Event-triggered Finite-time Stabilization of a Class of Uncertain Nonlinear Switched Systems with Delay

Mengxiao Deng, Yali Dong* and Mengying Ding

***Abstract*—In this paper, we investigate the problem of event-triggered finite-time stabilization for a class of uncertain nonlinear switched systems with time-varying delay. Firstly, we propose an event-triggered sampling mechanism and an event-triggered state feedback controller. By using the Lyapunov-Krasovskii functional method and the free-weight matrix method, we give new sufficient conditions of the finite-time stabilization for nonlinear switched systems under event-triggered state feedback control. The design method of controller gain matrix is given. Finally, a numerical example is provided to illustrate the effectiveness of the obtained results.**

***Index Terms*—Switched systems, finite-time stabilization, uncertain, time-varying delay, event-triggered controller.**

I. Introduction

THE switched system is composed of several subsystems and switched strategies acting on them. Stability and control design are very important topics in the theoretical study of switched systems, and many results have been reported [1-3]. In [2], Liu et al. investigated stability and stabilization of nonlinear switched systems under average dwell time. In [3], Dong et al. dealt with exponential stabilization and L_2 gain for uncertain switched nonlinear systems.

The research on stability of switched systems generally focuses on Lyapunov stability. Not much attention was paid to finite-time stability. However, for some systems that operate in a finite time, it is extremely important to study the finite time stability [4-8]. In [5], the problem of the finite time control for switched stochastic nonlinear systems was considered. In [6], Wang et al. studied the finite-time stability and stabilization of switched nonlinear systems with asynchronous switching. In [7], Chen et al. considered the finite time stability for switched systems with stable and unstable subsystems.

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Because of external disturbances, time delays and uncertainties often exist in actual physical systems, which leads to instability and degradation of some performance. The stability and stabilization of switched systems with uncertainties, time delays and external disturbances have become important research topics [8-12]. In [8], the authors designed the finite-time control for switched linear systems with interval time-delay. The stability of a class of switched linear systems with uncertainties and averaged well time switching was considered in [9]. In [10], Shen et al. gave the stability analysis for uncertain switched neural networks with time-varying delay. In [11], Lin investigated the problem of stabilization for LTI switched systems with input time delay.

An event-triggered mechanism strategy is different from the time trigger mechanism: it controls the system by designing an event-triggered controller. The advantage of event-triggered controller is that it can effectively improve the utilization rate of system resources by reducing the sampling update and network communication frequency of controller. It can also save system resources by reducing unnecessary sampling and calculation. In recent years, scholars have become more and more interested in switched systems and event-triggered control. Therefore, a number of research results have been reported [13-15]. In [13], Ma et al. considered the finite-time event-triggered H_∞ control for switched systems with time-varying delay. In [14], Liu et al. designed the event-triggered controller for nonlinear systems with state quantization. In [15], Zhang et al. studied the event-triggered control for networked switched fuzzy system based on average dwell time. However, to the fullest of our authors' knowledge, the problem of event-triggered finite-time stabilization for uncertain nonlinear switched systems with time-varying delay has yet to be explored.

In this paper, we investigate the problem of finite-time stabilization for nonlinear uncertain switched systems with time-varying delay by constructing an event-triggered controller. We give the sufficient conditions for the finite-time stabilization of switched system and design the event-triggered controller gain by using matrix inequality technique and averaged dwell time method.

The remainder of this paper is organized as follows. Section II gives the model description and preliminaries. In Section III, the new criteria are obtained to ensure the finite-time stabilization of uncertain switched systems with time-varying delay. Section IV gives a numerical example to illustrate the validity of the results. Section V summarizes the paper.

Notation. N represents a set of natural numbers. $R^{n\times n}$ denotes the set of $n \times n$ real matrices. A^T and A^{-1} are the transpose and inverse of A, respectively. $X<0$ ($X \le 0$) means X is a negative definite (semi-negative definite) matrix. * is the terms below the main diagonal of a symmetric matrix.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following switched nonlinear system with time-varying delay:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + A_{d\sigma(t)}(t)x(t-\tau(t)) + B_{\sigma(t)}u(t) \\ \qquad + f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t-\tau(t))), \\ x(t) = \varphi(t) \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input vector, $\tau(t)$ represents the time-varying delay which satisfies

$$0 < \tau(t) < \tau, \quad 0 < \dot{\tau}(t) < \hat{\tau} < 1.$$

$\varphi(t) \in C^1([0,\tau], R^n)$ is a continuous initial function. $\sigma(t):[0,\infty) \to M = \{1,2,\cdots,\bar{N}\}$ is the switching signal. $\{(i_0,t_0),(i_1,t_1),\cdots,(i_k,t_k),\cdots,| i_k \in M, k=0,1,\cdots\}$ is a switching sequence, where t_0 is the initial time instant, and t_k is the kth switching instant. When $t \in [t_k, t_{k+1})$, the $i_k th$ subsystem is activated.

$$A_{\sigma(t)}(t) = A_{\sigma(t)} + \Delta A_{\sigma(t)}(t),$$
$$A_{d\sigma(t)}(t) = A_{d\sigma(t)} + \Delta A_{d\sigma(t)}(t),$$

where A_i, A_{di}, B_i are known real-valued matrices, and the matrix B_i has a full column rank. $\Delta A_{\sigma(t)}(t)$ and $\Delta A_{d\sigma(t)}(t)$ are the time-varying parametric uncertainties satisfying

$$\left[\Delta A_i(t) \quad \Delta A_{di}(t)\right] = H_i F_i(t)[E_{1i} \quad E_{2i}], \tag{2}$$

where H_i, E_{1i}, E_{2i} are known constant matrices. $F_i(t)$ is an unknown real-valued matrix function which satisfies

$$F_i^T(t)F_i(t) \le I, \quad \forall t > 0.$$

$f_i(x(t))$ and $g_i(x(t-\tau(t)))$ $(i \in M)$ are nonlinear functions with $f_i(0)=0,\ g_i(0)=0$ $(i \in M)$, and satisfy

$$\begin{aligned} \left\| f_i(\gamma) - f_i(\beta) \right\| &\le \left\| U_i(\gamma-\beta) \right\|, \forall \gamma, \beta \in R^n, \\ \left\| g_i(\gamma) - g_i(\beta) \right\| &\le \left\| V_i(\gamma-\beta) \right\|, \forall \gamma, \beta \in R^n, \end{aligned} \tag{3}$$

where $U_i, V_i \in R^{n \times n}$ are known constant matrices.

