A Non-Dimensional Mathematical Model of Shoreline Evolution with a Groin Structure Using an Unconditionally Stable Explicit Finite Difference Technique

Surasak Manilam, and Nopparat Pochai

***Abstract*—Coastal erosion is a natural phenomenon that occurs when sediment transport away from the coast is not countered by the formation of new material on the shoreline. This is indeed a problem that is driving the erosion of coastal areas. A sea wall and a groin were created to prevent coastal erosion and floods. The future topography of the beach is being investigated using shoreline evolution analysis. Erosion, accretion, and sea level changes are basic stages that have a significant impact on the coastal structure. A qualitative analysis of the model coastal behavior in relation to the controlling process is required to research beach erosion and beach deposition. When stated in terms of non-dimensional variables, all are mathematically equivalent. In general, the models do not have to be dimensionally different. Those might just be modifications of the same problems. One can solve a wide range of models with a single solution to the related non-dimensional equation. In this research, we provide a governing equation when a groin is introduced to a one-dimensional shoreline growth model. A non-dimensional shoreline evolution model with a groin structure model is provided. The model now has the ability to manipulate physical parameters. When groin structural effects are present, the initial condition setting method and boundary condition approaches are also given. To approximate the incremental model in each year, the forward time-centered space technique and the unconditionally stable Saulyev finite difference methods are used. The Saulyev finite difference approach can handle numerical solutions in almost any scenario since the stability requirements are not restricted. The Saulyev finite difference technique can be very useful for computing a practical conceptual design of shoreline evolution since the number of grids has increased. The numerical models offered provide a viable simulation for evaluating long-term coastal development. The proposed modeling may be used to forecast the effectiveness of constructing a groin system on a local beach.**

***Index Terms*—shoreline evolution, groin structure, non-dimension, mathematical model, finite difference method**

Manuscript received August 21, 2021; revised April 19, 2022.

This paper is supported by Centre of Excellence in Mathematics, Ministry of Higher Education, Science, Research and Innovation, Bangkok, Thailand.

S. Manilam is a Postgraduate Student of Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (e-mail: surasak.manilam915@gmail.com).

N. Pochai is an Assistant Professor of Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (corresponding author to provide phone: 662329-8400; fax: 662-329-8400; e-mail: nop_math@yahoo.com).

INTRODUCTION

Coastal erosion is a natural process that occurs when sediment movement away from the coast is not counterbalanced by fresh material growth along the shoreline. This is certainly a problem that is related to coastal erosion. A sea wall and groin were built to prevent coastal erosion and floods. The beach's future topography is analyzed using shoreline evolution research. Basic phrases that have a substantial influence on the coastal structure are erosion, accretion, and sea-level variations.

The partial differential equation represents several disciplines, including mass, heat, energy, velocity, and vorticity (see for example [1-2]). In [3-8] these papers, the diffusion equation has been used to solve a variety of engineering problems, including pollutant and salinity transport in rivers and streams, and groundwater and contaminant dispersion in shallow lakes. In [9-11], they presented a case study of water level forecasting, a water quality evaluation based on probabilistic echo state networks, and the fluid dynamics of nonaqueous phase pollutants in groundwater.

Understanding the ideal shorelines' responses to the governing processes is important in the study of beach actions. A model for describing realistic situations involving general shoreline configuration settings and time-varying waves in more detail is proposed. As a result, numerical methods of shoreline evolution are preferred to analytical methods. In 1966, this paper [12] introduces modern logical design guidelines for groin structures. They are organized into three basic categories: Coastal processes, Functional design and Structural design. In [13] this paper expands on both the theoretical and practical concepts of mathematical models related to coastal behaviors: Theoretically, the influence of diffraction behind the groin is used to determine computer programming; Practically, the coastal constant in the theoretical model of the coast is expressed in terms of wave height and SVASEK's theoretical wave direction. In [14] this paper describes the development of the governing equations in general form and describes the assumptions and techniques used to obtain more than 25 analytical solutions. Solution for shoreline evolution with and without the influence of coastal structures. It covers situations involving beach filling of initial shape, sand mining, river discharge, groin and jetty and breakwater etc. The wind wave-driven longshore sediment transport rate and shoreline change are evaluated using a numerical model based on one-line theory in [15].

The model transforms waves from deep water into the surf zone and calculates their breaking characteristics. The model [16] provides complete, time-dependent simulations of shoreline evolution for coastlines driven by structures and a variety of boundary conditions that are both practical and reliable. This research looks at two case studies: New Jersey's Sea Isle City Beach and Egypt's Nile Delta Coast.
The purpose of [17] is to quantify the changes in the shoreline along with the sand reclamation for the Sultan Mahmud Airport runway in Kuala Terengganu. Littoral Processes and Coastline Kinetics (LITPACK) numerical model is the numerical device employed to solve the shoreline problem. This research [18] describes a numerical modelling framework called GENESIS that is used to simulate long-term shoreline change caused by spatial and temporal variations in longshore sand transport at coastal engineering projects. The modelling system is managed via an organized and user-friendly interface, which reduces the need for the operator to get concerned with computer code specifics. The modelling-system application method is described from the viewpoint of engineers and planners involved in the evaluation of shore-protection projects [19-21].

