Overview of Long Memory for Economic and Financial Time Series Dataset and Related Time Series Models: A Review Study

Mohd Tahir Ismail and Remal Shaher Al-Gounmeein

Abstract—**Verifying the existence of long-memory feature is a crucial activity performed during the development process of the autoregressive integrated moving average (ARIMA) model. The verifying step will determine whether a researcher needs to use the ARIMA model or the autoregressive fractionally integrated moving average (ARFIMA) model, which depends on the estimated value of the fractional difference (d). This study focuses on analytical techniques for verifying the long-memory feature (graphs and statistical tests); determines estimation methods/functions for approximating long-memory parameters (i.e., d), limitations, extensions, comparisons, applications, and performs an in-depth review of the recent literature on the ARFIMA model. The discussion will also include the hybrid method for forecasting in different fields. Although the validation of the existence of the long-memory feature and its estimation, limitations, extensions, comparisons, and applications has been extensively investigated, specific criteria should be considered to avoid obtaining invalid or wrong ARFIMA models remain unclear. We examine the literature to validate these issues and identify effective methods and tests to avoid these errors. Thus, the results of this study can provide an initial classification of the literature on long memory and the ARFIMA model that can be used as a basis for future work when the value of d is a non-integer number.**

Index Terms—**Long memory, Autoregressive fractionally integrated moving average, Individual and Hybrid models, Modeling and forecasting, Time series.**

I. Introduction

TIME series modeling and forecasting are essential methods used in different fields, especially in economic and financial trends, because of their ability to manage risk and increase investment in financial and industrial markets. Therefore, a nontraditional and accurate statistical technique called long memory must be used to describe changes in time series. Long memory is a phenomenon that may be encountered when investigating a time series dataset, where the long-term dependence between two points increases the amount between the points' distance [1]. Modeling of long-memory behavior for any time series is usually performed accurately by relying on autoregressive fractionally integrated moving average (ARFIMA) models because long-memory models are important in the study of time series data [2]. [3]created the ARFIMA model to capture the long-memory behavior of time series data. This model was fitted for the time series data to understand the dataset further or forecast future points in the series [4]-[5]. The ARFIMA model is suitable for linear time series but unsuitable for time series that contain nonlinear structures [6].

Economic and financial studies have shown the importance of the ARFIMA model in analyzing time series datasets [7]. In addition, numerous high-frequency financial time series data have exhibited the property of long-memory (i.e., presence of statistically significant correlations between observations that are considerably distant) [8]. Another distinguishing feature of many financial time series data is time-varying volatility or "heteroscedasticity" of datasets through the emergence of high volatility, followed by low volatility [8]. Thus, long-memory processes are applicable to economic and financial data. This attribute is what distinguishes our selection of this type of data and the ARFIMA models.

This study focuses on analytical techniques for verifying the long-memory feature in terms of graphs and statistical tests; determines estimation methods/functions for approximating long-memory parameters (i.e., value of the fractional difference), limitations, extensions, comparisons, applications, and performs an in-depth review of the recent literature on the ARFIMA model, including hybrid methods, for forecasting in different fields.

The remainder of this paper is organized as follows. Theoretical explanations of several graphs and statistical tests used to verify the long-memory feature in time series as well as some methods and functions used to estimate the long-memory parameter are provided in second section. The ARFIMA model and its advantages are also presented in this section. Limitations, extensions, comparisons with an overview of developments in the field of the ARFIMA methodology and its application in different areas are discussed in third section. Recent studies that have applied ARFIMA to forecasting time series are presented in fourth section. Finally, conclusions of this study are drawn in fifth section.

II. Long Memory

Long memory is a phenomenon that may be observed in time series data; it is evident when the distance between two points is further apart [1], [6] and it considerably affects the financial field [9], [6]. Experimental research on long memory processes dates back to [10], who studied the hydrological properties of the Nile Basin. However, interest in using long memory models for economic data series was elicited when [3]observed that many such series are nonstationary in terms of the mean value.

Here, we explain the concept of long memory in a time series as presented by [11]- [12]. Suppose that $\rho(h)$ is the

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Mohd Tahir Ismail is an associate professor of School of Mathematical Sciences, Universiti Sains Malaysia, Pulau Pinang, Malaysia (e-mail: m.tahir@usm.my).

