A Sequentially Updated Distributed Receding Horizon Control Scheme with Application to Irrigation Canal Systems

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***Abstract*—This study addresses a distributed receding horizon control strategy for a discrete linear system consisting of several subsystems. These subsystems are updated sequentially. The primary purpose of the study is to construct a control strategy such that all states converge to origin. We implement our proposed method to irrigation canal systems.**

***Index Terms*—receding horizon control, distributed control, model predictive control, irrigation canal systems, sequential control**

I. Introduction

DISTRIBUTED receding horizon control, also known as distributed model predictive control (DMPC), involves some local controllers optimizing their local objective to provide performances similar to those offered by centralized MPC. The idea of DMPC schemes is to decompose a large optimization problem into several subproblems which correspond to the designated local controllers and use MPC to design them. In this way, DMPC is preferable to control large-scale systems such as irrigation systems due to its capability in saving computational time (see [1]). Irrigation canal systems consist of such interacting components as gates, reaches or pools, and pumps which all work together to meet the public's water demand, see [2].

DMPC and any MPC-based controllers can be used to control water levels or flow in irrigation canal systems by manipulating the gates, reaches, and pumps (see [1]). There are some control techniques based on MPC for irrigation canal systems, like [3]–[8], etc. The authors in [1] summarized some MPC methods for irrigation canal systems. In [3], some DMPC algorithms based on Han's parallel method for the convex program are designed and implemented in canal systems. As mentioned in [3], although the simulation result has verified the method, some theoretical problems prevent its applicability. In [4], two DMPC schemes based on game theory are applied to an irrigation canal system with five reaches, a slight modification of irrigation canal system [3]. A coalitional-based MPC is established for water networks and is applied to the first 7 reaches of Dez irrigation canal by [5]. The authors in [6], and [7] proposed a simplified method of [8] to reduce the risk in the operation process of irrigation canals. In most literature, fixed setpoints are used as control targets in the design. [9] dan [10], however, established a strategy that enables the controllers to cope with the imbalance of water supply and demand, i.e., the changing setpoints.

In [11] and [12], a DMPC algorithm is designed for a continuous nonlinear system with m inputs. To update these inputs, m Lyapunov-based MPC controllers, also known as LMPCs, are designed for each input. These LMPCs are updated sequentially, starting from the LMPC m to LMPC 1. Each LMPC sends its optimal inputs trajectories and other trajectories it received from other subsystems to the subsystem right below it. When all the LMPCs have completed updating their inputs, new optimal states measurement are available to the system. In these methods, Lyapunov-based auxiliary control laws are used to guarantee the stability of each LMPC.

This paper discusses a DMPC algorithm for a discrete linear system. It is assumed that the system consists of M subsystems with input couplings. Apart from [11] and [12], we updated the subsystems sequentially, in the proposed method. We assume that the control design involves a fix updating sequence that starts from subsystem in the upstream to the ones in the downstream. Furthermore, the design only considers fixed setpoints. We give a simulation of our method to an irrigation canal system which consists of four reaches as discussed in [3]. In addition, we provide a comparison of our approach to Nash-Bargaining MPC [4]. Moreover, for this irrigation canal, we omit any conditions that may cause abrupt changes in the water levels, such as flood or water loss in the canal due to any of its permeability. Interested readers on the study of irrigation canals with impermeable soil layers are directed to [13] and the references therein.

The paper is organized as follows: In Section I we provide an introduction, the problem to be solved, and the method briefly. Next, the description of the Sequential Distributed Receding Horizon Control Scheme and the underlying assumptions used in the design are provided in Section II. In Section III, we discuss the properties of the proposed method. A brief overview of irrigation canal systems is addressed in Section IV, and simulation is in Section V. Lastly, we give the conclusion in Section VI.

Notation 1: $(p|k)$ denotes the prediction for time step $k+p$ made at time step k, where p and k are nonnegative integers. $X^i_{1|k}$ and $U^i_{0|k}$ represent state prediction vector and input for Subsysten i made at time step k, respectively.

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Moreover, $A = diag(a_{11}, a_{22}, \ldots, a_{nn})$ means that A is a $n \times n$ diagonal matrix with the main diagonal elements are $a_{ii}, i = 1, 2, \ldots, n$.

II. Sequential Distributed Receding Horizon Control Scheme

Considering a discrete linear time invariant system consists of M interconnected subsystems, it is assumed that the dynamics of each subsystem can be described as a discrete linear time invariant model as follows

$$\begin{cases} x_i(k+1) & = A_i x_i(k) + B_i u_i(k) + \sum_{j \neq i} B_{ij} u_j(k) \\ y_i(k) & = C_i x_i(k) \end{cases} \tag{1}$$

where $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ is the state of Subsystem i, while $u_i \in \mathcal{U}_i \subseteq \mathbb{R}^{m_i}$, and $y_i \in \mathcal{Y}_i \subseteq \mathbb{R}^{p_i}$ represent input and output of Subsystem i, respectively. Both $\mathcal{X}_i$ and $\mathcal{U}_i$ are convex and closed sets containing origin in its interior. From Eq. (1), it can be seen that all the subsystems are coupled through the inputs.

