Multistability in the Finance Chaotic System, Its Bifurcation Analysis and Global Chaos Synchronization via Integral Sliding Mode Control

Betty Subartini, Sukono, Sundarapandian Vaidyanathan, Aceng Sambas, and Sen Zhang

Abstract—**This paper reports the finding of a new 3-D finance chaotic system with two quadratic nonlinearities and a sextic nonlinearity. We also discover interesting properties of the new finance chaotic system such as symmetry, equilibrium points, bifurcation, multistability and Lyapunov exponents. Using integral sliding mode control, we derive new results for the global chaos synchronization of a pair of new finance chaotic systems taken as master-slave systems. We illustrate all the main results of this research work using MATLAB phase plots.**

Keywords: Chaos; chaotic systems; finance systems; sliding mode control; synchronization; multi-stability; bifurcations.

I. INTRODUCTION

CHAOS theory involves the study of nonlinear dynamical systems that exhibit a high degree of sensitivity with regard to even mild perturbations in the initial phases of the system [1]. In Science, chaotic systems are applicable in domains such as biology [2]–[6], neural networks [7]–[10], mechanical oscillators [11]–[14], chemical systems [15]–[18], jerk systems [19], [20], etc. In engineering, chaotic systems are applicable in domains such as memristors [21], [22], circuits [23]–[26], communication devices [27]–[30], etc.

In 2009, Gao and Ma introduced a new finance chaotic system and discussed its qualitative properties such as Hopf bifurcation [31]. In the chaos literature, many research studies dealt with modelling and control techniques for finance chaotic systems [32]–[35]. In this research work, we propose a new finance chaotic system and study its dynamic behavior with the help of bifurcation diagrams and Lyapunov exponents. We exhibit that the new finance chaotic system has three unstable balance points and that it has interesting properties such as rotation symmetry, multi-stability, etc. A nonlinear chaotic system is called *multistable* if it possesses coexisting chaotic attractors for the same set of parameter values but different sets of initial states [36]–[38].

Manuscript received June 21, 2021; revised September 7, 2021. This research was funded Ministry of Research and Higher Education, Republik of Indonesia through Penelitian Dasar Unggulan Perguruan Tinggi (Grant No. 1207/UN6.3.1/PT.00/2021).

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Some important applications of chaos theory are the feedback control and stabilization of nonlinear chaotic systems [1]. Researchers have worked on these problems using a variety of control strategies such as adaptive control [39]–[41], passive control [42], [43], fuzzy control [44], [45], active control [46], [47], sliding mode control [48]–[50], etc.

In this work, we use integral sliding mode control for the global chaos synchronization of the new finance chaotic system by considering a set of two finance systems as *master-slave* systems. Finally, we conclude this work with a summary of main findings.

II. A NEW FINANCE CHAOTIC SYSTEM

In [31], Gao and Ma proposed a new finance chaotic system, which is modelled by the 3-D dynamics:

$$\begin{cases} \dot{y}_1 &= y_3 + (y_2 - a)y_1 \\ \dot{y}_2 &= 1 - by_2 - y_1^2 \\ \dot{y}_3 &= -y_1 - y_3 \end{cases} \tag{1}$$

In the Gao-Ma system (1), y_1 denotes the interest rate, y_2 denotes the investment demand rate and y_3 represents the price exponent. The constant a stands for the household savings rate and b designates the investment cost. We assume that a and b are positive.

In [31], Gao and Ma showed that the system (1) has a chaotic attractor for the parameter values $a = 6$ and $b = 0.1$.

Using MATLAB, the Lyapunov characteristic exponents of the Gao-Ma financial system (1) with $(a, b) = (6, 0.1)$ and $Y(0) = (0.4, 0.2, 0.5)$ were estimated for $T = 1E5$ seconds as follows:

$$\mu_1 = 0.09004, \quad \mu_2 = 0, \quad \mu_3 = -0.39313 \tag{2}$$

This shows that the Gao-Ma financial system (1) is dissipative and chaotic with the largest Lyapunov exponent value as $\mu_1 = 0.09004$.

