customer, $PQ\left(c_i\right) = 0$ else, $PT\left(c_i\right) = 1$ and $PQ\left(c_i\right) = q_i$. Thus, the sum of the time and quantity priorities for each of the customers is given by:

$$\delta\left(c_i\right) = PT\left(c_i\right) + PQ\left(c_i\right) \qquad (17)$$

while the sum of priorities by all the customers, $c_1,\ c_2,\ \ldots,\ c_N$ is given as:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{n=1}^{4} PT(c_i) + \sum_{n=1}^{4} PQ(c_i) \qquad (18)$$

By extension, if the quantity demanded by the customer, $q\left(c_i\right)$ exceeds the carrying capacity of the vehicle then, $PQ(c_i)$ will involve split deliveries, $PSD(c_i)$. Where splitting is involved, the sum of the time, quantity and split delivery for such customer(s) is given by:

$$\delta\left(c_i\right) = PT\left(c_i\right) + PQ\left(c_i\right) + PSD(c_i) \qquad (19)$$

and the sum of priorities by all the customers, $c_1,\ c_2,\ \ldots,\ c_N$ is given as:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{n=1}^{4} PT(c_i) + \sum_{n=1}^{4} PQ(c_i) + \sum_{n=1}^{3} PSD(c_i) \qquad (20)$$

In the next section we consider the inclusion of the priorities into the formulated VRP objective function. Figure 4 shows the interplay among the three priorities.

Comment on the Flow Chart

In figure 5, at the depot, c_0, all the vehicles, V, loaded to their carrying capacities, $Q(v_k)$, are ready for dispatch along their various routes, R_i, to deliver to their first respective $M \leq N$ customers. To effect this, the priority based on time of each of the customers, (4), is first decided to ascertain