The following condition is assumed to apply to each subsystem in order to simplify the control design.

Assumption 1: $\forall i, i = 1, 2, \ldots, M$, full state x_i is available for all time steps.

Remember that without employing the Assumption 1, we need to involve a state estimation in the design, which is beyond the scope of this paper. The goal of control given to Subsystem (1) is to design a DMPC controller which steers the subsystem's trajectories to origin while addressing the constraints on input and state as well. To this end, we propose a DMPC algorithm that updates the subsystems sequentially. More precisely, at time step k, all subsystems have information of the state of other subsystems, namely $x(k)$, the state measurement at time step k. Subsystem i use this information to conduct an optimization which yields an optimal input sequence $U_i^*(0|k)$. Next, the first element of this sequence is chosen to implement the system. After that, Subsystem i send $U_i^*(0|k)$ and $U_j^*(0|k)$ where $j < i$ to Subsystem $i+1$. The same process is repeated until Subsystem M. In this way, Subsystem 1 does not have any information of any Subsystem j, where $j = 2, 3, \ldots, M$, whereas Subsystem M, does not send any information to the other subsystems. The unknown information then approximated using linear auxiliary control law $\kappa(x) = \begin{bmatrix} \kappa_1(x) & \ldots & \kappa_M(x) \end{bmatrix}$, which is assumed to be available at time k, where $\kappa_i(x)$ represents auxiliary control law of Subsystem i. This strategy also used in [11] and [12]. Another study on DMPC for systems with coupling inputs can be found in [14].

The proposed sequential DMPC is established using the following assumptions. Let $\mathcal{X}_{f,i}$ be a terminal constraints set for Subsystem i. The assumptions below apply to $\mathcal{X}_{f,i}$ and the auxiliary control law, which are useful to characterize the stability of the proposed method.

Assumption 2: $\mathcal{X}_{f,i} \subset X_i$, and $0 \in \mathcal{X}_{f,i}$.

Assumption 3: For each $x_i \in \mathcal{X}_{f,i}$, there exist an auxiliary feedback control law $\kappa_i(x) = \kappa_i x_i(k) \in \mathcal{U}_i$ such that $x_i^\kappa(k+1) = (A_i + B_i\kappa_i)x_i(k) + \sum_{j \neq i} B_{ij} u_j(k) \in \mathcal{X}_{f,i}$. In addition, $\kappa_i(x)$ is constant during two consecutive interval updates.

Assumption 3 declares the existence of auxiliary control law in $\mathcal{X}_{f,i}$ and the invariance property of $\mathcal{X}_{f,i}$ under this control law. These conditions are used to establish a stability property in the Subsystem i.

According to the implementation strategy mentioned earlier, the state of Subsystem i at time $k+1$ can be rewritten as

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j<i} B_{ij} u_j^*(k) + \sum_{j>i} B_{ij} \kappa_j(x_j(k)) \tag{2}$$

where $u_j^*(k)$ and $\kappa_j(x_j(k))$ denote optimal input and auxiliary feedback control law of Subsystem j at time step k, respectively.

Remark 1: In Eq. (2), if Subsystem i is not affected by Subsystem j, then $B_{ij} = 0$. Thus, the interconnection matrix B_{ij} plays a role in determining whether the optimal control $u_j^*(k)$ and the auxiliary control law $\kappa_j(x_j(k))$ affecting the dynamics of Subsystem i, where $j \neq i$.

To determine the optimal sequential DMPC control, at each time step k, Subsystem i solves the following optimization sequentially starting from Subsystem 1, and so on until Subsystem M. In this paper, we assumed that the prediction and control horizon lengths are equal to $N \in \mathbb{N}$. The problem need to be solved by all subsystems sequentially is described as,

$$\mathcal{P}_i : \quad \min_{u_i(p|k), p=0,1,\ldots,N-1} J_i(k) \tag{3}$$

$$\text{s.t} \quad x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j \neq i} B_{ij} u_j(k), \tag{4}$$

$$u_j(k) = u_j^*(0|k), j = 1, 2, \ldots, i-1, \tag{5}$$

$$u_j(k) = \kappa_j(x_j(k)), j = i+1, i+2, \ldots, M, \tag{6}$$

$$x_i(k+N) \in \mathcal{X}_{f,i}, \tag{7}$$

where at the beginning of each optimization it is assumed that the initial states and inputs of all subsystems are known to the system. In addition, terminal constraint set $\mathcal{X}_{f,i}$ can be determined using function 'oinfsetcl' in Invariant Set Toolbox [15].

