One Dimensional Steady Thermal Stresses in a Rotating Thick-walled Sphere made of Functionally Graded Material

Pooja Rani, Kuldip Singh and Renu Muwal

ABSTRACT - In this study, a general analysis of one-dimensional steady-state thermal stresses in a hollow rotating thick-walled sphere under internal and external pressures is developed as a function of radial direction to an exact solution. Material properties, except Poisson's ratio, are assumed to depend on radius 'r' and the Poisson's ratio remains constant. The distributions of the thermal stresses are obtained for different values of the powers of the modulus of elasticity. The results have been computed numerically and presented graphically.

Index Terms - functionally Graded Material, rotation, thermal stresses, thick-walled sphere

I INTRODUCTION

Functionally Graded Materials (FGMs) are composite materials which are designed to present a particular spatial variation of their properties. The concept of FGM was first considered in Japan in 1984 during a space plane project. Functionally graded materials have the properties of the two raw materials which are mixed together and the component distribution is graded continuously. The problems of rotating annular disks, spheres or cylinders have been investigated under various assumptions and conditions. One-dimensional steady-state thermal and mechanical stresses for a hollow thick sphere made of functionally graded material was discussed by Eslami et al. [1]. Guven and Baykara [2] considered a functionally graded isotropic hollow sphere with spherical symmetry subjected to internal pressure and this sphere considered as a multimaterial sphere composed of great number of concentric homogeneous spheres of the different elasticity moduli. Li et al. [3] determined the elastic field of a functionally graded hollow spherical vessel with spherically isotropy and spherically transversely isotropy.

Given the assumption that the material had a graded modulus of elasticity with constant Poisson's ratio.

Manuscript received June 07, 2021; revised September 01, 2021.

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Applying the Frobenius method, Lutz and Zimmerman [4] found a way round the problem of uniform heating of a spherical body whose elastic moduli and thermal expansion coefficient varied linearly with radius. Nayak et al. [5] disscussed the general analytical solution of a functionally graded thick spherical vessel with the consideration that the material properties vary with the power law of radius and Poisson's ratio remains constant. Nejad et al. [6] derived exact closed form solutions for stresses and displacements in thick spherical shells made of functionally graded materials. Good agreement was found between the analytical solutions and the solutions carried out through the Finite Element Method.

Pawar et al. [7] determined the thermal stresses in FGM hollow sphere due to non-uniform internal heat generation with radially varying properties by using theory of elasticity. The boundary value problem was converted to a Fredholm Integral Equation to obtain the radial displacement and thermal stresses in a functionally graded hollow cylinder as Peng and Li [8]. Thermal stresses in a hollow rotating thick-walled cylinder made of functionally graded material under internal and external pressure as a function of radial direction to an exact solution by using the theory of elasticity was studied by Rahimi and Nejad [9]. Rani et al. [10] studied thermal stresses of a functionally graded hollow thick cylinder due to non-uniform internal heat generation and obtained analytical solutions with radially varying properties.

Using plane elasticity theory and complementary function method, axisymmetric displacements and stresses in functionally graded hollow cylinders, disks and spheres subjected to uniform internal pressure were determined by Tutuncu and Temel [11]. Complementary function method reduced the boundary value problem to an initial value problem which could be solved accurately by one of many efficient methods such as Runge-kutta method. Yildirim [12] conducted the thermal analysis of functionally graded thick-walled spherical vessel and an infinite cylinderical vessel or a circular annulus by the steady-state one dimensional Fourier heat conduction theory under Dirichlet's boundary conditions. A parametric study was performed with hypothetic inhomogeneity indexes for varying aspect ratios. Yildirim [13] presented the thermo-mechanical analysis of sphere made of non-homogeneous isotropic materials and proposed the closed form formulas for the elastic fields in a simple-power-law graded spheres subjected to steady-state thermal and internal/external pressure loads.

