Measuring Risk utilizing Credible Monte Carlo Value at Risk and Credible Monte Carlo Expected Tail Loss

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***Abstract*—This paper proposes two new methods to measure the risk of individual stocks, which construct a portfolio, namely Credible Monte Carlo Value at Risk (CMC VaR) and Credible Monte Carlo Expected Tail Loss (CMC ETL). The CMC VaR is developed by combining the concept of Credible Value at Risk (Cr VaR) with Monte Carlo VaR (MC VaR). Meanwhile, CMC ETL is constructed by mixing Credible ETL (Cr ETL) and MC ETL. The new method's performance is empirically verified to evaluate the individual risk of each asset developing three portfolios. The analyzed portfolios are designed by Indonesian five stocks indexed by LQ 45, four stocks traded in New York Stock Exchange (NYSE), two stocks indexed by NASDAQ, and two stocks indexed by London Stock Exchange. We also assess the accuracy of the CMC VaR by Kupiec Backtesting. The empirical results of this paper implied that two novel methods are effective in measuring the risk at 80 percent, 90 percent, and 95 percent confidence levels. The proposed methods can also overcome the drawback of VaR and ETL, which do not contemplate the risk among assets grouped in a portfolio.**

***Index Terms*—Conditional-Value-at-Risk, Monte-Carlo, premium, VaR-algorithm.**

I. Introduction

EFFECTIVE risk management has become a prominent aspect of financial investors. This aspect should consider the potential returns or manage the investment risk encountered by investors. Determining an appropriate method to manage the risk of an individual asset or a portfolio asset among several risk measurement methods developed by researchers is a challenging problem [1].

Two risk measurement methods employed widely in finance are Value at Risk (VaR) and Expected Tail Loss (ETL). VaR at a confidence level (cl) $1 - a$ is defined as a threshold of loss such that the loss probability exceeding the threshold is not more than a [2]. Meanwhile, ETL at a confidence level $1 - a$ is the expectation of loss, which is greater than VaR [2].

Numerous researchers have developed VaR and ETL to measure the risk of an asset or a portfolio asset ([3]; [4]). Several risk measure methods elaborated from the two methods that can be chosen to assess the financial risk had been proposed by Mina and Ulmer [5], Castelacci and Siclary [6], Hong, Hu, and Zhang [7], Hong, Hu, and Liu [8], Wang et al. [9], Tzeng, Beaumont, and Ökten [10], and Martins-Filho and Yao [11]. They constructed VaR and ETL based on Monte Carlo Simulation concept that indicated well performance in measuring the risk of several assets (portfolios).

Then, in 2016, Pitselis [12] also developed VaR and ETL by presenting credible risk measurement methods, namely Credible VaR (Cr VaR) and Credible ETL (Cr ETL). Both methods were constructed by mixing a concept that was previously popular to determine the insurance premium, namely, the Bühlmann Credibility concept with classical VaR/ETL employed to measure the investment risk.

Cr VaR and Cr ETL, presented by Pitselis [12], are new types of risk measures. Both of them are a combination of credibility theory utilized extensively in insurance and risk measures, namely VaR chosen by the Basel Committee on Banking Supervision as a standard to measure the risk for capital requirements [13] and ETL as a complementary measure of VaR [14]. Pitselis [12] claimed that both methods provide more information than the classical VaR and ETL, because they are able to capture the risk of individual assets and portfolios constructed by similar but not identical asset returns conjoined in sharing the risk.

In this paper, we are interested in developing two new risk measures, namely Credible Monte Carlo VaR (CMC VaR) and Credible Monte Carlo ETL (CMC ETL) constructed by combining the basic notion of Cr VaR and Cr ETL proposed by [12] with MC VaR employed in many research stated in the prior paragraph.

The rest of this paper is organized as follows. The second section explains the basic concept of risk measures and MC VaR. In the third section, there is a derivation of CMC VaR. Then, the formulation of CMC ETL is presented in the fourth section. Meanwhile, in the fifth and in the sixth section, CMC VaR and CMC ETL are examined and evaluated successively from the empirical application. The application of the two methods is utilized to measure the risk of four portfolios which consist of stocks traded in the LQ 45 Index, New York Stock Exchange (NYSE), NASDAQ, and London Stock Exchange. The fifth section also shows the CMC VaR accuracy in measuring risk. Then, in the sixth section, we also give the application of CMC ETL for the individual stocks constructing the four portfolios. In the last section, we provide some conclusions.

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II. Review of Risk Measures

This section provides a brief theory of coherent risk measure proposed by Artzner et al. [15]. Furthermore, the definition of VaR and ETL and MC VaR, which becomes a basic concept in developing the novel methods, are briefly presented here.

Definition 2.1 Given a set of random variables which are real-valued. A function $\wp : \mathcal{A} \rightarrow \mathcal{R}$ is a coherent risk measure if it fulfills the four axioms as follows ([15]; [12]):

a. Monotonicity Axiom. For all $X \in \mathcal{A}$, then $\wp(X) \leq 0$.
b. Positive Homogeneity Axiom. For all $h \geq 0$, if $X \in \mathcal{A}, Xh \in \mathcal{A}$ then $\wp(hX) = h\wp(X)$
c. Translation Invariance Axiom. For all $X \in \mathcal{A}, X \geq 0$ and all $r \in \mathcal{R}$, then $\wp(X + r) = \wp(X) - r$.
d. Subadditivity Axiom. For all $X_1, X_2 \in \mathcal{A}$, if $X_1+X_2 \in \mathcal{A}$, then $\wp(X_1 + X_2) \leq \wp(X_1) + \wp(X_2)$.

Next, we provide a quantile definition at a confidence level, $1 - a$, stated in Definition 2.2, which will be a basic concept in VaR defined in Definition 2.3.

Definition 2.2 Consider a data set $X_1, X_2, \ldots, X_n$, define $F(X) = P[X \leq x]$ as a cumulative distributive function, which continues everywhere and differentiable. The quantile, q, at a confidence level, $1 - a$, of distribution is denoted by [12]

$$q = F^{-1} = \inf \{x : F(x) \geq 1 - a\} .$$

Definition 2.3 VaR is defined as a $1 - a$ quantile of profit/loss distribution for a specified confidence level $1 - a$. [12] .

VaR has been used extensively as a risk measure to manage financial risk over a specified (commonly relatively short) time period. VaR can be a coherent risk measure when gains or losses (return) are normally distributed. Normal distribution of return is difficult to be discovered in financial and insurance data distribution, which is frequently skewed distributed [16]. In other words, VaR provides restricted guidance to investigate the function of the tail [17]. Due to the VaR limitedness, we present a coherent risk measure, namely ETL defined by Definition 2.4.

Definition 2.4 ETL at a confidence level (cl) $1 - a$ is the loss expectation exceeding the VaR [2]. The ETL definition for the corresponding VaR given in Definition 2.3 is written by the following expression [18]

$$ETL(X) = E\left(X \mid X \geq VaR_{1-a}(X)\right).$$

ETL considers only the tail of the distribution [19]. In addition, Klugman, Panjer, and Willmot [16] asserted that ETL explains the distribution tail of return more than employing only VaR in measuring risk. Hence, we proposed CMC ETL to cope with the drawback of VaR.

Several methods to compute VaR have been administered. Ammann and Reich [20] divided VaR into parametric methods and nonparametric methods. One of the nonparametric methods implemented to calculate VaR is Monte Carlo (MC) VaR. MC VaR is VaR examining risk based on MC simulation. MC simulation has become a popular and flexible method utilized in several applications such as economics and finance ([21]; [10]). MC is regarded as a flexible and simple method to implement in many cases [10]. It does not also require the assumption of asset distribution [22]. Although there are several algorithms in estimating VaR based on MC simulation, this method basically simulates data by generating independent random variables with the same distribution based on data characteristics ([23], [24]). VaR based on MC simulation for an individual asset can be derived using several steps provided by Algorithm 1 [23].