Suppose that $U_i^*(0|k)$ be the optimal solution of Problem $\mathcal{P}_i$ at time step k, where $U_i^*(0|k) = \begin{bmatrix} u_i^*(0|k)^T & u_i^*(1|k)^T & \ldots & u_i^*(N-1|k)^T \end{bmatrix}^T$. In each optimization, by following the receding horizon principle, the first element of each optimal input sequence, i.e., $u_i^*(0|k)$ is chosen as the optimal input to be applied to Subsystem i. Using these optimal inputs, each subsystem can calculate its optimal state. These optimal states are then used as initial values for the optimization at time step $k+1$. This whole process is repeated max_iter times, where max_iter is a positive integer chosen by the designer.

We use the following form of objective function since we solve a regulation problem,

$$J_i(k) = \sum_{j=1}^{N-1} \|x_i(j|k)\|_{Q_i}^2 + \sum_{j=0}^{N-1} \|u_i(j|k)\|_{R_i}^2 + V_{f,i}\left(x_i(N|k)\right), \tag{8}$$

where Q_i and R_i are symmetric positive definite matrices, $V_{f,i}(\cdot)$ is a terminal cost of the form as given below

$$V_{f,i}\left(x_i(N|k)\right) = x_i(N|k)^T P_i x_i(N|k) \tag{9}$$

where P_i is a symmetric positive definite matrix and is a solution of the following equation

$$(A_i + B_i\kappa_i)^T P_i (A_i + B_i\kappa_i) - P_i = -\left(Q_i + \kappa_i^T R_i \kappa_i\right). \quad (10)$$

This matrix P_i is also known as a Riccati matrix.

We use the following assumption on the terminal cost of each subsystem.

Assumption 4: For every Subsystem i with $x_i(k)$ and $x_i(k+1) \in \mathcal{X}_{f,i}$, the following inequality holds

$$V_{f,i}(x_i(k+1)) - V_{f,i}(x_i(k)) \leq -\|x_i(0|k)\|_{Q_i}^2 - \|u_i(0|k)\|_{R_i}^2,$$

for all $k \geq 0$, and for all $i = 1, 2, \ldots, M$.

Assumption 4 states that $V_{f,i}$ is a nonincreasing function in $\mathcal{X}_{f,i}$, for all $i = 1, 2, \ldots, M$.

To synthesize the sequential DMPC controller, the processes in each subsystem are in general similar to those for centralized MPC algorithms; for example, see [16]. Thus, we derive the model prediction of each subsystem, transform the objective function into a compact form. Then, we write the resulting optimization problem into a suitable form as required by the solver to be used in the optimization. To begin with, for horizon prediction length equal to N, the model prediction of Subsystem i can be derived recursively as follows, $i = 1, 2, \ldots, M$. Note that quantities from the other subsystems are constant during these N time steps; thus, we have

$$\begin{aligned}
x_i(1|k) &= A_i x_i(0|k) + B_i u_i(0|k) + B_{i1} u_1^*(k) + \cdots + \\
&\quad B_{i(i-1)} u_{i-1}^*(k) + B_{i(i+1)} \kappa_{i+1}(x_{i+1}(0|k)) \\
&\quad + \cdots + B_{iM} \kappa_M (x_M(0|k)), \\
x_i(2|k) &= A_i^2 x_i(0|k) + A_i B_i u_i(0|k) + B_i u_i(1|k) + \\
&\quad (A_i B_{i1} + B_{i1})\, u_1^*(0|k) + \cdots + \\
&\quad \left(A_i B_{i(i-1)} + B_{i(i-1)}\right) u_{i-1}^*(0|k) + \\
&\quad \left(A_i B_{i(i+1)} + B_{i(i+1)} \kappa_{i+1}\right)(x_{i+1}(0|k)) \\
&\quad + \cdots + (A_i B_{iM} + B_{iM})\, \kappa_M (x_M(0|k)) \\
&\ \ \vdots \\
x_i(N|k) &= A_i^N x_i(0|k) + A_i^{N-1} B_i u_i(0|k) + \cdots + \\
&\quad B_i u_i(N-1|k) + \sum_{p=0}^{N-1} A_i^{N-1-p} B_{i1} u_1^*(0|k) \\
&\quad + \cdots + \sum_{p=0}^{N-1} A_i^{N-1-p} B_{i(i-1)} u_{i-1}^*(0|k) + \\
&\quad \sum_{p=0}^{N-1} A_i^{N-1-p} B_{i(i+1)} \kappa_{i+1}(x_{i+1}(0|k)) + \cdots + \\
&\quad \sum_{p=0}^{N-1} A_i^{N-1-p} B_{iM} \kappa_M (x_M(0|k)).
\end{aligned}$$

