Array Cyclic (3*,4)-Cycle Design

Khaled Matarneh, Ahmad Abubaker*, and Suha Al-Shaikh

***Abstract*— This paper aims to propose a new design known as array cyclic $(k^*, 4)$-$cycle$ design. The development of this design is by forming a combination between a cyclic (k_1, k_2)-$cycle\ system$ and a near-two-factor that will be constructed. To do so, we need to introduce a new cycle system, namely a $(3^*, 4)$-$cycle$ system. Thereafter, we present an analysis for the case, $v \equiv 8, 4 \pmod{12}$.**

***Index Terms*— Near-two-factor, simple cyclic, difference set, cyclic $(k_1, \dots, k_r)$-cycle system.**

I. Introduction

Throughout this paper, all graphs will be finite and undirected. A k-$cycle$, written $C_k = (b_0, b_1, \dots, b_{k-1})$, consists of k distinct vertices $b_0, b_1, \dots, b_{k-1}$, and k edges $\{b_i, b_{i+1}\}$, $0 \leq i \leq k-2$ and $\{b_0, b_{k-1}\}$. Let $k_1, \dots, k_r$ be integers greater than two, a $(k_1, \dots, k_r)$-$cycle$ is the union of edge-disjoint k_i-$cycles$ for $1 \leq i \leq r$. A $(k_1, \dots, k_r)$-$cycle\ system$ of a graph G is a pair $(\mathcal{V}, \mathcal{C})$, where $\mathcal{V}$ is the vertex set of G and $\mathcal{C}$ is a collection of $(k_1, \dots, k_r)$-$cycles$ whose edges partition the edges of G. If $G = K_v$, the complete graph with v vertices, then such a $(k_1, \dots, k_r)$-$cycle\ system$ is called a $(k_1, \dots, k_r)$-$cycle\ system\ of\ order\ v$. In particular, if $k_1 = \cdots = k_r = k$, this is known as a k-$cycle\ system$ of a graph G, or (G, C_k)-design. A k-$cycle\ system$ is Hamiltonian if $k = |V|$. A trivial counting show that the number of cycles of a Hamiltonian cycle system of K_v is $(v-1)/2$. Hence, a necessary condition for its existence is that v must be odd [1, 2, 3].

Given a k-cycle $C_k = (b_0, b_1, \dots, b_{k-1})$, by $C_k + j$ we mean $(b_0 + j, b_1 + j, \dots, b_{k-1} + j)$, where $j \in Z_v$. Analogously, if $\mathcal{C} = \{C_{k_1}, \dots, C_{k_1}\}$ is a $(k_1, \dots, k_r)$-$cycle$, we use $\mathcal{C} + j$ instead of $\{C_{k_1} + j, \dots, C_{k_1} + j\}$. A $(k_1, \dots, k_r)$-cycle system of order v, $(\mathcal{V}, \mathcal{C})$, is said to be m-$cyclic$ if $\mathcal{V} = Z_v$ and for $m \in Z_v$, $C + m \in \mathcal{C}$ whenever $C \in \mathcal{C}$ and is said to be simple when its cycles are all distinct. In particular, if $m = 1$, then it is simply called *cyclic*. A cyclic $(k_1, \dots, k_r)$-$cycle\ system$, of course, is also an m-$cyclic$ $(k_1, \dots, k_r)$-$cycle\ system$ for $m \in Z_v$. A set of cycles that generates the cyclic $(k_1, \dots, k_r)$-cycle system of K_v by repeated addition of 1 modular v is called a $starter\ set$.

The existence problem of $k - cycle$ systems of the complete multigraph λK_v, a graph where any two vertices are joined by λ distinct edges, has received much attention in recent years. For the important case of $\lambda = 1$, this existence problem has been completely solved by Alspach and Gavlas [4] for k odd, by Sajna [5] for k even; and by Alspach et al. [6] for the case $\lambda = 2$. The necessary and sufficient conditions for the existence of a k-$cycle\ system$ of λK_v have been established by Bryant et al. in [7] for all values of λ. More general results, such as the existence problem for decomposing λK_v into cycles of varying lengths, have been presented in [8, 9]. Furthermore, the necessary and sufficient conditions for the existence of cyclic v-cycle system of λK_v and for the existence of simple cyclic p-cycle system of λK_p, where p is a prime, have been provided by Buratti et al. [10].

A k-factor in a graph G is a subgraph of G each of whose vertices has degree k, while a near-k factor is a subgraph of G in which all but one vertex has degree k with the remaining vertex having degree 0 (isolated vertex). Note that an almost 2-regular graph is equivalent to a near-2-factor [11].

The partition of an edge set of a graph G into k-factor (respectively, near- k-factor) called a k-factorisation (respectively, near- k-factorisation). The decomposition of λK_v into $near - \lambda - factor$ for $\lambda \in \{2, 4\}$ and $v \equiv 2, 9, 10 (mod\ 12)$ has been recently constructed in [12, 13, 14].

The main concern of the literature is limited to the existence problem of cyclic k-$cycle\ system$ of λK_v with $\lambda > 1$, which lacks a complete solution given by Colbourn and Colbourn [15] for the very special case of $k = 3$. In this paper, we propose a new design that is called an array cyclic $(3^*, 4)$-$cycle$ design denoted by $ACC((3^*, 4), 2K_v)$, which is obtained by merging an m-$cyclic$ $(k_1, \dots, k_r)$-$cycle$ system of a graph $G = 2K_v$ for $k_i = 4$, except for $k_1 = 3$ and near-two-factor. In addition, $ACC((3^*, 4), 2)$ is an $\left(v \times \frac{v}{4}\right)$ array design that satisfies the following conditions:

1. The cycles in row r form a near-two-factor with focus r.
2. The cycle associated with the rows contains no repetitions.

II. Preliminary Definitions

The main results of this paper will be obtained by using the method of difference set that we are going to explain in this section.

Let G be a group of order v, with the operation +. A k-subset D of G, is a (v, k, λ) difference set of G if each non-identity element $g \in G$ can be written in precisely λ different ways in the form of $x - y$ for $x, y \in D$, where λ is constant.

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K. Matarneh is an Assistant Professor in mathematics of Faculty of Computer Studies, Arab Open University, Saudi Arabia (e-mail: k.matarneh@arabou.edu.sa).