In this work, we propose a new finance chaotic system with two quadratic nonlinearities and a sextic nonlinearity, which is modelled by the 3-D dynamics:

$$\begin{cases} \dot{y}_1 &= y_3 + (y_2 - a)y_1 \\ \dot{y}_2 &= 1 - b(y_2 + y_1^2) - cy_1^6 \\ \dot{y}_3 &= -y_1 - y_3 \end{cases} \tag{3}$$

We shall establish that the new finance system (3) exhibits a chaotic attractor for the parameter values

$$a = 7.6, \quad b = 0.1, \quad c = 0.2 \tag{4}$$

For the MATLAB simulations, we consider the initial state of the new finance system (3) as

$$y_1(0) = 0.4, \quad y_2(0) = 0.2, \quad y_3(0) = 0.5 \tag{5}$$

Using MATLAB, the Lyapunov characteristic exponents of the new finance system (3) with $(a, b, c) = (7.6, 0.1, 0.2)$ and $Y(0) = (0.4, 0.2, 0.5)$ were estimated for $T = 1E5$ seconds as follows:

$$\mu_1 = 0.12299, \quad \mu_2 = 0, \quad \mu_3 = -0.39478 \tag{6}$$

This shows that the new finance system (3) is dissipative and chaotic with the largest Lyapunov exponent value as $\mu_1 = 0.12299$. Comparing the largest Lyapunov exponent values of the Gao-Ma finance chaotic system (1) and the new finance chaotic system (3), we observe that the new finance chaotic system (3) has a larger value of μ_1 than the Gao-Ma finance chaotic system (1).

The new finance chaotic system (3) is invariant under the transformation of coordinates

$$(y_1, y_2, y_3) \mapsto (-y_1, y_2, -y_3) \tag{7}$$

for all values of the parameters. This shows that the new finance chaotic system (3) has rotation symmetry about the y_2-axis.

Next, we calculate the balance points of the new finance chaotic system (3) for the values of the parameters as in the chaotic case, *viz.* $(a, b, c) = (7.6, 0.1, 0.2)$. Thus, we consider solving the following the system of equations:

$$y_3 + (y_2 - 7.6)y_1 = 0 \tag{8a}$$

$$1 - 0.1(y_2 + y_1^2) - 0.2y_1^6 = 0 \tag{8b}$$

$$-y_1 - y_3 = 0 \tag{8c}$$

From Eq. (8c), it is clear that

$$y_3 = -y_1 \tag{9}$$

Using (9), we can simplify the equations (8a) and (8b) as follows:

$$y_1(y_2 - 8.6) = 0 \tag{10a}$$

$$1 - 0.1(y_2 + y_1^2) - 0.2y_1^6 = 0 \tag{10b}$$

We have two cases to consider: (A) $y_1 = 0$ and (B) $y_1 \neq 0$.

In Case (A), we suppose that $y_1 = 0$. Since $y_3 = -y_1$, it is immediate that $y_3 = 0$.

From (10b), we obtain $y_2 = 10$.

Thus, in Case (A), we have one balance point $E_0 = (0, 10, 0)$ of the new finance chaotic system (3).

In Case (B), we suppose that $y_1 \neq 0$. From Eq. (10a), we get $y_2 = 8.6$.

Thus, Eq. (10b) can be simplified as follows:

$$0.2y_1^6 + 0.1y_1^2 - 0.14 = 0 \tag{11}$$

Solving Eq. (11), we get two real roots, *viz.* $y_1 = \pm 0.8388$. Since $y_3 = -y_1$, we deduce that $y_3 = \mp 0.8388$.

Thus, in Case (B), we have two balance points $E_1 = (0.8388, 8.6, -0.8388)$ and $E_2 = (-0.8388, 8.6, 0.8388)$ of the new finance chaotic system (3).

The Jacobian matrix of the new finance chaotic system (3) at $E_0 = (0, 10, 0)$ is found as follows:

$$W_0 = \begin{bmatrix} 2.4 & 0 & 1 \\ 0 & -0.1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \tag{12}$$

The spectral values of W_0 are determined using MATLAB as

$$\alpha_1 = -0.1, \quad \alpha_2 = -0.6748, \quad \alpha_3 = 2.0748 \tag{13}$$

This establishes that E_0 is a saddle point and an unstable balance point for the new finance chaotic system (3).

The Jacobian matrix of the new finance chaotic system (3) at $E_1 = (0.8388, 8.6, -0.8388)$ is found as follows:

$$W_1 = \begin{bmatrix} 1 & 0.8388 & 1 \\ -0.6660 & -0.1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \tag{14}$$

The spectral values of W_1 are determined using MATLAB as

$$\alpha_{1,2} = 0.2639 \pm 0.9057i, \quad \alpha_3 = -0.6277 \tag{15}$$

This establishes that E_1 is a saddle-focus and an unstable balance point for the new finance chaotic system (3).

The Jacobian matrix of the new finance chaotic system (3) at $E_2 = (-0.8388, 8.6, 0.8388)$ is found as follows:

$$W_2 = \begin{bmatrix} 1 & -0.8388 & 1 \\ 0.6660 & -0.1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \tag{16}$$

The spectral values of W_2 are determined using MATLAB as

$$\alpha_{1,2} = 0.2639 \pm 0.9057i, \quad \alpha_3 = -0.6277 \tag{17}$$

This establishes that E_2 is a saddle-focus and an unstable balance point for the new finance chaotic system (3).