These equations can be rewritten in a matrix form as

$$X_{1|k}^i = \mathcal{A}_i x_i(0|k) + \mathcal{B}_i U_{0|k}^i + \sum_{j=1}^{i-1} \mathcal{B}_{ij} u_j^*(0|k) + \sum_{j=i+1}^{M} \mathcal{B}_{ij} \kappa_j (x_j(0|k)) \quad (11)$$

which is a model prediction of Subsystem i for N time steps, where

$$X_{1|k}^i = \begin{bmatrix} x_i(1|k) \\ x_i(2|k) \\ \vdots \\ x_i(N|k) \end{bmatrix}, \quad \mathcal{A}_i = \begin{bmatrix} A_i^1 \\ A_i^2 \\ \vdots \\ A_i^N \end{bmatrix}, \quad \mathcal{B}_i = \begin{bmatrix} B_i & 0 & \ldots & 0 \\ A_i B_i & B_i & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_i^{N-1} B_i & A_i^{N-2} B_i & \ldots & B_i \end{bmatrix}, \quad U_{0|k}^i = \begin{bmatrix} u_i(0|k) \\ u_i(1|k) \\ \vdots \\ u_i(N-1|k) \end{bmatrix},$$

$$\mathcal{B}_{ij} = \begin{bmatrix} B_{ij} \\ A_i B_{ij} + B_{ij} \\ \vdots \\ \sum_{p=0}^{N-1} A_i^{N-1-p} B_{ij} \end{bmatrix}.$$

Following [16], by having this prediction model, we proceed to transform the objective function given in Eq. (8) into a suitable compact form as required by the Active Set Method, namely

$$\begin{aligned}
J_i(k) &= \sum_{p=1}^{N-1} \|x_i(p|k)\|_{Q_i}^2 + \sum_{p=0}^{N-1} \|u_i(p|k)\|_{R_i}^2 + \\
&\quad x_i(N|k)^T P_i x_i(N|k), \\
&= \left(X_{1|k}^i\right)^T \mathcal{Q}_i X_{1|k}^i + \left(U_{0|k}^i\right)^T \mathcal{R}_i U_{0|k}^i, \quad (12)
\end{aligned}$$

where $\mathcal{Q}_i = diag\,(Q_i, Q_i, \ldots, P_i)$, and $\mathcal{R}_i = diag\,(R_i, R_i, \ldots, R_i)$. The matrix Riccati P_i can be determined using function 'dlqr' in MATLAB for the corresponding A_i, B_i, Q_i and R_i, $i = 1, 2, \ldots, M$.

We summarize our proposed control technique in the following algorithm:

Algorithm 1: Computation of Sequential DMPC Controllers

Require: N prediction horizon, M number of subsystems, A_i, B_i, B_{ij}, Q_i, R_i, $x_i(0|k)$ initial state of Subsystem i, $i = 1, 2, \ldots, M$, max_iter maximum iteration

for $k = 1$ **to** max_iter **do**

  for i = 1 **to** M **do**

    1. solve $\mathcal{P}_i$ in (3)-(7) to obtain $U_i^*(0|k)$,
    2. apply the first element of $U_i^*(0|k)$ to get optimal state of Subsystem i for $k+1$,
    3. send $U_j^*(0|k)$ to Subsystem $i+1$, $j = 1, 2, \ldots, i$,
    4. $i \leftarrow i + 1$

  end for

    5. $x_i(0|k) \leftarrow x_i(1|k)$,
    6. $k \leftarrow k + 1$

end for

Notice that, at every time step, our proposed method works to update all subsystems sequentially. Specifically, the optimization of Subsystem $i+1$ will only be held once the optimization Subsystem i is completed.

After discussing the problem and the underlying assumptions, we provide the method's properties in the next section.

III. Properties of The Proposed Method

This section provides two properties of our sequential DMPC strategy: the feasibility and stability of each subsystem to the origin inside its terminal set. The following theorem states the feasibility property.

Theorem 3.1: Assume that Problem $\mathcal{P}_i$ given by Eqs. (3) - (7) is feasible at time step k with the optimal input $U_i^*(0|k)$. Then, Problem $\mathcal{P}_i$ is feasible for all $k \geq 0$, $i = 1, 2, \ldots, M$.

Proof: Using similar steps discussed in [17] of using the tail of previous optimization as a feasible solution for the next one, we firstly note that from the feasible input at time step k, we can form a feasible input for time step $k+1$. Let the optimal input sequence at time step k, namely $U_i^*(0|k)$, can be written as $\{u_i^*(0|k), u_i^*(1|k), \ldots, u_i^*(N-1|k)\}$, and the corresponding optimal state is given by $X_{1|k}^{i,*} = \{x_i^*(0|k), x_i^*(1|k), x_i^*(2|k), \ldots, x_i^*(N|k)\}$. From Eq. 7, we have $x_i^*(N|k) \in \mathcal{X}_{f,i}$. In addition, based on Assumption 3, it follows that $\kappa_i(x_i^*(N|k)) \in \mathcal{U}_i$ and $x_i^\kappa(x_i^*(N|k)) \in \mathcal{X}_{f,i}$. Thus, it is clearly that the input sequence $U_i^+(k+1) = \{u_i^*(1|k), \ldots, u_i^*(N-1|k), \kappa_i(x_i^*(N|k))\}$ is a feasible input sequence for time step $k+1$. Using similar argument, we can prove the feasibility for time step $k+2$, and so on. Therefore, Problem $\mathcal{P}_i$ is feasible for all $k \geq 0$, $i = 1, 2, \ldots, M$. ∎

We provide the stability of the closed-loop system under our proposed method in the following theorem.