A. Abubaker is an Assistant Professor in mathematics of Faculty of Computer Studies, Arab Open University, Saudi Arabia (e-mail: a.abubaker@arabou.edu.sa).

S. Al-Shaikh is an Assistant Professor in mathematics of Faculty of Computer Studies, Arab Open University, Saudi Arabia (e-mail: s.alshaikh@arabou.edu.sa).

∗Corresponding author: Ahmad Abubaker.

To construct cyclic designs or difference families, we will use the following notations: The vertices of K_v will always be understood as elements of Z_v. Let $Z_v^* = Z_v - \{0\}$ and arithmetic $mod v$ as follows: for $x \neq y \in Z_v^*$, the difference d of a pair $\{x, y\}$ is defined as $d = min\{y - x, x - y\}$. Arithmetic $mod v$ so $1 \leq d \leq \lfloor v/2 \rfloor$. the orbit of the pairs corresponding to the difference d is $\{\{x, x+d\}: x \in Z_v\}$.

Note that the orbit of any edge $\{x, y\}$ has full length v apart from the case that v is even and $x - y = v/2$, the case in which Orbit $\{x, y\}$ has length $(v/2)$. For this reason, we say that an edge of K_v is full or short according to whether its orbit has length v or $v/2$, respectively. Of course, if Orbit $\{x, y\}$ has length $(v/2)$ and full, then any edge in a multi-set $\{\{x, x + v/2\}: x \in Z_v\}$ appears twice.

Now, if $G = 2K_v$, then each difference d appears twice except for the middle difference $v/2$ which appears once.

III. ARRAY CYCLIC $(\mathbf{3^*, 4})$-CYCLE DESIGN

In this section, we provide some definitions and results of $ACC((3^*, 4), 2K_v)$, which will be needed in the sequel.

Definition 1 A $(3^*, 4)$-cycle system of a graph $2K_v$ is an m-*cyclic* $(k_1, \ldots, k_r)$-*cycle system* of a graph $G = 2K_v$ for $k_i = 4 \; if \; i = 2, \ldots, \frac{v}{4}$ and $k_1 = 3$.

Definition 2 Let $B_i = \{(b_{i1}, b_{i2}, b_{i3}, b_{i4}) | \; i = 2, \ldots, \frac{v}{4}\}$ and $B^* = \{(b_1^*, b_2^*, b_3^*)\}$ be cycles with vertices in Z_v, the list of differences from $\mathrm{B_i}$ and $\mathrm{B^*}$ is the multi-set.

Definition 3 Let $\mathrm{Ƒ} = \{B_1^*, B_2, \ldots, B_{v/4}\}$ be a set of cycles of λK_v for λ and v are even, Ƒ is called a $(\lambda K_v,$ Ƒ$)$-difference set $(D(Z_v))$, if the multi-set $D(\mathrm{Ƒ}) = (\bigcup_{i=2}^{v/4} D(B_i) \cup D(B_1^*))$ covers each non-zero element of $Z_{\frac{v+2}{2}}$ exactly λ times except for the middle difference $v/2$ which appears $\frac{\lambda}{2}$ times.

The following lemma is a consequence of the theory developed in [16]. Accordingly, it will be crucial to prove our main results.

Lemma 1 Let Ƒ be a multi-set of cycles of λK_v for λ and v are even. Then, Ƒ is a starter of cyclic $(k_1, \ldots, k_r)$-*cycle system* of λK_v, if and only if Ƒ is a $(\lambda K_v,$ Ƒ$)$-difference set.

Definition 4 The array cyclic $(3^*, 4)$-*cycle* design of a graph $G = 2K_v$, denoted by $ACC((3^*, 4), 2K_v)$, is an $\left(v \times \frac{v}{4}\right)$ array design that satisfies the following conditions:

1) The cycles in row r form a near-two-factor with focus r.
2) The set of cycles in the first row that generates all the cycles in $\left(v \times \frac{v}{4}\right)$ array by repeated addition of 1 modular(v).
3) The cycle associated with the rows contains no repetitions.

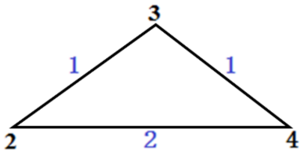
Now, we will present the following example to illustrate the construction of $ACC((3^*, 4), 2K_v)$, when $v = 4$.

Example 1 Let $G = 2K_4$, $B^* = \{(2, 3, 4)\}$ and $\mathrm{B_i} = \emptyset$, the list of differences from B_i and B^* is the multi-set $\Delta B_i = \emptyset$, $\Delta B^* = \{\pm(b_i^* - b_{i-1}^*) \mid i = 1, \ldots, 4\}$ and $\mathrm{D(B^*)} = \{\min\{|\mathrm{b_i^*} - \mathrm{b_{i-1}^*}|, \mathrm{v} - |\mathrm{b_i^*} - \mathrm{b_{i-1}^*}|\} \mid \mathrm{i} = 1, \ldots, 4\}$ where $b_1^* = b_4^* = 2$.

Ƒ $= B^* = \{(2, 3, 4)\}$, ΔƑ $= \Delta B^* = \{1, 3, 1, 3, 2, 2\}$ and $D(B^*) = \mathrm{D(Ƒ)} = \{d_1{}^*, d_2{}^*, d_3{}^*\} = \{1, 1, 2\}$ see Table I.

TABLE I
ΔB^* OF $2K_4$

Focus $\boldsymbol{i} = \mathbf{1}$



-	3	2
3	0	1
2	3	0

$d_1{}^* = 1 = min\{1, 3\}$

-	4	3
4	0	1
3	3	0

$d_2{}^* = 1 = min\{1, 3\}$

-	2	4
2	0	2
4	2	0

$d_3{}^* = 2 = min\{2, 2\}$

Difference set $(1, 1, 2)$
Starter cycles $(2, 3, 4)$

It can be seen from Table I that each non-zero integer 1, 2, 3 in Z_4 occurs exactly twice in the off-diagonal position. In addition, D(Ƒ) covers each non-zero element of Z_2 exactly twice except for the middle difference $(4/2) = 2$ which appears once.

Now consider the graph $G = 2K_4$ of 4 vertices and one is focus. The starter on Table I which is a near-two-factor, has a C_3 cycle and any difference in D(Ƒ) appears twice in the cycle edges, except for the middle difference (2) which appears once. It follows then, that a $ACC((3^*, 4), 2K_4)$ is a (4×1) array design and the starter cycles $(2, 3, 4)$ in the first row generate all the cycles in the (4×1) array by repeated addition of 1 modular(4) (see Table II).