TABLE I
Algorithm 1. Derivation of Monte Carlo VaR for the individual asset

Stage	Process
1	Calculating the return of an asset.
2	Determining the parameter value(s) of the return.
3	Determining confidence level(s) and time period(s).
4	Simulating return by generating individual return asset randomly with the specified parameter provided in the third step for N times.
5	Computing the estimation of maximum loss at a confidence level $1 - a$ as an a^{th} quantile of the empirical distribution of return derived from step 5.
6	Counting VaR at a confidence level $1 - a$ over t period.
7	Repeating step 2 until step 6 for M times.
8	Computing the mean of results of step 7.

III. Credible Monte Carlo Value at Risk

This section presents the derivation of CMC VaR, a proposed method in this paper, to assess the investment risk. Similar to Cr VaR, CMC VaR is also developed based on the credibility theory. The theory is considered as a remarkable method in estimating the insurance premium for a group of insurance contracts when it possesses various claim experiences for the group and several experiences for a larger group of contracts that are similar but not identical [25].

Bühlmann's credibility model focusing on VaR is constructed by some assumptions as presented in Pitselis [13]. Assumptions used in the proposed model (CMC VaR) are developed by adjusting assumptions of Cr VaR in Pitselis [25]. The assumptions for CMC VaR are given as follows ([25];[26]):

1) A portfolio comprising of m assets is given. Then, a random vector indicating MC VaR of the j^{th} assets at period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$ is denoted by $w_j^{'} = (\mathtt{N_{1,j}}, \ldots, \mathtt{N_{n,j}})$.
2) A random variable $\mathtt{N_{i,j}}, \ldots, \mathtt{N_{n,j}}$ is assumed identically distributed with mean $E(\mathtt{N_{i,j}}) = \mu$ and variance $Var(\mathtt{N_{i,j}}) = \sigma^2$.
3) Each asset risk forming a portfolio is represented by a random variable S assumed unknown distributed. Meanwhile, a random variable $\mathtt{N_{1,j}}, \ldots, \mathtt{N_{n,j}}$ is assumed conditional i.i.d for a fixed S with $E\left(\mathtt{N_{i,j}} \mid S = s\right) = \mu(s)$ and $Var(\mathtt{N_{i,j}} \mid S = s) = \tau(s)$ for $i = 1, 2, \ldots, n$.

Then, adopting Bühlmann credibility and MC simulation to risk estimation concept, VaR based on the credibility theory and MC simulation of the j^{th} assets in the next period that latter will be called as CMC VaR will be derived by implementing Theorem 3.1 as follows

Theorem 3.1 Under the aforementioned assumptions, the linear estimator of CMC VaR of the j^{th} asset can be denoted by

$$\Psi_{MC}(s) = \bar{\mathtt{N}}_j Z_{MC} + (1 - Z_{MC})\mu(s), \tag{1}$$

where $E\left(\mathtt{N_j}\right) = \bar{\mathtt{N}}_j$ is the mean of MC VaR for the j^{th} asset over the observed periods, $\mu(s)$ is the mean of $E\left(\mathtt{N_j}\right)$ for $j = 1, 2, \ldots, m$, and Z_{CMCVaR} is a risk factor of CMC VaR expressed as Equation (2) as follows

$$Z_{MC} = \frac{nVar(\mu(s))}{E\left[\tau(s)\right] + nVar\left[\mu(s)\right]}. \tag{2}$$

Proof. Performance of any estimators $h(X_j)$ of $\mathtt{N_j}$ is measured by using estimation of squared error. Firstly, given a linear Bayes estimator of $E\left[\mathtt{N_j}\right]$ as follows

$$h_j\left(g_0^j, g_l^j, \widehat{\mathtt{N}_l}\right) = g_0^j + \sum_{l=1}^{m} g_l^j \widehat{E\left[\mathtt{N}_l\right]},$$

where g_0^j and g_l^j are given to minimize the expected square error for the estimator, namely

$$R = E\left[\mu(s) - g_0^j - \sum_{l=1}^{m} g_l^j \widehat{E\left[\mathtt{N}_l\right]}\right]^2. \tag{3}$$

After that, taken a derivative of Equation (3) which is relative to g_0^j, and $g_{l'}^j$, so that it can be obtained the following equation

$$Cov\left[\mu(s), \widehat{E\left[\mathtt{N}_{l'}\right]}\right] = g_l^j Var\left[\widehat{E\left[\mathtt{N}_j\right]}\right]$$

and

$$g_{l,j} = \frac{Cov\left[\mu(s), \widehat{E\left[\mathtt{N}_j\right]}\right]}{Var\left[\widehat{E\left[\mathtt{N}_j\right]}\right]} \tag{4}$$

$$= \frac{E\left[Cov\left(\mu(s), \widehat{E\left[\mathtt{N}_j\right]} \mid s\right)\right]}{Var\left(\widehat{E\left[\mathtt{N}_j\right]}\right)} + \frac{Cov\left[E\left(\mu(s) \mid s\right), E\left[\widehat{E\left[\mathtt{N}_j\right]} \mid s\right]\right]}{Var\left(\widehat{E\left[\mathtt{N}_j\right]}\right)} \tag{5}$$

$$\tag{6}$$

which results Z_{MC} provided in Equation (2).

Risk assessment using CMC VaR requires some information, which are $E\left[\mathtt{N}_j\right], \mu(s)$, $E\left[Var(\mathtt{N}_{i,j} \mid s)\right] = E\left[\tau(s)\right]$, and $Var\left[E(\mathtt{N}_{i,j} \mid s)\right] = Var\left[\mu(s)\right]$. In line with the three assumptions, $E\left[\mathtt{N}_j\right], \mu(s), E[\tau(s)]$, and $Var\left[\mu(s)\right]$ are estimated using the mean sample formula of the data. The unbiased estimator of $E\left[\mathtt{N}_j\right], \mu(s), E\left[\tau(s)\right]$, and $Var\left[\mu(s)\right]$ are provided in Equation (7), Equation (8), Equation (9), and Equation (10), respectively.

$$\widehat{E\left[\mathtt{N}_j\right]} = \frac{1}{n}\sum_{i=1}^{n} \mathtt{N}_{i,j}, \tag{7}$$

$$\widehat{\mu(s)} = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n} \mathtt{N}_{i,j}, \tag{8}$$

$$\widehat{E\left[\tau(s)\right]} = \frac{1}{m(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(\mathtt{N}_{i,j} - \bar{\mathtt{N}}_j)^2 \tag{9}$$

$$\widehat{Var\left[\mu(s)\right]} = \frac{1}{m-1}\sum_{j=1}^{m}(\bar{\mathtt{N}}_j - \widehat{E\left[\tau(s)\right]})^2 - \frac{\widehat{E\left[\tau(s)\right]}}{n}, \tag{10}$$

TABLE II
ALGORITHM 2. RECURSIVE PROCESS OF CMC VAR DERIVATION

Stage	Process
1	Fixing a number of assets constructing a portfolio, denoted by m.
2	Fixing the observed period n, where $n \geq 2$,
3	Enumerating the MC VaR for the j^{th} asset at period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$ symbolized as $\mathtt{N_{i,j}}$. This stage comprises of several steps related to Algorithm 1 as follows: (a) Calculating the return of the j^{th} asset (b) Determining the parameter value(s) of the return of the j^{th} asset. (c) Determining the confidence level(s) and time period(s). (d) Simulating the j^{th} asset return by generating asset return randomly with the specified parameter provided in step (b) for N times. (e) Computing the estimation of maximum loss at a confidence level $1 - a$ as an a^{th} quantile of the empirical distribution of j^{th} asset return derived from step (d). (f) Counting MC VaR at a specified confidence level. (g) Repeating step (b) until step (f) for M times. (h) Computing the mean of MC VaR resulted from step (f) and step (g).
4	Counting the estimated mean of MC VaR for the j^{th} asset during n observed periods,
5	Computing the estimated mean of MC VaR mean for the m assets during n observed periods, denoted by $\widehat{E\left[\mathtt{N}_j\right]}$.
6	Calculating the estimated MC VaR variance mean of assets during observed periods, represented by $\widehat{E\left[\tau(s)\right]}$.
7	Enumerating the estimated variance of MC VaR mean for the assets during observed periods, symbolized as $\widehat{Var\left[\mu(s)\right]}$.
8	Calculating the estimated value of $n\widehat{Var\left[\mu(s)\right]}$.
9	Counting the estimated Z_{MC} for each asset.
10	Calculating the CMC VaR of the j^{th} asset.