Remal Shaher Al-Gounmeein is a PhD candidate of School of Mathematical Sciences, Universiti Sains Malaysia, Pulau Pinang, Malaysia (e-mail: ralgounmeein@yahoo.com).

autocovariance function at $lag(h)$ of a stationary process $y_t : t \in \mathbb{Z}$. Then, y_t exhibits long memory if the autocovariance sequence decays extremely slowly such that it is not absolutely summable, as follows:

$$\sum_{h=-\infty}^{\infty} |\rho(h)| = \infty \ , \tag{1}$$

where

$$\rho(h) = E(y_t, y_{t+h}) \tag{2}$$

Otherwise, y_t exhibits short memory if the following formula is verified:

$$\sum_{h=-\infty}^{\infty} |\rho(h)| < \infty \ . \tag{3}$$

However, several graphs and statistical tests are used to verify the long memory feature. These graphs and tests are given below.

A. Verifying the Long Memory Feature by Using Graphs

Many graphs provide an indication of the existence of the long memory feature. They include the autocorrelation function (ACF), range over the standard deviation (R/S), variance, variogram, spectral density function, and Higuchi plots.

1) Autocorrelation Function Plot: A long memory phenomenon can be specified when ACF decays more slowly than exponential decay [1], as reported by [11], in accordance with the following formula:

$$\rho(h) = \frac{\Gamma(1-d)\Gamma(h+d)}{\Gamma(d)\Gamma(1+h-d)}. \tag{4}$$

It can also be written using another formula, as follows:

$$\rho(h) \sim \frac{\Gamma(1-d)}{\Gamma(d)}\ h^{2d-1}, \tag{5}$$

where $-0.5\ < d < 0.5$ and $h \to \infty$.

ACF shows the correlation between observations for different periods. An ACF diagram is frequently used as a primary diagnostic tool in studying time series applications. It is considered necessary in highlighting some of the crucial characteristics of a time series, particularly in verifying the presence of long memory in a time series.

2) R/S Plot: The R/S graph was described by [13] to have the following steps.

1) $Q = R/S$ is calculated for all possible values of time t and $lag(k)$.
2) $Log(Q)$ versus $\log(k)$ is plotted.
3) A straight line $y = a \pm b \log(k)$ that corresponds to the ultimate behavior of the data is drawn. Coefficients a and b can be estimated, e.g., via the least-squares method.

The slope of this straight line is considered a measure for distinguishing between short and long memory processes. In particular, the slope of this straight line is greater than 0.5 for operations involving long memory and tends to be 0.5 for most short memory operations.

3) Variance Plot: The variance of the sample mean of a long memory process based on m observations was explained by [11] by calculating the following formula:

$$Var(\bar{y}_m) \sim cm^{2d-1}, \tag{6}$$

where

$$Var(\bar{y}_m) = \frac{1}{m}\left[2\sum_{j=1}^{m-1}\left(1-\frac{j}{m}\right)\gamma(j) + \gamma(0)\right], \tag{7}$$

and c is a positive constant. Consequently, by dividing a sample with size n into k blocks with size m, we obtain

$$\log\left(Var(\bar{y}_j)\right) \sim c + (2d-1)\log(j) \tag{8}$$

for $j = 1, 2, \ldots, k$ and $(\bar{y}_j)$ is the mean of the j^{th} block. That is,

$$\bar{y}_j = \frac{1}{m}\sum_{t=(j-1)\times m+1}^{j\times m} y_t. \tag{9}$$

Thus, for a long memory process, the slope of the line described by Equation (8) should be greater than -1. By contrast, the slope of the line should be -1 for a short memory process.

4) Variogram Plot: [14] defined the variogram for the lag distance k formula as follows:

$$V\left(k\right) = \frac{1}{2}\ E\left[\left(X_t - X_{t-k}\right)^2\right], \tag{10}$$

where t denotes all possible locations.

Thus, the presence of long memory in data can be inferred through the behavior of the variogram in terms of the slow ascent and non-zigzagging of the plot in accordance with Equation (10), i.e., it is opposite to that in the ACF plot. Notably, this type of graph requires numerous observations to present the correct behavior.

5) Spectral Density Function Plot: Another graph used to distinguish between short and long memory processes is the spectral density function plot. As [13] and [15] reported, the spectral density function needs to be formulated before constructing the plot. The formulation is given as follows:

A covariance stationary process X_t is a long memory process if its spectral density function f at $\lambda \to 0^+$ is approximated by

$$f(\lambda) \sim C_f(\lambda) \cdot |\lambda|^{-2d}, \qquad d \in (0, \frac{1}{2}) \tag{11}$$

where

$$C_f(\lambda) = \frac{\sigma_\epsilon^2 |\psi(1)|^2}{2\pi|\phi(1)|^2}. \tag{12}$$

The function in Equation (12) gradually changes to zero at frequency zero. That is, most of the data are centered around zero. Meanwhile, $\psi(1)$ and $\phi(1)$ are the moving average and autoregression polynomials, respectively, when the backward shift operator is 1. This phenomenon is explained in detail in Section II-D.