TABLE II
$ACC((3^*, 4), 2K_4)$

Focus	$ACC((3^*, 4), 2K_4)$
$i = 1$	$(2, 3, 4)$
$i = 2$	$(3, 4, 1)$
$i = 3$	$(4, 1, 2)$
$i = 4$	$(1, 2, 3)$

In Table II, we can see that every edge in K_4 appears two times and is able to generate all cycles by addition of modular 4. In the next part, we will be able to find the solution for a general case of $v = 12n + 4$.

Lemma 2 There exists a $ACC((3^*, 4), 2\, K_{12n+4})$.

Proof: Let the starter cycles be $ACC((3^*, 4), 2\, K_{12n+4})$, as shown in Fig. 1:

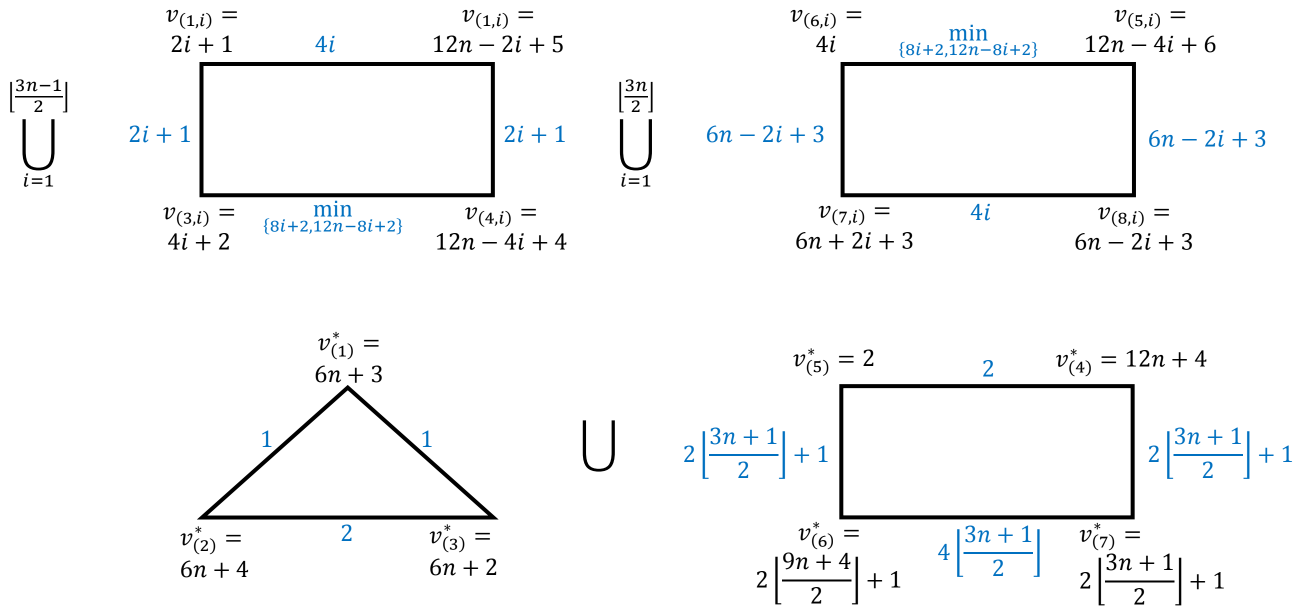


Fig. 1. Starter $ACC((3^*,4),2K_{12n+4})$

Let us breakdown the proof into five parts as follows:

Part 1: We will calculate the vertices.

$$v_j = \begin{cases} \bigcup_{i=1}^{\left\lfloor\frac{3n-1}{2}\right\rfloor} v_{(j,i)} & \text{if } j=\{1,2,3,4\} \\ \bigcup_{i=1}^{\left\lfloor\frac{3n}{2}\right\rfloor} v_{(j,i)} & \text{if } j=\{5,6,7,8\} \end{cases}$$

and $\{v_j^* : j = 1, \dots, 7\}$ then

- $v_1 = \begin{cases} \{12n+3, 12n+1, \dots, 9n+7\} & \text{if } n \text{ even} \\ \{12n+3, 12n+1, \dots, 9n+6\} & \text{if } n \text{ odd} \end{cases}$
- $v_2 = \begin{cases} \{3, 5, \dots, 3n-1\} & \text{if } n \text{ even} \\ \{3, 5, \dots, 3n\} & \text{if } n \text{ odd} \end{cases}$
- $v_3 = \begin{cases} \{6, 10, \dots, 3n-2, \dots, 6n-2\} & \text{if } n \equiv 0 (\text{mod } 4) \\ \{6, 10, \dots, 3n-1, \dots, 6n\} & \text{if } n \equiv 1 (\text{mod } 4) \\ \{6, 10, \dots, 3n, \dots, 6n-2\} & \text{if } n \equiv 2 (\text{mod } 4) \\ \{6, 10, \dots, 3n+1, \dots, 6n\} & \text{if } n \equiv 3 (\text{mod } 4) \end{cases}$
- $v_4 =$
  $\begin{cases} \{12n, 12n-4, \dots, 9n+4, \dots, 6n+8\} & \text{if } n \equiv 0 (\text{mod } 4) \\ \{12n, 12n-4, \dots, 9n+3, \dots, 6n+6\} & \text{if } n \equiv 1 (\text{mod } 4) \\ \{12n, 12n-4, \dots, 9n+2, \dots, 6n+8\} & \text{if } n \equiv 2 (\text{mod } 4) \\ \{12n, 12n-4, \dots, 9n+1, \dots, 6n+6\} & \text{if } n \equiv 3 (\text{mod } 4) \end{cases}$
- $v_5 =$
  $\begin{cases} \{12n+2, 12n-2, \dots, 9n+6, \dots, 6n+6\} & \text{if } n \equiv 0 (\text{mod } 4) \\ \{12n+2, 12n-2, \dots, 9n+5, \dots, 6n+8\} & \text{if } n \equiv 1 (\text{mod } 4) \\ \{12n+2, 12n-2, \dots, 9n+4, \dots, 6n+6\} & \text{if } n \equiv 2 (\text{mod } 4) \\ \{12n+2, 12n-2, \dots, 9n+3, \dots, 6n+8\} & \text{if } n \equiv 3 (\text{mod } 4) \end{cases}$
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- $v_6 = \begin{cases} \{4, 8, \dots, 3n, \dots, 6n\} & \text{if } n \equiv 0 (\text{mod } 4) \\ \{4, 8, \dots, 3n+3, \dots, 6n-2\} & \text{if } n \equiv 1 (\text{mod } 4) \\ \{4, 8, \dots, 3n-2, \dots, 6n\} & \text{if } n \equiv 2 (\text{mod } 4) \\ \{4, 8, \dots, 3n-1, \dots, 6n-2\} & \text{if } n \equiv 3 (\text{mod } 4) \end{cases}$
- $v_7 = \begin{cases} \{6n+5, 6n+7, \dots, 9n+3\} & \text{if } n \text{ even} \\ \{6n+5, 6n+7, \dots, 9n+2\} & \text{if } n \text{ odd} \end{cases}$
- $v_8 = \begin{cases} \{6n+1, 6n-1, \dots, 3n+3\} & \text{if } n \text{ even} \\ \{6n+1, 6n-1, \dots, 3n+4\} & \text{if } n \text{ odd} \end{cases}$
- $v_1^* = (6n+3)$, $v_2^* = (6n+4)$, $v_3^* = (6n+2)$, $v_4^* = (12n+4)$, $v_5^* = (2)$,
- $v_6^* = \left(2 \times \left\lfloor\frac{9n+4}{2}\right\rfloor + 1\right) = \begin{cases} 9n+5 & \text{if } n \text{ even} \\ 9n+4 & \text{if } n \text{ odd} \end{cases}$
- $v_7^* = \left(2 \times \left\lfloor\frac{3n+1}{2}\right\rfloor + 1\right) = \begin{cases} 3n+1 & \text{if } n \text{ even} \\ 3n+2 & \text{if } n \text{ odd} \end{cases}$.