Lemma 1 [16]. Let X, U be real matrices with appropriate dimensions, and a matrix $F(t)$ satisfies $F^T(t)F(t) \le I$. Then for any $v>0$, the following inequality holds

$$XF(t)U + \left[XF(t)U\right]^T \le v^{-1}XX^T + vU^TU.$$

Definition 1. Given time instants $\underline{T}$ and $\bar{T}$ with $0 \le \underline{T} \le \bar{T}$, let $N_\sigma(\underline{T},\bar{T})$ indicate the switching number of $\sigma(t)$ over $(\underline{T},\bar{T})$. If

$$N_\sigma\left(\underline{T},\bar{T}\right) \ge N_0 + (\bar{T}-\underline{T})/\tau_a,$$

holds for constants $N_0 \ge 0, \tau_a \ge 0$, then τ_a is called an average dwell time and N_0 is called the chattering bound. Without loss of generality, we choose $N_0 = 0$ in this paper.

Consider the following system

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + A_{d\sigma(t)}(t)x(t-\tau(t)) \\ \qquad + f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t-\tau(t))), \\ x(t) = \varphi(t) \quad t \in [-\tau, 0], \end{cases} \tag{4}$$

Definition 2. [17] Given a positive definite matrix R, and three positive constants T, c_1, c_2 with $c_1 < c_2$, and a switching signal $\sigma(t)$. If

$$\begin{aligned} &\sup_{-\tau \le s \le 0} \{x^T(s)Rx(s), \dot{x}^T(s)R\dot{x}(s)\} \le c_1 \\ &\Rightarrow \quad x^T(t)Rx(t) < c_2, \forall t \in [0,T], \end{aligned} \tag{5}$$

holds, then system (4) is said to be finite-time stable with respect to (c_1, c_2, T, R, σ).

III. MAIN RESULTS

We develop an event-triggered mechanism:

$$\|e(t)\|^2 \ge \eta \|x(t)\|^2, \tag{6}$$

where $e(t) = x(\hat{t}_s) - x(t)$, $0<\eta<1$ is a given event threshold valve. $\{\hat{t}_s\}_{k=0}^\infty$ is a sequence of event-triggered instants with $\hat{t}_s < \hat{t}_{s+1}$. The event-triggered instant is determined by the designed event-triggered mechanism (6). We denote $x(\hat{t}_s)$ as the sampling state of the triggered instant $\hat{t}_s$, then the next sampling instant is determined by

$$\hat{t}_{s+1} = \inf\{t > \hat{t}_s \mid \|e(t)\|^2 \ge \eta \|x(t)\|^2\}. \tag{7}$$

For any $t \in [t_k, t_{k+1})$, the state feedback controller can be designed as follows:

$$u(t) = K_{\sigma(t)} x(\hat{t}_s), \tag{8}$$

where K_i is the control gain of the ith subsystem. Suppose that there is no Zeno behavior.

For the convenience of discussion, let

$$\bar{P}_i = R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}, \bar{S}_i = R^{-\frac{1}{2}} S_i R^{-\frac{1}{2}}, \bar{R}_i = R^{-\frac{1}{2}} R_i R^{-\frac{1}{2}},$$
$$\bar{Z}_i = R^{-\frac{1}{2}} Z_i R^{-\frac{1}{2}}, \lambda_1 = \lambda_{\min}(\bar{P}_i), \lambda_2 = \lambda_{\max}(\bar{P}_i),$$
$$\lambda_3 = \lambda_{\max}(\bar{R}_i), \lambda_4 = \lambda_{\max}(\bar{S}_i), \lambda_5 = \lambda_{\max}(\bar{Z}_i).$$

Now, we have the following theorem.

Theorem 1. Given a positive definite matrix R, and positive scalars c_1, c_2, T with $c_1 < c_2$, the system (4) is finite-time stable with respect to $(c_1, c_2, T, R, \sigma(t))$ for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln(\lambda_1 c_2) - \ln(c_1(\lambda_2 + \tau\lambda_3 + \tau\lambda_4 + \frac{\tau^2}{2}\lambda_5)) - \alpha T}, \tag{9}$$

if there exist positive definite matrices $P_i, R_i, S_i, Z_i, i \in M$, any matrices $W_{1i}, W_{2i}, T_{1i}, T_{2i}$, and positive scalars $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \varepsilon_1, \varepsilon_2, \mu \ge 1,\ \forall i, j \in M$ such that

$$\Sigma_i = \begin{bmatrix} \Sigma_i^{11} & \Sigma_i^{12} \\ * & \Sigma_i^{22} \end{bmatrix} < 0, \tag{10}$$

$$P_i \le \mu P_j, R_i \le \mu R_j, S_i \le \mu S_j, Z_i \le \mu Z_j, \forall i, j \in M, \tag{11}$$

$$c_1(\lambda_2 + \tau\lambda_3 + \tau\lambda_4 + \frac{\tau^2}{2}\lambda_5) < \lambda_1 c_2 e^{-\alpha T}, \tag{12}$$

where

$$\Sigma_i^{11}=\begin{bmatrix}\bar{\Sigma}_i^{11} & A_i^T(t)P_i & \bar{\Sigma}_i^{13} & -T_{1i} & P_i\\ * & -2P_i & P_iA_{di}(t) & 0 & P_i\\ * & * & \bar{\Sigma}_i^{33} & -T_{2i} & 0\\ * & * & * & -R_i & 0\\ * & * & * & * & -\varepsilon_1 I\end{bmatrix},$$

$$\Sigma_i^{12}=\begin{bmatrix}P_i & 0 & 0 & 0 & W_{1i} & T_{1i}\\ P_i & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & W_{2i} & T_{2i}\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Sigma_i^{22}=diag(-\varepsilon_2 I,-\frac{\alpha}{\tau}R_i,-\frac{\alpha}{\tau}S_i,\bar{\Sigma}_i^{99},-\frac{1}{\tau}Z_i,-\frac{1}{\tau}Z_i),$$

$$\bar{\Sigma}_i^{11}=P_iA_i(t)+A_i^T(t)P_i+R_i+S_i+W_{1i}+W_{1i}^T+\varepsilon_1U_i^TU_i-\alpha P_i,$$

$$\bar{\Sigma}_i^{13}=P_iA_{di}(t)-W_{1i}+W_{2i}^T+T_{1i},$$

$$\bar{\Sigma}_i^{33}=-(1-\hat{\tau})S_i-W_{2i}^T-W_{2i}+T_{2i}^T+T_{2i}+\varepsilon_2V_i^TV_i,$$

$$\bar{\Sigma}_i^{99}=-\frac{2\alpha}{\tau^2}Z_i.$$

Proof. Choose the following multiple Lyapunov-Krasovskii functionals:

$$V_i(t)=V_{1i}(t)+V_{2i}(t)+V_{3i}(t), \tag{13}$$

where

$$V_{1i}(t)=x^T(t)P_ix(t)$$

$$V_{2i}(t)=\int_{t-\tau}^{t}x^T(s)R_ix(s)ds+\int_{t-\tau(t)}^{t}x^T(s)S_ix(s)ds$$

$$V_{3i}(t)=\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_i\dot{x}(s)dsd\theta.$$