6) Higuchi Plot: The Higuchi plot is another method for verifying the existence of long memory by using graphs. As reported in [16]-[20], this technique calculates the fractal dimension (D) of a finite time series at regular intervals $j = 1, 2, \ldots, N$.

$$X(1),\ X(2),\ X(3), \ldots, X(N) \tag{13}$$

From the series in Equation (13), we construct a new time series for a fixed $(k \leq \frac{N}{64})$, X_k^m, which is defined as follows:

$$X_k^m : X(m), X(m+k), X(m+2k), \ldots, X(m+\left[\frac{N-m}{k}\right]*k), \tag{14}$$

with $m = 1,\ 2,\ \ldots, k$, where m and k are integers that indicate the initial time value and the interval time (lag), respectively. Moreover, the symbol [...] denotes Gauss' notation (i.e., the larger integer). Then, [16] defined the length of the curve associated with each time series for X_k^m as follows:

$$L_m(k) = \frac{1}{k}\left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]} |X(m+ik) - X(m+(i-1)*k)|\right)\left(\frac{N-1}{\left[\frac{N-m}{k}\right]*k}\right). \tag{15}$$

The term $(\left[\frac{N-m}{k}\right]*k)$ represents the normalization factor for $L_m(k)$. Thereafter, Higuchi calculated the length of all the curves for the time interval k in Equation (15) as follows:

$$L(k) = \frac{1}{k}\left(\sum_{m=1}^{k} L_m(k)\right). \tag{16}$$

Finally, if the expected value for Equation (16) follows a power law, namely,

$$E(L(k)) \sim k^{-D}, \tag{17}$$

then the time series in Equation (13) achieves long memory operation with dimension D. We use the log of Equation (17) to illustrate this phenomenon via plotting. If $\log(E(L(k)))$ is plotted against $\log(k)$, then data should fall on a straight line with a slope $-D$. Thus, we verify the presence of the long memory feature in the time series.

B. Verifying the Long memory Feature Using Statistical Tests

Numerous statistical methods are used to verify the existence of the long memory feature [18]. These methods include R/S analysis, Higuchi, aggregated variance, and structural break methods. These methods are explained in detail in the succeeding sections.

1) R/S Analysis: In particular, the range divided via R/S analysis is commonly used in these methods. [21] found that R/S analysis exhibited better properties than ACF and variance time function analyses. [22] modified R/S analysis and determined that it is a powerful tool for non-normal distributions, short-range dependence, and conditional heteroscedasticity under a null hypothesis. The formula of R/S analysis is as follows [21]- [22]:

$$\frac{R_{(n)}}{S_{(n)}} = \frac{\max\limits_{1\leq k\leq n}\sum_{i=1}^{k}\left(X_i - \bar{X_n}\right) - \min\limits_{1\leq k\leq n}\sum_{i=1}^{k}(X_i - \bar{X_n})}{\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X_n})^2\right)^{\frac{1}{2}}}, \tag{18}$$

where

$$\bar{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{19}$$

and n is the sample size.

2) Higuchi Method: The method proposed by [16] involves calculating the length L or the fractal dimension D of a fractional Brownian motion path obtained by taking the cumulative sum of fractional Gaussian noise. The fractal dimension D is linked to $\alpha \in (1, 3)$ in accordance with [16] and [23] through the following relationship:

$$D = \frac{5-\alpha}{2}, \tag{20}$$

where α is considered the index of a power law spectrum [24]. [25] reported that the method involves the creation of subseries in succeeding iterations with different initial samples m and time intervals K. Notably, this method can yield D values, such that $2 < D < 2.1$ for $\alpha < 1$, as mentioned by [23]. [16] reported that this method could obtain a more precise and stable characteristic time scale. Moreover, this method could be applied to the observational data of natural phenomena and is useful for analyzing nonperiodic and irregular time series. In addition, the Higuchi method is a time-domain method that is useful for nonstationary series [25]. The aforementioned features represent the advantages of the Higuchi method over other methods. The mathematical formula of the Higuchi method is presented in Section II-A6.

3) Aggregated Variance Method: To apply this technique, the time series X_i must have length N. Thus, the corresponding aggregated series as reported by [13] and [26] can be defined by

$$X^{(m)}(k) = \frac{1}{m}\left(\sum_{i=m(k-1)+1}^{km} X(i)\right), \tag{21}$$

where $k = 1, 2, \ldots, \frac{N}{m}$ for successive values of m.