Part 2: We need to prove that the $\left(\left(\bigcup_{j=1}^{8}(v_j)\right) \cup \left(\bigcup_{j=1}^{7}(v_j^*)\right)\right)$ covers all vertices in K_{12n+4}, except for the focus one.

$$v_5^* \cup (v_6 \cup v_3) \cup (v_3^* \cup v_2^*) \cup (v_5 \cup v_4) \cup v_4^* = \begin{cases} \{2, 4, \dots, 3n, \dots, 6n+2, \dots, 9n+4, \dots, 12n+4\} \text{ if } n \text{ even} \\ \{2, 4, \dots, 3n+1, \dots, 6n+2, \dots, 9n+3, \dots, 12n+4\} \text{ if } n \text{ odd} \end{cases} \quad (1)$$

$$(v_2 \cup v_7^* \cup v_8 \cup v_1^* \cup v_7 \cup v_6^* \cup v_1) = \begin{cases} \{3, 5, \dots, 3n+1, \dots, 6n+3, \dots, 9n+5, \dots, 12n+3\} \text{ if } n \text{ even} \\ \{3, 5, \dots, 3n+2, \dots, 6n+3, \dots, 9n+4, \dots, 12n+3\} \text{ if } n \text{ odd} \end{cases} \quad (2)$$

We will use (1) and (2)

$\left(\left(\bigcup_{j=1}^{8}(v_j)\right) \cup \left(\bigcup_{j=1}^{7}(v_j^*)\right)\right) = \{2, 3, \dots, 12n+3, 12n+4\}$.

Part 3: We will check for the difference $D = \{d_{(j,i)} : j = 1, \dots, 8\} \cup \{d_j^* : j = 1, \dots, 7\}$

- $d_{(1,i)} = min\{|v_{(1,i)} - v_{(2,i)}|, 12n+4 - |v_{(1,i)} - v_{(2,i)}|\}$ since $(1) \le i \le \left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(1,i)} = (12n+4) - (12n-4i+4) = 4i$.
- $d_{(2,i)} = min\{|v_{(3,i)} - v_{(2,i)}|, 12n+4 - |v_{(3,i)} - v_{(2,i)}|\}$ since $(1) \le i \le \left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(2,i)} = (2i+1)$.
- $d_{(3,i)} = min\{|v_{(4,i)} - v_{(3,i)}|, 12n+4 - |v_{(4,i)} - v_{(3,i)}|\}$ Since $(1) \le i \le \left\lfloor\frac{3n-1}{2}\right\rfloor$ then
  $d_{(3,i)} = \begin{cases} 8i+2 & \text{if } i \le \frac{3n}{4} \\ 12n-8i+2 & \text{if } i > \frac{3n}{4} \end{cases} = \min \{8i+2, 12n-8i+2\}$.
- $d_{(4,i)} = min\{|v_{(1,i)} - v_{(4,i)}|, 12n+4 - |v_{(1,i)} - v_{(4,i)}|\}$ since $(1) \le i \le \left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(4,i)} = (2i+1)$.
- $d_{(5,i)} = min\{|v_{(5,i)} - v_{(6,i)}|, 12n+4 - |v_{(5,i)} - v_{(6,i)}|\}$ since $(1) \le i \le \left\lfloor\frac{3n}{2}\right\rfloor$ then

$d_{(5,i)} = \begin{cases} 8i-2 & if\ i \le \frac{3n+2}{4} \\ 12n-8i+6 & if\ i > \frac{3n+2}{4} \end{cases}$ = min $\{8i-2, 12n-8i+6\}$.