In this paper, we have studied the thermal stresses based on uncoupled thermoelasticity in a functionally graded hollow rotating thick-walled sphere under internal and external pressures and obtained the expressions for radial and tangential stresses by using the theory of elasticity. The variation of thermal stresses are also shown graphically for different values of the modulus of elasticity in the plane strain conditions.

II MATERIALS AND METHODS

Consider a thick-walled functionally graded sphere with an inner radius 'a' and outer radius 'b', subjected to an internal pressure P_i and external pressure P_e that are axisymmetrtic and rotating at a constant angular velocity ω about its axis.

The properties in spherical coordinatres θ and ϕ are identical. The material properties are assumed to vary as power function in radial direction. The sphere is graded in the radial direction so that the material properties of modulus of elasticity, thermal expansion coefficient, thermal conductivity and density through the wall thickness are functions of 'r' only.

The following power law functions of radius in the radial directions are assumed to vary as

$$E = E_0 r^{m_1} \tag{1}$$

$$k = k_0 r^{m_2} \tag{2}$$

$$\alpha = \alpha_0 r^{m_3} \tag{3}$$

$$\rho = \rho_0 r^{m_4} \tag{4}$$

where E, k, α and ρ are modulus of elasticity, thermal conductivity, linear expansion coefficient and density respectively . E_0, k_0 , α_0 and ρ_0 are the material constants and m_1, m_2, m_3 and m_4 are the power law indices of the material.

It is assumed that for plane strain $\varepsilon_{\phi\phi} = 0$. The radial strain ε_{rr} and tangential strain $\varepsilon_{\theta\theta}$ are related to the radial displacement u by

$$\varepsilon_{rr} = \frac{du}{dr} \tag{5}$$

$$\varepsilon_{\theta\theta} = \frac{u}{r} \tag{6}$$

The equilibrium equation in the radial direction in the absence of body force is given as under:

$$r\frac{d\sigma_{rr}}{dr} + 2(\sigma_{rr} - \sigma_{\theta\theta}) = -\rho r\omega^2 \tag{7}$$

where σ_{rr} and $\sigma_{\theta\theta}$ are the radial and tangential stress components.

In the steady-state condition, for one-dimensional problem in polar coordinates, the heat conduction equation may be obtained as under:

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 k \frac{dT}{dr}\right] = 0, \qquad a \le r \le b \tag{8}$$

Boundary conditions of temperature are as follows:

$$T = T_a \quad \text{at} \quad r = a \tag{9}$$

$$T = T_b \quad \text{at} \quad r = b \tag{10}$$

where T_a and T_b are the temperature at the inner and outer surface.

The general solution of equation (8) by considering the relation of thermal transfer coefficient equation (2) and boundary conditions (9) and (10) is:

For $m_2 \neq -1$

$$T(r) = \frac{1}{b^{-m_2-1} - a^{-m_2-1}}[(T_a - T_b)r^{-m_2-1} + T_b a^{-m_2-1} - T_a b^{-m_2-1}] \tag{11}$$

For $m_2 = -1$

$$T(r) = \frac{(T_a - T_b)}{\ln\frac{a}{b}}\ln r + \frac{T_b \ln a - T_a \ln b}{\ln\frac{a}{b}} \tag{12}$$

The stress components σ_{rr} and $\sigma_{\theta\theta}$ in the radial and tangential directions in the terms of strain components ε_{rr} and $\varepsilon_{\theta\theta}$ for plane strain condition are given by

For $m_2 \neq -1$

$$\sigma_{rr} = A_1 r^{m_1}\varepsilon_{rr} + A_2 r^{m_1}\varepsilon_{\theta\theta} + A_3 r^{m_1+m_3-m_2-1} + A_4 r^{m_1+m_3} \tag{13}$$

$$\sigma_{\theta\theta} = A_5 r^{m_1}\varepsilon_{rr} + A_6 r^{m_1}\varepsilon_{\theta\theta} + A_3 r^{m_1+m_3-m_2-1} + A_4 r^{m_1+m_3} \tag{14}$$