Then, if a groin is added to a one-dimensional shoreline evolution model, we propose a governing equation for this research. It is provided with a non-dimensional shoreline evolution model with a groin construction model. The model can now be used to manipulate physical parameters.

I. Governing Equation

A. Shoreline Evolution Model

The beach profile is supposed to travel landward and seaward while maintaining the same form in the one-line model, meaning that all bottom contours are parallel. As a result, specifying the horizontal location of the profile with respect to the baseline is sufficient under this assumption, and one contour line may be used to represent changes in the beach plan shape and volume as the beach reduces and accretes. Sand is carried alongshore between two well-defined limiting heights on the profile, according to the model's main assumption. If there is a variation in the alongshore sand transport rate at the lateral sides of the section and the related sand continuity, it contributes to the volume change.

At all times, the laws of mass conservation must be applied to the system. The following differential equation for shoreline evolution is produced using the given definitions:

$$\frac{\partial y}{\partial t}=\frac{1}{D_B+D_C}\left(-\frac{\partial Q}{\partial x}\right) , \tag{1}$$

where x is the alongshore coordinate (m), y is the shoreline positions (m) and perpendicular to x-axis, t is time (day), Q is the long-shore sand transport rate (m^3/day), D_B is the average berm height (m) and D_C is the average closure depth (m).

In order to solve Eq.(1), an equation for the longshore sand transport rate Q must be specified. This quantity is thought to be created by a wave that strikes the coastline obliquely. [22] provided a general expression for the long-shore sand transport rate,

$$Q=Q_0\sin\left(2\alpha_b\right) , \tag{2}$$

where Q_0 is the amplitude of the long-shore sand transport rate. The empirical predictive formula for the amplitude of the long-shore sand transport rate is [23]:

$$Q_0=\frac{\rho}{16}\left(H_b^2c_{gb}\right)\frac{K}{(\rho_s-\rho)(1-n)} , \tag{3}$$

where the subscript b represent the value at the point breaking, ρ is the density of sea water (kg/m^3), ρ_s is the density of the sediment (kg/m^3), n is the porosity, K is the dimensionless coefficient which is a function of particle size, H is the wave height and c_g is the wave group velocity.

The quantity α_b the impact angle between breaking wave crests angle with local shoreline, and may be written as,

$$\alpha_b=\alpha_0-\tan^{-1}\left(\frac{\partial y}{\partial x}\right) , \tag{4}$$

where α_0 is the angle between breaking wave crests and the x-axis. For beaches with a slight slope, the breaking wave angle to the coastline is likely to be minimal. Assuming that,

$$\sin\left(2\alpha_b\right)\approx 2\alpha_b ,$$

and

$$\tan^{-1}\left(\frac{\partial y}{\partial x}\right)\approx\left(\frac{\partial y}{\partial x}\right).$$

Substituting Eq.(4) into Eq.(2), and assuming the beach with mild slope yields,

$$Q=Q_0\left(2\alpha_0-2\frac{\partial y}{\partial x}\right) , \tag{5}$$

Substituting Eq.(5) into Eq.(1), and neglecting the sources or sinks along the coast gives,

$$\frac{\partial y}{\partial t}=D\frac{\partial^2 y}{\partial x^2} , \tag{6}$$

for all $(x,t)\in[0,L]\times[0,\tau]$, where $D=\dfrac{2Q_0}{D_B+D_C}$.

B. The Initial and Boundary Conditions of the One-Dimensional Model

a) The Initial Condition

Groin system that is impermeable and straight. The shoreline of the initials is considered to be parallel to the x-axis.

Assume α_0 is the braking wave angle to the beach as show in Fig. 3. As a result, the rate of sand transport along the beach is homogeneous. As seen in Fig. 1, the groin is inserted instantly at $x = 0$. As a result, the initial conditions become,

$$y(x,0) = 0 \ , \tag{7}$$

for all $x \in [0, L]$.

b) The Left Boundary Condition

The left boundary condition is defined by the interpolation function of the measured evolutionary data. It is shoreline evolution with the left-hand side groin system. The boundary conditions are assumed to be as follows,

$$y(0,t) = g(t) \ , \tag{8}$$

for all $t \in [0, \tau]$, where $g(t)$ is a given interpolation function of the measured evolutionary data at the left-hand side groin system.

c) The Right Boundary Condition

The right boundary condition is defined by the interpolation function of the measured evolutionary data.
It is shoreline evolution with the right-hand side groin system. The boundary conditions are assumed to be as follows,

$$y(L,t) = h(t) \ , \tag{9}$$

for all $t \in [0, \tau]$, where $h(t)$ is a given interpolation function of the measured evolutionary data at the right-hand side groin system.

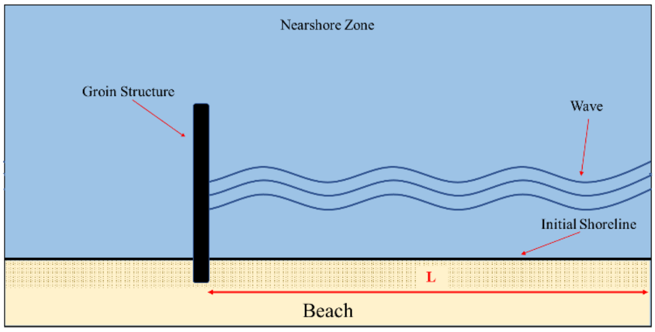

Fig. 1. Initial shoreline with configuration straight impermeable groins.