- $d_{(6,i)} = min\{|v_{(7,i)} - v_{(6,i)}|, 12n+4-|v_{(7,i)} - v_{(6,i)}|\}$ since $(1) \le i \le \left\lfloor \frac{3n}{2} \right\rfloor$ then $d_{(6,i)} = (6n-2i+3)$.
- $d_{(7,i)} = min\{|v_{(7,i)} - v_{(8,i)}|, 12n+4-|v_{(7,i)} - v_{(8,i)}|\}$ since $(1) \le i \le \left\lfloor \frac{3n}{2} \right\rfloor$ then $d_{(7,i)} = (4i)$.
- $d_{(8,i)} = min\{|v_{(5,i)} - v_{(8,i)}|, 12n+4-|v_{(5,i)} - v_{(8,i)}|\}$ since $(1) \le i \le \left\lfloor \frac{3n}{2} \right\rfloor$ then $d_{(8,i)} = (6n-2i+3)$.
- $d_1^* = min\{|v_1^* - v_3^*|, 12n+4-|v_1^* - v_3^*|\} = 1$.
- $d_2^* = min\{|v_2^* - v_1^*|, 12n+4-|v_2^* - v_1^*|\} = 1$.
- $d_3^* = min\{|v_2^* - v_3^*|, 12n+4-|v_2^* - v_3^*|\} = 2$.
- $d_4^* = min\{|v_4^* - v_5^*|, 12n+4-|v_4^* - v_5^*|\} = 2$.
- $d_5^* = min\{|v_6^* - v_5^*|, 12n+4-|v_6^* - v_5^*|\} = \begin{cases} 3n+1 & if\ n \equiv 0,2 (mod\ 4) \\ 3n+2 & if\ n \equiv 1,3 (mod\ 4) \end{cases} = 2 \times \left\lfloor \frac{(3n+1)}{2} \right\rfloor + 1$.
- $d_6^* = min\{|v_6^* - v_7^*|, 12n+4-|v_6^* - v_7^*|\} = \begin{cases} (6n) & if\ n \equiv 0,2 (mod\ 4) \\ (6n+2) & if\ n \equiv 1,3 (mod\ 4) \end{cases} = \left(4 \times \left\lfloor \frac{(3n+1)}{2} \right\rfloor\right)$.
- $d_7^* = min\{|v_4^* - v_7^*|, 12n+4-|v_4^* - v_7^*|\} = \begin{cases} (3n+1) & if\ n \equiv 0,2 (mod\ 4) \\ (3n+2) & if\ n \equiv 1,3 (mod\ 4) \end{cases} = 2 \times \left\lfloor \frac{(3n+1)}{2} \right\rfloor + 1$.

Part 4: We will calculate the difference $D = \{d_{(j,i)} : j = 1, \dots, 8\} \cup \{ d_j^* : j = 1, \dots, 7\}$

Suppose $d_j = \begin{cases} \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} d_{(j,i)} & \text{if } j = \{1,2,3,4\} \\ \bigcup_{i=1}^{\left\lfloor \frac{3n}{2} \right\rfloor} d_{(j,i)} & \text{if } j = \{5,6,7,8\} \end{cases}$ then

- $d_1 = \begin{cases} \{4, 8, \dots, 6n-4\} & if\ n \equiv 0,2 (mod\ 4) \\ \{4, 8, \dots, 6n-2\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$
- $d_2 = \begin{cases} \{3, 5, \dots, 3n-1\} & if\ n \equiv 0,2 (mod\ 4) \\ \{3, 5, \dots, 3n\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$.
- $d_3 = \begin{cases} \{10, 18, \dots, 6n+2\} \cup \{6n-6, \dots, 18, 10\} & if\ n \equiv 0 (mod\ 4) \\ \{10, 18, \dots, 6n-4\} \cup \{6n, \dots, 14, 6\} & if\ n \equiv 1 (mod\ 4) \\ \{10, 18, \dots, 6n-2\} \cup \{6n-2, \dots, 18, 10\} & if\ n \equiv 2 (mod\ 4) \\ \{10, 18, \dots, 6n\} \cup \{6n-4, \dots, 14, 6\} & if\ n \equiv 3 (mod\ 4) \end{cases}$
- $d_4 = \begin{cases} \{3, 5, \dots, 3n-1\} & if\ n \equiv 0,2 (mod\ 4) \\ \{3, 5, \dots, 3n\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$.
- $d_5 = \begin{cases} \{6, 14, \dots, 6n-2\} \cup \{6n-2, \dots, 14, 6\} & if\ n \equiv 0 (mod\ 4) \\ \{6, 14, \dots, 6n\} \cup \{6n-4, \dots, 18, 10\} & if\ n \equiv 1 (mod\ 4) \\ \{6, 14, \dots, 6n+2\} \cup \{6n-6, \dots, 14, 6\} & if\ n \equiv 2 (mod\ 4) \\ \{6, 14, \dots, 6n-4\} \cup \{6n, \dots, 18, 10\} & if\ n \equiv 3 (mod\ 4) \end{cases}$
- $d_6 = \begin{cases} \{6n+1, 6n-1, \dots, 3n+3\} & if\ n \equiv 0,2 (mod\ 4) \\ \{6n+1, 6n-1, \dots, 3n+5\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$.
- $d_7 = \begin{cases} \{4, 8, 12, \dots, 6n\} & if\ n \equiv 0,2 (mod\ 4) \\ \{4, 8, 12, \dots, 6n-2\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$.
- $d_8 = \begin{cases} \{6n+1, 6n-1, \dots, 3n+3\} & if\ n \equiv 0,2 (mod\ 4) \\ \{6n+1, 6n-1, \dots, 3n+5\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$.

Part 5: We need to prove that each difference in $Z^*_{6n+3} = \{1, 2, \dots, 6n+1, 6n+2\}$ appears two times in $D = \left(\left(\bigcup_{j=1}^{8}(d_j)\right) \cup \left(\bigcup_{j=1}^{7}(d_j^*)\right)\right)$, except for the middle difference $(6n+2)$ which appears once.

$$(d_3^* \cup d_7 \cup (d_5 \cup d_3) \cup d_1 \cup d_6^* \cup d_4^*) = \begin{cases} \{2, 4, 6, \dots, 6n, 6n+2\} \cup \{6n, 6n-2, \dots, 6, 4, 2\} & if\ n \equiv 0,2 (mod\ 4) \\ \{2, 4, 6, \dots, 6n-2, 6n\} \cup \{6n+2, 6n, \dots, 6, 4, 2\} & if\ n \equiv 1,3 (mod\ 4) \end{cases} \quad (3)$$

$$(d_1^* \cup d_4 \cup d_7^* \cup d_8) = (d_2^* \cup d_2 \cup d_5^* \cup d_6) = \begin{cases} \{1, 3, 5, \dots, 3n+1, \dots, 6n+1\} & if\ n \equiv 0,2 (mod\ 4) \\ \{1, 3, 5, \dots, 3n+2, \dots, 6n+1\} & if\ n \equiv 1,3 (mod\ 4) \end{cases} \quad (4)$$

We will use (3) and (4).

$$(d_1^* \cup d_4 \cup d_7^* \cup d_8) \cup (d_3^* \cup d_7 \cup d_5 \cup d_3 \cup d_1 \cup d_6^* \cup d_4^*) \cup (d_2^* \cup d_2 \cup d_5^* \cup d_6) = \begin{cases} \{1, 2, \dots, 6n+1, 6n+2\} \cup \{6n, 6n+1, \dots, 2, 1\} & if\ n \equiv 0,2 (mod\ 4) \\ \{2, 4, 6, \dots, 6n, 6n+1\} \cup \{6n+2, 6n+1, \dots, 2, 1\} & if\ n \equiv 1,3 (mod\ 4) \end{cases}$$

It can then be seen that every difference in Z^*_{6n+3} appears twice in $D = \left(\left(\bigcup_{j=1}^{8}(d_j)\right) \cup \left(\bigcup_{j=1}^{7}(d_j^*)\right)\right)$, except for the middle difference $(6n+2)$ which appears once.