Then, we propose Algorithm 2 provided in Table II, consisting of a recursive process in order to derive CMC VaR. The algorithm is developed using the Bühlmann credibility concept and MC simulation.

IV. CREDIBLE MONTE CARLO ETL

Besides CMC VaR, we also proposed CMC ETL to overcome the issue of subadditivity axiom that is not fulfilled in CMC VaR. In this section, we present the derivation of the CMC ETL formula in estimating asset risk. The formulation of CMC ETL will utilize the similar concept of CMC VaR formulation elaborated in the previous section. Several assumptions required in CMC ETL derivation are provided as follows:

1) m assets constructing a portfolio are given. Then, a random vector representing MC ETL of the j^{th} assets at period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$, is expressed by $w_j^{'*} = (Q_{1,j}, \ldots, Q_{n,j})$.
2) A random variable $Q_{1,j}, \ldots, Q_{n,j}$ is assumed identically distributed with mean $E(Q_{i,j}) = \mu^*$ and variance $E(Q_{i,j}) = \sigma^{*2}$
3) The risk of each asset comprising a portfolio is characterized by a random variable $\mathcal{S}$ which is assumed unknown distributed while a random variable $Q_{1,j}, \ldots, Q_{n,j}$ is assumed conditional i.i.d for a fixed $\mathcal{S}$ with $E(Q_{i,j} \mid \mathcal{S} = s) = \mu^*$ and $Var(Q_{i,j} \mid \mathcal{S} = s) = \tau^*(s)$, for $i = 1, 2, \ldots, n$.

The assumptions will be utilized in Theorem 3.2 to formulate CMC ETL inspired by Credible Conditional Tail Expectation proposed by Pitselis [12].

Theorem 3.2 Due to the assumptions explained previously, the linear estimator of CMC ETL of asset can be formulated by [13]

$$\Upsilon_{MC}(s) = E(Q_j)Z^*_{MC} + (1 - Z^*_{MC})\mu^*(s), \tag{11}$$

where $E(Q_j)$ is the mean of MC ETL for the j^{th} asset, $\mu^*(s)$ is the mean of $E(Q_j)$ for $j = 1, 2, \ldots, m$, and CMC ETL is a risk factor of CMC ETL denoted as Equation (12) as follows

$$Z^*_{MC} = \frac{nVar(\mu^*(s))}{E\left[\tau^*(s)\right] + nVar\left[\mu^*(s)\right]}. \tag{12}$$

Proof. Performance of any estimators $h^*(X_j)$ of Q_j is measured by using estimation of squared error. Firstly, given a linear Bayes estimator of $E\left[Q_j\right]$ as follows

$$h_j^*\left(g_0^{*j}, g_l^{*j}, \widehat{Q_l}\right) = g_0^{*j} + \sum_{l=1}^{m} g_l^{*j}\widehat{E\left[Q_l\right]},$$

where g_0^{*j} and g_l^{*j} are given to minimize the expected square error for the estimator, namely

$$R = E\left[\mu^*(s) - g_0^{*j} - \sum_{l=1}^{m} g_l^{*j}\widehat{E\left[Q_l\right]}\right]^2. \tag{13}$$

After that, take a derivative of Equation (13) which is relative to g_0^{*j} and $g_{l'}^{*j}$, so that it can be obtained the following equation

$$Cov\left[\mu^*(s), \widehat{E\left[Q_{l'}\right]}\right] = g_l^{*j} Var\left[\widehat{E\left[Q_j\right]}\right]$$

and

$$g^*_{l,j} = \frac{Cov\left[\mu^*(s), \widehat{E\left[Q_j\right]}\right]}{Var\left[\widehat{E\left[Q_j\right]}\right]} \tag{14}$$

$$= \frac{E\left[Cov\left(\mu^*(s), \widehat{E\left[Q_j\right]} \mid s\right)\right]}{Var\left(\widehat{E\left[Q_j\right]}\right)} + \frac{Cov\left[E\left(\mu^*(s) \mid s\right), E\left[\widehat{E\left[Q_j\right]} \mid s\right]\right]}{Var\left(\widehat{E\left[Q_j\right]}\right)} \tag{15}$$

$$\tag{16}$$

which results Z^*_{MC} ETL provided in Equation (12).

Risk assessment using CMC ETL requires some information, namely $E(Q_j), \mu^*(s), E\left[Var(Q_{i,j} \mid S = s)\right] = E(\tau^*(s))$, and $Var\left(\mu^*(s)\right)$. In line with the three assumptions, $E(Q_j), \mu^*(s), E\left[\tau^*(s)\right]$, and $Var(\mu^*(s))$ are estimated by the mean sample formula of the data. The unbiased estimator of $E(Q_j), \mu^*(s), E\left[\tau^*(s)\right]$, and $Var(\mu^*(s))$ is presented in Equation (17), Equation (18), Equation (19), and Equation (20), respectively.

$$\widehat{E\left[Q_j\right]} = \frac{1}{n}\sum_{i=1}^{n} Q_{i,j}, \tag{17}$$

$$\widehat{\mu^*(s)} = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n} Q_{i,j}, \tag{18}$$

$$E\widehat{\left[\tau^*(s)\right]} = \frac{1}{m(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(Q_{i,j} - \bar{Q}_j)^2 \tag{19}$$

$$Var\widehat{\left[\mu^*(s)\right]} = \frac{1}{m-1}\sum_{j=1}^{m}(\bar{Q}_j - E\widehat{\left[\tau^*(s)\right]})^2 - \frac{\widehat{E\left[\tau(s)\right]}}{n}, \tag{20}$$

The CMC ETL for the j^{th} asset which develops a portfolio is obtained by completing the CMC ETL recursive process. The recursive process is elaborated in Algorithm 3 presented in Table III.

V. Application of CMC VaR

This section implements theories provided in the preceding sections to analyze the individual asset risks compiled in a portfolio.The portfolio analyzed in this study consists of four portfolios for a one-day holding period and a ten-day holding period. The first portfolio was constructed by Indonesian five stocks, namely PT. Bank Central Asia Tbk (BBCA), PT. Telekomunikasi Indonesia Tbk (TLKM), PT. Aneka Tambang Tbk (ANTAM), Semen Indonesia (SMGR) Tbk, and PT. Indofood Sukses Makmur Tbk (INDF). The five stocks were indexed by LQ 45 from February 2018 until July 2018. The second portfolio comprised of four stocks was indexed by New York Stock Exchange (NYSE). The four stocks were Barrick Gold Corporation (GOLD), AT&T Inc. (T), Unilever PLC (UL), and Newmont Corporation (NEM). The third portfolio consisted of two stocks indexed by NASDAQ. The two stocks were Advanced Micro Devices,

TABLE III
ALGORITHM 3. RECURSIVE PROCESS IN CMC ETL DERIVATION

Stage	Process
1	Enumerating the MC ETL corresponding with the MC VaR of the asset at period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$, symbolized as $Q_{i,j}$.
2	Counting the estimated mean of MC ETL for the j^{th} assets during n observed periods,
3	Computing the mean of the MC ETL mean for the m assets during n observed periods, denoted by $\widehat{\mu^*(s)}$
4	Calculating the estimated mean of the MC ETL variance for m assets during n observed periods, represented by $E\left[\widehat{\tau^*(s)}\right]$.
5	Enumerating the estimated variance of the MC ETL mean for m assets during n observed periods, symbolized as $Var\left[\widehat{\mu^*(s)}\right]$.
6	Calculating the estimated value of $nVar\left[\widehat{\mu^*(s)}\right]$.
7	Counting the estimated Z^*_{MC}.
8	Calculating the estimated CMC ETL of the j^{th} asset.