Theorem 3.2: Let $U_i^*(0|k)$ denotes the optimal solution at time step k of Problem $\mathcal{P}_i$ given in Eqs. 3 - 7, and $u_i^*(0|k)$ is its first element. Then, the resulting closed loop system is asymptotically stable in $\mathcal{X}_{f,i}$, $i = 1, 2, \ldots, M$.

Proof: Suppose that the state and the optimal cost correspond to $U_i^*(0|k)$ are denoted by $X_{1|k}^{i,*}$ and $J_i^*(k)$, respectively. From Theorem 3.1, we have that $U_{1|k}^{i,+}$ is a feasible solution for optimization at time step $k+1$. Note that the state associates to this input is $X_{1|k}^{i,+} = \{x_i^*(1|k), x_i^*(2|k), \ldots, x_i^*(N|k), x_i^{*,\kappa}(N+1|k)\}$, where $x_i^{*,\kappa}(N+1|k)$ is the state for input $\kappa_i(x_i^*(N|k))$. As stated in [17], these two input sequences correspondingly result in the cost functions with only differences in some terms, namely

$$\begin{aligned} J_i^*(k) = &\|x_i^*(0|k)\|_{Q_i}^2 + \sum_{p=1}^{N-1} \|x_i^*(p|k)\|_{Q_i}^2 + \\ &\|u_i^*(0|k)\|_{R_i}^2 + \sum_{p=1}^{N-1} \|u_i^*(p|k)\|_{R_i}^2 + \\ &V_{f,i}(x_i^*(N|k)), \text{and} \\ J_i^+(k+1) = &\sum_{p=1}^{N-1} \|x_i^*(p|k)\|_{Q_i}^2 + \|x_i^*(N|k)\|_{Q_i}^2 + \\ &\sum_{p=1}^{N-1} \|u_i^*(p|k)\|_{R_i}^2 + \|\kappa_i(x_i^*(N|k))\|_{R_i}^2 \\ &+ V_{f,i}\left(x_i^{*,\kappa}(N+1|k)\right). \end{aligned}$$

Therefore, we have

$$\begin{aligned} &J_i^+(k+1) - J_i^*(k) \\ &\quad = -\|x_i^*(0|k)\|_{Q_i}^2 - \|u_i^*(0|k)\|_{R_i}^2 - \\ &\quad V_{f,i}(x_i^*(N|k)) + \|x_i^*(N|k)\|_{Q_i}^2 \\ &\quad + \|\kappa_i(x_i^*(N|k))\|_{R_i}^2 + \\ &\quad V_{f,i}\left(x_i^{*,\kappa}(N+1|k)\right). \end{aligned} \tag{13}$$

According to Assumption 4, the sum of the last four terms on the right-hand side is less than or equal to zero. Thus, from Eq.(13), we can conclude that the following inequalities hold

$$\begin{aligned} J_i^+(k+1) - J_i^*(k) &\leq -\|x_i^*(0|k)\|_{Q_i}^2 - \|u_i^*(0|k)\|_{R_i}^2 \\ &\leq 0, \end{aligned}$$

or, equivalently stated as

$$J_{i,+}(k+1) \leq J_{i,*}(k).$$

Hence, for $U_i^*(1|k)$ the optimal solution at time step $k+1$ with the corresponding optimal cost $J_i^*(k+1)$, the following property is satisfied

$$J_i^*(k+1) \leq J_i^+(k+1) \leq J_i^*(k),$$

which states that the optimal cost of Problem $\mathcal{P}_i$ is nonincreasing along the state trajectories. Notice that, the strict inequality is satisfied for $x_i(k) \neq 0$, meaning the optimal cost is decreasing along non zero state trajectories. Furthermore, $J_i(k)$ is bounded below by 0. Thus, $J_i(k)$ converge to 0 and $J_i(k) = 0$ if and only if $x_i(k) = 0$. Therefore, we can conclude that $x_i(k)$ converge to 0 as time increases, $i = 1, 2, \ldots, M$. ∎

Our proposed method is applied to an irrigation canal system with a particular configuration as discussed in [3]. Therefore, we have briefly overviewed the system in the next section.

IV. Irrigation Canal System

An irrigation canal system is a large-scale system consisting of several connected canal reaches, each of which is equipped with gates regulating the water flow through the canals (see [18]). A pump at the very end downstream canal is to release water to the users or consumers. There are several configurations of irrigation canal systems, like the one discussed in [2] which is made up of eight reaches, four reaches [3], five reaches [4], and so on.