Lemma 3 There exists a $ACC((3^*, 4), 2\ K_{12n-4})$.

Proof: Let the starter cycles be $ACC((3^*, 4), 2\ K_{12n-4})$, as shown in Fig. 2:

If we breakdown the proof into five parts as follows:

Part 1: We will calculate the vertices $\{v_{(j,i)} : j = 1, \dots, 8\}$ and $\{v_j^* : j = 1, 2, 3\}$:

Suppose $v_j = \begin{cases} \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} v_{(j,i)} & \text{if } j = \{1,2,3,4\} \\ \bigcup_{i=1}^{\left\lfloor \frac{3n-2}{2} \right\rfloor} v_{(j,i)} & \text{if } j = \{5,6,7,8\} \end{cases}$ then

- $v_1 = \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} v_{(1,i)} = \begin{cases} \{12n-4, 12n-6, \dots, 9n+2, 9n\} & if\ n\ even \\ \{12n-4, 12n-6, \dots, 9n+1, 9n-1\} & if\ n\ odd \end{cases}$
- $v_2 = \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} v_{(2,i)} = \begin{cases} \{2, 4, 6, \dots, 3n-4, 3n-2\} & if\ n\ even \\ \{2, 4, 6, \dots, 3n-3, 3n-1\} & if\ n\ odd \end{cases}$
- $v_3 = \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} v_{(3,i)} = \begin{cases} \{12n-5, 12n-7, \dots, 9n+1, 9n-1\} & if\ n\ even \\ \{12n-5, 12n-7, \dots, 9n, 9n-2\} & if\ n\ odd \end{cases}$
- $v_4 = \bigcup_{i=1}^{\left\lfloor \frac{3n-1}{2} \right\rfloor} v_{(4,i)} = \begin{cases} \{3, 5, 7, \dots, 3n-3, 3n-1\} & if\ n\ even \\ \{3, 5, 7, \dots, 3n-2, 3n\} & if\ n\ odd \end{cases}$
- $v_5 = \bigcup_{i=1}^{\left\lfloor \frac{3n-2}{2} \right\rfloor} v_{(5,i)} = \begin{cases} \{6n+2, 6n+4, \dots, 9n-4, 9n-2\} & if\ n\ even \\ \{6n+2, 6n+4, \dots, 9n-5, 9n-3\} & if\ n\ odd \end{cases}$
- $v_6 = \bigcup_{i=1}^{\left\lfloor \frac{3n-2}{2} \right\rfloor} v_{(6,i)} = \begin{cases} \{6n-4, 6n-6, \dots, 3n+2, 3n\} & if\ n\ even \\ \{6n-4, 6n-6, \dots, 3n+3, 3n+1\} & if\ n\ odd \end{cases}$
- $v_7 = \bigcup_{i=1}^{\left\lfloor \frac{3n-2}{2} \right\rfloor} v_{(7,i)} = \begin{cases} \{6n+1, 6n+3, \dots, 9n-5, 9n-3\} & if\ n\ even \\ \{6n+1, 6n+3, \dots, 9n-6, 9n-4\} & if\ n\ odd \end{cases}$
- $v_8 = \bigcup_{i=1}^{\left\lfloor \frac{3n-2}{2} \right\rfloor} v_{(8,i)} = \begin{cases} \{6n-3, 6n-5, \dots, 3n+3, 3n+1\} & if\ n\ even \\ \{6n-3, 6n-5, \dots, 3n+4, 3n+2\} & if\ n\ odd \end{cases}$
- $v_1^* = (6n-1),\ v_2^* = (6n-2),\ v_3^* = (6n)$.

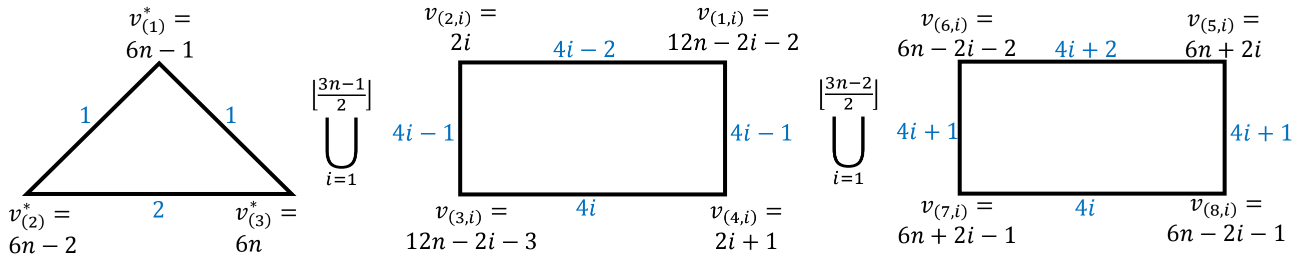


Fig. 2. Starter $ACC\big((3^*,4),2K_{12n-4}\big)$

Part 2: We need to prove that $\left(\left(\bigcup_{j=1}^{8}(v_j)\right)\cup\left(\bigcup_{j=1}^{7}(v_j^*)\right)\right)$ covers all vertices in K_{12n-4} except for the focus one.