For $m_2 = -1$

$$\sigma_{rr} = A_1 r^{m_1}\varepsilon_{rr} + A_2 r^{m_1}\varepsilon_{\theta\theta} + A_7 r^{m_1+m_3}\ln r + A_8 r^{m_1+m_3} \tag{15}$$

$$\sigma_{\theta\theta} = A_5 r^{m_1}\varepsilon_{rr} + A_6 r^{m_1}\varepsilon_{\theta\theta} + A_7 r^{m_1+m_3}\ln r + A_8 r^{m_1+m_3} \tag{16}$$

where A_1, A_2, A_3, A_4, A_5, A_6, A_7 and A_8 are

$$\begin{aligned}
A_1 &= \frac{E_0(1-\sigma)}{(1+\sigma)(1-2\sigma)} \\
A_2 &= \frac{2E_0\sigma}{(1+\sigma)(1-2\sigma)} \\
A_3 &= \frac{E_0\alpha_0(T_a - T_b)(ab)^{m_2+1}}{(a^{m_2+1} - b^{m_2+1})(1-2\sigma)} \\
A_4 &= \frac{E_0\alpha_0(T_a b^{m_2+1} - T_b a^{m_2+1})}{(a^{m_2+1} - b^{m_2+1})(1-2\sigma)} \\
A_5 &= \frac{E_0\sigma}{(1+\sigma)(1-2\sigma)} \\
A_6 &= \frac{E_0}{(1+\sigma)(1-2\sigma)}
\end{aligned} \tag{17}$$

$$A_7 = \frac{E_0\alpha_0(T_a - T_b)}{(1-2\sigma)\ln\dfrac{b}{a}}$$

$$A_8 = \frac{E_0\alpha_0(T_b\ln a - T_a\ln b)}{(1-2\sigma)\ln\dfrac{b}{a}}$$

Using equations (1)-(10) and (13)-(17), the Navier's equation in terms of radial displacement becomes as under:

For $m_2 \neq -1$

$$\begin{aligned} &r^2\frac{d^2u}{dr^2} + (m_1+2)r\frac{du}{dr} + 2(nm_1-1)u \\ &= B_1 r^{3+m_4-m_3} + B_2 r^{m_3-m_2} + B_3 r^{m_3+1} \end{aligned} \tag{18}$$

For $m_2 = -1$

$$\begin{aligned} &r^2\frac{d^2u}{dr^2} + (m_1+2)r\frac{du}{dr} + 2(nm_1-1)u \\ &= B_1 r^{3+m_4-m_3} + B_4 r^{m_2+1}\ln r + B_5 r^{m_3+1} \end{aligned} \tag{19}$$

where $n = \dfrac{\sigma}{1-\sigma}$

and B_1, B_2, B_3, B_4, B_5 are

$$\begin{aligned} B_1 &= \frac{-\rho_0\omega^2(1+\sigma)(1-2\sigma)}{E_0(1-\sigma)} \\ B_2 &= \frac{\alpha_0(1+\sigma)(m_1-m_2+m_3-1)(T_a-T_b)(ab)^{m_2+1}}{(1-\sigma)(b^{m_2+1}-a^{m_2+1})} \\ B_3 &= \frac{\alpha_0(1+\sigma)(m_1+m_3)(T_b b^{m_2+1} - T_a a^{m_2+1})}{(1-\sigma)(b^{m_2+1}-a^{m_2+1})} \\ B_4 &= \frac{\alpha_0(1+\sigma)(m_1+m_3)(T_b-T_a)}{(1-\sigma)\ln\dfrac{b}{a}} \\ B_5 &= \frac{\alpha_0(1+\sigma)[(T_b-T_a)+(m_1+m_3)(T_a\ln b - T_b\ln a)]}{(1-\sigma)\ln\dfrac{b}{a}} \end{aligned} \tag{20}$$

Eqs. (18) - (19) are the non-homogeneous Cauchy-Euler equations.