Inc. (AMD) and eBay Inc. (EBAY). Then, the fourth portfolio comprised of two stocks indexed by London Stock Exchange. The two stocks were Associated British Foods plc (ABF.L) and Antofagasta plc (ANTO.L).

The analysis was begun by breaking up the whole daily stock price data observed over ten years, from June 2008 to May 2018. The ten-year data accessed from https://finance.yahoo.com/ was split into different ten observed periods presented in Table IV.

TABLE IV
OBSERVED DATA OVER TEN PERIODS

Period (i)	Duration
1	1 June 2008-31 May 2009
2	1 June 2009-31 May 2010
3	1 June 2010-31 May 2011
4	1 June 2011-31 May 2012
5	1 June 2012-31 May 2013
6	1 June 2013-31 May 2014
7	1 June 2014-31 May 2015
8	1 June 2015-31 May 2016
9	1 June 2016-31 May 2017
10	1 June 2017-31 May 2018

Then, the analysis of portfolio was continued by computing the log return of the assets of the daily close stock price over the ten-period data. The log return data later was employed to quantify the MC VaR of the stocks constructing the four portfolios for each observed period. Table V summarizes the one-day MC VaR of each asset constructing Portfolio I and II, at 80, 90, 95, and 99 percent of confidence level with the simulation number (M) at 10.000 and generated data (N) for each asset and period, which are specified based on the number of log return data corresponding to the number of real log return data of each asset for every period. The one-day MC VaR for each analyzed asset at several confidence levels joined at Portfolio III and Portfolio IV is not presented here for brevity. Then, the one-day MC VaR mean and the ten-day MC VaR mean for each analyzed asset constructing Portfolio I, II, III, and IV of the ten observed periods at several confidence levels are tabulated in Table VI. The interpretation of the results in this article is analog with the result interpretation in [26] and [27] because we are also using Cr VaR framework.

Table VI reveals that the mean of maximum potential loss with a confidence level at 90 percent within one-day holding period over ten years of ANTM, BBCA, INDF, SMGR, and TLKM constructing Portfolio I are 3.5486 percent, 2.2691 percent, 2.7218 percent, 2.9234 percent, and 2.3832 percent, respectively which are relative to each asset price on a preceding day. The other values in Table VI can be interpreted analogously.

Measuring the risk using CMC VaR requires not only the estimated parameter provided in Table VI but also other components derived from some steps given in Algorithm 2 (Table II). Components listed in Table V and Table VI are required to obtain the estimated parameters of $\mu(s)$ counted by Equation (8), $E[\tau(s)]$ calculated by Equation (9), $Var(\mu(s))$ counted by Equation (10) and the corresponding Z_{MC} necessitated in CMC VaR calculation for each asset constructing Portfolio I, II, III, and IV. This process is conducted by constructing an R Program to obtain the four components provided in Table VII.

From Table VII, it can be implied that the mean of maximum potential loss for one-day MC VaR mean for the five assets constructing Portfolio I over the ten periods at a confidence level of 90 percent for each asset which forms the portfolio is 2.7692 percent that is relative to the asset price at the previous one day. Furthermore, the mean of one-day MC VaR variance at the corresponding confidence level for each asset in the portfolio is 0.000102, and the corresponding estimated variance of $\widehat{\mu(s)}$ that represents the variability in the tail distribution for this case of each asset is 0.000016. Interpretation of the other components in Table VII is not provided for brevity.

Then, the estimators of $\mu(s), E(\tau(s))$, and $Var(\mu(s))$ are employed to compute a credible risk factor of CMC VaR. Utilizing Equation (2), we obtain the corresponding estimator of Z_{MC} at 0.603625 for 90 percent of confidence level. Next, administering the precedence information in Table VI and Table VII, the CMC Var of the j^{th} asset is counted directly using Equation (1), and the results are summarized in Table VIII.

Table VIII states that the maximum potential losses measured by CMC VaR for the holder of ANTM, BBCA, INDF, SMGR, and TLKM constructing Portfolio I in the next period of investment when the stock is held for one day at a confidence level of 90 percent are 3.2397 percent, 2.4673 percent, 2.7406 percent, 2.8623 percent, and 2.5362 percent, respectively. Furthermore, when we compared the estimation of loss for each asset using the conventional MC VaR Mean shown in Table VI and MC VaR Mean of the whole assets listed in Table VII, it can be identified that the estimation of maximum potential loss using one-day CMC VaR for the five assets at 90 percent of confidence level is likely to approach the mean of MC VaR for each asset. It is because of a risk factor of one-day CMC VaR, Z_{CMC}, at 0.603625, which is relatively big. Hence, based on Equation (1), the weight given to $\widehat{\mu(s)}$ is smaller than the weight given to the MC VaR

TABLE V
ESTIMATED MC VAR

cl = 80 percent

Period (i)	$\widehat{\mathtt{N}^{MC}_{i,ANTM}}$	$\widehat{\mathtt{N}^{MC}_{i,BBCA}}$	$\widehat{\mathtt{N}^{MC}_{i,INDF}}$	$\widehat{\mathtt{N}^{MC}_{i,SMGR}}$	$\widehat{\mathtt{N}^{MC}_{i,TLKM}}$	$\widehat{\mathtt{N}^{MC}_{i,GOLD}}$	$\widehat{\mathtt{N}^{MC}_{i,NEM}}$	$\widehat{\mathtt{N}^{MC}_{i,T}}$	$\widehat{\mathtt{N}^{MC}_{i,UL}}$
1	0.047558	0.029248	0.033418	0.031253	0.027641	0.044426	0.040276	0.026544	0.024520
2	0.027074	0.018278	0.019917	0.017875	0.015216	0.020288	0.018821	0.009454	0.010976
3	0.015840	0.015480	0.015108	0.015357	0.016974	0.014704	0.014334	0.006962	0.009969
4	0.018904	0.014010	0.018974	0.018156	0.014090	0.018496	0.016780	0.008870	0.012223
5	0.016832	0.012911	0.012145	0.013266	0.013621	0.023525	0.017695	0.008087	0.006594
6	0.023935	0.016557	0.019852	0.023688	0.019106	0.023900	0.023319	0.008228	0.008428
7	0.017357	0.009149	0.010973	0.013751	0.010632	0.023700	0.018859	0.023745	0.008503
8	0.029001	0.013490	0.022299	0.024335	0.012701	0.030964	0.026493	0.007511	0.010210
9	0.021726	0.008524	0.013470	0.016282	0.012802	0.024217	0.020637	0.007807	0.012012
10	0.017241	0.008733	0.011819	0.017519	0.012746	0.013922	0.011415	0.011768	0.009213

cl = 90 percent

Period (i)	$\widehat{\mathtt{N}^{MC}_{i,ANTM}}$	$\widehat{\mathtt{N}^{MC}_{i,BBCA}}$	$\widehat{\mathtt{N}^{MC}_{i,INDF}}$	$\widehat{\mathtt{N}^{MC}_{i,SMGR}}$	$\widehat{\mathtt{N}^{MC}_{i,TLKM}}$	$\widehat{\mathtt{N}^{MC}_{i,GOLD}}$	$\widehat{\mathtt{N}^{MC}_{i,NEM}}$	$\widehat{\mathtt{N}^{MC}_{i,T}}$	$\widehat{\mathtt{N}^{MC}_{i,UL}}$
1	0.070935	0.044813	0.049799	0.047629	0.041801	0.067429	0.061387	0.039430	0.036540
2	0.040694	0.028707	0.031605	0.028293	0.023103	0.031050	0.028860	0.014338	0.016931
3	0.024304	0.024131	0.023805	0.023596	0.025880	0.022541	0.021817	0.011089	0.015492
4	0.027650	0.021351	0.028499	0.027929	0.021452	0.027725	0.021798	0.013634	0.018528
5	0.025779	0.020501	0.019382	0.021309	0.021658	0.034242	0.026096	0.012364	0.010681
6	0.036219	0.025263	0.029940	0.035581	0.029288	0.035806	0.034583	0.012538	0.012945
7	0.025482	0.014397	0.016783	0.020658	0.016399	0.035406	0.029038	0.035403	0.016472
8	0.044159	0.020328	0.033859	0.036143	0.019870	0.047774	0.040591	0.011670	0.015648
9	0.033255	0.013589	0.020977	0.024785	0.019818	0.036746	0.031455	0.011846	0.018665
10	0.026381	0.013829	0.017530	0.026414	0.019049	0.020783	0.017647	0.017474	0.013975