$(v_2\cup v_6)\cup(v_2^*\cup v_3^*)\cup(v_5\cup v_1) = \begin{cases}\{2,4,\dots,3n,\dots,6n-2,6n,6n+2,\dots,9n,\dots,12n-4\}\ \text{if } n \text{ even}\\ \{2,4,\dots,3n+1,\dots,6n-2,6n,6n+2,\dots,9n-1,\dots,12n-4\}\ \text{if } n \text{ odd}\end{cases}$ (5)

$(v_4\cup v_8)\cup(v_1^*)\cup(v_7\cup v_3) = \begin{cases}\{3,5,\dots,3n+1,\dots,6n-3,6n-1,6n+1,\dots,9n-1,\dots,12n-5\}\ \text{if } n \text{ even}\\ \{3,5,\dots,3n+2,\dots,6n-3,6n-1,6n+1,\dots,9n-2,\dots,12n-5\}\ \text{if } n \text{ odd}\end{cases}$ (6)

We will use (5) and (6)

$\left(\left(\bigcup_{j=1}^{8}(v_j)\right)\cup\left(\bigcup_{j=1}^{3}(v_j^*)\right)\right) = \{2,3,\dots,12n-5,12n-4\}$.

Part 3: We will check for the difference $D=\{d_{(j,i)} : j=1,\dots,8\}\cup\{d_j^* : j=1,\dots,7\}$

- $d_{(1,i)} = min\{|v_{(1,i)}-v_{(2,i)}|, 12n-4-|v_{(1,i)}-v_{(2,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(1,i)}=(12n-4)-(12n-2-4i)=4i-2$.
- $d_{(2,i)} = min\{|v_{(3,i)}-v_{(2,i)}|, 12n-4-|v_{(3,i)}-v_{(2,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(2,i)}=(12n-4)-(12n-3-4i)=4i-1$.
- $d_{(3,i)} = min\{|v_{(3,i)}-v_{(4,i)}|, 12n-4-|v_{(3,i)}-v_{(4,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(3,i)}=(12n-4)-(12n-4-4i)=4i$.
- $d_{(4,i)} = min\{|v_{(1,i)}-v_{(4,i)}|, 12n-4-|v_{(1,i)}-v_{(4,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-1}{2}\right\rfloor$ then $d_{(4,i)}=(12n-4)-(12n-3-4i)=4i-1$.
- $d_{(5,i)} = min\{|v_{(5,i)}-v_{(6,i)}|, 12n-4-|v_{(5,i)}-v_{(6,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-2}{2}\right\rfloor$ then $d_{(5,i)}=4i+2$.
- $d_{(6,i)} = min\{|v_{(7,i)}-v_{(6,i)}|, 12n-4-|v_{(7,i)}-v_{(6,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-2}{2}\right\rfloor$ then $d_{(6,i)}=4i+1$.
- $d_{(7,i)} = min\{|v_{(7,i)}-v_{(8,i)}|, 12n-4-|v_{(7,i)}-v_{(8,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-2}{2}\right\rfloor$ then $d_{(7,i)}=4i$.
- $d_{(8,i)} = min\{|v_{(5,i)}-v_{(8,i)}|, 12n-4-|v_{(5,i)}-v_{(8,i)}|\}$ since $(1)\le i\le\left\lfloor\frac{3n-2}{2}\right\rfloor$ then $d_{(8,i)}=4i+1$.
- $d_1^* = min\{|v_1^*-v_2^*|, 12n-4-|v_1^*-v_2^*|\}=1$.
- $d_2^* = min\{|v_3^*-v_2^*|, 12n-4-|v_3^*-v_2^*|\}=2$.
- $d_3^* = min\{|v_3^*-v_1^*|, 12n-4-|v_3^*-v_1^*|\}=1$.

Part 4: We will calculate the difference $d_j = \begin{cases}\bigcup_{i=1}^{\lfloor\frac{3n-1}{2}\rfloor} d_{(j,i)} & \text{if } j=\{1,2,3,4\}\\ \bigcup_{i=1}^{\lfloor\frac{3n-2}{2}\rfloor} d_{(j,i)} & \text{if } j=\{5,6,7,8\}\end{cases}$ then

- $d_1 = \bigcup_{i=1}^{\lfloor\frac{3n-1}{2}\rfloor} d_{(1,i)} = \begin{cases}\{2,6,10,\dots,6n-10,6n-6\} & if\ n\ even\\ \{2,6,10,\dots,6n-8,6n-4\} & if\ n\ odd\end{cases}$
- $d_2 = \bigcup_{i=1}^{\lfloor\frac{3n-1}{2}\rfloor} d_{(2,i)} = \begin{cases}\{3,7,11,\dots,6n-9,6n-5\} & if\ n\ even\\ \{3,7,11,\dots,6n-7,6n-3\} & if\ n\ odd\end{cases}$
- $d_3 = \bigcup_{i=1}^{\lfloor\frac{3n-1}{2}\rfloor} d_{(3,i)} = \begin{cases}\{4,8,12,\dots,6n-8,6n-4\} & if\ n\ even\\ \{4,8,12,\dots,6n-6,6n-2\} & if\ n\ odd\end{cases}$
- $d_4 = \bigcup_{i=1}^{\lfloor\frac{3n-1}{2}\rfloor} d_{(4,i)} = \begin{cases}\{3,7,11,\dots,6n-9,6n-5\} & if\ n\ even\\ \{3,7,11,\dots,6n-7,6n-3\} & if\ n\ odd\end{cases}$
- $d_5 = \bigcup_{i=1}^{\lfloor\frac{3n-2}{2}\rfloor} d_{(5,i)} = \begin{cases}\{6,10,14,\dots,6n-6,6n-2\} & if\ n\ even\\ \{6,10,14,\dots,6n-8,6n-4\} & if\ n\ odd\end{cases}$
- $d_6 = \bigcup_{i=1}^{\lfloor\frac{3n-2}{2}\rfloor} d_{(6,i)} = \begin{cases}\{5,9,13,\dots,6n-7,6n-3\} & if\ n\ even\\ \{5,9,13,\dots,6n-9,6n-5\} & if\ n\ odd\end{cases}$
- $d_7 = \bigcup_{i=1}^{\lfloor\frac{3n-2}{2}\rfloor} d_{(7,i)} = \begin{cases}\{4,8,12,\dots,6n-8,6n-4\} & if\ n\ even\\ \{4,8,12,\dots,6n-10,6n-6\} & if\ n\ odd\end{cases}$
- $d_8 = \bigcup_{i=1}^{\lfloor\frac{3n-2}{2}\rfloor} d_{(8,i)} = \begin{cases}\{5,9,13,\dots,6n-7,6n-3\} & if\ n\ even\\ \{5,9,13,\dots,6n-9,6n-5\} & if\ n\ odd\end{cases}$.