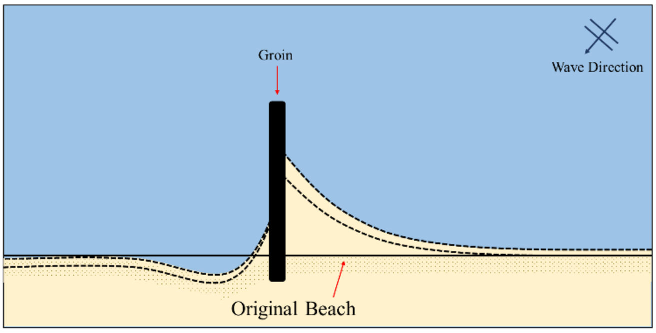

Fig. 2. Shoreline evolution with a groin structure.

C. Physical Parameters

Physical parameters of the model can be illustrated as shown in Figs. 3-4. that are listed below.

α_0 is the impact angle between breaking wave crests angle with the x-axis.

Q_0 is the amplitude of the long-shore sand transport rate (m^3/day).

D_B is the average berm height (m).

D_C is the average closure depth (m).

L is the length of alongshore (m).

τ is time of simulation (day).

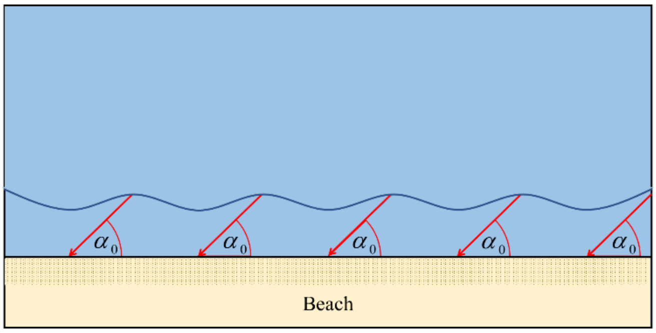

Fig. 3. Breaking wave crests impact angle.

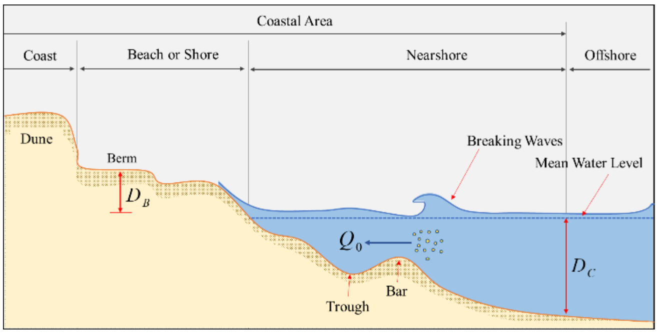

Fig. 4. Beach profile and shoreline physical parameters.

II. A NON-DIMENSIONAL SHORELINE EVOLUTION MODEL

A. A Non-Dimensional Shoreline Evolution Model

Taking non-dimensional technique [24] into Eq.(6), we obtain the following,

$$\frac{\partial Y}{\partial T} = \frac{\partial^2 Y}{\partial X^2} \ , \tag{10}$$

for all $(X,T) \in [0,1] \times [0,\Gamma]$, where L is the length of alongshore, Y_* is the expected shoreline evolution,

$Y = \dfrac{y}{Y_*}$, $X = \dfrac{x}{L}$, and $T = \dfrac{Dt}{L^2}$.

Equation (10) is similar to the one-dimensional heat equation which has a thermal conductivity coefficient of 1, so in order to solve a problem it is necessary to define the initial conditions and the boundary conditions.

B. The Initial and Boundary Conditions of the Non-Dimensional Model

The initial and boundary conditions for a non-dimensional model can be simply defined under the known initial and boundary conditions according to Eqs.(7)-(9) for a one-dimensional model, so we can define the necessary conditions as the follows. These means that the initial condition becomes,

$$Y(X,0) = F(X) \qquad \text{for all } X \in [0,1] \ , \tag{11}$$

Boundary conditions are also defined by:

$$Y(0,T) = G(T) \qquad \text{for all } T \in [0,\Gamma] , \tag{12}$$

and

$$Y(1,T) = H(T) \qquad \text{for all } T \in [0,\Gamma] , \tag{13}$$

where $F(X) = 0$, $G(T) = \dfrac{g(t)}{Y_*}$ and $H(T) = \dfrac{h(t)}{Y_*}$.

III. Numerical Techniques

A. Grid Spacing

We now discretize the domain of Eq.(10) by dividing the interval $[0,1]$ into M subintervals such that $M\Delta X = 1$ and the time interval $[0,\Gamma]$ into N subintervals such that $N\Delta T = \Gamma$. We then approximate $Y(X_m, T_n)$ by Y_m^n , at the point $X_m = m\Delta X$ and $T_n = n\Delta T$, where $0 \le m \le M$ and $0 \le n \le N$ in which M and N are positive integers.