Calculating the derivative of (13) along the trajectory of (4), we have

$$\begin{aligned}\dot{V}_i(t)-\alpha V_i(t)\le{}& x^T(t)(P_iA_i(t)+A_i^T(t)P_i)x(t)\\ &+x^T(t)P_iA_{di}(t)x(t-\tau(t))+x^T(t-\tau(t))A_{di}^T(t)P_i\\ &\times x(t)+2x^T(t)P_if(x(t))+2x^T(t)P_ig(x(t-\tau(t)))\\ &+x^T(t)R_ix(t)-x^T(t-\tau)R_ix(t-\tau)+x^T(t)S_ix(t)\\ &+\tau\dot{x}^T(t)Z_i\dot{x}(t)-(1-\hat{\tau})x^T(t-\tau(t))S_ix(t-\tau(t))\\ &-\alpha x^T(t)P_ix(t)-\int_{t-\tau}^{t}\dot{x}^T(s)Z_i\dot{x}(s)ds\\ &-\alpha\int_{t-\tau}^{t}x^T(s)R_ix(s)ds-\alpha\int_{t-\tau(t)}^{t}x^T(s)S_ix(s)ds\\ &-\alpha\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_i\dot{x}(s)dsd\theta.\end{aligned} \tag{14}$$

For any matrices $W_{1i},W_{2i},T_{1i},T_{2i}$ with appropriate dimensions, one can get

$$\begin{aligned}&2[x^T(t)W_{1i}+x^T(t-\tau(t))W_{2i}]\\ &\quad\times[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]=0,\end{aligned} \tag{15}$$

$$\begin{aligned}&2[x^T(t)T_{1i}+x^T(t-\tau(t))T_{2i}]\\ &\quad\times[x(t-\tau(t))-x(t-\tau)-\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds]=0.\end{aligned} \tag{16}$$

In addition, note that

$$\begin{aligned}-\int_{t-\tau}^{t}\dot{x}^T(s)Z_i\dot{x}(s)ds={}&-\int_{t-\tau(t)}^{t}\dot{x}^T(s)Z_i\dot{x}(s)ds\\ &-\int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)Z_i\dot{x}(s)ds.\end{aligned} \tag{17}$$

From (3), one has

$$\begin{aligned}&f_i^T(x(t))f_i(x(t))\le x^T(t)U_i^TU_ix(t),\\ &g_i^T(x(t-\tau(t)))g_i(x(t-\tau(t)))\le x^T(t-\tau(t))V_i^TV_ix(t-\tau(t)).\end{aligned} \tag{18}$$

It then follows from Jensen's inequality and (14)-(18) that

$$\begin{aligned}\dot{V}_i(t)-\alpha V_i(t)\le{}& x^T(t)(P_iA_i(t)+A_i^T(t)P_i)x(t)\\ &+x^T(t)P_iA_{di}(t)x(t-\tau(t))+x^T(t-\tau(t))A_{di}^T(t)P_i\\ &\times x(t)+2x^T(t)P_if_i(x(t))+x^T(t)R_ix(t)\\ &+2x^T(t)P_ig_i(x(t-\tau(t)))-x^T(t-\tau)R_ix(t-\tau)\\ &+x^T(t)S_ix(t)-(1-\hat{\tau})x^T(t-\tau(t))S_ix(t-\tau(t))\\ &-\int_{t-\tau(t)}^{t}\dot{x}^T(s)Z_i\dot{x}(s)ds-\int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)Z_i\dot{x}(s)ds\\ &-\alpha x^T(t)P_ix(t)-\frac{\alpha}{\tau}\int_{t-\tau}^{t}x^T(s)dsR_i\int_{t-\tau}^{t}x(s)ds\\ &-\frac{\alpha}{\tau}\int_{t-\tau(t)}^{t}x^T(s)dsS_i\int_{t-\tau(t)}^{t}x(s)ds+\tau\dot{x}^T(t)Z_i\dot{x}(t)\\ &-\frac{2\alpha}{\tau^2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta Z_i\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta\\ &+2[x^T(t)W_{1i}+x^T(t-\tau(t))W_{2i}][x(t)\\ &-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]+2[x^T(t)T_{1i}\\ &+x^T(t-\tau(t))T_{2i}][x(t-\tau(t))-x(t-\tau)\\ &-\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds]+2\dot{x}^T(t)P_i[A_i(t)x(t)\\ &+A_{di}(t)x(t-\tau(t))+f_i(x(t))+g_i(x(t-\tau(t)))\\ &-\dot{x}(t)]+\varepsilon_1(x^T(t)U_i^TU_ix(t)-f_i^T(x(t))f_i(x(t)))\\ &+\varepsilon_2(x^T(t-\tau(t))V_i^TV_ix(t-\tau(t))\\ &-g_i^T(x(t-\tau(t)))g_i(x(t-\tau(t))))\\ \le{}&\xi^T(t)(\Gamma_i+\tau W_iZ_i^{-1}W_i^T+\tau T_iZ_i^{-1}T_i^T)\xi(t)\\ &-\frac{\alpha}{\tau}\int_{t-\tau}^{t}x^T(s)dsR_i\int_{t-\tau}^{t}x(s)ds\\ &-\frac{\alpha}{\tau}\int_{t-\tau(t)}^{t}x^T(s)dsS_i\int_{t-\tau(t)}^{t}x(s)ds\\ &-\frac{2\alpha}{\tau^2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta Z_i\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta\\ &-\int_{t-\tau(t)}^{t}[\xi^T(t)W_i+\dot{x}^T(s)Z_i]Z_i^{-1}[W_i^T\xi(t)\\ &+Z_i\dot{x}(s)]ds-\int_{t-\tau}^{t-\tau(t)}[\xi^T(t)T_i+\dot{x}^T(s)Z_i]Z_i^{-1}\\ &\times[T_i^T\xi(t)+Z_i\dot{x}(s)]ds,\end{aligned} \tag{19}$$