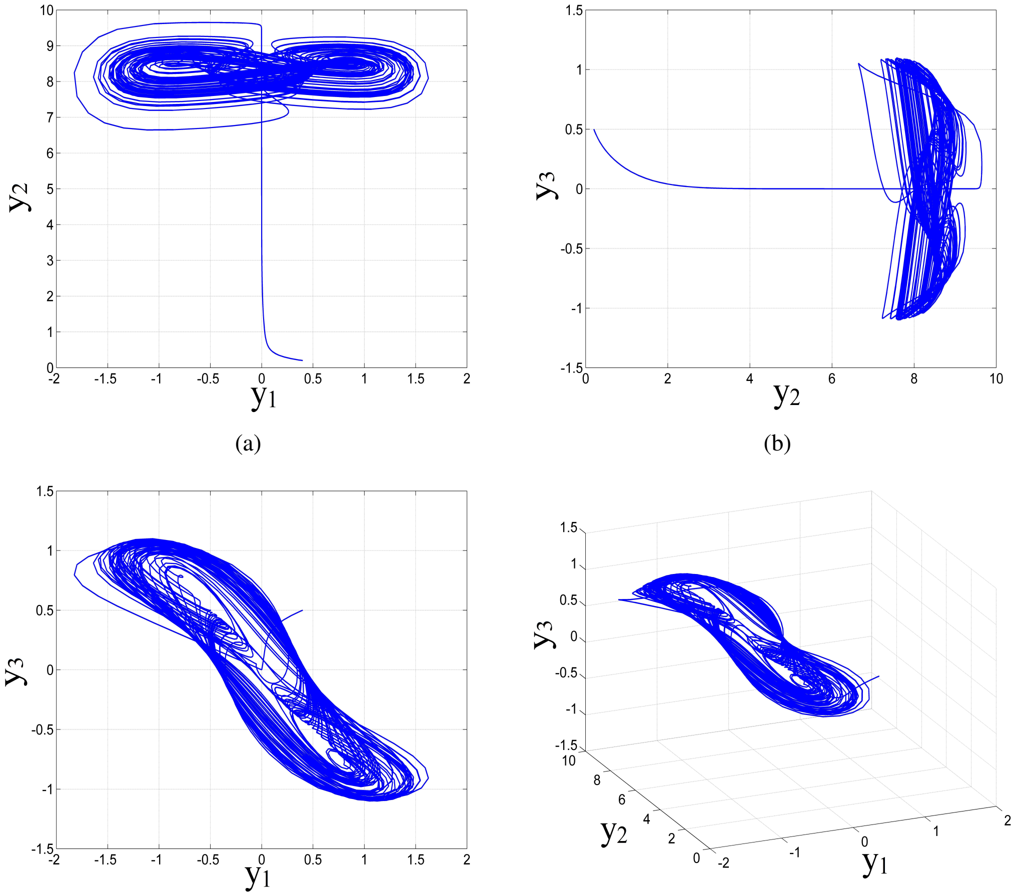
The signal plots of the 3-D new finance system (3) for the constant parameters $(a, b, c) = (7.6, 0.1, 0.2)$ (chaotic case) and the initial state $Y(0) = (0.4, 0.2, 0.5)$ are simulated in Figure 1.

III. DYNAMIC STUDY OF THE NEW FINANCE CHAOTIC SYSTEM

A. Bifurcation analysis

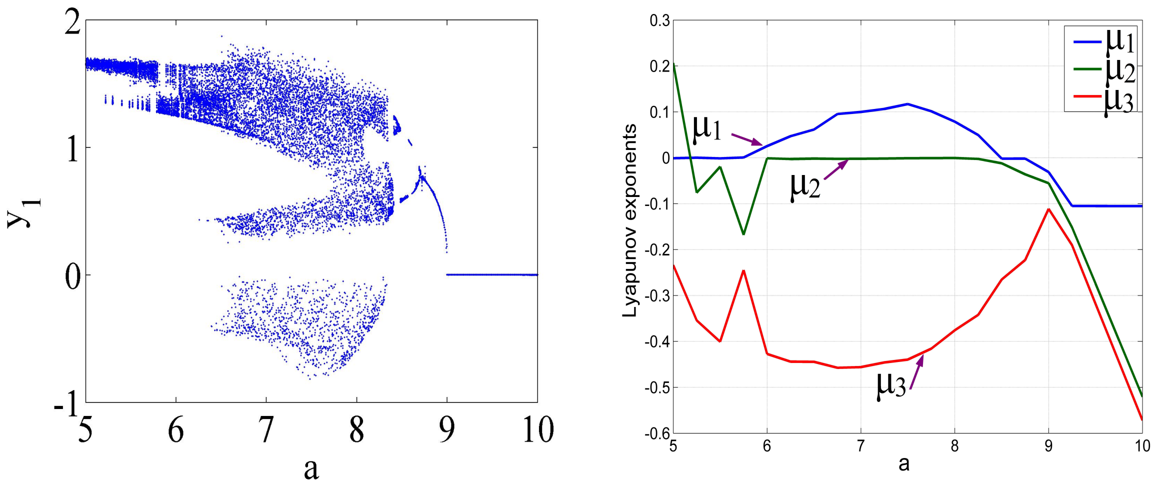
First, we assume that the finance system parameters are taken as $b = 0.1$, $c = 0.2$, and the initial condition is fixed as $Y(0) = (0.4, 0.2, 0.5)$. When changing the parameter a from 5 to 10, the bifurcation diagram of the state variable y_1 and the corresponding Lyapunov exponents are shown in Figures 2 (a) and 2 (b), respectively. From Figure 2, it can be seen that the new finance chaotic system (3) is in periodic state at first, then goes into chaos, next drops out of chaos via period-doubling route and finally tends to a fixed point.