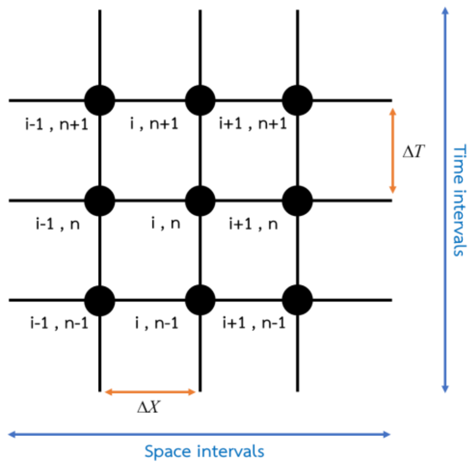

Fig. 5. Grid spacing.

B. The Traditional Forward Time Centered Space (FTCS) Techniques

The forward time centered space (FTCS) Technique is employed. Consequently, the finite difference approximation [19],[25] becomes,

$$Y(X_m, T_n) \cong Y_m^n , \tag{14}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T} , \tag{15}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^n}{2\Delta X} , \tag{16}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - 2Y_m^n + Y_{m-1}^n}{(\Delta X)^2} , \tag{17}$$

Substituting Eqs.(14)-(17) into Eq.(10), we obtain,

$$\frac{Y_m^{n+1} - Y_m^n}{\Delta T} \cong \frac{Y_{m+1}^n - 2Y_m^n + Y_{m-1}^n}{(\Delta X)^2} , \tag{18}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. Equation (18) can be written in an explicit form of finite difference as follows,

$$Y_m^{n+1} \cong \mu Y_{m+1}^n + (1-2\mu) Y_m^n + \mu Y_{m-1}^n , \tag{19}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. where $\mu = \dfrac{\Delta T}{(\Delta X)^2}$.

C. An Unconditionally Saulyev Finite Difference Techniques

The Saulyev finite difference Technique will be also employed. We can obtain that the finite difference approximatetion [19],[25] becomes,

$$Y(X_m, T_n) \cong Y_m^n , \tag{20}$$

$$\frac{\partial Y}{\partial T} \cong \frac{Y_m^{n+1} - Y_m^n}{\Delta T} , \tag{21}$$

$$\frac{\partial Y}{\partial X} \cong \frac{Y_{m+1}^n - Y_{m-1}^{n+1}}{2\Delta X} , \tag{22}$$

$$\frac{\partial^2 Y}{\partial X^2} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{(\Delta X)^2} , \tag{23}$$

Substituting Eqs.(20)-(23) into Eq.(10), we obtain,

$$\frac{Y_m^{n+1} - Y_m^n}{\Delta T} \cong \frac{Y_{m+1}^n - Y_m^n - Y_m^{n+1} + Y_{m-1}^{n+1}}{(\Delta X)^2} , \tag{24}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. Equation (24) can be written in an explicit form of finite difference as follows,

$$Y_m^{n+1} \cong (1+\mu)^{-1} \cdot \left(\mu Y_{m+1}^n + (1-\mu) Y_m^n + \mu Y_{m-1}^{n+1} \right) , \tag{25}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. where $\mu = \dfrac{\Delta T}{(\Delta X)^2}$.

IV. Error Measurement

A simple measure used to measure the difference between actual and approximate values is the absolute error method. The absolute error formula be as follows,

$$E_m^n = \left| \tilde{y}_m^n - y_m^n \right| , \tag{26}$$

where $\tilde{y}_m^n$ is the analytical solution of shoreline evolution and y_m^n is the approximate solution of shoreline evolution.

V. Numerical Experiment

In order to investigate the shoreline evolution in the long-term scale. Assuming that the length of considered shoreline L is 5000 m, the amplitude of the long-shore transport rate Q_0 is 7500 m^3/day, the averaged berm height D_B is 2 m, the averaged closure depth D_C is 28 m, the breaking wave impact angle α_0 is 0.02, and the expected shoreline evolution Y_* is 20 m. The simulation setting is illustrated in Fig. 6-7.

We will employ the traditional forward time centered space (FTCS) techniques Eq.(19), and the Saulyev finite difference techniques Eq.(25), to approximate the model solution.

The analytical solution of the simulation is [26]:

$$\tilde{y}(x,t) = \tan\alpha_0 \sqrt{\frac{4Dt}{\pi}} \left(e^{-\frac{x^2}{4Dt}} - \frac{x\sqrt{\pi}}{2\sqrt{Dt}} erfc\left(\frac{x}{2\sqrt{Dt}}\right)\right), \quad (27)$$

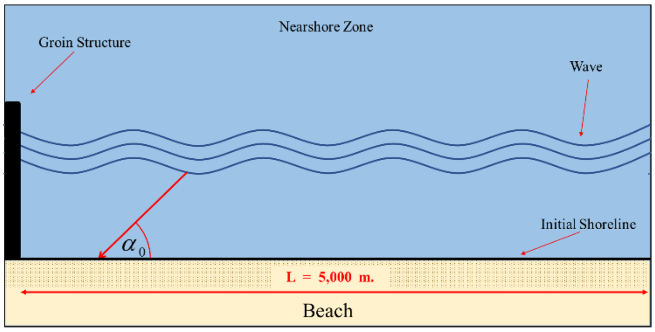

Fig. 6. Initial shoreline.