For a given k, the sample variance of $X(k)$ is used as a plausible estimator of $Var(\widehat{X^{(m)}}(k))$, as follows:

$$Var(\widehat{X^{(m)}}(k)) = \frac{1}{N/m}\left(\sum_{k=1}^{N/m}\left(X^{(m)}(k) - \bar{X^{(m)}}\right)^2\right), \tag{22}$$

where $\bar{X^{(m)}}$ denotes the sample mean of $X^{(m)}$. On this basis,

$$Var\left(X^{(m)}\right) \sim \sigma^2 m^{\beta}, \tag{23}$$

because $m \to \infty$ and $\beta = 2H - 2 < 0$, where H is Hurst exponent, and σ is the scale parameter. Lastly, Equation (22) should be asymptotically proportional to $m^{2H-2} = m^{\beta}$ for large N/m and m, and the resulting points should form a straight line with slope β, where $-1 \leq \beta < 0$.

4) Structural Breaks: Long memory characteristics and features are also generated by a nonstationary structural break or by regime-switching models [27], [6]. Therefore, these breaks of a time series should be verified because they determine whether long memory is actually present or merely imaginary, as pointed out by [27]- [29], [6]. In [30], Chow introduced the single break test, which had been modified as the Quandt likelihood ratio (QLR) test as reported by [6]. This test is performed to determine the break between two times (t_0 and t_1), also known as the supremum F-statistic [31], [6] which is given by the following expression:

$$Sup\ F = \max\ \{F(t_0),\ F(t_0+1),\ \ldots\ ,\ F(t_1)\}, \quad (24)$$

where Sup F-statistic is the highest among the given values. In addition, if the P-value of the F-statistic is less than 0.05, then the test rejects the null hypothesis (see [6]).

C. Estimations

Meanwhile, several methods and functions have been used to estimate long memory parameters [the value of the fractional difference (d)]. These methods and functions are the Hurst exponent, Geweke and Porter–Hudak's (GPH's) estimator, the smoothed periodogram (Sperio), and fractionally differenced (Fracdiff), which were presented by [11], [13], [18], [32]- [34]. These methods and functions are described in the following sections.

1) Hurst Exponent: This method was proposed by [10] and then reviewed by [35], as mentioned by [33]. The range $R^*{}_{(n)}$ of the subtotals are used to deviate the values from their mean in a time series and divided by the standard deviation $S^*_{(n)}$. The full formula is denoted by $Q_{(n)}$ and written as follows:

$$Q_{(n)} = \frac{R^*_{(n)}}{S^*_{(n)}} = \frac{\max\limits_{1\leq k\leq n} \sum\limits_{i=1}^{k} (X_i - \bar{X}) - \min\limits_{1\leq k\leq n} \sum_{i=1}^{k}(X_i - \bar{X})}{\left(\frac{1}{n}\sum\limits_{i=1}^{n}(X_i - \bar{X})^2\right)^{\frac{1}{2}}}. \quad (25)$$

2) Geweke and Porter-Hudak's (GPH's) Estimator: On the basis of the regression equation Y_i, Geweke and Porter–Hudak (1983) proposed the estimation for parameter $\hat{d}_n$ in accordance with the following equation:

$$\hat{d}_n = -\left(\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right)^{-1}\left(\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)\right), \quad (26)$$

where

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \quad (27)$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i. \quad (28)$$

3) The Smoothed Periodogram (Sperio): The Sperio function is simply a parameter used in R software. This function is used to estimate fractional differences d value [33], and it is denoted by $f_s(w)$ by using the Parzen lag window as follows:

$$f_s(w_j) = \frac{1}{2\pi}\sum_{-m}^{m} k\left(\frac{s}{m}\right) R(s)\cos(sw_j), \quad (29)$$

where

$$k(u) = \left\{\begin{array}{lll} 1-6u^2+6|u|^3 & , & |u| \leq 1/2 \\ 2(1-|u|)^3 & , & -1/2 < u \leq 1 \\ 0 & , & |u| > 1 \end{array}\right\} \quad (30)$$

$k(u)$ is called the Parzen lag window generator (i.e., the Parzen lag window is selected due to its feature of always yielding positive estimates of the spectral density), m is the parameter that commonly corresponds to the truncation point, and

$$R(s) = \tfrac{1}{n}\left(\sum_{i=1}^{n-s}(X_i - \bar{X})(X_{i+s} - \bar{X}\ )\right), \quad s = 0, \pm 1, \ldots, \pm(n-1), \quad (31)$$

is the sample autocovariance function.