cl = 95 percent

Period (i)	$\widehat{\mathtt{N}^{MC}_{i,ANTM}}$	$\widehat{\mathtt{N}^{MC}_{i,BBCA}}$	$\widehat{\mathtt{N}^{MC}_{i,INDF}}$	$\widehat{\mathtt{N}^{MC}_{i,SMGR}}$	$\widehat{\mathtt{N}^{MC}_{i,TLKM}}$	$\widehat{\mathtt{N}^{MC}_{i,GOLD}}$	$\widehat{\mathtt{N}^{MC}_{i,NEM}}$	$\widehat{\mathtt{N}^{MC}_{i,T}}$	$\widehat{\mathtt{N}^{MC}_{i,UL}}$
1	0.090102	0.057486	0.063069	0.061006	0.053352	0.086093	0.078479	0.049954	0.046455
2	0.051780	0.037270	0.041179	0.036834	0.029563	0.039817	0.037030	0.018335	0.021771
3	0.031242	0.031151	0.030925	0.030374	0.033002	0.028928	0.027903	0.014466	0.0199634
4	0.034831	0.027327	0.036349	0.035804	0.027446	0.035244	0.032024	0.017507	0.023669
5	0.033001	0.026703	0.025325	0.027839	0.028177	0.043106	0.032955	0.015888	0.014039
6	0.046277	0.032449	0.038214	0.045257	0.037588	0.045396	0.043719	0.016082	0.016625
7	0.032086	0.018700	0.021555	0.045766	0.021078	0.044914	0.037276	0.044967	0.016467
8	0.056510	0.025974	0.043302	0.026327	0.025689	0.061380	0.052112	0.015066	0.020037
9	0.042689	0.017750	0.027057	0.031648	0.025467	0.046996	0.040282	0.015147	0.024032
10	0.033809	0.017965	0.022166	0.033672	0.024106	0.026355	0.022700	0.022133	0.017879

cl = 99 percent

Periode (i)	$\widehat{\mathtt{N}^{MC}_{i,ANTM}}$	$\widehat{\mathtt{N}^{MC}_{i,BBCA}}$	$\widehat{\mathtt{N}^{MC}_{i,INDF}}$	$\widehat{\mathtt{N}^{MC}_{i,SMGR}}$	$\widehat{\mathtt{N}^{MC}_{i,TLKM}}$	$\widehat{\mathtt{N}^{MC}_{i,GOLD}}$	$\widehat{\mathtt{N}^{MC}_{i,NEM}}$	$\widehat{\mathtt{N}^{MC}_{i,T}}$	$\widehat{\mathtt{N}^{MC}_{i,UL}}$
1	0.123936	0.080071	0.086711	0.084636	0.073858	0.119421	0.108887	0.068552	0.435147
2	0.071384	0.052334	0.058034	0.051936	0.040962	0.055444	0.051555	0.025398	0.476369
3	0.043456	0.043703	0.043520	0.042324	0.045825	0.040281	0.038804	0.020420	0.328491
4	0.047529	0.037932	0.050068	0.049948	0.038031	0.048545	0.044227	0.024445	0.340959
5	0.045869	0.037627	0.035766	0.039358	0.039771	0.058639	0.045044	0.022079	0.358304
6	0.064049	0.044974	0.052804	0.062295	0.052223	0.062455	0.059920	0.022265	0.511200
7	0.043834	0.026262	0.029976	0.036376	0.029472	0.061978	0.052103	0.061996	0.514470
8	0.078314	0.035914	0.059975	0.062956	0.036133	0.085818	0.072446	0.021108	0.656439
9	0.059342	0.025029	0.037887	0.043992	0.035537	0.065090	0.055944	0.020983	0.711411
10	0.047074	0.025375	0.030433	0.046570	0.033179	0.036312	0.031701	0.030413	1.364214

TABLE VI
ESTIMATED MC VAR MEAN

Portfolio I (One-Day Holding Period)				Portfolio I (Ten-Day Holding Period)			
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,ANTM})}$	$\widehat{E(\mathtt{N}^{MC}_{i,BBCA})}$	$\widehat{E(\mathtt{N}^{MC}_{i,INDF})}$	$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,ANTM})}$	$\widehat{E(\mathtt{N}^{MC}_{i,BBCA})}$	$\widehat{E(\mathtt{N}^{MC}_{i,INDF})}$
0.80	0.023547	0.014638	0.017797	0.80	0.074244	0.046160	0.056488
0.90	0.035486	0.022691	0.027218	0.90	0.111937	0.071576	0.086438
0.95	0.045233	0.029277	0.034914	0.95	0.142563	0.092313	0.110839
0.99	0.062479	0.040922	0.048517	0.99	0.196935	0.129054	0.154139
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,SMGR})}$	$\widehat{E(\mathtt{N}^{MC}_{i,TLKM})}$		$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,SMGR})}$	$\widehat{E(\mathtt{N}^{MC}_{i,TLKM})}$	
0.80	0.019148	0.015553		0.80	0.060432	0.049057	
0.90	0.029234	0.023832		0.90	0.092234	0.075185	
0.95	0.037453	0.030547		0.95	0.118170	0.096403	
0.99	0.039015	0.052039		0.99	0.164092	0.123288	

Portfolio II (One-Day Holding Period)				Portfolio II (Ten-Day Holding Period)			
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,GOLD})}$	$\widehat{E(\mathtt{N}^{MC}_{i,NEM})}$	$\widehat{E(\mathtt{N}^{MC}_{i,T})}$	$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,GOLD})}$	$\widehat{E(\mathtt{N}^{MC}_{i,NEM})}$	$\widehat{E(\mathtt{N}^{MC}_{i,T})}$
0.80	0.023814	0.020863	0.011898	0.80	0.076989	0.066025	0.032531
0.90	0.035950	0.031327	0.017979	0.90	0.113649	0.100105	0.049281
0.95	0.045823	0.040448	0.022954	0.95	0.144928	0.127898	0.06296
0.99	0.063398	0.056063	0.031766	0.99	0.170574	0.177322	0.087181
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,UL})}$			$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,UL})}$		
0.80	0.011265			0.80	0.035631		
0.90	0.017587			0.90	0.054489		
0.95	0.022094			0.95	0.069859		
0.99	0.569700			0.99	0.097191		

Portfolio III (One-Day Holding Period)				Portfolio III (Ten-Day Holding Period)			
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,AMD})}$	$\widehat{E(\mathtt{N}^{MC}_{i,EBAY})}$		$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,AMD})}$	$\widehat{E(\mathtt{N}^{MC}_{i,EBAY})}$	
0.80	0.030878	0.016334		0.80	0.097597	0.051640	
0.90	0.047087	0.025049		0.90	0.148940	0.079254	
0.95	0.060328	0.032165		0.95	0.190808	0.101681	
0.99	0.083758	0.044785		0.99	0.265100	0.141602	

Portfolio IV (One-Day Holding Period)				Portfolio IV (Ten-Day Holding Period)			
$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,ABF})}$	$\widehat{E(\mathtt{N}^{MC}_{i,ANTO})}$		$1-\alpha$	$\widehat{E(\mathtt{N}^{MC}_{i,ABF})}$	$\widehat{E(\mathtt{N}^{MC}_{i,ANTO})}$	
0.80	0.011513	0.023037		0.80	0.011513	0.023037	
0.90	0.017734	0.035096		0.90	0.017734	0.035096	
0.95	0.022821	0.044891		0.95	0.022821	0.044891	
0.99	0.031836	0.062351		0.99	0.031836	0.062351	

mean for each asset over the ten periods. Furthermore, Table VIII also implies that the higher confidence level yields the greater CMC VaR.