where

$$\begin{aligned}\xi(t)=[&x^T(t),\dot{x}^T(t),x^T(t-\tau(t)),x^T(t-\tau),f_i^T(x(t)),\\ &g_i^T(x(t-\tau(t))),\int_{t-\tau}^{t}x^T(s)ds,\int_{t-\tau(t)}^{t}x^T(s)ds,\\ &\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta]^T,\end{aligned}$$

$$W_i=[W_{1i}^T,0,W_{2i}^T,0,0,0,0,0,0]^T,$$

$$T_i=[T_{1i}^T,0,T_{2i}^T,0,0,0,0,0,0]^T,$$

$$\Gamma_i=\begin{bmatrix}\Gamma_i^{11} & \Gamma_i^{12}\\ * & \Gamma_i^{22}\end{bmatrix},$$

$$\varGamma_i^{12}=\begin{bmatrix} P_i & 0 & 0 & 0 \\ P_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\varGamma_i^{11}=\begin{bmatrix} \bar{\varSigma}_i^{11} & A_i^T(t)P_i & \bar{\varSigma}_i^{13} & -T_{1i} & P_i \\ * & -2P_i & P_iA_{di}(t) & 0 & P_i \\ * & * & \bar{\varSigma}_i^{33} & -T_{2i} & 0 \\ * & * & * & -R_i & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix},$$

$$\varGamma_i^{22}=diag(-\varepsilon_2 I,-\frac{\alpha}{\tau}R_i,-\frac{\alpha}{\tau}S_i,-\frac{2\alpha}{\tau^2}Z_i).$$

Using Schur complement, it follows from (10) that

$$\Gamma_i+\tau W_iZ_i^{-1}W_i^T+\tau T_iZ_i^{-1}T_i^T<0.$$

So, we have

$$\dot{V}_i(t)-\alpha V_i(t)<0. \tag{20}$$

Suppose that at the switching instant t_k, $\sigma(t_k)=i$, $\sigma(t_k^-)=j$, $\forall i,j\in M$. It follows from (11) and (13) that,

$$V(t_k)\le \mu V(t_k^-).$$

When $t\in[t_k,t_{k+1})$, multiplying (20) both sides by $e^{-\alpha t}$, one has

$$e^{-\alpha t}\dot{V}_i(t)<e^{-\alpha t}\alpha V_i(t). \tag{21}$$

Integrating (21) from t_k to t, yields

$$\begin{aligned} V(t)&\le e^{\alpha(t-t_k)}V(t_k)\\ &\le \mu e^{\alpha(t-t_k)}V(t_k^-)\\ &\le\cdots\\ &\le \mu^N e^{\alpha T}V(0), \end{aligned} \tag{22}$$

where N represents the switching number of $\sigma(t)$ over $[0,T]$. Since $N\ge T/\tau_a$, it follows that

$$V(t)<\mu^{\frac{T}{\tau_a}}e^{\alpha T}V(0).$$

From (13), one has

$$V(t)\ge\lambda_{\min}(\bar{P}_i)x^T(t)Rx(t)=\lambda_1x^T(t)Rx(t), \tag{23}$$

and

$$\begin{aligned} V(0)&=x^T(0)P_ix(0)+\int_{-\tau}^0 x^T(s)R_ix(s)ds\\ &\quad+\int_{-\tau(0)}^0 x^T(s)S_ix(s)ds+\int_{-\tau}^0\int_\theta^0 \dot{x}^T(s)Z_i\dot{x}(s)dsd\theta\\ &\le\lambda_{\max}(\bar{P}_i)x^T(0)Rx(0)+\tau\lambda_{\max}(\bar{S}_i)\sup_{-\tau\le\theta\le0}\left\{x^T(\theta)Rx(\theta)\right\}\\ &\quad+\tau\lambda_{\max}(\bar{R}_i)\sup_{-\tau\le\theta\le0}\left\{x^T(\theta)Rx(\theta)\right\}\\ &\quad+\frac{\tau^2}{2}\lambda_{\max}(\bar{Z}_i)\sup_{-\tau\le\theta\le0}\left\{\dot{x}^T(\theta)R\dot{x}(\theta)\right\}\\ &\le(\lambda_2+\tau\lambda_3+\tau\lambda_4+\frac{\tau^2}{2}\lambda_5)c_1. \end{aligned} \tag{24}$$

From (22), (23) and (24), we get

$$\begin{aligned} x^T(t)Rx(t)&<\frac{\mu^{T/\tau_a}e^{\alpha T}V(0)}{\lambda_1}\\ &<\frac{\mu^{T/\tau_a}e^{\alpha T}(\lambda_2+\tau\lambda_3+\tau\lambda_4+\frac{\tau^2}{2}\lambda_5)c_1}{\lambda_1}. \end{aligned} \tag{25}$$

When $\mu=1$, from (12), we have

$$\begin{aligned} x^T(t)Rx(t)&\le\frac{V(t)}{\lambda_1}\\ &<\frac{e^{\alpha T}(\lambda_2+\tau\lambda_3+\tau\lambda_4+\frac{\tau^2}{2}\lambda_5)c_1}{\lambda_1}<c_2. \end{aligned} \tag{26}$$

While $\mu>1$, from (9), we have

$$\frac{T}{\tau_a}<\frac{\chi}{\ln\mu}. \tag{27}$$

Therefore, it can be obtained that

$$\begin{aligned} x^T(t)Rx(t)&<\frac{(\lambda_2+\tau\lambda_3+\tau\lambda_4+\frac{\tau^2}{2}\lambda_5)c_1}{\lambda_1}e^{\alpha T}\\ &\quad\times\frac{\lambda_1c_2}{(\lambda_2+\tau\lambda_3+\tau\lambda_4+\frac{\tau^2}{2}\lambda_5)c_1}e^{-\alpha T}=c_2. \end{aligned} \tag{28}$$

Based on Definition 2, the system (4) is finite-time stable with respect to $(c_1,c_2,T,R,\sigma(t))$. This completes the proof.

Remark 1. The lower bound of the average dwell time can be estimated by the condition (9) in Theorem 1.