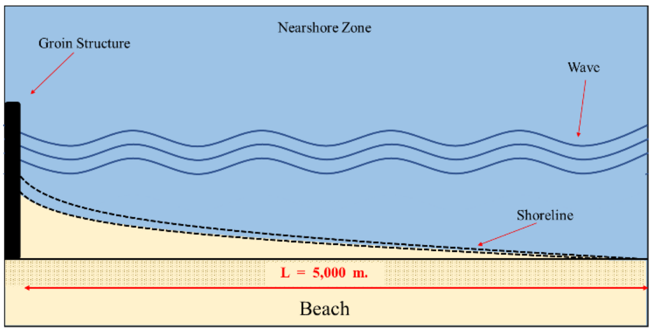

Fig. 7. The evolution from initial shoreline.

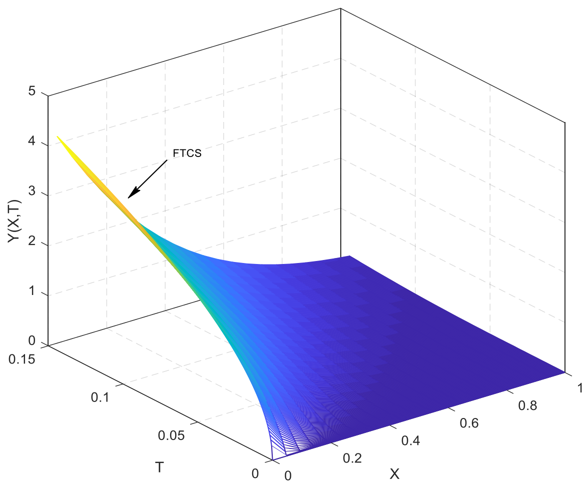

Fig. 8. Approximated shoreline evolution under the non-dimensional equations in 20 years when the FTCS technique is used.

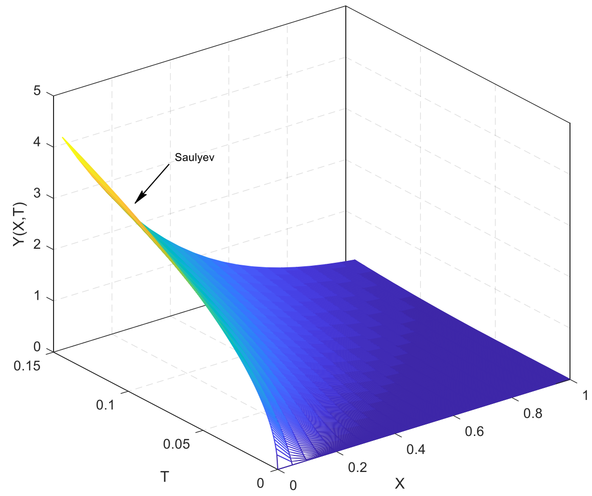

Fig. 9. Approximated shoreline evolution under the non-dimensional equations in 20 years when the Saulyev technique is used.

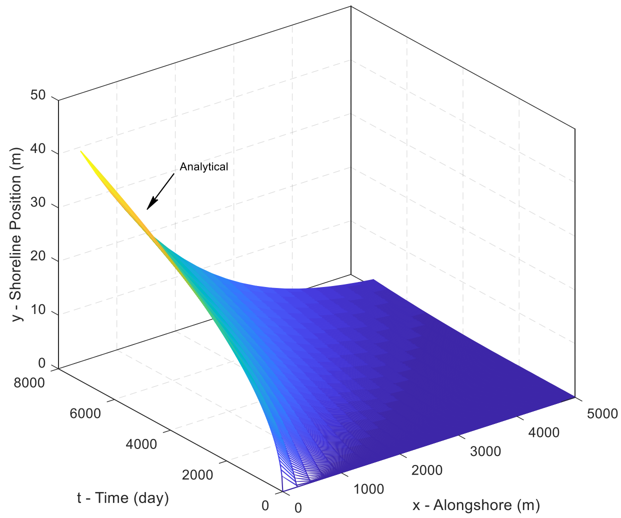

Fig. 10. Analytical shoreline evolution in 20 years.

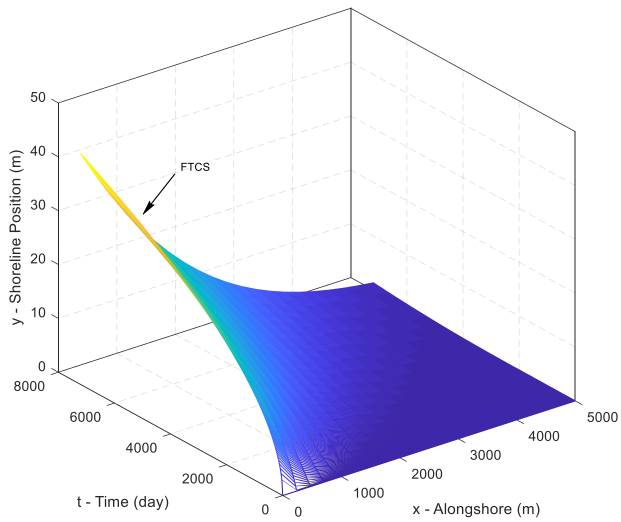

Fig. 11. Approximated shoreline evolution under the one-dimensional equations in 20 years when the FTCS technique is used.