We also assessed the performance of CMC VaR using Kupiec Backtesting introduced by [28]. Kupiec Backtesting is applied to verify the accuracy of CMC VaR as a new risk measure. In Table IX, we show the result of Kupiec Backtesting at the specified confidence levels for the proposed method, in which NL abbreviates the losses number exceeding CMC VaR and the percentage of NL is abbreviated by PNL. NL also implies the number of tail losses (loss(es) that occurred in the tail distribution). Meanwhile, for the ten-day holding period of CMC VaR, based on Table IX, CMC VaR for ten-day holding periods performs accurately at a confidence level of 80 percent, 90 percent, and 95 percent. The method's effectiveness can be verified by checking the P- Value in TableIX. A measure risk method possesses a good performance when the P-Value of Binomial statistic is not smaller than a. Table IX also shows that ten-day CMC VaR for this case has performed consistently at a 99 percent of a confidence level. This result is not in line with [12] and CMC VaR for a one-day holding period that is inconsistent.

VI. APPLICATION OF CMC ETL

CMC ETL is computed to quantify the tail loss information, which cannot be provided by CMC VaR. Firstly, the MC ETL of the five log-returns is calculated and summarized in Table X.

Table X shows that in the first period, the average estimation of 10 percent worst lost exceeding the CMC VaR for ANTM in Portfolio I in the next investment period when the stock is held for one day is 6.8176 percent of the asset price on the previous day. The other interpretations of the other values presented in Table X are not explained for the conciseness. Then, we calculated the mean of the estimated MC ETL for each observed asset constructing Portfolio I, II, III, and IV at several confidence levels. The calculation results are summarized in Table XI.

According to the result of the analysis listed in Table XI, it can be noticed that the estimated mean of the 10

TABLE VII
ESTIMATION PARAMETERS OF CMC VAR

Portfolio I (One-Day Holding Period)					Portfolio I (Ten-Day Holding Period)				
$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$	$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$
0.80	0.018137	0.000046	0.000008	0.625289	0.80	0.057276	0.000480	0.000074	0.608124
0.90	0.027692	0.000102	0.000016	0.603625	0.90	0.087474	0.001013	0.000155	0.604891
0.95	0.035485	0.000165	0.000024	0.593447	0.95	0.112058	0.001629	0.000238	0.593845
0.99	0.048594	0.000321	0.000057	0.638226	0.99	0.153501	0.004772	0.000400	0.456110
Portfolio II (One Day Holding Period)					Portfolio II (Ten-Day Holding Period)				
$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$	$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$
0.80	0.016960	0.000035	0.000054	0.864245	0.80	0.052794	0.000496	0.000439	0.898427
0.90	0.025711	0.000075	0.000124	0.857636	0.90	0.079381	0.001147	0.000928	0.890042
0.95	0.032830	0.000127	0.000200	0.863327	0.95	0.101412	0.001865	0.001503	0.889638
0.99	0.180232	0.065210	0.023881	0.964672	0.99	0.133067	0.003815	0.001871	0.830650
Portfolio III (One-Day Holding Period)					Portfolio III (Ten-Day Holding Period)				
$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$	$1-\alpha$	$\widehat{\mu(s)}$	$\widehat{E(\tau(s))}$	$\widehat{Var(\mu(s))}$	$\widehat{Z_{MC}}$
0.80	0.023606	0.000068	0.000099	0.936043	0.80	0.074619	0.000677	0.000988	0.935895
0.90	0.036068	0.000151	0.000228	0.937966	0.90	0.114097	0.001507	0.002277	0.937933
0.95	0.046247	0.000243	0.000372	0.938647	0.95	0.046247	0.000243	0.000372	0.938647
0.99	0.064272	0.000462	0.000713	0.939225	0.99	0.203351	0.004643	0.007161	0.939111

TABLE VIII
ESTIMATION OF CMC VAR FOR EACH STOCK

Portfolio I (One-Day Holding Period)					Portfolio I (Ten-Day Holding Period)				
CMC VaR	ANTM	BBCA	INDF	SMGR	CMC VaR	ANTM	BBCA	INDF	SMG
$\widehat{\Psi_{MC,0.99}}$	0.057456	0.043698	0.048545	0.050793	$\widehat{\Psi_{MC,0.99}}$	0.173312	0.142351	0.153792	0.158332
$\widehat{\Psi_{MC,0.95}}$	0.041270	0.031801	0.035146	0.036653	$\widehat{\Psi_{MC,0.95}}$	0.130173	0.100332	0.111334	0.115688
$\widehat{\Psi_{MC,0.90}}$	0.032397	0.024673	0.027406	0.028623	$\widehat{\Psi_{MC,0.90}}$	0.102272	0.077858	0.086848	0.090353
$\widehat{\Psi_{MC,0.80}}$	0.021520	0.015949	0.017925	0.018769	$\widehat{\Psi_{MC,0.80}}$	0.067595	0.050516	0.056797	0.059195
CMC VaR	TLKM				CMC VaR	TLKM			
$\widehat{\Psi_{MC,0.99}}$	0.042480				$\widehat{\Psi_{MC,0.99}}$	0.139721			
$\widehat{\Psi_{MC,0.95}}$	0.032554				$\widehat{\Psi_{MC,0.95}}$	0.102761			
$\widehat{\Psi_{MC,0.90}}$	0.025362				$\widehat{\Psi_{MC,0.90}}$	0.080041			
$\widehat{\Psi_{MC,0.80}}$	0.016521				$\widehat{\Psi_{MC,0.80}}$	0.052278			
Portfolio II (One-Day Holding Period)					Portfolio II (Ten-Day Holding Period)				
CMC VaR	GOLD	NEM	T	UL	CMC VaR	GOLD	NEM	T	UL
$\widehat{\Psi_{MC,0.99}}$	0.067526	0.060450	0.037011	0.555941	$\widehat{\Psi_{MC,0.99}}$	0.164222	0.169828	0.094952	0.103267
$\widehat{\Psi_{MC,0.95}}$	0.044047	0.039407	0.024304	0.023561	$\widehat{\Psi_{MC,0.95}}$	0.140126	0.124975	0.067205	0.073342
$\widehat{\Psi_{MC,0.90}}$	0.034492	0.030528	0.019079	0.018744	$\widehat{\Psi_{MC,0.90}}$	0.109881	0.097826	0.052591	0.057226
$\widehat{\Psi_{MC,0.80}}$	0.022884	0.020333	0.012585	0.012038	$\widehat{\Psi_{MC,0.80}}$	0.074531	0.064681	0.034590	0.037374
Portfolio III (One-Day Holding Period)					Portfolio III (Ten-Day Holding Period)				
CMC VaR	AMD	EBAY			CMC VaR	AMD	EBAY		
$\widehat{\Psi_{MC,0.99}}$	0.082574	0.045969			$\widehat{\Psi_{MC,0.99}}$	0.261340	0.145362		
$\widehat{\Psi_{MC,0.95}}$	0.059464	0.033029			$\widehat{\Psi_{MC,0.95}}$	0.188078	0.104411		
$\widehat{\Psi_{MC,0.90}}$	0.046404	0.025732			$\widehat{\Psi_{MC,0.90}}$	0.146777	0.081416		
$\widehat{\Psi_{MC,0.80}}$	0.030413	0.016799			$\widehat{\Psi_{MC,0.80}}$	0.096124	0.053113		
Portfolio IV (One-Day Holding Period					Portfolio IV (Ten-Day Holding Period)				
CMC VaR	ABF	ANTO			CMC VaR	ABF	ANTO		
$\widehat{\Psi_{MC,0.99}}$	0.033045	0.061141			$\widehat{\Psi_{MC,0.99}}$	0.106238	0.192619		
$\widehat{\Psi_{MC,0.95}}$	0.023706	0.044006			$\widehat{\Psi_{MC,0.95}}$	0.076256	0.138800		
$\widehat{\Psi_{MC,0.90}}$	0.018431	0.034398			$\widehat{\Psi_{MC,0.90}}$	0.059313	0.108368		
$\widehat{\Psi_{MC,0.80}}$	0.011989	0.022561			$\widehat{\Psi_{MC,0.80}}$	0.038601	0.071076		

