New Results on a Single-Species Model with Nonlinear Harvesting and Feedback Control on Time Scales

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Abstract—**A single-species model with nonlinear harvesting and feedback control on time scales is studied. By using some differential inequalities on time scales and constructing a suitable Lyapunov function, some new conditions which guarantee the permanence and uniformly asymptotical stability of the model are obtained, respectively. Our results indicate that feedback term is irrelevant to the permanence of this model which improve and complement some existing ones. Numeric simulations are carried out to show the feasibility of the main results.**

Index Terms—**Permanence, Uniformly asymptotical stability, Single-species model, Feedback control, Time scales.**

I. Introduction

As an effective tool to depict real ecological system, mathematical ecological model has become more and more important in the study of modern applied mathematics. Differential equations and difference equations are two main tools for the description of species relationship. However, due to the different concepts, theoretical knowledge and research methods, differential equations and difference equations always appear separately and people need to study twice for a complete and comprehensive understanding for systems. Furthermore, only using differential equations or difference equations is ineffective for describing the law of those species whose development process are both continuous and discrete in the real world [1, 2]. In order to unify both differential and difference analysis, Hilger [12] introduced the theory of time scales in his Ph.D. thesis. After then, many researchers pay attentions to the study of dynamic equations on time scales, such as permanence [5, 14, 15], global attractivity [11, 18], periodic solution and almost periodic solution [4, 7, 16, 19–21] and so on. In particular, Li, Yang and Zhang [16] considered the following single-species model with nonlinear harvesting and feedback control on time scales $\mathbb{T}$

$$\begin{cases} x^{\Delta}(t) = a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)}{1+m\exp\{x(t)\}} \\ \qquad\qquad - r(t)u(t), \\ u^{\Delta}(t) = -\alpha(t)u(t) + \beta(t)\exp\{x(t)\}, \end{cases} \tag{1}$$

where $m > 0$ is a constant, $\{a(t)\}$, $\{b(t)\}$, $\{c(t)\}$, $\{r(t)\}$, $\{\alpha(t)\}$ and $\{\beta(t)\}$ are all bounded nonnegative functions on $\mathbb{T}$ such that

$$0 < a^l \le a(t) \le a^u, \quad 0 < b^l \le b(t) \le b^u,$$
$$0 < c^l \le c(t) \le c^u, \quad 0 < \beta^l \le \beta(t) \le \beta^u,$$
$$0 < \alpha^l \le \alpha(t) \le \alpha^u, \quad 0 < \beta^l \le \beta(t) \le \beta^u,$$

where we using the following notations:

$$h^l = \inf_{t\in\mathbb{T}} h(t), \quad h^u = \sup_{t\in\mathbb{T}} h(t),$$

for any $h(t)$ which is a continuous bounded function defined on $\mathbb{T}$. We also suppose that $1-\mu(t)\alpha(t) > 0$ ($\mu(t)$ is defined in Section II) and there exists a positive constant L such that $\mu(t) \le L$. Based on the following initial conditions of system (1)

$$x(0) > 0, u(0) > 0, \tag{2}$$

by using the time scale calculus theory, Li, Yang and Zhang [16] got the following permanent result for system (1):

Theorem A ([16]). *Assume*

$$-b^u, -b^l, -\alpha^u, -\alpha^l \in \mathcal{R}^+, a^u > b^l \tag{H_1}$$

and

$$a^l - c^u - r^u u^* > b^u \tag{H_2}$$

hold, where $u^* = \frac{\beta^u \exp\{x^*\}}{\alpha^l}$ *and* $x^* = \frac{a^u - b^l}{b^l}$, *then system (1) is permanent.*

According to Theorem A, feedback term can affect the permanence of system (1). However, some results (see such as [8–10, 14, 15] and so on) have shown that feedback term has no impact on the permanence of ecological system. In particular, by using some differential inequalities on time scales, Wang and Fan [15] showed that feedback term is irrelevant to the permanence of a Nicholson's blowflies model with feedback control on time scales. Their results motivated us to consider the permanence of system (1) again. In fact, in this paper, by utilizing the analytical skills of Wang and Fan [15], we ultimately get the following result:

Theorem B. *Assume*

$$a^l - c^u > 0 \tag{A_1}$$

holds, then system (1) *is permanent.*

One can easily find that (A_1) in Theorem B is weaker than (H_1) and (H_2) in Theorem A and feedback term is harmless to the permanence of system (1), hence our results improve those in [16]. For more similar problems, one could refer to [3, 6, 17, 22–24] and references therein.

The organization of this paper is as follows. In Section II, we give some foundational definitions and results on time scales. The permanence and global attractivity are discussed

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in Section III and IV. Then, in Section V, our results are verified by one example with numerical simulations. Finally, we conclude in Section VI.

II. Preliminaries

In this section, we shall present some foundational definitions and results on time scales and one can refer to [13] for more detail.

Definition 2.1. ([13]) *A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of the real numbers $\mathbb{R}$. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.*

Definition 2.2. ([13]) *For $t \in \mathbb{T}$, the forward jump operator, the backward jump operator $\sigma, \rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, +\infty)$ are defined by*

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\},$$

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

$$\mu(t) = \sigma(t) - t,$$

respectively. If $t < \sup \mathbb{T}$ and $\sigma(t) = t$, then t is called right-dense, and if $t > \inf \mathbb{T}$ and $\rho(t) = t$, then t is called left-dense.

Definition 2.3. ([13]) *A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions is denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.*

Definition 2.4. ([13]) *Suppose $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^{\Delta}(t)$, the delta-derivative of f at t, to be the number (provided it exists) with the property that, given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t-\delta, t+\delta) \cap \mathbb{T}$) for some $\delta > 0$ such that*

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \leq \varepsilon|\sigma(t) - s|, \text{ for all } s \in U.$$

Thus, f is said to be delta-differentiable if its delta-derivative exists. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.5. ([13]) *A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\triangle}(t) = f(t)$, for all $t \in \mathbb{T}$. Then, we write*

$$\int_r^s f(t)\Delta t = F(s) - F(r), \text{ for all } s, r \in \mathbb{T}.$$

Definition 2.6. ([13]) *A function $f : \mathbb{T} \to \mathbb{R}$ is regressive if $1 + \mu(t)f(t) \neq 0$ for all $t \in \mathbb{T}$ and is positively regressive if $1 + \mu(t)f(t) > 0$ for all $t \in \mathbb{T}$. Denote by $\mathcal{R}$ and $\mathcal{R}^+$ the set of regressive and positively regressive functions from $\mathbb{T}$ to $\mathbb{R}$, respectively. If $p \in \mathcal{R}$, we define the exponential function by*

$$e_p(a, b) = \exp\Big\{\int_b^a \xi_{\mu(t)}(p(t))\Delta t\Big\}, \ a, b \in \mathbb{T},$$

where the cylinder transformation $\xi_\mu(z) = (1/\mu)\log(1 + z\mu)$, for $\mu > 0$ and $\xi_0(z) = z$, for $\mu = 0$.

Lemma 2.1. ([13]) *Suppose that $p, q \in \mathcal{R}^+$; then for all $a, b \in \mathbb{T}$,*

(i) $e_p(a, b) > 0$;
(ii) *if $p(a) \leq q(a)$ for all $a \in \mathbb{T}$, then $e_p(a, b) \leq e_q(a, b)$ for all $a \geq b$.*

Lemma 2.2. ([13])

(i) $(\nu_1 f + \nu_2 g)^{\Delta} = \nu_1 f^{\Delta} + \nu_2 g^{\Delta}$, *for any constants ν_1, ν_2;*
(ii) *if $f^{\Delta} \geq 0$, then f is nondecreasing.*