In general, the steps to synthesize our sequential DMPC control for all of the canal structures above are similar. Hence, we consider an irrigation canal system with configuration as described in [3]. This irrigation canal system consists of four reaches and a pump. Each canal reach and its upstream gate constitute a subsystem, except for reach 4, gate 4, and the pump, which build Subsystem 4 (see [3]). Thus, each subsystem has a local controller, except Subsystem 4, which has two local controllers that correspondingly manipulate the gate and the pump. The control given to the system aims to maintain the water level in each reach to meet specific targets while respecting any boundaries on gates, reaches, and the pump.

The dynamics of canal reaches represents water level changes, which can be written mathematically as (see, for example [2], [3])

$$h_i(k+1) - h_i(k) = \frac{T_s}{A_{i,s}}\left(Q_{i,in}(k) - Q_{i,out}(k)\right),$$

where $h_i(k)$ is the water level at reach i, $Q_{i,in}(k)$ and $Q_{i,out}(k)$ are inflow and outflow through reach i at time step k, respectively, while T_s is the sampling time, and $A_{i,s}$ is the area of reach i, $i = 1, 2, 3, 4$. Using the mass conservation law, it follows that $Q_{i,out} = Q_{i+1,in}$, $i = 1, 2, 3$, whereas for reach 4, $Q_{4,out} = p_4$, where q_i and p_4 denote the flows through gate i and pump 4, respectively, $i = 1, 2, 3$ (see [3]).

Furthermore, inline with [3], we use the water level and flow as the state and input of each subsystem, consecutively, as described below

$$x_i(k) = h_i(k),$$
$$u_i(k) = \begin{cases} q_i(k), & i = 1,2,3 \\ \begin{bmatrix} q_i(k) \\ p_i(k) \end{bmatrix}, & i = 4. \end{cases}$$

Notice that, in this configuration, there are five inputs involved in this system. Four of them correspond to the inflow to each reach, while the rest is the outflow through the pump in reach 4.

Since an irrigation canal system involves interaction between reaches, the dynamics of each subsystem can be represented using a state-space model discussed in [19] by taking $x_{ii} = h_i(k)$. This model, also known as a composite model, incorporates decentralized dynamics and the interaction between subsystems. Thus, the composite model for each subsystem is given by (see [19])

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j \neq i} W_{ij} u_j(k) \quad (14)$$

where $x_i(k) = \begin{bmatrix} x_{i1}(k)^T & x_{i2}(k)^T & x_{i3}(k)^T & x_{i4}(k)^T \end{bmatrix}^T$, $x_{ii}(k)$ denotes water level of reach i at time step k, $x_{ij}(k)$ is the effect of water changes in Subsystem j to Subsystem i at time step k, $W_{ij} = \begin{bmatrix} 0 & \ldots & B_{ij}^T & \ldots & 0 \end{bmatrix}^T$, A_i is a square matrix with ii-th element equals to $A_{ii} = 1$ and zero elsewhere, $i, j = 1,2,3,4$, $B_{ii} = \frac{T_s}{A_i}, i = 1,2,3$, $B_{44} = \begin{bmatrix} \frac{T_s}{A_{4,s}} & -\frac{T_s}{A_{4,s}} \end{bmatrix}$, $B_{i(i+1)} = -\frac{T_s}{A_{i,s}}, i = 1,2$, $B_{34} = \begin{bmatrix} -\frac{T_s}{A_{4,s}} & 0 \end{bmatrix}$, $B_{ij} = 0, j \notin \{i, i+1\}$ (see [3]), T_s is sampling time, $A_{i,s}$ is the reach area in m^2 of Subsystem i; $A_{1,s} = 397$, $A_{2,s} = 653$, $A_{3,s} = 503$, and $A_{4,s} = 1530$, see [2].

The following section implements the proposed method to the irrigation canal system.

V. Simulation

Our proposed method is applied to an irrigation canal system in [3] which has 4 reaches (thus $M = 4$), with sampling time $T_s = 240s$, and prediction horizon length $N = 10$ steps. We use the reach area $A_{i,s}$ as given in Table 2.3 in [2] at low flow for $i = 1,2,3,4$.

In the objective function of each subsystem, we use $Q_i = 100$, $R_i = 40$ and the following constraints similar to [4],

$$0 \leq u_i(k+j) \leq 672, \; i = 1,2,3, \text{and}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq u_i(k+j) \leq \begin{bmatrix} 672 \\ 480 \end{bmatrix}, \; i = 4,$$

which represent the maximum bound on gates and pump sampled every $T_s = 240s$, where $j \geq 0$, $k = 0,1,\ldots,N$. Moreover, we assume that the target water level in each reach is $5m$, whereas the initial water levels are $h_i(0) = 3$, $i = 1,2,3$, and $h_4(0) = 0$ (see [4]). Since we are solving a regulation problem, then these values are transformed as $h_i^{reg}(0) = -2$, $i = 1,2,3$, and $h_4^{reg}(0) = -5$ for computational purposes. The problem will be solved using MATLAB involving 'fmincon' and Active Set Algorithm. In addition, the constraints are transformed into suitable forms required by the solver using invariant set toolbox [15]. Particularly, we employ functions 'defineq', 'normalise', and 'aug2std'.