The general solution of Eqs. (18)-(19) are obtained by adding particular solution to the complementary solution of homogeneous form. The complimentary function u_c is taken as

$$u_c(r) = Xr^f \quad \text{where X is constant} \tag{21}$$

Using (21) in homogeneous form of (18)-(19), we get

For $m_2 \neq -1$

$$u_c(r) = X_1 r^{f_1} + X_2 r^{f_2} \tag{22}$$

For $m_2 \neq -1$

$$u_c(r) = X_3 r^{f_1} + X_4 r^{f_2} \tag{23}$$

where X_1, X_2, X_3, X_4 are constants and

$$f_1 = \frac{-(m_1+1)+\sqrt{m_1^2-(8n-2)m_1+9}}{2},$$

$$f_2 = \frac{-(m_1+1)-\sqrt{m_1^2-(8n-2)m_1+9}}{2} \tag{24}$$

The particular solutions for Eqs. (18)-(19) are considered as

For $m_2 \neq -1$

$$u_p(r) = C_1 r^{3+m_4-m_1} + C_2 r^{m_3-m_2} + C_3 r^{m_3+1} \tag{25}$$

For $m_2 \neq -1$

$$u_p(r) = C_1 r^{3+m_4-m_1} + C_4 r^{m_3+1}\ln r + C_5 r^{m_3+1} \tag{26}$$

where C_1, C_2, C_3, C_4, C_5 are constants.

Using Eq. (25) in (18); (26) in (19) and equating the coefficients of identical powers, we obtain

$$\begin{aligned} C_1 &= \frac{B_1}{(3+m_4-m_1)(4+m_4)+2(nm_1-1)} \\ C_2 &= \frac{B_2}{(m_3-m_2)(m_1+m_3-m_2+1)+2(nm_1-1)} \\ C_3 &= \frac{B_3}{(1+m_3)(m_1+m_3+2)+2(nm_1-1)} \\ C_4 &= \frac{B_4}{(1+m_3)(m_1+m_3+2)+2(nm_1-1)} \\ C_5 &= \frac{B_5 - C_4(m_1+2m_3+3)}{(1+m_3)(m_1+m_3+2)+2(nm_1-1)} \end{aligned} \tag{27}$$

The complete solutions u(r) of Eqs. (18)-(19) are obtained as

For $m_2 \neq -1$

$$u(r) = X_1 r^{f_1} + X_2 r^{f_2} + C_1 r^{3+m_4-m_1} + C_2 r^{m_3-m_2} + C_3 r^{m_3+1} \tag{28}$$

For $m_2 = -1$

$$u(r) = X_3 r^{f_1} + X_4 r^{f_2} + C_1 r^{3+m_4-m_1} + C_4 r^{m_3+1}\ln r + C_5 r^{m_3+1} \tag{29}$$

By substituting Eqs. (28)-(29) into Eqs. (5)-(6) and on using these in Eqs. (13)-(16), we get the stresses as under:

For $m_2 \neq -1$

$$\begin{aligned} \sigma_{rr} &= X_1[A_1 f_1 + A_2]r^{m_1+f_1-1} + X_2[A_1 f_1 + A_2]r^{m_1+f_2-1} \\ &+ C_1[A_1(3+m_4-m_1)+A_2]r^{m_4+2} \\ &+ C_2[A_1(m_3-m_2)+A_2]r^{m_1+m_3-m_2-1} \\ &+ C_3[A_1(1+m_3)+A_2]r^{m_1+m_3} + A_3 r^{m_1+m_3-m_2-1} + A_4 r^{m_1+m_3} \end{aligned} \tag{30}$$

$$\begin{aligned} \sigma_{\theta\theta} &= X_1[A_5 f_1 + A_6]r^{m_1+f_1-1} + X_2[A_5 f_1 + A_6]r^{m_1+f_2-1} \\ &+ C_1[A_5(3+m_4-m_1)+A_6]r^{m_4+2} \\ &+ C_2[A_5(m_3-m_2)+A_6]r^{m_1+m_3-m_2-1} \\ &+ C_3[A_5(1+m_3)+A_6]r^{m_1+m_3} + A_3 r^{m_1+m_3-m_2-1} + A_4 r^{m_1+m_3} \end{aligned} \tag{31}$$