When $\Delta A_{\sigma(t)}(t)=\Delta A_{d\sigma(t)}(t)=0$, the switched delay system (4) can be written as follows:

$$\begin{cases} \dot{x}(t)=A_{\sigma(t)}x(t)+A_{d\sigma(t)}x(t-\tau(t))+f_{\sigma(t)}(x(t))\\ \qquad+g_{\sigma(t)}(x(t-\tau(t))),\\ x(t)=\varphi(t),\quad t\in[-\tau,0]. \end{cases} \tag{29}$$

Corollary 1. Given a positive definite matrix R, and positive scalars c_1,c_2,T with $c_1<c_2$, the system (29) is finite-time stable with respect to $(c_1,c_2,T,R,\sigma(t))$ for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying (9), if there exist positive definite matrices P_i,R_i,S_i,Z_i, $i\in M$, any matrices $W_{1i},W_{2i},T_{1i},T_{2i}$, and positive scalars $\alpha,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\varepsilon_1,\varepsilon_2,\mu\ge1$, such that (11), (12) and the following inequality hold for any $i,j\in M$,

$$\hat{\Sigma}_i=\begin{bmatrix}\hat{\Sigma}_i^{11} & \hat{\Sigma}_i^{12}\\ * & \hat{\Sigma}_i^{22}\end{bmatrix}<0, \tag{30}$$

where

$$\hat{\varSigma}_i^{11}=\begin{bmatrix} \tilde{\varSigma}_i^{11} & A_i^TP_i & \tilde{\varSigma}_i^{13} & -T_{1i} & P_i \\ * & -2P_i & P_iA_{di} & 0 & P_i \\ * & * & \bar{\varSigma}_i^{33} & -T_{2i} & 0 \\ * & * & * & -R_i & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix},$$

$$\hat{\Sigma}_i^{12} = \begin{bmatrix} P_i & 0 & 0 & 0 & W_{1i} & T_{1i} \\ P_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_{2i} & T_{2i} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\hat{\Sigma}_i^{22} = diag(-\varepsilon_2 I, -\frac{\alpha}{\tau}R_i, -\frac{\alpha}{\tau}S_i, -\frac{2\alpha}{\tau^2}Z_i, -\frac{1}{\tau}Z_i, -\frac{1}{\tau}Z_i),$$

$$\tilde{\Sigma}_i^{11} = P_i A_i + A_i^T P_i + R_i + S_i + W_{1i} + W_{1i}^T + \varepsilon_1 U_i^T U_i - \alpha P_i,$$

$$\tilde{\Sigma}_i^{13} = P_i A_{di} - W_{1i} + W_{2i}^T + T_{1i},$$

$$\bar{\Sigma}_i^{33} = -(1-\hat{\tau})S_i - W_{2i}^T - W_{2i} + T_{2i} + T_{2i}^T + \varepsilon_2 V_i^T V_i.$$

Because Theorem 1 contains uncertainties, it is inconvenient to apply. Next, an improved result of Theorem 1 is given.

Theorem 2. Given a positive definite matrix *R*, and positive scalars c_1, c_2, T with $c_1 < c_2$, the system (4) is finite-time stable with respect to $(c_1, c_2, T, R, \sigma(t))$ for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying (9), if there exist positive definite matrices P_i, R_i, S_i, Z_i, any matrices $W_{1i}, W_{2i}, T_{1i}, T_{2i}$, and positive scalars $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$, $\varepsilon_1, \varepsilon_2, \delta_1, \delta_2, \mu \geq 1$, such that (11), (12) and the following inequality hold for any $i, j \in M$,

$$\Lambda_i = \begin{bmatrix} \Lambda_i^{11} & \Lambda_i^{12} \\ * & \Lambda_i^{22} \end{bmatrix} < 0, \qquad (31)$$

where

$$\Lambda_i^{11} = \begin{bmatrix} \tilde{\Sigma}_i^{11} & A_i^T P_i & \tilde{\Sigma}_i^{13} & -T_{1i} & P_i & P_i \\ * & -2P_i & P_i A_{di} & 0 & P_i & P_i \\ * & * & \bar{\Sigma}_i^{33} & -T_{2i} & 0 & 0 \\ * & * & * & -R_i & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix},$$

$$\Lambda_i^{12} = \begin{bmatrix} 0 & 0 & 0 & W_{1i} & T_{1i} & E_{1i}^T & P_i H_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_i H_i \\ 0 & 0 & 0 & W_{2i} & T_{2i} & E_{2i}^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_i^{22} = (-\frac{\alpha}{\tau}R_i, -\frac{\alpha}{\tau}S_i, -\frac{2\alpha}{\tau^2}Z_i, -\frac{1}{\tau}Z_i, -\frac{1}{\tau}Z_i, -(\delta_1+\delta_2)^{-1}I, -\delta_1 I, -\delta_2 I),$$

$$\tilde{\Sigma}_i^{11} = P_i A_i + A_i^T P_i + R_i + S_i + W_{1i} + W_{1i}^T + \varepsilon_1 U_i^T U_i - \alpha P_i,$$

$$\tilde{\Sigma}_i^{13} = P_i A_{di} - W_{1i} + W_{2i}^T + T_{1i},$$

$$\bar{\Sigma}_i^{33} = -(1-\hat{\tau})S_i - W_{2i}^T - W_{2i} + T_{2i} + T_{2i}^T + \varepsilon_2 V_i^T V_i.$$

Proof. From (2), the condition $\Sigma_i < 0$ in (10) can be rewritten as

$$\begin{aligned} \Sigma_i = \hat{\Sigma}_i + \Omega_{1i}^T F_i(t)\Phi_i + \Phi_i^T F_i^T(t)\Omega_{1i} \\ + \Omega_{2i}^T F_i(t)\Phi_i + \Phi_i^T F_i^T(t)\Omega_{2i} < 0, \end{aligned} \qquad (32)$$

where

$$\Omega_{1i}^T = [H_i^T P_i^T, 0,0,0,0,0,0,0,0,0,0,0]^T,$$

$$\Omega_{2i}^T = [0, H_i^T P_i^T, 0,0,0,0,0,0,0,0,0,0]^T,$$

$$\Phi_i = [E_{1i}, 0, E_{2i}, 0,0,0,0,0,0,0,0,0].$$

Using Lemma 1, the inequality (32) holds if

$$\hat{\Sigma}_i + \delta_1^{-1}\Omega_{1i}^T\Omega_{1i} + \delta_2^{-1}\Omega_{2i}^T\Omega_{2i} + (\delta_1+\delta_2)\Phi_i^T\Phi_i < 0. \quad (33)$$

Furthermore, by using Schur complement, (33) holds if

$$\begin{bmatrix} \hat{\Sigma}_i + \delta_1^{-1}\Omega_{1i}^T\Omega_{1i} + \delta_2^{-1}\Omega_{2i}^T\Omega_{2i} & \Phi_i^T \\ * & -(\delta_1+\delta_2)^{-1} \end{bmatrix} < 0. \quad (34)$$

Then, by Schur complement and (31), we can obtain (34). Therefore, according to Theorem 1, the system (4) is finite-time stable with respect to $(c_1, c_2, T, R, \sigma(t))$.