To implement the proposed sequential updating scheme, the dynamics of each subsystem given by Eq. (14) is rewritten into the form in Eq. (2). Since $B_{ij} = 0, j \notin \{i, i+1\}$ (see [3]), thus we get

$$\begin{aligned} x_1(k+1) &= A_1 x_1(k) + B_1 u_1(k) + W_{12} u_2(k) \\ &= A_1 x_1(k) + B_1 u_1(k) + W_{12} \kappa_2(x_2(k)), \quad (15) \\ x_2(k+1) &= A_2 x_2(k) + B_2 u_2(k) + W_{23} u_3(k) \\ &= A_2 x_2(k) + B_2 u_2(k) + W_{23} \kappa_3(x_3(k)), \quad (16) \\ x_3(k+1) &= A_3 x_3(k) + B_3 u_3(k) + W_{34} u_4(k) \\ &= A_3 x_3(k) + B_3 u_3(k) + W_{34} \kappa_4(x_4(k)), \quad (17) \\ x_4(k+1) &= A_{44} x_4(k) + B_4 u_4(k), \quad (18) \end{aligned}$$

where κ_i is computed using 'dlqr' in MATLAB for the corresponding Q_i and R_i, $i = 1,2,3,4$. Furthermore, $A_1 = diag(1,0,0,0)$, $A_2 = diag(0,1,0,0)$, $A_3 = diag(0,0,1,0)$, and $A_4 = diag(0,0,0,1)$. Recall Eq. (2), notice that, for each Subsystem i with dynamics given by Eqs. (15)-(18), the terms $u_j(k) = u_j^*(0|k), j = 1,2,\ldots,i-1$, are not involved in these expressions. This is due to the abscence of interaction from Subsystem j to Subsystem i where $j = 1,2,\ldots,i-1$, thus $B_{ij} = 0$. As a result, we have that $W_{ij} = 0$, for all $i = 1,2,3,4$, and $j = 1,2,\ldots,i-1$.

Since the prediction horizon is $N = 10$, the model prediction of each subsystem is obtained using Eqs. (15-18) and (11) for $N = 10$. Thus, we have

$$\begin{aligned} X_{1|k}^1 &= \mathcal{A}_1 x_1(0|k) + \mathcal{B}_1 U_{0|k}^1 + \mathcal{W}_{12} \kappa_2(x_2(k)), \\ X_{1|k}^2 &= \mathcal{A}_2 x_2(0|k) + \mathcal{B}_2 U_{0|k}^2 + \mathcal{W}_{23} \kappa_3(x_3(k)), \\ X_{1|k}^3 &= \mathcal{A}_3 x_3(0|kk) + \mathcal{B}_3 U_{0|k}^3 + \mathcal{W}_{34} \kappa_4(x_4(k)), \\ X_{1|k}^4 &= \mathcal{A}_4 x_4(0|k) + \mathcal{B}_4 U_{0|k}^4, \end{aligned}$$

while their corresponding objective functions are given by

$$\begin{aligned} J_1(k) &= \left(X_{1|k}^1\right)^T \mathcal{Q}_1 X_{1|k}^1 + \left(U_{0|k}^1\right)^T \mathcal{R}_1 U_{0|k}^1, \\ J_2(k) &= \left(X_{1|k}^2\right)^T \mathcal{Q}_2 X_{1|k}^2 + \left(U_{0|k}^2\right)^T \mathcal{R}_2 U_{0|k}^2, \\ J_3(k) &= \left(X_{1|k}^3\right)^T \mathcal{Q}_3 X_{1|k}^3 + \left(U_{0|k}^3\right)^T \mathcal{R}_3 U_{0|k}^3, \\ J_4(k) &= \left(X_{1|k}^4\right)^T \mathcal{Q}_4 X_{1|k}^4 + \left(U_{0|k}^4\right)^T \mathcal{R}_4 U_{0|k}^4. \end{aligned}$$

Since we only consider the input contraints in the controller design, then the terminal set contraints $\mathcal{X}_{f,i}$ is not computed, $i = 1,2,3,4$. Hence, only the computation of terminal cost is involved.

The simulation is performed using MATLAB for 30 iterations ($max\_iter = 30$). Moreover, the optimization for each subsystem is solved using 'fmincon' in MATLAB. In addition, we also provided a comparison to the Nash Bargaining MPC (NB-MPC) method in [4] where each subsystem is weighted by w_i; $w_1 = 0.3871, w_2 = 0.2354, w_3 = 0.3056, w_4 = 0.4002$. Note that, in our proposed strategy, at every time step, we updated these four subsystems sequentially starting from Subsystem 1 until Subsystem 4. In contrast, in NB-MPC [4], these subsystems are updated simultaneously at every time step. We obtained the following results as depicted in the figures below.