TABLE IX
KUPIEC BACKTESTING FOR ESTIMATED CMC VAR

	$1-a$	j	NL	PNL	P-Value		$1-a$	j	NL	PNL	P-Value
Portfolio I	0.80	ANTM	427	16.486490	0.999997	Portfolio I	0.80	ANTM	40	1.544402	1
(One-Day	0.90		197	7.606178	0.999985	(Ten-Day	0.90		5	0.193050	1
Holding Period)	0.95		127	4.903475	0.566387	Holding Period)	0.95		2	0.077220	1
	0.99		767	2.586873	0		0.99		1	0.038610	1
	0.80	BBCA	350	13.513510	1		0.80	BBCA	39	1.505792	1
	0.90		204	7.876448	0.999885		0.90		9	0.347490	1
	0.95		118	4.555985	0.839347		0.95		1	0.038610	1
	0.99		60	2.316602	0		0.99		0	0	1
	0.80	INDF	385	14.864860	1		0.80	INDF	41	1.583012	1
	0.90		182	7.027027	1		0.90		10	0.386100	1
	0.95		114	4.401544	0.913803		0.95		2	0.077220	1
	0.99		67	2.586873	0		0.99		1	0.038610	1
	0.80	SMGR	389	15.019310	1		0.80	SMGR	30	1.158301	1
	0.90		199	7.683398	0.999973		0.90		6	0.231660	1
	0.95		149	5.752896	0.037949		0.95		3	0.115830	1
	0.99		47	1.814672	0.000059		0.99		2	0.077220	1
	0.80	TLKM	412	15.907340	1		0.80	TLKM	24	0.926641	1
	0.90		207	7.992278	0.999743		0.90		6	0.231660	1
	0.95		92	3.552124	0.999762		0.95		2	0.077220	1
	0.99		70	2.702703	0		0.99		0	0	1
Portfolio II	0.80	GOLD	420	16.73307	0.999984	Portfolio II	0.80	GOLD	39	1.553785	1
(One-Day	0.90		229	9.123506	0.925108	(Ten-Day	0.95		4	0.159363	1
Holding Period)	0.95		135	5.378486	0.179248	Holding Period)	0.99		1	0.039841	1
	0.99		48	1.912351	0.000014		0.99		1	0.039841	1
	0.80	NEM	418	16.65339	0.999990		0.80	NEM	38	1.513944	1
	0.90		215	8.565737	0.991926		0.90		9	0.358566	1
	0.95		127	5.059761	0.422141		0.95		3	0.119522	1
	0.99		48	1.912351	0.000014		0.99		0	0	1
	0.80	T	287	11.43426	1		0.80	T	40	1.593625	1
	0.90		143	5.697211	1		0.90		9	0.358566	1
	0.95		91	3.625498	0.999424		0.95		5	0.199203	1
	0.99		36	1.434263	0.014912		0.99		0	0	1
	0.80	UL	367	14.62151	1		0.80	UL	35	1.394422	1
	0.90		169	6.733068	1		0.90		11	0.438247	1
	0.95		103	4.103586	0.980346		0.95		5	0.199203	1
	0.99		0	0	1		0.99		0	0	1
Portfolio III	0.80	AMD	127	5.059761	1	Portfolio III	0.80	AMD	45	1.792829	1
(One-Day	0.90		45	1.792829	1	(Ten-Day	0.90		10	0.398406	1
Holding Period)	0.95		24	0.9561753	1	Holding Period)	0.95		1	0.039841	1
	0.99		10	0.3984064	0.999475		0.99		1	0.039841	1
	0.80	EBAY	362	14.42231	1		0.80	EBAY	33	1.314741	1
	0.90		178	7.091633	1		0.90		10	0.398406	1
	0.95		101	4.023904	0.987968		0.95		2	0.0793651	1
	0.99		45	1.792829	0.000106		0.99		2	0.0796813	1
Portfolio IV	0.80	ANTO	418	16.5873	0.999993	Portfolio IV	0.80	ANTO	43	1.706349	1
(One-Day	0.90		234	9.285714	0.878064	(Ten-Day	0.90		5	0.1984127	1
Holding Period)	0.99		66	2.619048	0	Holding Period)	0.95		3	0.1190476	0
	0.80	ABF	393	15.59524	1		0.80	ABF	22	0.8730159	1
	0.90		186	7.380952	0.999997		0.90		9	0.3571429	1
	0.95		108	4.285714	0.947677		0.95		3	0.1190476	1
	0.99		34	1.349206	0.036393		0.99		2	0.0793651	1

TABLE X
ESTIMATED MC ETL AT SPECIFIED CONFIDENCE LEVELS

	cl = 80 percent				cl = 90 percent			
Period (i)	$\widehat{Q^{MC}_{i,ANTM}}$	$\widehat{Q^{MC}_{i,BBCA}}$	$\widehat{\eta^{MC}_{i,INDF}}$	$\widehat{Q^{MC}_{i,SMGR}}$	$\widehat{Q^{MC}_{i,ANTM}}$	$\widehat{Q^{MC}_{i,BBCA}}$	$\widehat{\eta^{MC}_{i,INDF}}$	$\widehat{Q^{MC}_{i,SMGR}}$
1	0.055139	0.039952	0.040528	0.0473051	0.068176	0.047162	0.053603	0.065726
2	0.039058	0.031620	0.033091	0.032594	0.048789	0.038121	0.042460	0.044905
3	0.031729	0.027375	0.032384	0.032176	0.042244	0.034744	0.045311	0.041199
4	0.034221	0.028505	0.033554	0.032887	0.059571	0.034347	0.045260	0.040470
5	0.030108	0.027748	0.024905	0.030316	0.044014	0.038861	0.043810	0.041725
6	0.038248	0.030597	0.037467	0.038247	0.051122	0.039284	0.045429	0.050699
7	0.034796	0.031012	0.025022	0.033283	0.046800	0.044416	0.061502	0.044539
8	0.041649	0.028440	0.035254	0.035801	0.054313	0.033418	0.049567	0.043732
9	0.032704	0.028190	0.028304	0.028878	0.048871	0.034237	0.039781	0.036633
10	0.031092	0.025543	0.027277	0.029576	0.040686	0.033522	0.034897	0.037738

	cl = 95 percent				cl = 99 percent			
Period (i)	$\widehat{Q^{MC}_{i,ANTM}}$	$\widehat{Q^{MC}_{i,BBCA}}$	$\widehat{\eta^{MC}_{i,INDF}}$	$\widehat{Q^{MC}_{i,SMGR}}$	$\widehat{Q^{MC}_{i,ANTM}}$	$\widehat{Q^{MC}_{i,BBCA}}$	$\widehat{\eta^{MC}_{i,INDF}}$	$\widehat{Q^{MC}_{i,SMGR}}$
1	0.074634	0.054877	0.06689443	0.083280	0.093187	0.064538	0.079481	0.116479
2	0.061181	0.047321	0.04795885	0.053765	0.072949	0.055049	0.057339	0.061757
3	0.048310	0.045707	0.04923523	0.049059	0.072568	0.063406	0.059422	0.067900
4	0.065627	0.043791	0.05341525	0.045041	0.074360	0.055901	0.069056	0.061530
5	0.054444	0.044513	0.05341873	0.047952	0.065960	0.059499	0.060625	0.086593
6	0.059163	0.045425	0.05305398	0.047271	0.073604	0.054275	0.060868	0.068703
7	0.060381	0.048410	0.0615022	0.049730	0.069920	0.051236	0.061502	0.063982
8	0.060188	0.038380	0.05675451	0.056297	0.069849	0.055188	0.064254	0.072683
9	0.055799	0.040477	0.06435904	0.046838	0.069664	0.047695	0.064359	0.000000
10	0.048840	0.036902	0.04163014	0.041841	0.057987	0.000000	0.057377	0.051825

percent worst loss, which is larger than CMC VaR for each asset of the ten observed periods, is between 3.7230 percent until 5.0459 percent when the assets of Portfolio I were held for one day. The results presented in Table XI would be utilized to count the four-parameter estimations required in the CMC ETL computation for each asset. The estimators of $\mu^*(s)$, $E\left[\tau^*(s)\right]$, $Var\left(\mu^*(s)\right)$, and the corresponding estimator of Z^*_{MC} for Portfolio I, II, III, and IV are revealed in Table XII. From Table XII, it can be interpreted that the estimated loss average, which is greater than CMC VaR at confidence level 90 percent from the five assets constructing Portfolio I, is about 4.3280 percent that is relative to the close-price asset in the previous day. Furthermore, the estimated expectation of the CMC ETL variance from each asset is about 0.000048, and the estimated variance of the average CMC ETL for every asset is 0.000027. The estimators of $\mu^*(s)$, $E\left[\tau^*(s)\right]$, and $Var\left(\mu^*(s)\right)$ are employed to count the risk factor of CMC ETL based on Equation (12). Then, we can obtain an estimated risk factor of Portfolio I, $Z^*_{MC} = 0.851777$, at a 90 percent of confidence level.