Lemma 2.3. ([14]) *Suppose $A, B > 0$ and $x(0) > 0$, further assume that*

(i)
$$x^{\Delta}(t) \leq B - A\exp\{x(t)\}, \ \forall\, t \geq 0,$$
then
$$\limsup_{t \to +\infty} x(t) \leq BL + \ln\frac{B}{A}.$$

(ii)
$$x^{\Delta}(t) \geq B - A\exp\{x(t)\}, \ \forall\, t \geq 0,$$
and there exists a constant $M > 0$, such that $\limsup_{t \to +\infty} x(t) < M$, then
$$\liminf_{t \to +\infty} x(t) \geq (B - A\exp\{M\})L + \ln\frac{B}{A}.$$

Lemma 2.4. ([14]) *Assume that $C(t)$, $D(t) > 0$ are bounded and rd-continuous functions, $-C \in \mathcal{R}^+$ and $C^l > 0$. Further suppose that*

(i)
$$x^{\Delta}(t) \leq -C(t)x(t) + D(t), \ \forall\, t \geq T_0,$$
then there exists a constant $T_1 > T_0$, such that for $t > T_1$,
$$x(t) \leq x(T_1)e_{(-C)}(t, T_1) + \frac{D(t)}{C^l}.$$
Especially, if $D(t)$ is bounded above with respect to H_1, then
$$\limsup_{t \to +\infty} x(t) \leq \frac{H_1}{C^l}.$$

(ii)
$$x^{\Delta}(t) \geq -C(t)x(t) + D(t), \ \forall\, t \geq T_0,$$
then there exists a constant $T_2 > T_0$, such that for $t > T_2$,
$$x(t) \geq \Big(x(T_2) - \frac{D(T_2)}{C^u}\Big)e_{(-C)}(t, T_2) + \frac{D(T_2)}{C^u}.$$
Especially, if $D(t)$ is bounded below with respect to h_1, then
$$\liminf_{t \to +\infty} x(t) \geq \frac{h_1}{C^u}.$$

Definition 2.7. *System (1) is said to be permanent if for any solution $(x(t), u(t))^T$ of system (1), there exist four constants w, k, W and K such that*

$$w \leq \liminf_{t \to +\infty} x(t) \leq \limsup_{t \to +\infty} x(t) \leq W,$$

$$k \leq \liminf_{t \to +\infty} u(t) \leq \limsup_{t \to +\infty} u(t) \leq K.$$

III. Permanence

We shall investigate the permanence of system (1) in this part. Similarly to the proof of [15, Lemma 18], we can obtain the following result:

Lemma 3.1. *For any solution $(x(t), u(t))^T$ of system (1) with initial condition (2), we have*

$$\exp\{x(t)\} > 0,\ u(t) > 0,\ \forall t \in \mathbb{T}.$$

Lemma 3.2. *For any solution $(x(t), u(t))^T$ of system (1) with initial condition (2), we have*

$$\limsup_{t\to+\infty} x(t) \le W,\ \limsup_{t\to+\infty} u(t) \le K,$$

where $W = a^u L + \ln\frac{a^u}{b^l}$ and $K = \frac{\beta^u \exp\{W\}}{\alpha^l}$.

Proof. From the positivity of $u_1(t)$ and the first equation of system (1), we get

$$\begin{aligned} x^\Delta(t) \le& a(t) - b(t)\exp\{x(t)\} \\ \le& a^u - b^l \exp\{x(t)\}. \end{aligned}$$

According to Lemma 2.3 (i), we obtain

$$\limsup_{t\to+\infty} x(t) \le a^u L + \ln\frac{a^u}{b^l} \triangleq W. \tag{3}$$

Thus, for any $\varepsilon_0 > 0$, there exists a large enough $t_0 \in \mathbb{T}^+$, such that for all $t > t_0$,

$$x(t) \le W + \varepsilon_0.$$

Then, for $t > t_0$, we can get from the second equation of system (1) that

$$\begin{aligned} u^\Delta(t) =& -\alpha(t)u(t) + \beta(t)\exp\{x(t)\} \\ \le& -\alpha(t)u(t) + \beta^u \exp\{W + \varepsilon_0\}. \end{aligned}$$