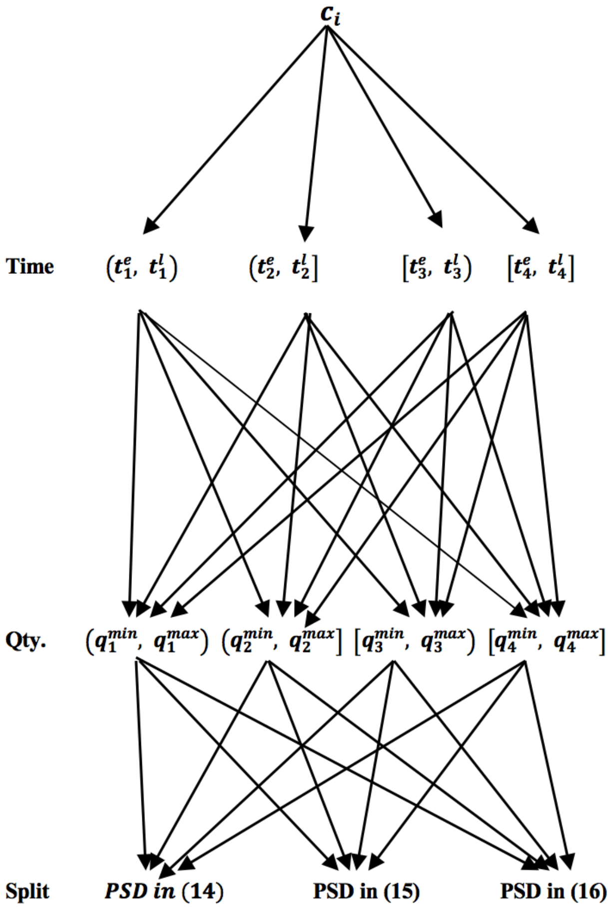

Fig. 4. Priorities Interplay Tree

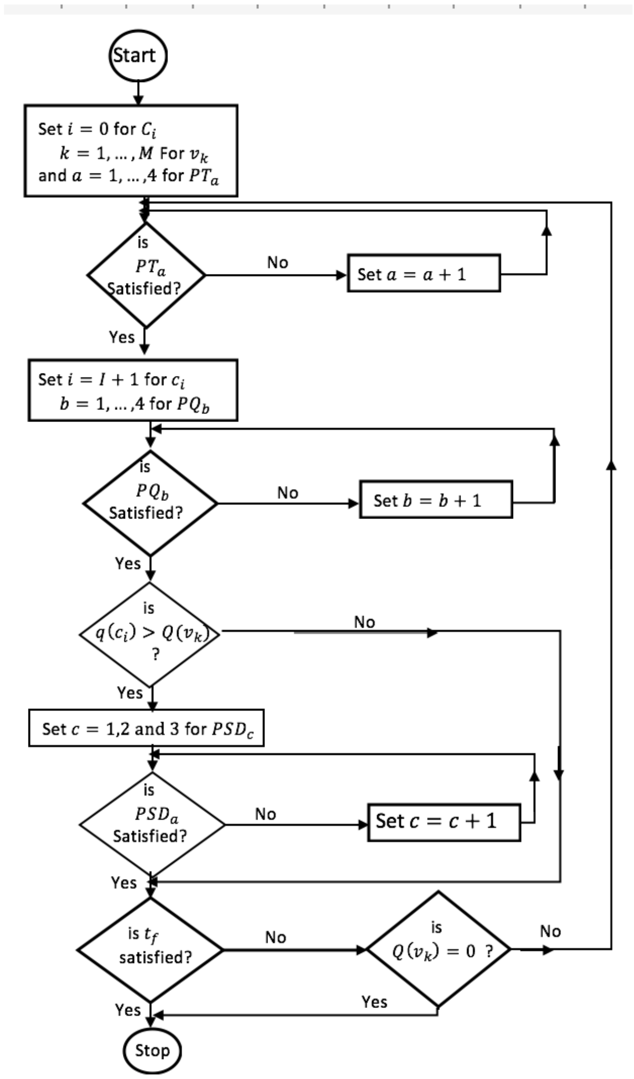

Fig. 5. Priorities Interplay Flow chart

which customer along the route is to be serviced first. Also, the quantity required, (5), by the customer is to be considered to assist in further decision making.

The dispatch manager evaluates the relationship between the $q(c_i)$ and $Q(v_k)$. If the customer's request, $q(c_i)$, is greater than the carrying capacity of the vehicle, $Q(v_k)$, then, splitting is sought for else, no need for splitting. Where splitting is necessary, a decision is made as to which of the splitting, (14), (15) or (16), will be most appropriate. After splitting or where a split is not involved after the first customer has been serviced, next is to check if the finish time of the vehicle, $T^f(v_k)$, has been reached. If yes then, the vehicle returns back to the depot else, checks again whether the carrying capacity of the vehicle, $Q(v_k)$, has been exhausted. If yes, it stops i.e. it returns back to the depot. Else, it returns back into the system to pick on the next customer, c_{i+1}. The loop stops when all the customers, $c_{i+1} = c_N$, have been serviced by one or more of the vehicles as the case may be.

IV. VRP Objective Function Formulation

Time Windows, Quantities and Split Deliveries priorities that arise in real-life situations in Vehicle Routing Problems is such a multi-objective problem with the series, $Min\ J = \sum_{x=1}^{3} J_x$.

In here, $Min\ J_1$ is meant to calculate the distance carrying cost of the customers, $Min\ J_2$ is aimed at computing the fixed cost of the vehicles and $Min\ J_3$ is targeted at evaluating the priorities of the customers.