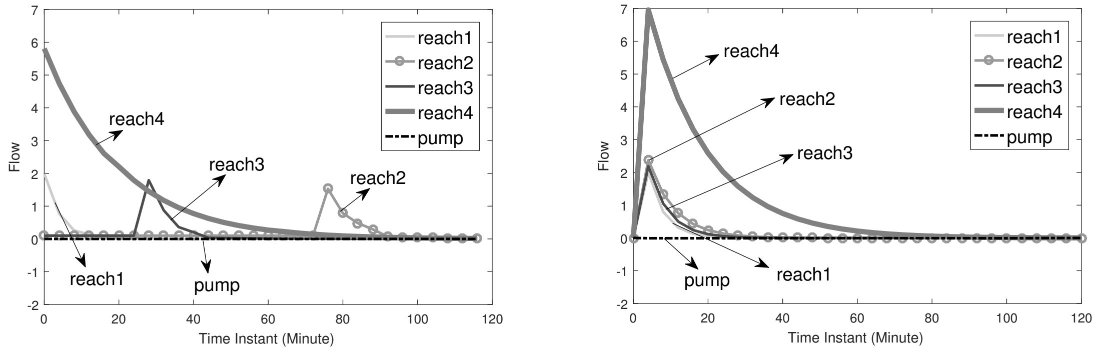

Fig. 1. Flows of each reach in irrigation canal system with configuration as in [3] using Nash-Bargaining MPC [4] (first column) and the proposed sequential DMPC (second column)

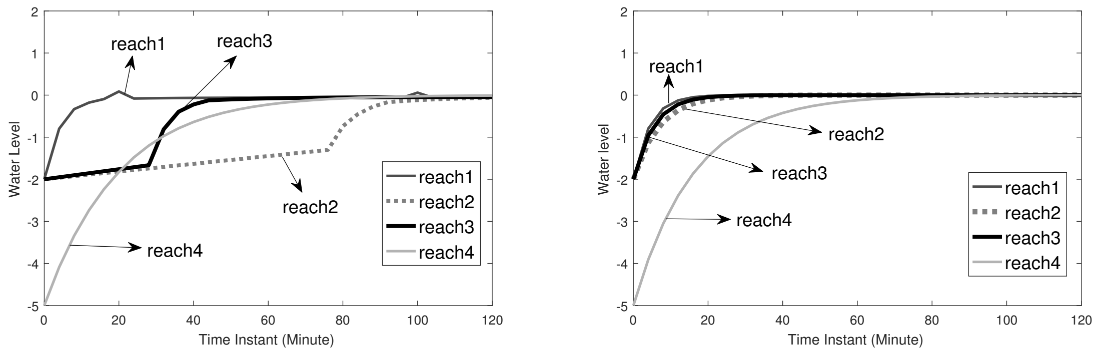

Fig. 2. Water levels of each reach in irrigation canal system with configuration as in [3] using Nash-Bargaining MPC [4] (first column) and the proposed sequential DMPC (second column)

Fig. 1 and Fig.2 show the flows and the water levels of the irrigation canal system in [3] when controlled using NB-MPC [4] and our sequential DMPC, respectively. The results of NB-MPC [4] are provided in the first column and our proposed method in the second. Based on the second column of Fig.2, it can be seen that the water level at each canal reach becomes zero or equivalent to $5m$, as time increases. The first three reaches need less than 12 steps or equal to 48 minutes for their water levels to converge to zero. Reach 4, on the other hand, takes a longer time, i.e., 25 steps or 100 minutes, to get the same level. Two reasons caused this behavior. First, reach 4 has the smallest initial water level compared to other reaches. In addition, there is an outflow from reach 4 through the pump. As a result, although the inflow in reach 4 is considered the largest, the existence of these two makes the reach 4 need a longer time to reach zero water level. In conclusion, our proposed method can steer the water level to zero in each canal. If we compared these results to those in the first column, it is clear that for the irrigation canal system in [3], our proposed method has produced better results.

VI. Conclusion

This paper discussed a sequential DMPC algorithm for a discrete linear time invariant system built by M interconnected subsystems. It is shown that the recursive feasibility of the resulting optimization problem is guaranteed by assuming the initial feasibility. Moreover, our proposed method stabilizes the system asymptotically. The proposed control strategy is implemented to irrigation canal system with particular configuration as given by [3]. In addition, a comparison to the Nash-Bargaining MPC strategy [4] has been made. It turns out that our proposed method stabilizes the system asymptotically relatively faster compared to the results from the Nash-Bargaining MPC strategy [4]. Hence, from the results of our simulations and comparisons, we can conclude that the approach we have developed has a potential for further development.

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