Using Lemma 2.4 (i), we further obtain

$$\limsup_{t\to+\infty} u(t) \le \frac{\beta^u \exp\{W + \varepsilon_0\}}{\alpha^l}. \tag{4}$$

Setting $\varepsilon_0 \to 0$, it follows from (4) that

$$\limsup_{t\to+\infty} u(t) \le \frac{\beta^u \exp\{W\}}{\alpha^l} \triangleq K.$$

The proof is completed. □

Lemma 3.3 *Assume*

$$a^l - c^u > 0, \tag{A_1}$$

then there exists two constants w and k such that

$$\liminf_{t\to+\infty} x(t) \ge w,\ \liminf_{t\to+\infty} u(t) \ge k,$$

where w and k can be found in the proof.

Proof. It follows from the second equation of system (1) that

$$u^\Delta(t) \le -\alpha(t)u(t) + \beta^u \exp\{x(t)\}.$$

By Lemma 2.4 (i), there exists a constant $t_1 > t_0$, such that for $t > t_1$,

$$u(t) \le u(t_1)e_{(-\alpha)}(t, t_1) + \frac{\beta^u \exp\{x(t)\}}{\alpha^l}.$$

Since $u(t_1)e_{(-\alpha)}(t, t_1) \to 0$ as $t \to +\infty$, then there exists a positive integer $t_2 > t_1$ such that

$$r^u u(t_1)e_{(-\alpha)}(t_2, t_1) \le \frac{1}{2}(a^l - c^u). \tag{5}$$

Fix t_2, for $t > t_2$, we have

$$u(t) \le u(t_1)e_{(-\alpha)}(t_2, t_1) + \frac{\beta^u \exp\{x(t)\}}{\alpha^l}. \tag{6}$$

One can get from (5), (7) and the first equation of system (1) that

$$\begin{aligned} x^\Delta(t) \ge& a^l - b^u \exp\{x(t)\} - c^u - r^u u(t) \\ \ge& a^l - b^u \exp\{x(t)\} - c^u \\ &- r^u\Big[u(t_1)e_{(-\alpha)}(t_2, t_1) + \frac{\beta^u \exp\{x(t)\}}{\alpha^l}\Big] \\ =& a^l - c^u - r^u u(t_1)e_{(-\alpha)}(t_2, t_1) \\ &- \Big(b^u + \frac{r^u \beta^u}{\alpha^l}\Big)\exp\{x(t)\} \\ \ge& \frac{1}{2}(a^l - c^u) - \Big(b^u + \frac{r^u \beta^u}{\alpha^l}\Big)\exp\{x(t)\}, \end{aligned} \tag{7}$$

for $t > t_2$. Using this and Lemma 2.3 (ii), we get

$$\begin{aligned} \liminf_{t\to+\infty} x(t) \ge& \Big[\frac{1}{2}(a^l - c^u) - \Big(b^u + \frac{r^u \beta^u}{\alpha^l}\Big)\exp\{W\}\Big]L \\ &+ \ln\frac{\alpha^l(a^l - c^u)}{2(\alpha^l b^u + r^u \beta^u)} \triangleq w. \end{aligned} \tag{8}$$

So for any $\varepsilon_1 > 0$, there exists enough large $t_3 > t_2$, such that for $t > t_3$,

$$x(t) \ge w - \varepsilon_1.$$

This together with the second equation of system (1) results in

$$u^\Delta(t) \ge -\alpha(t)u(t) + \beta^l \exp\{w - \varepsilon_1\},\ \text{for } t > t_3. \tag{9}$$

It follows from (9) and Lemma 2.4 (ii) that

$$\liminf_{t\to+\infty} u(t) \ge \frac{\beta^l \exp\{w - \varepsilon_1\}}{\alpha^u}. \tag{10}$$

Setting $\varepsilon_1 \to 0$, we get from (10) that

$$\liminf_{t\to+\infty} u(t) \ge \frac{\beta^l \exp\{w\}}{\alpha^u} \triangleq k. \tag{11}$$

The proof is completed. □

Theorem B can be obtained directly from Lemma 3.2 and Lemma 3.3.