If a vehicle, v_k, visits a customer $c_j = c_{i+1}$ immediately after visiting customer c_i then, $\xi_{ijk} = 1$ otherwise, $\xi_{ijk} = 0$. Such that:

$$Min\ J_1 = \alpha \sum_{i=0}^{N} \sum_{j=1}^{N} \left(d_{ij}(c_j) \sum_{k=1}^{M} \xi_{ijk} \right) \tag{21}$$

$$Min\ J_2 = \beta \sum_{k=1}^{M} \left(FC(v_k) \sum_{i=0}^{N} \sum_{j=1}^{N} \xi_{ijk} \right) \tag{22}$$

$$Min\ J_3 = \gamma \sum_{i=0}^{N} \left(\delta\left(c_i\right) \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \right) \tag{23}$$

where α, β and γ by [1] are specified constants for weighting the terms corresponding to each objective. On combining the three sub-objectives (21), (22) and (23) we obtain:

$$Min\ J = \sum_{x=1}^{3} J_x = J_1 + J_2 + J_3 \tag{24}$$

$$Min\ J = \alpha \sum_{i=0}^{N} \sum_{j=1}^{N} \left(d_{ij}(c_j) \sum_{k=1}^{M} \xi_{ijk} \right)$$

$$+\beta \sum_{k=1}^{M} \left(FC(v_k) \sum_{i=0}^{N} \sum_{j=1}^{N} \xi_{ijk} \right) \quad (25)$$

$$+ \gamma \sum_{i=0}^{N} \left(\delta\left(c_i\right) \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \right)$$

Subject to the following constraints:

$$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \leq 1, \ j = 1, \ldots, N \quad (26)$$

$$\sum_{i=0}^{N} \sum_{p=1}^{N} \sum_{k=1}^{M} \xi_{ipk} - \sum_{p=1}^{N} \sum_{j=2}^{N} \sum_{k=1}^{M} \xi_{pjk} = 0, \quad (27)$$

$$\sum_{i=0}^{N} \left(q\left(c_i\right) \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \right) \leq Q\left(v_k\right), \quad (28)$$

$$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} t_{ij} \xi_{ijk} \leq (T^f\left(v_k\right) - T^s\left(v_k\right)), \quad (29)$$

$$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \leq 1, \quad (30)$$

$$y_i - y_j + N \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \leq (N-1) \quad (31)$$

$$\xi_{ijk} \in \{0,1\} \ \ \forall \ i,j \ \ and \ \ k \quad (32)$$

$p = 1, \ \ldots, \ N$ and $\ i \neq j = 1, \ldots, N$
The constraint (26) states that a customer can be visited at most once by a vehicle. Constraint (27) states that if a vehicle visits a customer, it must also depart from it. Constraint (28) is the vehicle's carrying capacity on each route but where the $q(c_i) > Q(v_k)$, a split is required. The constraint (29) is the working time limit on each route. Constraint (30) states that a vehicle may be used at most once in a day. Where y_i is arbitrary, the relation (31) is the sub-tour-elimination condition derived for the Travelling Salesman Problem (TSP) by [26] and VRP as opined by [27] and [28]. This forces each route to pass through the depot. Constraint (32) is the integrality conditions.

If the aim of the VRP is to optimize priority alone then, the first and the second series in (24) will be set as zero while if the objective is to optimize the priority as well as the cost alone then, the second series in (24) is set as zero. But where all the objectives are to be computed then, (24) holds.

V. Conclusions and Future Work

Everyday economy situations characterized with multifarious changes have made it inevitable for Vehicle Routing Problems with Time Windows, Quantities and Split Deliveries (VRPTWQSD) to be encountered. So that, for a supplier, no customer will be lost to close competitors rather, more customers would be gained leading to increase in the profit margin and with a view to calculating the distance carrying cost, computing the fixed cost as well as evaluating the priorities so involved.

Solving this class of problems have become less difficult due to the advent and improvement in the information technology that brought about information interconnectivity between the depot, the vehicle, customers and other vehicles in the chain through the use of GSM, GPS and network facilities. Otherwise, it would have been a mirage and unattainable. Problems of these nature are aimed at being solved in subsequent publication to which we intend modifying existing methods with a view to improving the results, widening the range of method used, consolidating other researchers working on these classes of problems and translating these to our everyday life.

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