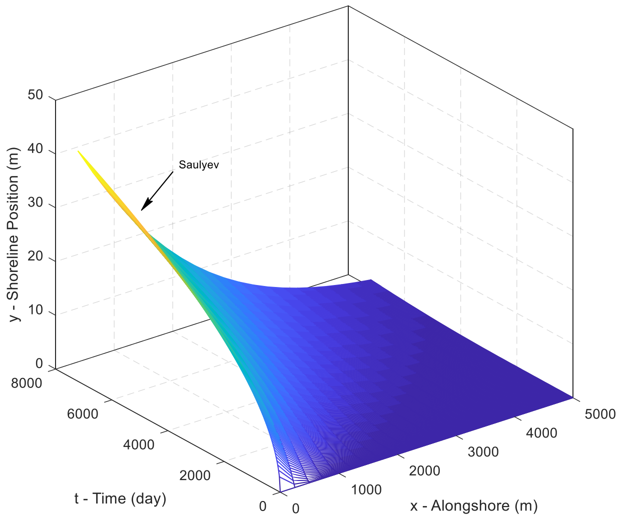

Fig. 12. Approximated shoreline evolution under the one-dimensional equations in 20 years when the Saulyev technique is used.

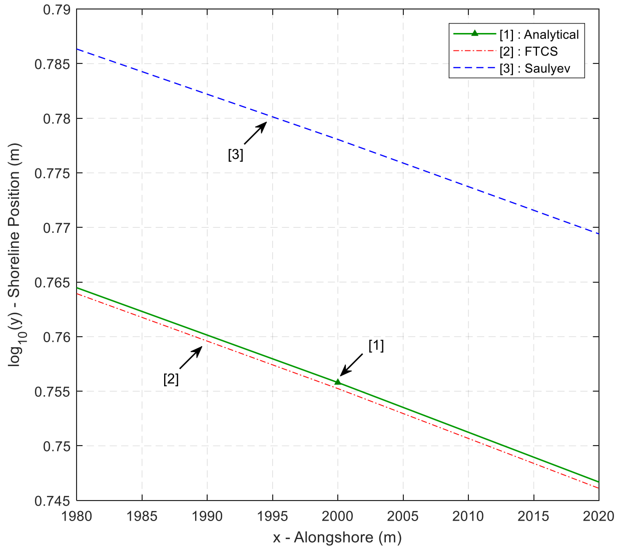

Fig. 15. Shoreline evolution in 10 years.

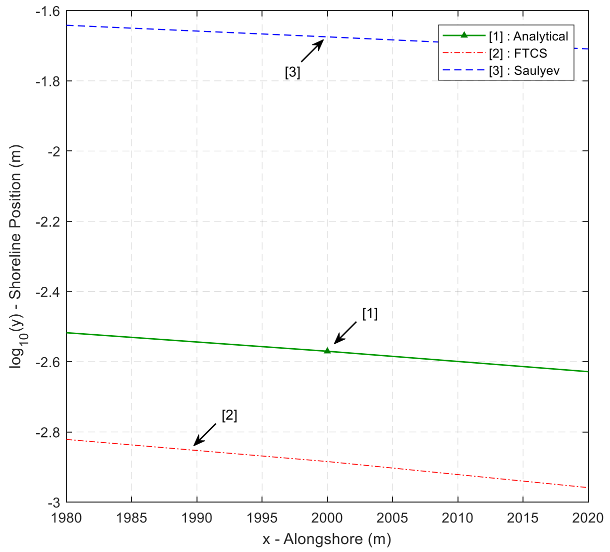

Fig. 13. Shoreline evolution in 1 year.

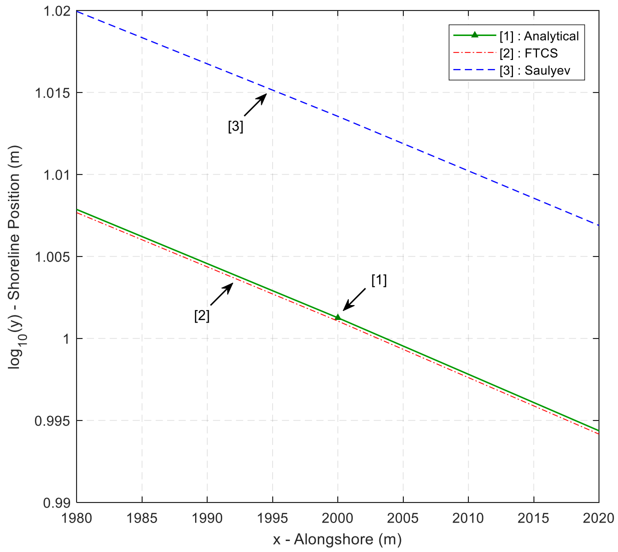

Fig. 16. Shoreline evolution in 15 years.