IV. Uniform Asymptotical Stability

In this part, we will investigate the uniform asymptotical stability of system (1) by the method of Lyapunov function.

Theorem 4.1. *Assume (A_1), further suppose that*

$$\alpha^l > r^u \text{ and } b^l - \frac{mc^u}{[1 + m\exp\{w\}]^2} > \beta^u \tag{A_2}$$

where w is defined in Lemma 3.3, then system (1) with initial conditions (2) is uniformly asymptotically stable.

Proof. It follows from (A_2) that there exists a small enough $\varepsilon > 0$ such that

$$\alpha^l - r^u > \varepsilon \tag{12}$$

and

$$\Big[b^l - \frac{mc^u}{[1 + m\exp\{w - \varepsilon\}]^2} - \beta^u\Big]\exp\{w - \varepsilon\} > \varepsilon. \tag{13}$$

Suppose $z_1(t) = (x(t), u(t))^T, z_2(t) = (x_*(t), u_*(t))^T$ are two solutions of system (1) with initial conditions (2). For above ε, according to Lemma 3.2 and Lemma 3.3, there exist a $t_4 > 0$, when $t > t_4$,

$$\begin{aligned} w-\varepsilon \le x(t) \le W+\varepsilon,\ \ k-\varepsilon \le u(t) \le K+\varepsilon, \\ w-\varepsilon \le x_*(t) \le W+\varepsilon,\ \ k-\varepsilon \le u_*(t) \le K+\varepsilon. \end{aligned} \tag{14}$$

Consider the following Lyapunov function

$$V(t,z_1,z_2) = |x(t)-x_*(t)| + |u(t)-u_*(t)|.$$

Calculating $D^+V^\Delta(t,z_1,z_2)$ of $V(t,z_1,z_2)$ along system (1) leads to

$$\begin{aligned} &D^+V^\Delta(t,z_1,z_2) \\ =&\mathrm{sgn}(x(t)-x_*(t))\Big[-b(t)(\exp\{x(t)\}-\exp\{x_*(t)\}) \\ &-c(t)\Big(\frac{1}{1+m\exp\{x(t)\}}-\frac{1}{1+m\exp\{x_*(t)\}}\Big) \\ &-r(t)(u(t)-u_*(t))\Big] \\ &+\mathrm{sgn}(u(t)-u_*(t))\Big[-\alpha(t)(u(t)-u_*(t)) \\ &+\beta(t)(\exp\{x(t)\}-\exp\{x_*(t)\})\Big] \\ =&\mathrm{sgn}(x(t)-x_*(t))\Big[-b(t)(\exp\{x(t)\}-\exp\{x_*(t)\}) \\ &+\frac{mc(t)(\exp\{x(t)\}-\exp\{x_*(t)\})}{[1+m\exp\{x(t)\}][1+m\exp\{x_*(t)\}]} \\ &-r(t)(u(t)-u_*(t))\Big]-\alpha(t)|u(t)-u_*(t)| \\ &+\mathrm{sgn}(u(t)-u_*(t))\beta(t)(\exp\{x(t)\}-\exp\{x_*(t)\}). \end{aligned} \tag{15}$$