Under the controller (8), the closed-loop system is given by:

$$\begin{aligned} \dot{x}(t) &= (A_{\sigma(t)}(t) + B_{\sigma(t)}K_{\sigma(t)})x(t) + f_{\sigma(t)}(x(t)) \\ &\quad + A_{d\sigma(t)}(t)x(t-\tau(t)) + g_{\sigma(t)}(x(t-\tau(t))) \\ &\quad + B_{\sigma(t)}K_{\sigma(t)}e(t). \end{aligned} \qquad (35)$$

Definition 3. Given positive constants c_1, c_2, T with $c_1 < c_2$, a positive definite matrix *R* and a switching signal $\sigma(t)$. If the following condition is satisfied

$$\sup_{-\tau \leq s \leq 0} \{x^T(s)Rx(s), \dot{x}^T(s)R\dot{x}(s)\} \leq c_1$$
$$\Rightarrow \quad x^T(t)Rx(t) < c_2, \forall t \in [0,T],$$

then system (1) under the event-triggered controller (8) is said to be finite-time stabilizable with respect to $(c_1, c_2, T, R, \sigma(t))$.

Theorem 3. Given a positive definite matrix *R*, and positive scalars c_1, c_2, T with $c_1 < c_2$, the system (1) is finite-time stabilizable with respect to $(c_1, c_2, T, R, \sigma(t))$ for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying (9), if there exist positive definite matrices P_i, R_i, S_i, Z_i, any matrices $X_i, W_{1i}, W_{2i}, T_{1i}, T_{2i}$, and positive scalars $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4$, $\lambda_5, \varepsilon_1, \varepsilon_2, \delta_1, \delta_2, \mu \geq 1$, such that (11), (12) and the following inequality hold for any $i, j \in M$,

$$\bar{\Lambda}_i = \begin{bmatrix} \Pi_i & \Theta_i \\ * & \Psi_i \end{bmatrix} < 0, \qquad (36)$$

where

$$\Pi_i = \Pi_i^T = (\Pi_i^{jk})_{8\times 8}, \Theta_i = \Theta_i^T = (\Theta_i^{jk})_{8\times 8},$$

$$\Psi_i = \Psi_i^T = (\Psi_i^{jk})_{8\times 8},$$

$$\begin{aligned} \Pi_i^{11} &= P_i A_i + A_i^T P_i + R_i + S_i + W_{1i} + W_{1i}^T + \varepsilon_1 U_i^T U_i \\ &\quad - \alpha P_i + B_i X_i^T + X_i B_i^T + 2\eta I \end{aligned}$$

$$\Pi_i^{12} = A_i^T P_i + X_i B_i^T, \Pi_i^{13} = P_i A_{di} - W_{1i} + W_{2i}^T + T_{1i},$$

$$\Pi_i^{14} = -T_{1i}, \Pi_i^{15} = P_i, \Pi_i^{16} = P_i, \Pi_i^{22} = -2P_i,$$

$$\Pi_i^{23} = P_i A_{di}, \Pi_i^{25} = P_i, \Pi_i^{26} = P_i, \Pi_i^{34} = -T_{2i},$$

$$\Pi_i^{33} = -(1-\hat{\tau})S_i - W_{2i}^T - W_{2i} + T_{2i} + T_{2i}^T + \varepsilon_2 V_i^T V_i,$$

$$\Pi_i^{44} = -R_i, \Pi_i^{55} = -\varepsilon_1 I, \Pi_i^{66} = -\varepsilon_2 I,$$

$$\Pi_i^{77} = -\frac{\alpha}{\tau}R_i, \Pi_i^{88} = -\frac{\alpha}{\tau}S_i, \Psi_i^{11} = -\frac{2\alpha}{\tau^2}Z_i,$$

$$\Psi_i^{22} = -\frac{1}{\tau}Z_i, \quad \Psi_i^{33} = -\frac{1}{\tau}Z_i, \quad \Psi_i^{44} = -(\delta_1+\delta_2)^{-1}I,$$

$$\Psi_i^{55} = -\delta_1 I, \quad \Psi_i^{66} = -\delta_2 I, \quad \Psi_i^{77} = -I, \quad \Psi_i^{88} = -I,$$
$$\Theta_i^{12} = W_{1i}, \quad \Theta_i^{13} = T_{1i}, \quad \Theta_i^{14} = E_{1i}, \quad \Theta_i^{15} = P_i H_i,$$
$$\Theta_i^{17} = B_i X_i^T, \quad \Theta_i^{26} = P_i H_i, \quad \Theta_i^{28} = B_i X_i^T,$$
$$\Theta_i^{32} = W_{2i}, \quad \Theta_i^{33} = T_{2i}, \quad \Theta_i^{34} = E_{2i}^T, \quad B_i^T P_i = \tilde{P}_i B_i^T,$$

and all other sub-blocks in Π_i, Θ_i and Ψ_i are 0. Furthermore, the controller gain is given by $K_i = \tilde{P}_i^{-T} X_i^T$.

Proof. Consider the multiple Lyapunov-Krasovskii functional (13). Calculating the derivative of (13) along the trajectory of (35), we have

$$\begin{aligned}
\dot{V}_i(t) - \alpha V_i(t) \le\ & \xi^T(t)(\Gamma_i + \tau W_i Z_i^{-1} W_i^T + \tau T_i Z_i^{-1} T_i^T)\xi(t) \\
& + x^T(t)(P_i B_i K_i + K_i^T B_i^T P_i)x(t) + x^T(t) P_i B_i K_i e(t) \\
& + e^T(t) K_i^T B_i^T P_i x(t) + \dot{x}^T(t) P_i B_i K_i e(t) \\
& + e^T(t) K_i^T B_i^T P_i \dot{x}(t) + \dot{x}^T(t) P_i B_i K_i x(t) \\
& + x^T(t) K_i^T B_i^T P_i \dot{x}(t) \\
=\ & \xi^T(t) \underline{\Delta}_i \xi(t),
\end{aligned}$$

where

$$\underline{\Delta}_i = \Delta_i + \tau W_i Z_i^{-1} W_i^T + \tau T_i Z_i^{-1} T_i^T,$$