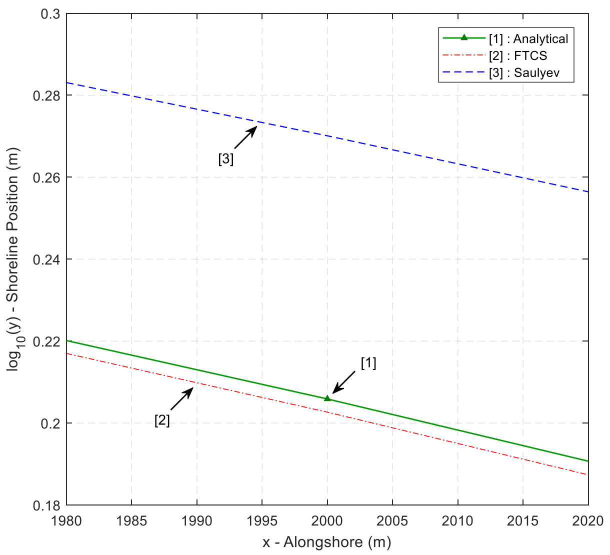

Fig. 14. Shoreline evolution in 5 years.

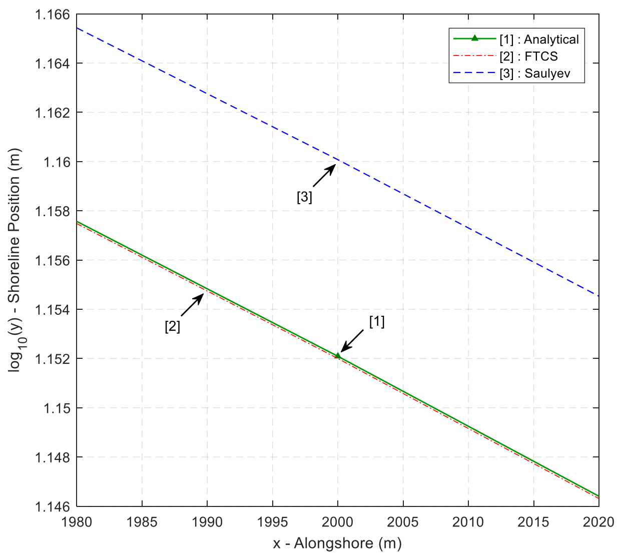

Fig. 17. Shoreline evolution in 20 years.

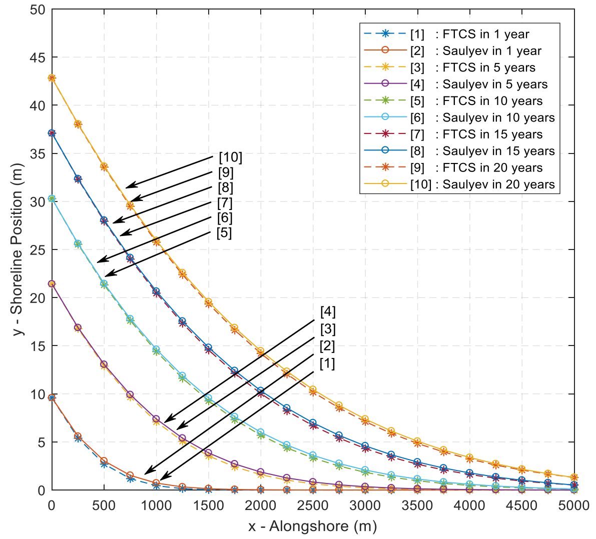

Fig. 18. Comparison of the approximate shoreline evolution results in 1, 5, 10, 15 and 20 years when the FTCS technique and the Saulyev technique are used.

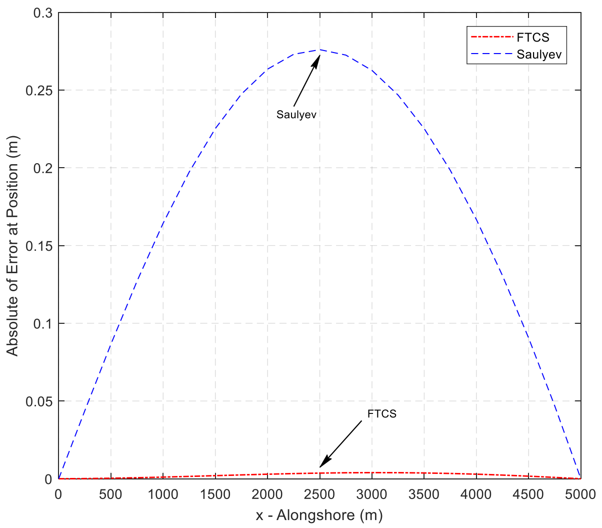

Fig. 19. Comparison of absolute error in 20 years between the FTCS technique and the Saulyev technique.

The analytical solution is illustrated in Table I and the approximate solutions of both the numerical techniques are illustrated in Tables II-III, respectively.