Using the mean value theorem, we get

$$\exp\{x(t)\}-\exp\{x_*(t)\} = \xi(t)(x(t)-x_*(t)), \tag{16}$$

where $\xi(t)$ lies between $\exp\{x(t)\}$ and $\exp\{x_*(t)\}$. We can obtain from (14), (15) and (16) that

$$\begin{aligned} &D^+V^\Delta(t,z_1,z_2) \\ \le&-b(t)\xi(t)|x(t)-x_*(t)| \\ &+\frac{mc(t)\xi(t)|x(t)-x_*(t)|}{[1+m\exp\{x(t)\}][1+m\exp\{x_*(t)\}]} \\ &+r(t)|u(t)-u_*(t)|-\alpha(t)|u(t)-u_*(t)| \\ &+\beta(t)\xi(t)|x(t)-x_*(t)| \\ =&\Big[-b(t)+\frac{mc(t)}{[1+m\exp\{x(t)\}][1+m\exp\{x_*(t)\}]} \\ &+\beta(t)\Big]\xi(t)|x(t)-x_*(t)| \\ &+(r(t)-\alpha(t))|u(t)-u_*(t)| \\ \le&-\Big[b^l-\frac{mc^u}{[1+m\exp\{w-\varepsilon\}]^2} \\ &-\beta^u\Big]\exp\{w-\varepsilon\}|x(t)-x_*(t)| \\ &-(\alpha^l-r^u)|u(t)-u_*(t)| \\ \le&-\varepsilon V(t,z_1,z_2),\ \text{for } t>t_4, \end{aligned} \tag{17}$$

where we also use (12) and (13). Therefore, $V(t,z_1,z_2)$ is non-increasing. Integrating (17) from t_4 to t ($t > t_4$) leads to

$$V(t,z_1,z_2)+\varepsilon\int_{t_4}^{t} V(s,z_1,z_2)\Delta s \le V(t_4,z_1,z_2) < +\infty.$$

Hence,

$$\int_{t_4}^{+\infty} V(s,z_1,z_2)\Delta s < +\infty,$$

which means that

$$\lim_{t\to+\infty}|x(t)-x_*(t)| = \lim_{t\to+\infty}|u(t)-u_*(t)| = 0.$$

Therefore, system (1) is uniformly asymptotically stable. □

Remark 4.1. *By constructing a different Lyapunov function with ours, Li, Yang and Zhang [16] established sufficient conditions on the uniformly asymptotical stability of the system (1) (see Theorem 3.2 in [16]) which are more complex than condition (A_2) in Theorem 4.1.*

V. EXAMPLE AND NUMERIC SIMULATION

In this part, we will give some numerical simulations to support our results.

Example 5.1. Consider the following system:

$$\begin{cases} x^\Delta(t) =0.35+0.02\sin(2t)-0.33\exp\{x(t)\} \\ \qquad\qquad -\dfrac{0.005+0.0003\cos(\sqrt{7}t)}{1+0.3\exp\{x(t)\}}-0.0002u(t), \\ u^\Delta(t) =-(0.5+0.03\cos(\sqrt{5}t))u(t)+0.08\exp\{x(t)\}, \end{cases} \tag{18}$$

we have

$$a^l - c^u = 0.3247 > 0 \text{ and } \alpha^l - r^u = 0.4698 > 0,$$

so (A_1) holds and system (18) is permanent according to Theorem B. However,

$$a^l - c^u - r^u u^* - b^u \approx -0.0053 < 0,$$

which implies that we can't judge the permanence by Theorem A since (H_2) does not hold and our results improved those in [16].

Moreover, if $\mathbb{T}=\mathbb{R}$, set $y(t)=\exp\{x(t)\}$, then system (18) reduces to the following continuous system:

$$\begin{cases} \dot{y}(t) =y(t)\Big[0.35+0.02\sin(2t)-0.33y(t) \\ \qquad\qquad -\dfrac{0.005+0.0003\cos(\sqrt{7}t)}{1+0.3y(t)}-0.0002u(t)\Big], \\ \dot{u}(t) =-(0.5+0.03\cos(\sqrt{5}t))u(t)+0.08y(t). \end{cases} \tag{19}$$

Since $\mu(t)\equiv 0$, we can choose $L=0$ for convenience. Thus, for system (14), we have

$$b^l - \frac{mc^u}{[1+m\exp\{w\}]^2} - \beta^u \approx 0.2488 > 0.$$

So all conditions in Theorem 4.1 are satisfied and system (19) is permanent and uniformly asymptotically stable which is supported by Fig. 1.