4) Fractionally-Differenced (Fracdiff): Another function in R software that is used to estimate the value of d is the Fracdiff function. This function uses the regression estimation method to estimate the fractional difference d of the ARFIMA model [36]. It is well-defined by using a binomial series as follows:

$$\begin{aligned}\nabla^d &= (1-B)^d \quad (32)\\ &= \sum_{k=0}^{\infty}\binom{d}{k}(-B)^k \\ &= 1 - dB - \frac{1}{2}d(1-d)B^2 - \frac{1}{6}d(1-d)(2-d)B^3 - \ldots\end{aligned}$$

D. Autoregressive Fractional Integrated Moving Average (ARFIMA)

The definition of the autoregressive integrated moving average (ARIMA) model was suggested by [37]. A stationary time series x_t is called an ARIMA model of order (p, d, q) and represented by (ARIMA(p, d, q)) if

$$\phi_p(B)\nabla^d x_t = \theta_q(B)\epsilon_t, \quad (33)$$

where

$$\phi_p(B) = \left(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p\right), \quad (34)$$

$$\theta_q(B) = \left(1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q\right), \quad (35)$$

$$\nabla^d = (1 - B\ )^d, \quad (36)$$

where $\phi_p(B)$ is a polynomial of the autoregression for the order p and denoted by $AR(p)$, $\theta_q(B)$ is a polynomial for the moving average of the order q and denoted by $MA(q)$, the integer number d is the nonseasonal difference order, B represents the backward shift operators defined by $B^k\ X_t = X_{t-k}$, ∇ represents the nonseasonal difference operators, and ϵ_t is a white noise process.

From the preceding definitions, the long memory property will be represented through the ARFIMA model. The ARFIMA model is similar to the ARIMA model except for the value of d. As reported in [13], if $d \in (0,\ 0.5)$, then the dataset has a long memory; if $d \in (-0.5, 0)$, then the memory is intermediate. Lastly, if $d = 0$, then the dataset has a short memory.

1) Advantages of ARFIMA Models: [18] explained the advantages of the $ARFIMA(p, d, q)$ model in accordance with the different values of d presented in Section II-D, as follows.

1) If $d > -\frac{1}{2}$ and all the roots of the polynomial $\theta_q(B)$ lie outside the unit root, then the series x_t is invertible and unbounded.
2) If $d < -\frac{1}{2}$ and all the roots of the polynomial $\phi_p(B)$ lie outside the unit root, then the series x_t is stationary.

3) If $-\frac{1}{2} < d < 0$, then the series x_t is invertible and nonpersistent.
4) If $0 < d < \frac{1}{2}$, then x_t is a stationary series.

In special cases, as mentioned by [18], $ARFIMA(0, d, 0)$ models are stationary and invertible if $d \in (-\frac{1}{2}, \frac{1}{2})$. Meanwhile, if $d = 0$, then $ARFIMA(0, d, 0)$ is a white noise.

III. Limitations, Extensions, Comparisons, and Applications of Arfima

Since its introduction, ARFIMA has been widely used in many research areas for analyzing time series [38], such as in crude oil prices by [1] and [31], gold prices by [39], traded securities by [40], financial markets by [41], air traffic passengers by [42], climate and financial econometrics by [43], air quality by [44], and electroencephalography (EEG) signals by [45].

However, the algorithm of the ARFIMA approach exhibits several limitations. First, it restricts the value of the fractional difference d in Equation (36), such that it lies in the stationary and invertible range, as detailed in Section II-D1. The second limitation is associated with the pre-assumed linear form of the ARFIMA model [46]. The third limitation is related to dealing with non-normal residual distribution for the individual ARFIMA model or the heteroscedasticity and ARCH effects for the model's residual [5]. To overcome the second and third limitations, [6] and [46] presented the use of hybrid methods based on the ARFIMA model by combining different methods or linear and nonlinear models, such as ARFIMA and Artificial neural networks (ANNs).

Choosing an appropriate forecasting model is essential in practical research and its application in different fields. Model fitting information criterion is commonly used in model selection. Typically applied information criteria are Akaike information criterion (AIC), Schwarz Bayesian information criterion (BIC), Corrected AIC (AICc), and Hannan–Quinn (HQ) information criterion [37], [47]- [49]. The optimal model demonstrates the minimum AIC, BIC, AICc, and HQ criterion values. [4]-[5], [50] utilized the two smallest values for the accuracy criteria, including AIC, and did not simply rely on a single value when choosing the optimal model for ARIMA, seasonal ARIMA (SARIMA), ARFIMA, ARFIMA with standard generalized autoregressive conditional heteroscedasticity (ARFIMA-sGARCH), ARFIMA with functional GARCH (ARFIMA-fGARCH), ARFIMA with exponential GARCH (ARFIMA-EGARCH), ARFIMA with threshold GARCH (ARFIMA-TGARCH), ARFIMA with integrated GARCH (ARFIMA-IGARCH), ARFIMA with absolute value GARCH (ARFIMA-AVGARCH), ARFIMA with nonlinear GARCH (ARFIMA-NGARCH), ARFIMA with nonlinear asymmetric GARCH (ARFIMA-NAGARCH), ARFIMA with asymmetric power ARCH (ARFIMA-APARCH), ARFIMA with Glosten, Jagannathan, and Runkle GARCH (ARFIMA-GJRGARCH), and ARFIMA with component standard GARCH (ARFIMA-csGARCH). The authors explained that the model with the best AIC value among the frameworks does not produce the most outstanding forecast model for the dataset. Thus, their studies used three models that demonstrate the minimum value for this criterion and compared their results to test this hypothesis.