$$\Delta_i = \begin{bmatrix} \Delta_i^{11} & \Delta_i^{12} \\ * & \Delta_i^{22} \end{bmatrix}, \Delta_i^{12} = \begin{bmatrix} P_i & 0 & 0 & 0 \\ P_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Delta_i^{11} = \begin{bmatrix} \tilde{\Delta}_i^{11} & \Delta_i^{12} & \bar{\Sigma}_i^{13} & -T_{1i} & P_i \\ * & \tilde{\Delta}_i^{22} & P_i A_{di}(t) & 0 & P_i \\ * & * & \bar{\Sigma}_i^{33} & -T_{2i} & 0 \\ * & * & * & -R_i & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix}$$

$$\Delta_i^{22} = diag(-\varepsilon_2 I, -\frac{\alpha}{\tau} R_i, -\frac{\alpha}{\tau} S_i, -\frac{2\alpha}{\tau^2} Z_i),$$

$$\begin{aligned}
\tilde{\Delta}_i^{11} =\ & P_i A_i(t) + A_i^T(t) P_i + R_i + S_i + W_{1i} + W_{1i}^T + \varepsilon_1 U_i^T U_i \\
& - \alpha P_i + P_i B_i K_i + K_i^T B_i^T P_i + 2\eta I + P_i B_i K_i K_i^T B_i^T P_i,
\end{aligned}$$

$$\tilde{\Delta}_i^{12} = A_i^T(t) P_i + K_i^T B_i^T P_i,$$

$$\tilde{\Delta}_i^{22} = -2P + P_i B_i K_i K_i^T B_i^T P_i.$$

According to Schur complement, we get $\underline{\Delta}_i < 0$ if and only if

$$\bar{\Delta}_i = \begin{pmatrix} \Delta_i & W_i & T_i \\ * & -\frac{1}{\tau} Z_i & 0 \\ * & * & -\frac{1}{\tau} Z_i \end{pmatrix} < 0.$$

We have that $\bar{\Delta}_i < 0$, if

$$\Theta_i = \hat{\Delta}_i + \delta_1^{-1} \Omega_{1i}^T \Omega_{1i} + \delta_2^{-1} \Omega_{2i}^T \Omega_{2i} + (\delta_1 + \delta_2) \Phi_i^T \Phi_i < 0,$$

where

$$\hat{\Delta}_i = \begin{bmatrix} \Upsilon_i^{11} & \hat{\Sigma}_i^{12} \\ * & \hat{\Sigma}_i^{22} \end{bmatrix},$$

$$\Upsilon_i^{11} = \begin{bmatrix} \bar{\Upsilon}_i^{11} & \bar{\Upsilon}_i^{12} & \tilde{\Sigma}_i^{13} & -T_{1i} & P_i \\ * & \tilde{\Delta}_i^{22} & P_i A_{di} & 0 & P_i \\ * & * & \bar{\Sigma}_i^{33} & -T_{2i} & 0 \\ * & * & * & -R_i & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix},$$

$$\begin{aligned}
\bar{\Upsilon}_i^{11} =\ & P_i A_i + A_i^T P_i + R_i + S_i + W_{1i} + W_{1i}^T + P_i B_i K_i K_i^T B_i^T P_i \\
& + \varepsilon_1 U_i^T U_i - \alpha P_i + P_i B_i K_i + K_i^T B_i^T P_i + 2\eta I,
\end{aligned}$$

$$\bar{\Upsilon}_i^{12} = A_i^T P_i + K_i^T B_i^T P_i.$$

According to Schur complement, we can get that $\Theta_i < 0$, if

$$\begin{bmatrix} \hat{\Delta}_i + \delta_1^{-1} \Omega_{1i}^T \Omega_{1i} + \delta_2^{-1} \Omega_{2i}^T \Omega_{2i} & \Phi_i^T \\ * & -(\delta_1 + \delta_2)^{-1} I \end{bmatrix} < 0. \qquad (37)$$

According to Schur complement, (37) holds, if

$$\hat{\Lambda}_i = \begin{bmatrix} \bar{\Pi}_i & \bar{\Theta}_i \\ * & \Psi_i \end{bmatrix} < 0,$$

where

$$\begin{aligned}
\bar{\Pi}_i^{11} =\ & P_i A_i + A_i^T P_i + R_i + S_i + W_{1i} + W_{1i}^T + \varepsilon_1 U_i^T U_i \\
& - \alpha P_i + P_i B_i K_i + K_i^T B_i^T P_i^T + 2\eta I,
\end{aligned}$$

$$\bar{\Pi}_i^{12} = A_i^T P_i + K_i^T B_i^T P_i^T,$$

$$\bar{\Theta}_i^{17} = P_i B_i K_i,$$

$$\bar{\Theta}_i^{28} = P_i B_i K_i,$$

and the other sub-blocks in $\bar{\Pi}_i, \bar{\Theta}_i$ are the same as those in Π_i, Θ_i. Let $X_i = K_i^T \tilde{P}_i$. From $B_i^T P_i = \tilde{P}_i B_i^T$ and (36), one has

$$\hat{\Lambda}_i < 0.$$

Therefore, it follows that

$$\dot{V}_i(t) - \alpha V_i(t) \le 0.$$

The other proofs are similar to that of Theorem 1, which are omitted here.

Remark 2. In [8], the finite-time control was considered, but the authors only studied the linear switched systems. In this paper, we investigate finite-time control for uncertain nonlinear switched systems. Compared with [8], the results presented in this paper have a wider range of applications.

Remark 3. In [12], the robust control for a class of nonlinear switched systems was investigated, but the finite-time stabilization problem was not concerned. In this paper, we consider the finite-time stabilization for nonlinear uncertain switched time-delay systems and give new criteria of finite-time stabilization.

Remark 4. In [6], the finite-time stabilization problem was investigated, but the event-triggered control was not considered. Compared with the traditional time-triggered control, event-triggered control can effectively improve the utilization rate of system resources by reducing the sampling update and network communication frequency of controller. In this paper, we solved the event-triggered finite-time stabilization problem for nonlinear uncertain switched systems with time-varying delay.