Part 5: We must prove that each difference in ${Z^*}_{6n-1} = \{1,2,\dots,6n-3,6n-2\}$ appears two times in $D=\left(\left(\bigcup_{j=1}^{8}(d_j)\right)\cup\left(\bigcup_{j=1}^{3}(d_j^*)\right)\right)$, except for the middle difference $(6n-2)$ which appears once.

$$d_1^*\cup d_1\cup d_2\cup d_3\cup d_6 = \begin{cases}\{1,2,3,4,5,\dots,6n-4,6n-3\} & if\ n\ even\\ \{1,2,3,4,5,\dots,6n-3,6n-2\} & if\ n\ odd\end{cases} \quad (7)$$

$$d_3^*\cup d_2^*\cup d_4\cup d_7\cup d_8\cup d_5 = \begin{cases}\{1,2,3,4,5,6,\dots,6n-3,6n-2\} & if\ n\ even\\ \{1,2,3,4,5,6,\dots,6n-4,6n-3\} & if\ n\ odd\end{cases} \quad (8)$$

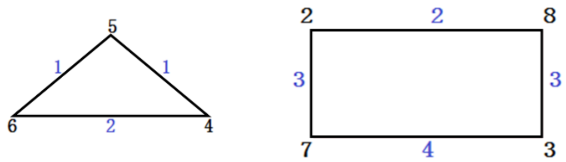
We will use (3) and (4).

$$(d_1^* \cup d_1 \cup d_2 \cup d_3 \cup d_6) \cup (d_3^* \cup d_2^* \cup d_4 \cup d_7 \cup d_8 \cup d_5) = \begin{cases} \{1,2,\dots,6n-4,6n-3\}\} \cup \{1,2,\dots,6n-3,6n-2\}\} & \text{if } n \text{ even} \\ \{1,2,\dots,6n-4,6n-2\}\} \cup \{1,2,\dots,6n-4,6n-3\}\} & \text{if } n \text{ odd} \end{cases}$$

It can then be seen that every difference in $Z^*{}_{6n-1}$ appears twice in $D = \left(\left(\cup_{j=1}^{8}(d_j)\right) \cup \left(\cup_{j=1}^{3}(d_j^*)\right)\right)$, except for the middle difference $(6n-2)$ which appears once.

Example 2 let $G = 2K_8$, By Lemma 3, $B^* = \{(4,5,6)\}$ and $B_1 = \{(8,2,7,3)\}$, the list of differences set from B_1 and B^* is the multi-set $D(B^*) = \{1,1,2\}$ and $D(B_1) = \{(2,3,4,3)\}$.
$\digamma = B^* \cup B_1 = \{(4,5,6),(2,3,4,3)\}, D(\digamma) = \{1,1,2,2,3,4,3\}$, see Table III.

TABLE III
$\Delta(\digamma)$ AND $D(\digamma)$ OF $2K_8$



-	5	6
5	0	7
6	1	0

$d^*{}_1 = 1$

-	5	4
5	0	1
4	7	0

$d^*{}_2 = 1$

-	2	8
2	0	2
8	6	0

$d_1 = 2$

-	7	2
7	0	5
2	3	0

$d_2 = 3$

-	6	4
6	0	2
4	6	0

$d^*{}_3 = 2$

-	3	7
3	0	4
7	4	0

$d_3 = 4$

-	8	3
8	0	5
3	3	0

$d_4 = 3$

Thence, a $ACC((3^*,4),2K_8)$ is an (8×2) array design and the starter cycles $(4,5,6),(2,3,4,3)$ in the first row generate all the cycles in (8×2) array by repeated addition of 1 modular(8) (see Table IV).

TABLE IV
$ACC\big((3^*,4),2K_8\big)$

Focus	$ACC\big((3^*,4),2K_8\big)$	
$i=1$	$(4,5,6)$	$(8,2,7,3)$
$i=2$	$(5,6,7)$	$(1,3,8,4)$
$i=3$	$(6,7,8)$	$(2,4,1,5)$
$i=4$	$(7,8,1)$	$(3,5,2,6)$
$i=5$	$(8,1,2)$	$(4,6,3,7)$
$i=6$	$(1,2,3)$	$(5,7,4,8)$
$i=7$	$(2,3,4)$	$(6,8,5,1)$
$i=8$	$(3,4,5)$	$(7,1,6,2)$

In Table IV, we can see that every edge in K_8 appears twice and is able to generate all cycles by addition of modular 8.

IV. CONCLUSION

This presents an analysis for array cyclic $(k^*,4)$-*cycle* design for case $v \equiv 8,4\ (mod\ 12)$. Furthermore, several definitions and concepts were formulated to construct $ACC((k^*,4),2K_v)$. The algorithm proposed in Lemma 2 and Lemma 3 will be a basis for further research in developing designs for v = 12n. However, we are unable find a method to construct $ACC((k^*,4),2K_v)$ in general.

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Khaled Matarneh is an assistant professor in the Faculty of Computer Studies at Arab Open University, KSA. He received his Ph.D. in mathematics at University Utara Malaysia, Malaysia in 2019. He received his Master's and B.Sc. in applied mathematics from Mutah University, Jordan in 2002, and 2007 respectively. His research interests include combinatorial designs, discrete mathematics, and graph theory.

Ahmad Abubaker is an assistant professor in the Faculty of Computer Studies at Arab Open University, KSA. He received his Ph.D. in applied mathematics at University Sains Malaysia, Malaysia in 2017. He received his Master's and B.Sc. in applied mathematics from Jordan University of Science and Technology, Jordan in 2005, and 2002 respectively. His research interests include single and multi-objective simulation optimization problems, simulated annealing, particle swarm optimization, clustering problems, inventory system, stochastic processes, stochastic optimization, image segmentation, and graph theory.

Suha AlShaikh is an assistant professor in the Faculty of Computer Studies at Arab Open University, KSA. She received her Ph.D. in Differential Geometry form King Saud University, Riyadh in July 2012. She received her B.Sc. degree in mathematics from Imam Abdulrahman bin Faisal University, Saudi Arabia, in 1999, and her Master's degree in Topology and Differential Geometry from King Saud University, Saudi Arabia, in 2003. Her research interest includes differential geometry, Manifolds, geometry of hypersurface, Killing vector fields, conformal vector field, and graph theory.