Many studies have developed an extension of the ARFIMA approach. The autoregressive tempered fractionally integrated moving average (ARTFIMA) model was introduced by [51] for wind speed data. [52] presented the ARFIMA with exogenous variables (ARFIMAX) model. [53] proposed the seasonal ARFIMA (SARFIMA) model.

Furthermore, many studies have compared ARFIMA with other Box–Jenkins approaches. [42] suggested using ARIMA and ARFIMA to model the time series dataset of domestic air passengers in India for the period of January 2012 until to December 2018. The forecast accuracy result of the ARFIMA(1,-0.347,1) model was better than that of the ARIMA(1,1,1) model. [45] confirmed that EEG signals could exhibit long-range dependencies, and ARFIMA models are more appropriate for capturing temporal correlations compared with conventional ARMA models by using the AIC as their metric.

[44] applied ARIMA, ARFIMA, and HW smoothing techniques to assess and predict air quality status in Chandigarh City from 2009 to 2010. The ARFIMA(2,0.3051,2) model was appropriate and even performed better than the other models. [39] found that the ARFIMA(1,1.05716,[3]) model outperforms the other ARFIMA models when applying this procedure by using gold price data in Indonesia.

While [54] showed that the time-series dataset of news sentiments exhibited self-similarity as well as a long memory feature, where the Hurst exponent and the long-range correlation exponent were greater than 0.55 over four orders of magnitude in time ranging from several minutes until to ten days, that when analyzed three-time series of news sentiments for companies traded on the London Stock Exchange, the New York Stock Exchange, and the Stock Exchange of Hong Kong.

IV. Forecasting Methods Based On Arfima

The individual model is a popular forecasting technique and a suitable methode used in many previous studies, such as the ARFIMA model in [6]. Another methode of obtaining accurate forecasts is using hybridization methods to determine the future movement of the dataset and overcome the weaknesses of individual models, such as dealing with non-normal residuals and with nonlinear structures [6]. Accordingly, these models are known as hybrid models as mentioned by [6].

Several studies have proposed using hybrid models and applied ARFIMA to time series forecasting. In these studies, ARFIMA is used as the primary model for hybridization or combination with other models and the hybridization of residual models. The forecasting model is utilized to forecast each component, and the predicated results are combined to obtain the final forecasted value of an original time series.

The literature provides several studies that use different modeling and forecasting methods, ranging from simple (individual models) to complex ones (hybrid models), to deal with other components of a time series. This section briefly presents some of the models used in previous studies in sequential order. [55] examined volatility models and their forecasting capability by using three types of petroleum future contracts, namely, West Texas Intermediate (WTI) crude oil, unleaded gasoline, and heating oil #2, traded in the New York Mercantile Exchange, particularly their volatility persistence (or long memory properties). The chosen models were ARIMA–GARCH, ARFIMA–GARCH, ARFIMA–IGARCH, and ARFIMA with fractionally integrated GARCH (ARFIMA–FIGARCH). Although the ARFIMA–FIGARCH model can capture the long memory characteristics of returns and volatilities more accurately than the other models, the out-of-sample analysis showed that no single model outperformed the others in the three types of petroleum future contracts. Consequently, investors should exercise caution when measuring and forecasting volatilities in petroleum future markets.

[44] used the ARFIMA, ARIMA, and Holt–Winters (HW) smoothing techniques to assess and predict air quality status in Chandigarh City. The ARFIMA(2,0.3051,2) model was appropriate and even performed better than the other models.

[31] examined numerous ARFIMA models to analyze and forecast crude oil prices using weekly WTI and Brent series. They reported that the WTI series revealed three breaks (in 1999, 2004, and 2008), while the Brent series also exhibited three breaks (in 1999, 2005, and 2009). Moreover, the ARFIMA(1,0.47,2) model was appropriate for the WTI series, while the ARFIMA(2,0.09,0) model was suitable for the Brent series.

[56] compared different techniques, namely, naïve forecast and neural networks, ARIMA, ARFIMA, Box–Cox transformation, ARMA errors, trend and seasonal components (BATS) forecast, trigonometric BATS forecast, Box–Cox forecast, random walk forecast, normal method, and HW. They were interested in introducing an appropriate model for forecasting the stock market prices of S&P 500. The ARIMA technique outperformed the other techniques and was even more accurate in the forecasted volatilities for the subsequent 10 days based on mean error (ME), root-mean-square error (RMSE), mean absolute error (MAE), mean percentage error (MPE), mean absolute percentage error (MAPE), and mean absolute scaled error (MASE).