For $m_2 = -1$

$$\sigma_{rr} = X_3[A_1 f_1 + A_2]r^{m_1+f_1-1} + X_4[A_1 f_1 + A_2]r^{m_1+f_2-1}$$
$$+C_1[A_1(3+m_4-m_1)+A_2]r^{m_4+2}$$
$$+C_4[A_1(1+(1+m_3)\ln r)+A_2\ln r]r^{m_1+m_3}$$
$$+C_5[A_1(1+m_3)+A_2]r^{m_1+m_3} + A_7 r^{m_1+m_3}\ln r + A_8 r^{m_1+m_3} \quad (32)$$

$$\sigma_{\theta\theta} = X_3[A_5 f_1 + A_6]r^{m_1+f_1-1} + X_4[A_5 f_1 + A_6]r^{m_1+f_2-1}$$
$$+C_1[A_5(3+m_4-m_1)+A_6]r^{m_4+2}$$
$$+C_4[A_5(1+(1+m_3)\ln r)+A_6\ln r]r^{m_1+m_3}$$
$$+C_5[A_5(1+m_3)+A_6]r^{m_1+m_3} + A_7 r^{m_1+m_3}\ln r + A_8 r^{m_1+m_3} \quad (33)$$

To determine the constants X_1, X_2, X_3 and X_4, consider the boundary conditions for stresses are given by

$$\sigma_{rr} = -P_i \text{ at } r = a \text{ and } \sigma_{rr} = -P_e \text{ at } r = b \quad (34)$$

Using boundary Eqs. (34) in (30) and (32), we get the values of constants X_1, X_2, X_3 and X_4 as under:

$$X_1 = \frac{g_4 g_5 - g_2 g_6}{g_1 g_4 - g_2 g_3}, \qquad X_2 = \frac{g_1 g_6 - g_3 g_5}{g_1 g_4 - g_2 g_3}$$
$$X_3 = \frac{g_4 g_7 - g_2 g_8}{g_1 g_4 - g_2 g_3}, \qquad X_4 = \frac{g_1 g_8 - g_3 g_7}{g_1 g_4 - g_2 g_3} \quad (35)$$

where the parameters $g_1, g_2, g_3, g_4, g_5, g_6, g_7$ and g_8 are given by

$$g_1 = (A_1 f_1 + A_2)a^{m_1+f_1-1}$$
$$g_2 = (A_1 f_1 + A_2)a^{m_1+f_2-1}$$
$$g_3 = (A_1 f_1 + A_2)b^{m_1+f_1-1}$$
$$g_4 = (A_1 f_1 + A_2)b^{m_1+f_2-1}$$
$$g_5 = -C_1[A_1(3+m_4-m_1)+A_2]a^{m_4+2}$$
$$-C_2[A_1(m_3-m_2)+A_2]a^{m_1+m_3-m_2-1}$$
$$+C_3[A_1(1+m_3)+A_2]a^{m_1+m_3} \quad (36)$$
$$-A_3 a^{m_1+m_3-m_2-1} - A_4 a^{m_1+m_3} - P_i$$
$$g_6 = -C_1[A_1(3+m_4-m_1)+A_2]b^{m_4+2}$$
$$-C_2[A_1(m_3-m_2)+A_2]b^{m_1+m_3-m_2-1}$$
$$+C_3[A_1(1+m_3)+A_2]b^{m_1+m_3}$$
$$-A_3 b^{m_1+m_3-m_2-1} - A_4 b^{m_1+m_3} - P_e$$
$$g_7 = -C_1[A_1(3+m_4-m_1)+A_2]a^{m_4+2}$$
$$-C_4[A_1(1+(1+m_3)\ln a)+A_2\ln a]a^{m_1+m_3}$$
$$-C_5[A_1(1+m_3)+A_2]a^{m_1+m_3}$$
$$-A_7 a^{m_1+m_3}\ln a - A_8 a^{m_1+m_3} - P_i$$
$$g_8 = -C_1[A_1(3+m_4-m_1)+A_2]b^{m_4+2}$$
$$-C_4[A_1(1+(1+m_3)\ln b)+A_2\ln b]b^{m_1+m_3}$$
$$-C_5[A_1(1+m_3)+A_2]b^{m_1+m_3}$$
$$-A_7 b^{m_1+m_3}\ln b - A_8 b^{m_1+m_3} - P_e$$