Next, we assume that the system parameters are taken as $a = 7.6$, $c = 0.2$, and the initial condition is fixed as $Y(0) = (0.4, 0.2, 0.5)$. When altering b in the region of [0, 0.2], the bifurcation diagram of the state variable y_1 and the corresponding Lyapunov exponents are depicted in Figures 3 (a) and 3 (b), respectively. From Figure 3, it is obvious to find that new finance chaotic system (3) exhibits different dynamical behaviors, such as period, chaos and fixed point.



(a) (b) (c) (d)

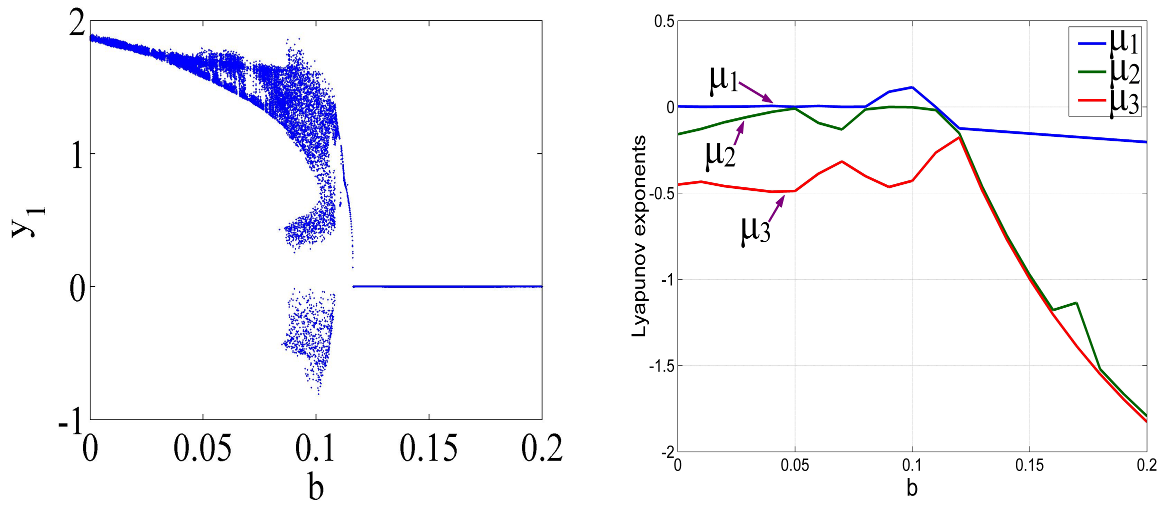
Fig. 1: Signal plots of the 3-D finance chaotic system (3) for $(a, b, c) = (7.6, 0.1, 0.2)$ and the initial state $Y(0) = (0.4, 0.2, 0.5)$: (a) (y_1, y_2) plane, (b) (y_2, y_3) plane, (c) (y_1, y_3) plane and (d) R^3.