Numerous techniques can be utilized to structure a nonlinear combination model (NCM), including the ARFIMA model with support vector machine (SVM) and the backpropagation neural network (BPNN) model. [57] showed that the forecasting performance evaluation of NCM was suitable and better compared with single and linear combination models when using the renminbi exchange rate against the US dollar (RMB/USD) and the Euro (RMB/EUR).

[58] show that the ARIMA(1,1,1) model was appropriate and performed better than the ARFIMA, ARIMA, and error correction models to model and forecast the monthly wholesale prices of mustard in the Sri Ganganagar District of Rajasthan based on a MAPE value.

[40] found that the ARFIMA(1,0.413,2) model outperformed and more accurate in forecasting compared with the other numerous individual ARFIMA and ARIMA models for modeling and forecasting the total value of the traded securities of the Arab Republic of Egypt based on RMSE, MAE, and MAPE values.

[59] used a state-space (SS) models, ARIMA, ARFIMA, artificial neural networks (ANN), an adaptive network-based fuzzy inference system, and their combined versions to study the sales forecasts of a global furniture retailer operating in Turkey. The experimental results demonstrated that most of the combined forecasts could achieve statistically significant increases, and their accuracy is better than the individual models.

[1] examined daily WTI data. The result of their study showed that the price of crude oil exhibits structural breaks.

[60] compared the ARFIMA model with the singular spectrum analysis (SSA) model to forecast the sales volume of motorcycles in Indonesia. The results showed that the second model outperformed the first model based on the MAPE value.

[61] found that the SVM model outperformed the feed-forward neural network (FFNN), ARIMA, ARFIMA, Markov-switching ARFIMA, and random walk models when using the monthly data of WTI and Brent oil prices.

[62] used cyclic regression with the ARFIMA–GARCH residual process to model and predict oil prices. Although the hybrid model can exhibit certain advantages and capture long memory and conditional heteroscedasticity, it also effectively captures periodicity. Furthermore, the first lag values of the squared standardized residuals were correlated.

[63] examined and compared numerous individual and hybrid models, namely, ARIMA, SARIMA, ARFIMA, HW, SSA, ARIMA–wavelet, ARFIMA–wavelet, SARIMA–wavelet, ARIMA with Kalman filter (ARIMA–KF), ARFIMA with Kalman filter (ARFIMA–KF), and SARIMA with Kalman filter (SARIMA–KF), for predicting the future workloads of CPU, RAM, and network. The SARIMA–KF hybrid model outperformed the other models and achieved extremely high forecasting accuracy based on the MAPE value.

[42] demonstrated that the forecast accuracy of the ARFIMA(1,-0.347,1) model was better than the ARIMA(1,1,1) model when modeling the data of domestic air passengers in India based on the RMSE, MAE, and MAPE values.

[39] found that the ARFIMA(1,1.05716,[3]) model outperformed the other ARFIMA models when using the gold price dataset of Indonesia.

[64] proposed a hybrid model that uses an adaptive neuro-fuzzy inference system, ARFIMA, and Markov switching models to forecast daily Brent oil prices. The experimental results showed that the hybrid model outperformed the individual models, namely, the adaptive network-based fuzzy inference system, ARFIMA, and Markov switching, based on RMSE, MAE, MAPE values. The researchers also performed the Diebold–Mariano test.

[65] suggested a new model based on generalized autoregressive score models theory and allowed the long memory parameter to change dynamically over time.

[66] showed that the proposed AFRIMA–LSTM hybrid model could minimize the volatility problem and overcome the overfitting problem of neural networks when compared a hybrid Pakistan Stock Exchange forecasting model with the ARIMA, long-short term memory (LSTM), and generalized regression radial basis neural network (i.e., a GRNN) models based on RMSE, MSE, and MAPE values.

[67] proposed a class of ARFIMA–GARCH models with level shift (LS)-type intervention that can capture the long-range dependence, volatility, and LS in the time series.

[41] applied the moving average cluster entropy approach to long-range correlated stochastic processes, such as ARFIMA and fractional Brownian motion. Thus, this

approach could capture detailed horizon dependence over relatively short horizons, highlighting its relevance in defining risk analysis indices. Also they proved that the horizon dependence of cluster entropy is related to long-range positive correlations in financial markets.

[43] found that the capability of the ARFIMA models achieved better forecasting performance compared with short-memory alternatives for all long memory generating mechanisms and forecast horizons, that's whenever the long memory of processes exhibits a high degree, regardless of the generated mechanism.