IV. NUMERICAL EXAMPLE

Consider the nonlinear uncertain switched systems (1) with the following parameters:

$$A_1=\begin{bmatrix}-0.02 & 0.01\\ -0.02 & 0.13\end{bmatrix},\quad A_2=\begin{bmatrix}-0.03 & 0.05\\ -0.02 & 0.04\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.05 & 0\\ 0 & 0.07\end{bmatrix},\quad B_2=\begin{bmatrix}-0.04 & 0\\ 0 & 0.03\end{bmatrix},$$
$$A_{d1}=\begin{bmatrix}0.03 & 0.02\\ -0.02 & 0.08\end{bmatrix},\quad A_{d2}=\begin{bmatrix}0.04 & 0.02\\ -0.03 & -0.05\end{bmatrix},$$
$$E_1=\begin{bmatrix}0.02 & 0.03\\ 0 & 0.08\end{bmatrix},\quad E_2=\begin{bmatrix}0.13 & 0\\ 0.03 & 0.04\end{bmatrix},$$
$$H_1=\begin{bmatrix}-0.01 & 0\\ 0 & -0.01\end{bmatrix},\quad H_2=\begin{bmatrix}-0.02 & 0\\ 0 & -0.05\end{bmatrix},$$
$$g_1(x(t-\tau(t)))=\begin{bmatrix}0.05\sin(x_1(t-\tau(t)))\\ 0.05\sin(x_2(t-\tau(t)))\end{bmatrix},$$
$$g_2(x(t-\tau(t)))=\begin{bmatrix}0.07\sin(x_1(t-\tau(t)))\\ 0.07\sin(x_2(t-\tau(t)))\end{bmatrix},$$
$$f_1(x(t))=\begin{bmatrix}0.04\sin(x_1(t))\\ 0.04\sin(x_2(t))\end{bmatrix},$$
$$f_2(x(t))=\begin{bmatrix}0.06\sin(x_1(t))\\ 0.06\sin(x_2(t))\end{bmatrix},$$
$$\tau(t)=|0.1\sin t|.$$

Take

$$c_1=0.001,\ c_2=4600,\ T=20,\ R=I,\ \alpha=0.5,$$
$$\tau=0.5,\ \hat{\tau}=0.3,\ \eta=0.1,\ \mu=1.001.$$

It is very easy to get

$$U_1=\begin{bmatrix}0.07 & 0\\ 0 & 0.07\end{bmatrix},\quad U_2=\begin{bmatrix}0.08 & 0\\ 0 & 0.08\end{bmatrix},$$
$$V_1=\begin{bmatrix}0.09 & 0\\ 0 & 0.09\end{bmatrix},\quad V_2=\begin{bmatrix}0.08 & 0\\ 0 & 0.08\end{bmatrix},$$

Using Matlab LMI control Toolbox to solve inequalities (11)，(12) and (36), we obtain

$$K_1=\begin{bmatrix}-6.5454 & -0.0375\\ 0.1037 & -5.1627\end{bmatrix},$$
$$K_2=\begin{bmatrix}8.1486 & 0.1770\\ -0.0071 & -11.1121\end{bmatrix},$$

and $\tau_a \geq \tau_a^* = 1.9803$. According to Theorem 3, the switched system (1) is finite-time stabilizable with respect to (c_1, c_2, T, R, σ). In this paper, we choose the average dwell time $\tau_a = 2$. Fig. 1 shows the switching signal. The state response of the switched system is shown in Fig. 2. Fig. 3 depicts the event-triggered instants

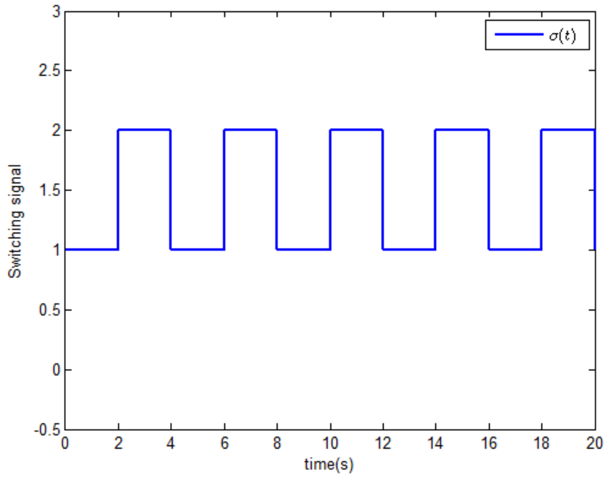

Fig. 1. The switching signal $\sigma(t)$.

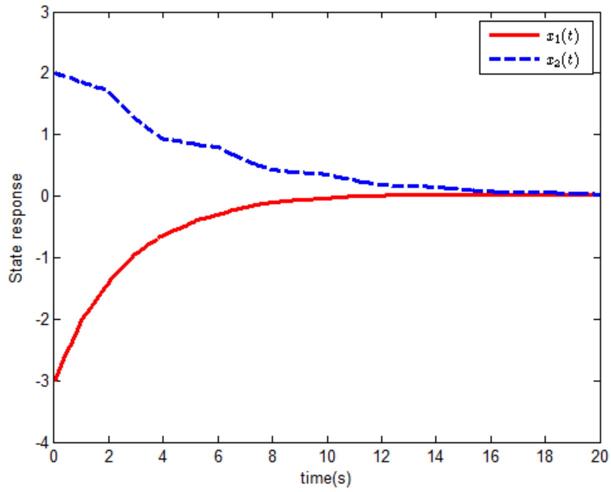

Fig. 2. The state trajectory of the switched system.

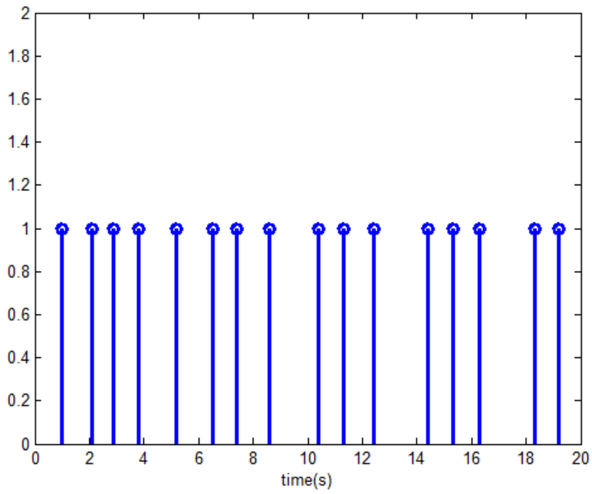

Fig. 3. Event-triggered instants: logical value is true when an event is triggering.

V. CONCLUSION

In this paper, we have investigated the problem of finite time stabilization of uncertain nonlinear switched system under the event-triggered control. An event-triggered sampling mechanism for nonlinear switched system with time-varying delay and uncertainties has been proposed, and an event-triggered controller has been designed based on the system state. The sufficient conditions of the finite-time stabilization for uncertain nonlinear switched system have been developed. Finally, a numerical example has been presented to demonstrate the effectiveness of the main results.

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