(a) (b)

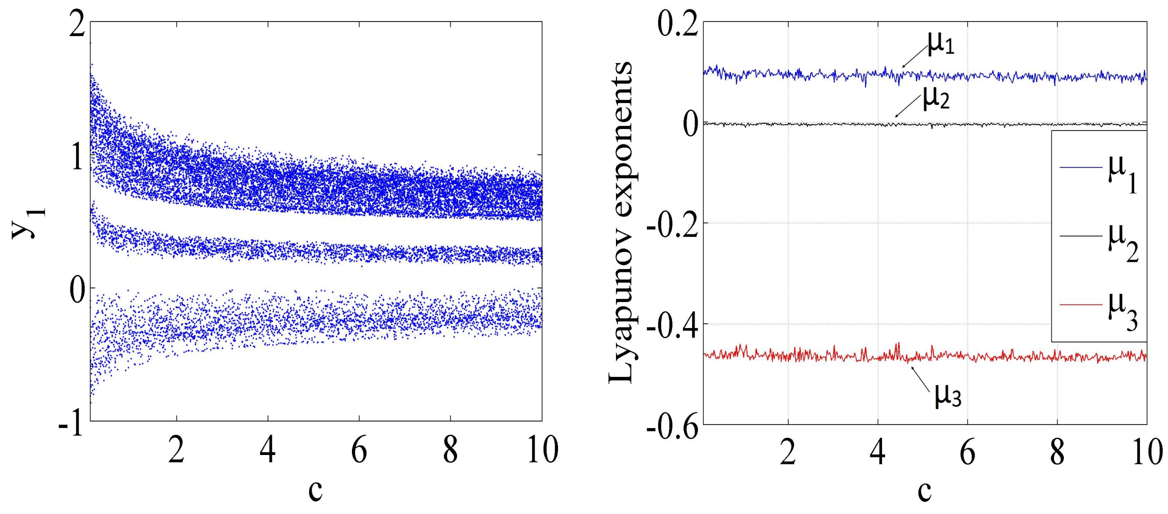
Fig. 2: Dynamics of the new finance chaotic system (3) with respect to a: (a) the bifurcation diagram; (b) the corresponding Lyapunov exponent spectrum

In the last case, we assume that the system parameters are taken as $a = 7.6$, $b = 0.1$, and the initial condition is fixed as $Y(0) = (0.4, 0.2, 0.5)$. As the parameter c varies in the region of $[0.1, 10]$, the bifurcation diagram of the state variable y_1 and the corresponding Lyapunov exponents are plotted in Figures 4 (a) and 4 (b), respectively. From Figure 4 (b), it is clear to see that the new finance chaotic system (3) exhibits a positive largest Lyapunov exponent in the whole region, indicating the emerging of constant Lyapunov exponent behavior which is important for chaos-based applications



(a) (b)

Fig. 3: Dynamics of the new finance chaotic system (3) with respect to b: (a) the bifurcation diagram; (b) the corresponding Lyapunov exponent spectrum.



(a) (b)

Fig. 4: Dynamics of the new finance chaotic system (3) with respect to c: (a) the bifurcation diagram; (b) the corresponding Lyapunov exponent spectrum.

B. Multistability

Mutistability is a fantastic nonlinear phenomenon, which can make a chaotic system exhibit great flexibility for potential engineering applications [36]–[38]. Fix the parameters $b = 0.1$, $c = 0.2$, and vary a in the region of $[5, 10]$ and the initial conditions are selected as $Y_0 = (0.4, 0.2, 0.5)$ and $Z_0 = (0.4, -0.2, -0.5)$. The coexisting bifurcation diagrams of the state variable y_1 are plotted in Figure 5. As can be seen from Figure 5, the new finance chaotic system (3) exhibits complex coexisting dynamics in the regions of $[5, 6.3]$ and $[8.36, 8.7]$, such as coexisting periodic attractors as well as coexisting chaotic attractors. For instance, for $a = 5$, the new finance chaotic system (3) shows coexisting periodic attractors; for $a = 6.2$, the system (3) shows coexisting chaotic attractors. These coexisting attractors are shown in Figures 6 and 7, respectively.

IV. INTEGRAL SLIDING MODE CONTROL FOR THE GLOBAL CHAOS SYNCHRONIZATION OF NEW FINANCE CHAOTIC SYSTEMS

As the master system, we consider the new 3-D finance chaotic system given by

$$\left\{\begin{array}{lcl} \dot{y}_1 & = & y_3 + (y_2 - a)y_1 \\ \dot{y}_2 & = & 1 - b(y_2 + y_1^2) - cy_1^6 \\ \dot{y}_3 & = & -y_1 - y_3 \end{array}\right. \tag{18}$$

where y_1, y_2, y_3 are the master states and a, b, c are positive parameters.