Then, using the precedence information, CMC ETL and its risk factor of the j^{th} asset in Portfolio I, II, III, and IV can be calculated directly using Equation (11) and Equation (12), and the results are shown in Table XIII. According to the result of the analysis, for Portfolio I, it can be concluded that the estimated mean of loss, which is bigger than the CMC VaR of asset ANTM, BBCA, INDF, SMGR, and TLKM, when the holding period is one day at a confidence level of 90 percent respectively, are 4.9395 percent, 3.8622 percent, 4.5735 percent, 4.4521 percent, and 3.8127 percent. In addition, when we compared the mean estimation of loss for each asset using the conventional MC ETL provided in Table X, and the loss average, which is greater than CMC VaR from each asset in Portfolio I, it can be identified that the estimation of loss using CMC ETL for the five assets is tended to approach the estimated mean of MC ETL for every asset over the ten periods. It is due to a risk factor of CMC ETL of Portfolio I, Z^*_{MC}, which is relatively big. Thus, based on Equation (11), the weight given to $\widehat{\mu^*(s)}$ is smaller than the weight granted to the mean of MC ETL for each asset in Portfolio I over the ten periods.

VII. CONCLUSION

CMC VaR and CMC ETL proposed in this paper are the new risk measurement methods which are designed by combining the concept of Credible VaR and Credible ETL with MC VaR and MC ETL. The new methods are able to cover joint utilization of asset information and other relevant information because of the other asset risks within a portfolio to estimate an individual asset risk. As described in the data

TABLE XI
THE ESTIMATED MEAN OF MC ETL FOR THE FIVE ASSETS

Portfolio I

$1-\alpha$	$E(\widehat{Q^{MC}_{i,ANTM}})$	$E(\widehat{Q^{MC}_{i,BBCA}})$	$E(\widehat{Q^{MC}_{i,INDF}})1-\alpha$	$E(\widehat{Q^{MC}_{i,SMGR}})$	$E(\widehat{Q^{MC}_{i,TLKM}})$
0.80	0.036874	0.029898	0.0317780.80	0.034106	0.029026
0.90	0.050459	0.037811	0.0461620.90	0.044737	0.037230
0.95	0.058857	0.044580	0.054822	0.053130	0.044
0.99	0.072005	0.050679	0.063428	0.065145	0.050898

Portfolio II

$1-\alpha$	$E(\widehat{Q^{MC}_{i,GOLD}})$	$E(\widehat{Q^{MC}_{i,NEM}})$	$E(\widehat{Q^{MC}_{i,T}})$	$E(\widehat{Q^{MC}_{i,UL}})$	
0.80	0.041068	0.035745	0.020896	0.020981	
0.90	0.053604	0.046662	0.027488	0.028991	
0.95	0.064560	0.044805	0.032345	0.033869	
0.99	0.089909	0.061548	0.023181	0.000000	

Portfolio III

$1-\alpha$	$E(\widehat{Q^{MC}_{i,AMD}})$	$E(\widehat{Q^{MC}_{i,EBAY}})$			
0.80	0.057862	0.030267			
0.90	0.076279	0.040127			
0.95	0.097286	0.049491			
0.99	0.120257	0.063592			

Portfolio IV

$1-\alpha$	$E(\widehat{Q^{MC}_{i,ABF}})$	$E(\widehat{Q^{MC}_{i,ANTO}})$			
0.80	0.021320	0.039952			
0.90	0.029029	0.039058			
0.95	0.035007	0.058480			
0.99	0.047679	0.068573			

TABLE XII
ESTIMATION PARAMETERS OF CMC ETL

$1-\alpha$	$\widehat{\mu^*(s)}$	$\widehat{E[\tau^*(s)]}$	$\widehat{Var(\mu^*(s))}$	$\widehat{Z^*_{MC}}$
Portfolio I				
0.80	0.032337	0.000029	0.000007	0.717151
0.90	0.043280	0.000048	0.000027	0.851777
0.95	0.051118	0.000063	0.000036	0.850691
0.99	0.060431	0.000326	0.000055	0.628463
Portfolio II				
0.80	0.029672	0.000103	0.000033	0.969164
0.90	0.039186	0.000164	0.000043	0.974147
0.95	0.043895	0.000202	0.000186	0.915712
0.99	0.043659	0.001548	0.000464	0.970864
Portfolio III				
0.80	0.044064	0.000038	0.000377	0.990046
0.90	0.058203	0.000077	0.000646	0.988182
0.95	0.073389	0.000170	0.001125	0.985096
0.99	0.091925	0.000248	0.001581	0.984579
Portfolio IV				
0.80	0.030636	0.000169	0.000042	0.976020
0.90	0.034043	0.000046	0.000047	0.907238
0.95	0.046743	0.000271	0.000049	0.982314
0.99	0.058126	0.000124	0.000939	0.569884

TABLE XIII
ESTIMATED CMC ETL FOR EACH STOCK

Portfolio I

CMC ETL	ANTM	BBCA	INDF	SMGR	TLKM
$\Upsilon_{\widehat{MC},0.99}$	0.067705	0.054302	0.062315	0.063394	0.054440
$\Upsilon_{\widehat{MC},0.95}$	0.057701	0.045556	0.054269	0.052830	0.045233
$\Upsilon_{\widehat{MC},0.90}$	0.049395	0.038622	0.045735	0.044521	0.038127
$\Upsilon_{\widehat{MC},0.80}$	0.035591	0.030588	0.031936	0.033606	0.029963

Portfolio II

CMC ETL	GOLD	NEM	T	UL	
$\Upsilon_{\widehat{MC},0.99}$	0.088562	0.061026	0.023777	0.001272	
$\Upsilon_{\widehat{MC},0.95}$	0.062818	0.044728	0.033319	0.034714	
$\Upsilon_{\widehat{MC},0.90}$	0.053231	0.046468	0.027790	0.029254	
$\Upsilon_{\widehat{MC},0.80}$	0.040717	0.035557	0.021167	0.021248	

Portfolio III

CMC ETL	AMD	EBAY			
$\Upsilon_{\widehat{MC},0.99}$	0.119821	0.064029			
$\Upsilon_{\widehat{MC},0.95}$	0.096930	0.049847			
$\Upsilon_{\widehat{MC},0.90}$	0.076066	0.040341			
$\Upsilon_{\widehat{MC},0.80}$	0.057724	0.030404			

Portfolio IV

CMC ETL	ABF	ANTO			
$\Upsilon_{\widehat{MC},0.99}$	0.052172	0.064079			
$\Upsilon_{\widehat{MC},0.95}$	0.035214	0.058273			
$\Upsilon_{\widehat{MC},0.90}$	0.029494	0.038593			
$\Upsilon_{\widehat{MC},0.80}$	0.021543	0.039729			

analysis in Section V and VI, it can be summarized that CMC VaR and CMC ETL are empirically effective chosen as alternatives to risk measures. Both risk measures also provide easiness to be implemented in the real data because the methods do not require a specified distribution of return assets.

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