Recently, many authors have demonstrated their interest in obtaining and estimating ARFIMA models to choose the best predictive model. [45] confirmed that EEG signals can exhibit long-range dependency and that ARFIMA models are more suitable for capturing temporal correlations than ARMA models based on AIC value.

[4] determined the modeling and forecasting of monthly Brent crude oil price and its volatility by comparing the ARFIMA-sGARCH models versus the ARFIMA-fGARCH models. The researchers noted that the ARFIMA(2,0.3589648,2)-sGARCH(1,1) and ARFIMA(2,0.3589648,2)-fGARCH(1,1) models under normal distribution with RMSE equal to 0.08808882 and optimal for these data (i.e., two-hybrid models of long-memory phenomenon [ARFIMA] were obtained with two members of the GARCH family [sGARCH and fGARCH] with the same accuracy in the RMSE value). These models outperformed several other models in modeling and forecasting the volatility. Notably, Hurst exponent method also demonstrated excellent results when constructing an appropriate hybridization model for predicting.

[5] compared symmetric and asymmetric effects of GARCH-type models to investigate the volatility of the ARFIMA model using the monthly Brent crude oil price series for the period of January 1979–July 2019. The ARFIMA(2,0.3589648,2)–IGARCH(1,1) model under normal distribution was selected as the optimal model based on AIC, BIC, and the minimum value for RMSE. The optimal model of volatility was determined by comparing 13 hybrid models of GARCH (sGARCH, fGARCH, EGARCH, TGARCH, IGARCH, AVGARCH, NGARCH, NAGARCH, APARCH, apARCH, GJRGARCH, gjrGARCH, and cs-GARCH) in terms of symmetric and asymmetric effects at the level of (1,1). The Hurst exponent method outperformed the other methods when constructing an appropriate hybridization model for the prediction.

[6] proposed a hybrid methodology that combines the ARFIMA model with multilayer perceptron (MLP) models to take the strength of these models in linear and nonlinear modeling. The researchers used the same period time in [5]. The empirical results pointed out the ARFIMA(1,0.3589648,0)–MLP(1,2,1) hybrid model outperforms the other models based on the RMSE and Ljung–Box test.

The main factor in choosing an appropriate model for any economic and financial time series is accuracy. Therefore, the long memory field of the time series has been focused in order to access accurate forecasting models. While many review papers focus on the use of long memory versus Box-Jenkins in a variety of areas, no study is concerned to focus on analytical techniques for verifying the long-memory feature as well as determining estimation methods/functions for approximating long-memory parameters, limitations, extensions,comparisons, and applications.

Following this gap, the authors found that this study added distinct importance to the modeling and forecasting research group by verifying the ARFIMA model's specific type. They also observed that this work integrated various traditional and modern methods of hybrid modeling on the basis of the ARFIMA models and tested these methods among individual and hybrid models in terms of performance accuracy. So, the ARFIMA model can be generalized to other commodities, not just the economic and financial time series data set. This is confirmed by the review in this study (e.g., [42], [44]-[45], [53]).Thus, the results highlight the relevance of verifying the existence of long-memory features in any time series.

On the other hand, the experimental results in [4-6] indicated our contributions to these works. First, these works integrated various traditional and modern methods of hybrid modeling on the basis of the ARFIMA models and tested these methods among individual and hybrid models in terms of performance accuracy. Second, the ARFIMA model with the best AIC value does not necessarily produce the most outstanding forecast model for the data set. Finally, the value of these studies lies in the suggestion to "anticipate potential obstacles and challenges when implementing steps to hybridize ARFIMA models the moment they arise or avoid them altogether".

Consequently, by advancing our understanding of modern modeling, researchers' selection of the long memory technique can be incentivized to increase both the value captured by the experimental analysis and the modeling in this study. Building on this, our paper discusses the value of the extent to which this technique is worth disseminating and popularizing.

This study offers several strategic suggestions and recommendations, including the need for further investigations in determining effective hybridization methods based on ARFIMA models, for improving its performance. Thus, future research directions may involve applying new hybridization methods (e.g., using empirical mode decomposition), and conducting a comparative study on the methods mentioned in this work to improve forecasting accuracy. The overview in this study can also help researchers further understand the ARFIMA model.

V. Conclusion

Analytical techniques for verifying the long-memory feature (graphs and statistical tests) and estimation methods/functions for approximating long-memory parameters (value of the fractional difference [d]) in time series datasets are comprehensively discussed in this study. Limitations, extensions, comparisons, and applications of the ARFIMA model are also presented in detail. A review of the recent literature discusses the application of ARFIMA, including hybrid methods, to time series forecasting in different fields. The results of this study help to clarify all these points.

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