As the slave system, we take the new 3-D finance system with sliding controls given by

$$\left\{\begin{array}{lcl} \dot{z}_1 & = & z_3 + (z_2 - a)z_1 + v_1 \\ \dot{z}_2 & = & 1 - b(z_2 + z_1^2) - cz_1^6 + v_2 \\ \dot{z}_3 & = & -z_1 - z_3 + v_3 \end{array}\right. \tag{19}$$

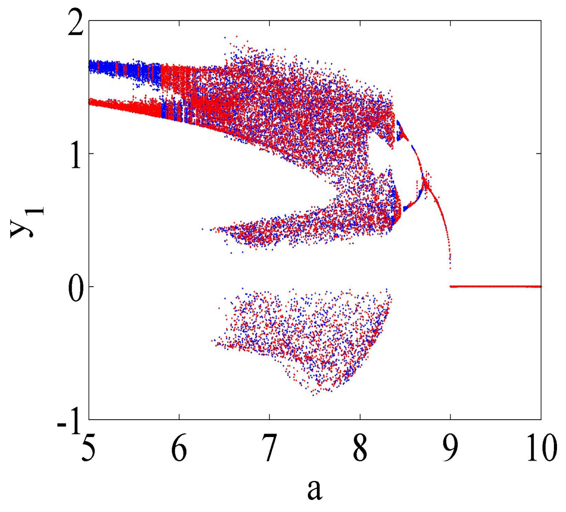


Fig. 5: Coexisting bifurcation diagrams of the new finance chaotic system (3) with respect to a.

where z_1, z_2, z_3 are the slave states.

The synchronization error between the systems (18) and (19) is given by

$$\epsilon_i = z_i - y_i, \text{ for } i = 1, 2, 3 \tag{20}$$

The synchronization error dynamics is derived as follows:

$$\begin{cases} \dot{\epsilon}_1 &= \epsilon_3 - a\epsilon_1 + z_2 z_1 - y_2 y_1 + v_1 \\ \dot{\epsilon}_2 &= -b(\epsilon_2 + z_1^2 - y_1^2) - c(z_1^6 - y_1^6) + v_2 \\ \dot{\epsilon}_3 &= -\epsilon_1 - \epsilon_3 + v_3 \end{cases} \tag{21}$$

Based on the sliding mode control theory, the integral sliding surface of each error variable ϵ_i, $(i = 1, 2, 3)$ is defined as follows:

$$\begin{aligned} \sigma_1 &= \epsilon_1 + \alpha_1 \int_0^t \epsilon_1(\tau)\, d\tau \\ \sigma_2 &= \epsilon_2 + \alpha_2 \int_0^t \epsilon_2(\tau)\, d\tau \\ \sigma_3 &= \epsilon_3 + \alpha_3 \int_0^t \epsilon_3(\tau)\, d\tau \end{aligned} \tag{22}$$

Differentiating each equation in (22), we get

$$\begin{aligned} \dot{\sigma}_1 &= \dot{\epsilon}_1 + \alpha_1 \epsilon_1 \\ \dot{\sigma}_2 &= \dot{\epsilon}_2 + \alpha_2 \epsilon_2 \\ \dot{\sigma}_3 &= \dot{\epsilon}_3 + \alpha_3 \epsilon_3 \end{aligned} \tag{23}$$

The Hurwitz condition will hold if $\alpha_i > 0$ for $i = 1, 2, 3$.

Based on the exponential reaching law, we set

$$\begin{aligned} \dot{\sigma}_1 &= -\beta_1 \text{sgn}(\sigma_1) - \mu_1 \sigma_1 \\ \dot{\sigma}_2 &= -\beta_2 \text{sgn}(\sigma_2) - \mu_2 \sigma_2 \\ \dot{\sigma}_3 &= -\beta_3 \text{sgn}(\sigma_3) - \mu_3 \sigma_3 \end{aligned} \tag{24}$$

where $\beta_1, \beta_2, \beta_3, \mu_1, \mu_2, \mu_3$ are positive constants.

Comparing the equations (23) and (24), we get

$$\begin{aligned} -\beta_1 \text{sgn}(\sigma_1) - \mu_1 \sigma_1 &= \dot{\epsilon}_1 + \alpha_1 \epsilon_1 \\ -\beta_2 \text{sgn}(\sigma_2) - \mu_2 \sigma_2 &= \dot{\epsilon}_2 + \alpha_2 \epsilon_2 \\ -\beta_3 \text{sgn}(\sigma_3) - \mu_3 \sigma_3 &= \dot{\epsilon}_3 + \alpha_3 \epsilon_3 \end{aligned} \tag{25}$$

With the help of the error dynamics (21), we can rewrite (25) as

$$\begin{aligned} -\beta_1 \text{sgn}(\sigma_1) - \mu_1 \sigma_1 &= \epsilon_3 - a\epsilon_1 + z_2 z_1 - y_2 y_1 \\ &\quad + v_1 + \alpha_1 \epsilon_1 \\ -\beta_2 \text{sgn}(\sigma_2) - \mu_2 \sigma_2 &= -b(\epsilon_2 + z_1^2 - y_1^2) + v_2 \\ &\quad - c(z_1^6 - y_1^6) + \alpha_2 \epsilon_2 \\ -\beta_3 \text{sgn}(\sigma_3) - \mu_3 \sigma_3 &= -\epsilon_1 - \epsilon_3 + v_3 + \alpha_3 \epsilon_3 \end{aligned} \tag{26}$$

Using (26), we state the main result of this section.