III VALIDATION

In the final equations, if we substitute zero for indices m_1, m_2, m_3, and m_4, we get the expressions for an isotropic and homogeneous sphere. This fact can be used as a partial validation of the final equations (30)-(31).

Substituting $m_1 = m_2 = m_3 = m_4 = 0$ in Eqs. (1)-(4); E, α, k and ρ become E_0, α_0, k_0 and ρ_0, which are modulus of elasticity, linear expansion coefficient, thermal conductivity and density respectively for an isotropic and homogeneous material.

On taking $C_1 = 0$, the results obtained for thermal stresses in the expressions (30)-(33) coincide with the results of Nayak et al. [5] and Pawar et al. [7].

IV NUMERICAL RESULTS AND DISCUSSION

The mathematical thermoelastic model of a functionally graded hollow thick-walled sphere can be constructed by a considering a thermal gradient through its radial direction.

A thick-walled spherical vessel of inner radius a=1.0 meter and outer radius b= 1.2 meter is considered, which is rotating at the constant angular velocity of $\omega = 10\, rad\, \sec^{-1}$. It is assumed that the Poisson's ratio, σ has a constant value of 0.3. Material constants of thermal expansion coefficient, modulus of elasticity and density are taken as $\alpha_0 = 10\times10^{-6}$, $E_0 = 200\, Gpa$ and $\rho_0 = 7810\, kgm^{-3}$ respectively. For boundary conditions, the inner and outer surfaces of the sphere are taken to be under the pressure of $P_i = 40\, MPa$ and $P_e = 0\, MPa$ respectively. In addition, the temperature of the inner and outer surfaces are considered constant as $T_a = 10^0 C$ and $T_b = 0^0 C$. Furthermore, it is assumed that $m_1 = m_2 = m_3 = m_4 = m$.

The range $-2 \le m \le 2$ is used in the present study which consists of all the values which has widely been used in the references cited earlier. Various values of m are used to demonstrate the effect of homogeneity on the stress distribution.

The variation of the radial displacement along the radius is shown in fig.1. There is increase in the value of the radial displacement as m increases. Figs. 2 and 3 shows the distribution of radial and tangential stresses in the radial direction. As m increases, so does the magnitude of the radial stress. For $-2 \le m \le 1$, the tangential stress decreases as the radius increases whereas for m = 2, the tangential stress along the radius increases.