When $\mathbb{T}=\mathbb{Z}$, if we also set $y(t)=\exp\{x(t)\}$, then system (18) reduces to the following discrete system:

$$\begin{cases} y(t+1) =y(t)\exp\Big[0.35+0.02\sin(2t)-0.33y(t) \\ \qquad\qquad -\dfrac{0.005+0.0003\cos(\sqrt{7}t)}{1+0.3y(t)}-0.0002u(t)\Big], \\ \Delta u(t) =-(0.5+0.03\cos(\sqrt{5}t))u(t)+0.08y(t), \end{cases} \tag{20}$$

Since $\mu(t)\equiv 1$, we choose $L=1$ for convenience. Thus, we have

$$b^l - \frac{mc^u}{[1+m\exp\{w\}]^2} - \beta^u \approx 0.2487 > 0,$$

so all conditions in Theorem 4.1 are satisfied, system (20) is permanent and uniformly asymptotically stable. Our numerical simulation also supports this result (see Fig. 2).

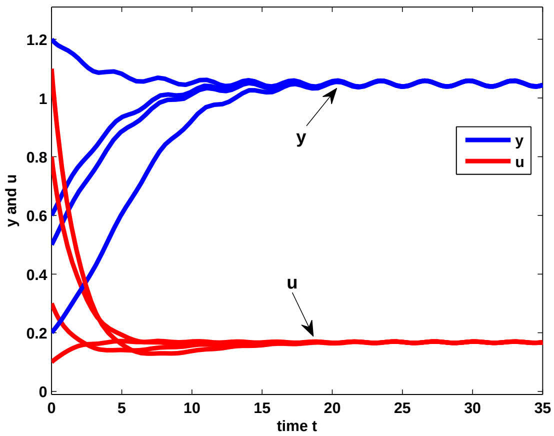

Fig. 1. Numeric simulations of system (19) with the initial condition $(y(0), u(0))^T = (0.5, 0.3)^T$, $(1.2, 0.1)^T$, $(0.6, 0.8)^T$ and $(0.2, 1.1)^T$, respectively.

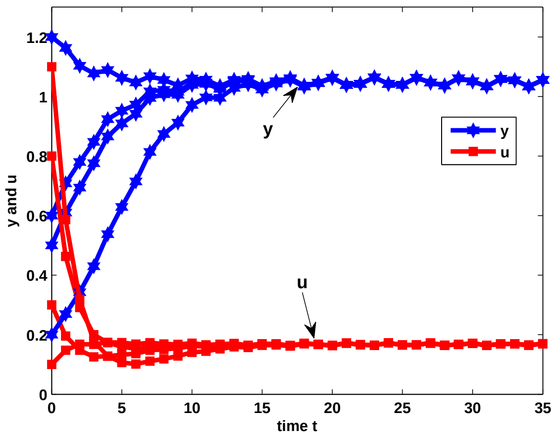

Fig. 2. Numeric simulations of system (20) with the initial condition $(y(0), u(0))^T = (0.5, 0.3)^T$, $(1.2, 0.1)^T$, $(0.6, 0.8)^T$ and $(0.2, 1.1)^T$, respectively.

VI. CONCLUSION

In this paper, we consider a single-species model with nonlinear harvesting and feedback control on time scales which was investigated by Li, Yang and Zhang [16]. By using some differential inequalities on time scales, we obtain some new conditions on the permanence of system (1) which are weaker than those in [16]. This result shows that feedback term has no influence on the permanence of the model. By constructing a different Lyapunov function with Li, Yang and Zhang [16], we established some new sufficient conditions on the uniformly asymptotical stability of the model which are more simpler and easier to verify then those in [16]. Therefore, our results improve and complement those in Li, Yang and Zhang [16].

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