Theorem 1: The master-slave finance chaotic systems represented by (18) and (19) are globally and asymptotically synchronized for all initial conditions $\mathbf{y}(0), \mathbf{z}(0) \in \mathbf{R}^3$ by the integral sliding mode control law

$$\begin{aligned} v_1 &= -\epsilon_3 + a\epsilon_1 - z_2 z_1 + y_2 y_1 \\ &\quad - \beta_1 \text{sgn}(\sigma_1) - \mu_1 \sigma_1 - \alpha_1 \epsilon_1 \\ v_2 &= b(\epsilon_2 + z_1^2 - y_1^2) + c(z_1^6 - y_1^6) \\ &\quad - \beta_2 \text{sgn}(\sigma_2) - \mu_2 \sigma_1 - \alpha_2 \epsilon_2 \\ v_3 &= \epsilon_1 + \epsilon_3 - \beta_3 \text{sgn}(\sigma_3) - \mu_3 \sigma_3 - \alpha_3 \epsilon_3 \end{aligned} \tag{27}$$

where α_i, β_i, μ_i, $(i = 1, 2, 3)$ are all positive constants.

Proof: The result is proved using Lyapunov stability theory [48].

We start the proof by taking the following Lyapunov function

$$W(\sigma_1, \sigma_2, \sigma_3) = \frac{1}{2} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \tag{28}$$

where $\sigma_i, (i = 1, 2, 3)$ are as defined in (22).

It is obvious that W is a positive definite function on $\mathbf{R}^3$. We also note that W is radially unbounded on $\mathbf{R}^3$.

We determine the time-derivative of W as follows:

$$\dot{W} = \sigma_1 \dot{\sigma}_1 + \sigma_2 \dot{\sigma}_2 + \sigma_3 \dot{\sigma}_3 \tag{29}$$

Substituting from (24) into (29), we get

$$\dot{W} = \sum_{i=1}^{3} s_i[-\eta_i \text{sgn}(\sigma_i) - \mu_i \sigma_i] \tag{30}$$

Simplifying (30), we get

$$\dot{W} = -\beta_1|\sigma_1| - \mu_1 \sigma_1^2 - \beta_2|\sigma_2| - \mu_2 \sigma_2^2 - \beta_3|\sigma_3| - \mu_3 \sigma_3^2 \tag{31}$$

which is negative definite on $\mathbf{R}^3$.

Thus, by Lyapunov stability theory [48], it follows that the error system (21) is globally asymptotically stable under the action of the integral sliding mode control (27). ■

For MATLAB simulations, we fix the parameters of the new finance chaotic systems as in the chaos case, *viz.* $(a, b, c) = (7.6, 0.1, 0.2)$.

We take the sliding constants as $\alpha_i = 0.1$, $\beta_i = 0.2$, and $\mu_i = 12$ for $i = 1, 2, 3$.

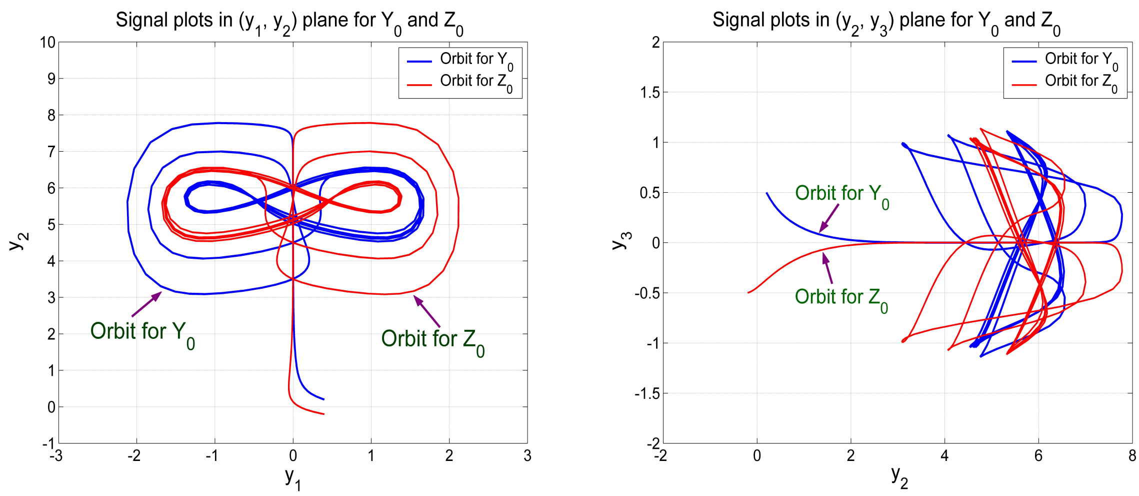
As the initial conditions of the master system (18), we choose