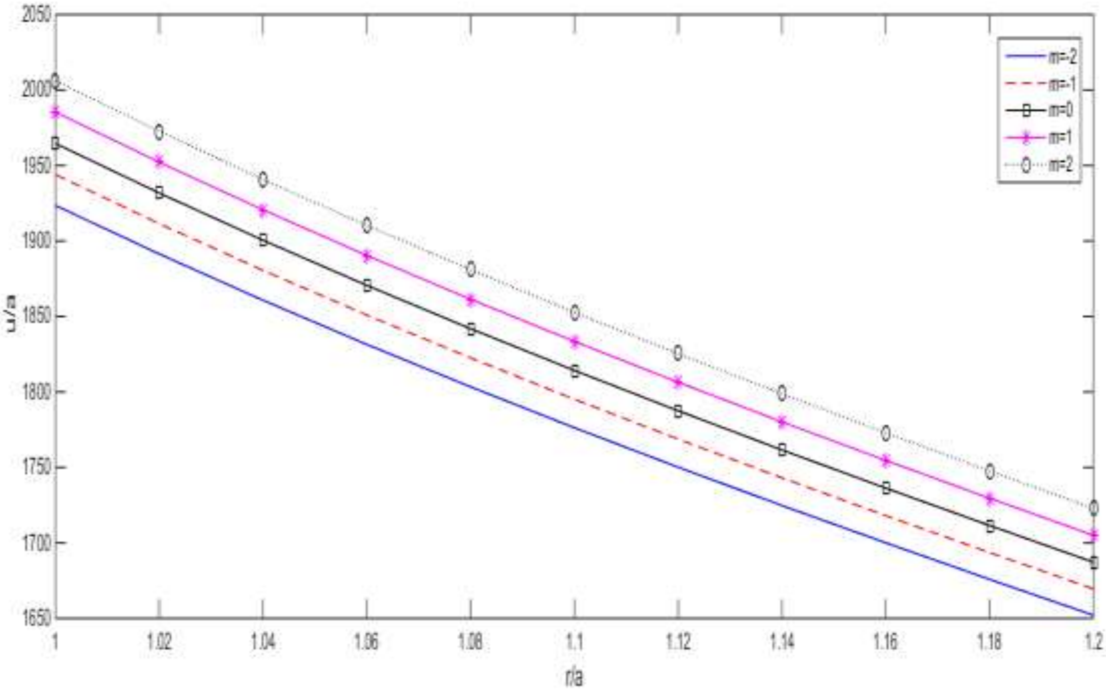

Fig. 1. Variation of radial displacement u with radius of sphere

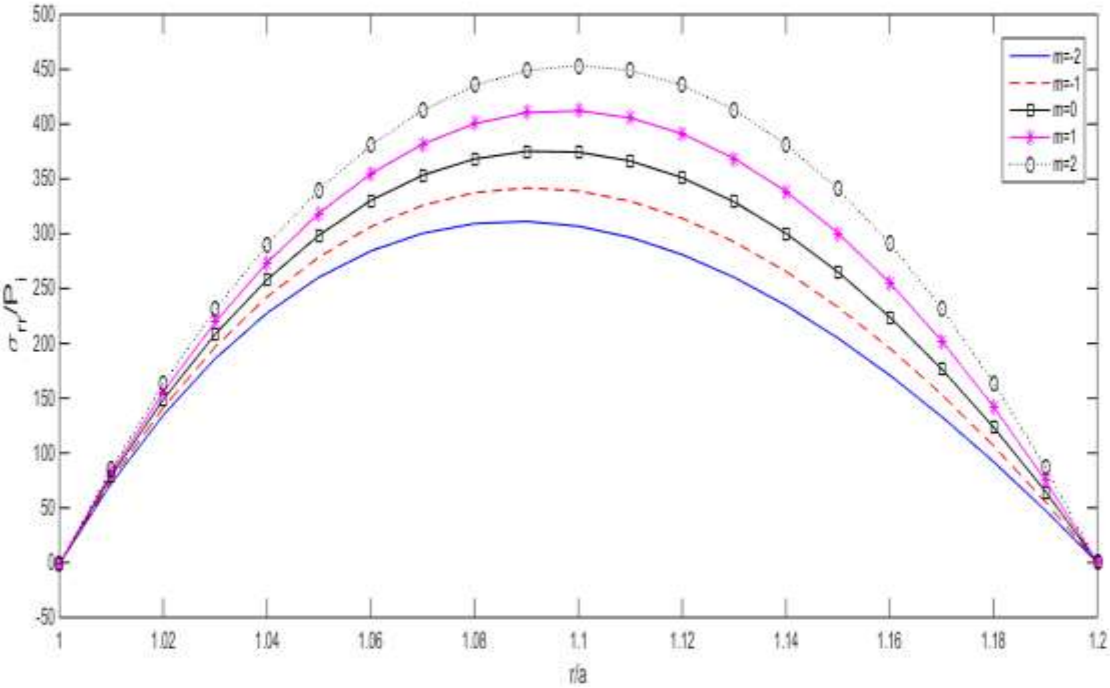

Fig. 2. Variation of radial stress σ_{rr} with radius of sphere

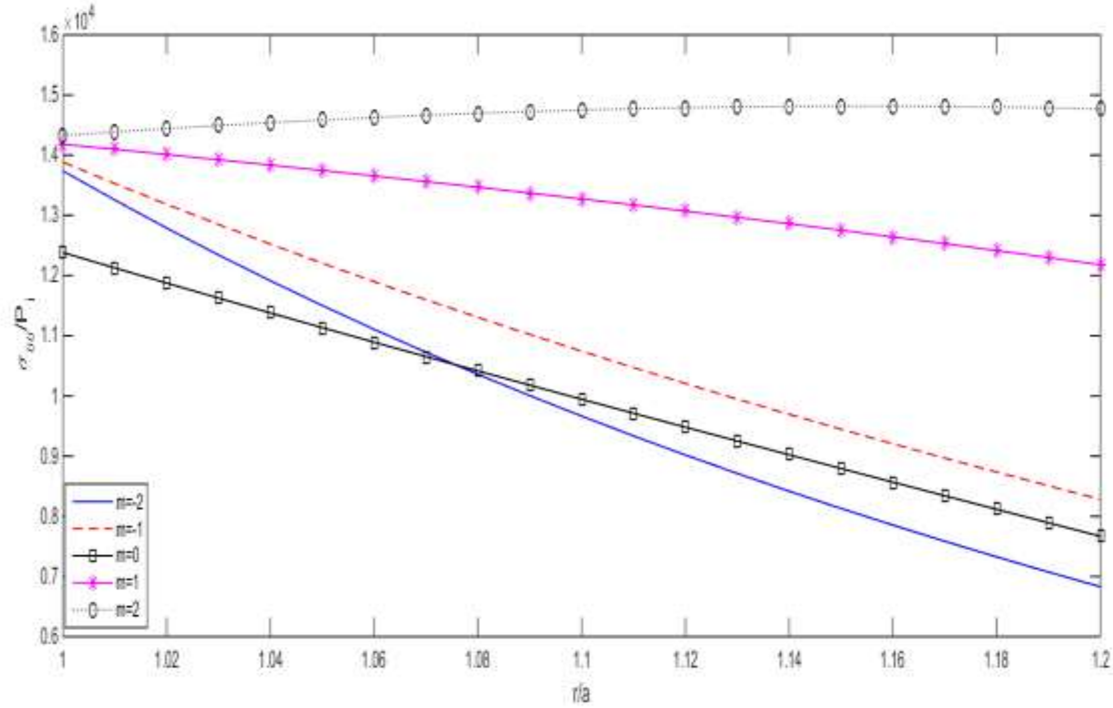

Fig. 3. Variation of tangential stress $\sigma_{\theta\theta}$ with radius of sphere

V CONCLUSION

In the present study, one dimensional steady-state thermal stresses are obtained for functionally graded rotating sphere. As a special case, Mathematical model is constructed for rotating sphere with material properties specified in the numerical calculations. In this study, it is observed that the radial displacement and the radial stress, both increase with the increase in power law index m, whereas the tangential stress decreases as the radius increases for $-2 \le m \le 1$ and for $m = 2$, the tangential stress along the radius increases. The results can be generalized for other parameters values.

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