$$y_1(0) = 1.8, \quad y_2(0) = 4.3, \quad y_3(0) = 2.7 \tag{32}$$

As the initial conditions of the slave system (19), we choose

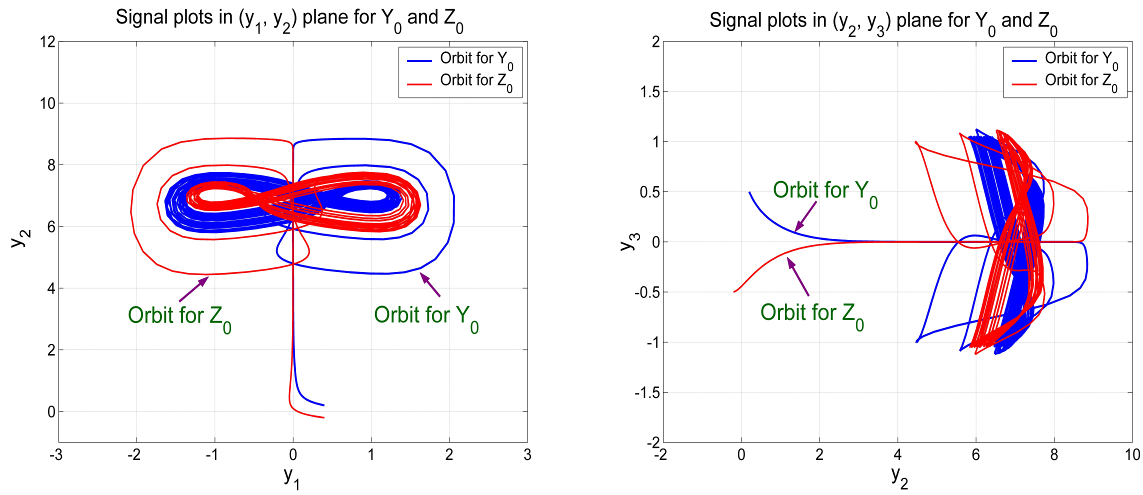
$$z_1(0) = 6.3, \quad z_2(0) = 1.5, \quad z_3(0) = 4.2 \tag{33}$$

Figure 8 shows the time-history of the synchronization error ϵ_1, ϵ_2 and ϵ_3 between the master finance system (18) and the slave finance system (19).



(a) (b)

Fig. 6: Phase portraits of the coexisting periodic attractors with $a = 5$ in (a) the $y_1 - y_2$ plane; (b) the $y_2 - y_3$ plane



(a) (b)

Fig. 7: Phase portraits of the coexisting periodic attractors with $a = 5$ in (a) the $y_1 - y_2$ plane; (b) the $y_2 - y_3$ plane.

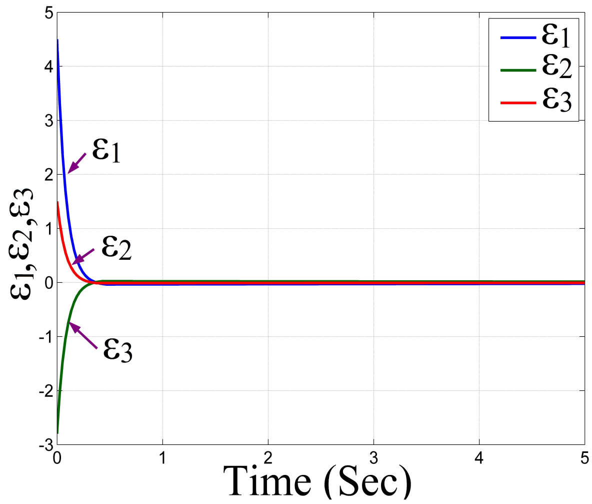


Fig. 8: Time-history of the synchronization error ϵ_1, ϵ_2 and ϵ_3 between the master finance system (18) and the slave finance system (19)

V. CONCLUSION

In this paper, we reported a new 3-D finance chaotic system with two quadratic nonlinearities and a sextic nonlinearity. We showed the signal plots of the new finance chaotic system and discussed its interesting properties such as symmetry, equilibrium points, bifurcation, multistability and Lyapunov exponents. Using integral sliding mode control and Lyapunov stability theory, we derived new results for the global chaos synchronization of a pair of new finance chaotic systems taken as master-slave systems. We illustrated all the main results of this research work using MATLAB phase plots.

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