TABLE I
ANALYTICAL SOLUTION OF SHORELINE EVOLUTION

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.476	0.003	0.000	0.000	0.000
5	21.412	7.096	1.607	0.237	0.022	0.001
10	30.282	14.390	5.688	1.844	0.480	0.099
15	37.087	20.467	10.029	4.314	1.614	0.521
20	42.825	25.762	14.194	7.107	3.213	1.304

TABLE II
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE (FTCS) TECHNIQUE

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.446	0.001	0.000	0.000	0.000
5	21.412	7.089	1.595	0.230	0.020	0.001
10	30.282	14.387	5.692	1.836	0.475	0.099
15	37.078	20.465	10.024	4.308	1.610	0.521
20	42.825	25.761	14.191	7.103	3.210	1.304

TABLE III
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING AN UNCONDITIONALLY SAULYEV FINITE DIFFERENCE TECHNIQUE

Time (Years)	Distance (m.) 0	1000	2000	3000	4000	5000
0	0.000	0.000	0.000	0.000	0.000	0.000
1	9.576	0.719	0.021	0.000	0.000	0.000
5	21.412	7.374	1.863	0.344	0.047	0.001
10	30.282	14.612	6.000	2.075	0.593	0.099
15	37.087	20.655	10.317	4.582	1.773	0.521
20	42.825	25.926	14.457	7.370	3.380	1.304

In order to be able to analyze the computational efficiency, the absolute error and comparisons of stability are illustrated in Tables IV-V, respectively.

TABLE IV
ABSOLUTE ERROR OF SHORELINE EVOLUTION WHEN USING THE FTCS TECHNIQUES AND SAULYEV TECHNIQUES

Time (Years)	Distance (m.)	Absolute Error FTCS	Absolute Error Saulyev
1	1000	0.2941×10^{-1}	0.24318
	2000	0.1385×10^{-2}	0.1846×10^{-1}
	3000	0.1283×10^{-5}	0.3314×10^{-3}
	4000	0.3588×10^{-14}	0.3425×10^{-5}
	5000	0.1233×10^{-31}	0.1233×10^{-31}
5	1000	0.7894×10^{-2}	0.2778
	2000	0.1187×10^{-1}	0.2560
	3000	0.6303×10^{-2}	0.1072
	4000	0.1533×10^{-2}	0.2482×10^{-1}
	5000	0.1822×10^{-16}	0.1822×10^{-16}
10	1000	0.3204×10^{-2}	0.2224
	2000	0.7287×10^{-2}	0.2997
	3000	0.7667×10^{-2}	0.2310
	4000	0.4505×10^{-2}	0.1134
	5000	0.4163×10^{-15}	0.4163×10^{-15}
15	1000	0.1791×10^{-2}	0.1888
	2000	0.4616×10^{-2}	0.2878
	3000	0.5833×10^{-2}	0.2644
	4000	0.4126×10^{-2}	0.1592
	5000	0.1110×10^{-15}	0.1110×10^{-15}
20	1000	0.1063×10^{-2}	0.1644
	2000	0.2933×10^{-2}	0.2634
	3000	0.3948×10^{-2}	0.2626
	4000	0.2950×10^{-2}	0.1667
	5000	0.2443×10^{-14}	0.2443×10^{-14}

TABLE V
COMPARISON OF THE STABILITY WHEN CHANGING GRID SPACING SIZES

ΔT	Δt	ΔX	Δx	Stability	
				FTCS	Saulyev
0.0006	30	0.005	25	U	S
		0.010	50	U	S
		0.020	100	U	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0003	15	0.005	25	U	S
		0.010	50	U	S
		0.020	100	U	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0002	10	0.005	25	U	S
		0.010	50	U	S
		0.020	100	S	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S
0.0001	5	0.005	25	U	S
		0.010	50	U	S
		0.020	100	S	S
		0.050	250	S	S
		0.100	500	S	S
		0.200	1000	S	S

Note: S represents Stable, U represents Unstable.

VI. Discussion

Using the traditional forward time centered space (FTCS) techniques and the Saulyev finite difference techniques, the annual evolution of the shoreline can be determined.

In the calculations, when we compared the approximated shoreline evolution solutions under the non-dimensionless model which occurred between the traditional forward times centered space (FTCS) technique and the Saulyev finite difference technique, they were found to be close together as shown in Figs. 8-9. For this reason, the approximated shoreline evolution solutions obtained when converted back to the solutions under the one-dimensional model are also close as shown in Figs. 11-12. Therefore, we can see that the approximate solutions as shown in Figs. 11-12 are close to the analytical solution as shown in Fig. 10.

As demonstrated in Tables II, III, and Fig. 13, the distance from the furthest shoreline evolution after 1 year is 9.576 m. The smallest distance from the evolution of the shoreline is 0.000 m.

As demonstrated in Tables II, III, and Fig. 14, the distance from the furthest shoreline evolution after 5 years is 21.412 m. The smallest distance from the evolution of the shoreline is 0.001 m.

As demonstrated in Tables II, III, and Fig. 15, the distance from the furthest shoreline evolution after 10 years is 30.282 m. The smallest distance from the evolution of the shoreline is 0.099 m.

As demonstrated in Tables II, III, and Fig. 16, the distance from the furthest shoreline evolution after 15 years is 37.087 m. The smallest distance from the evolution of the shoreline is 0.521 m.

As demonstrated in Tables II, III, and Fig. 17, the distance from the furthest shoreline evolution after 20 years is 42.825 m. The smallest distance from the evolution of the shoreline is 1.304 m.

In Fig.18, it is clearly confirmed that both numerical techniques produce shoreline evolution solutions close to each other even after 1, 5, 10, 15, and 20 years.

In terms of accuracy, the traditional forward time centered space (FTCS) technique is more accurate than the Saulyev finite difference techniques as shown in Table IV and Fig. 19, but the solution cannot be handled in some cases when the time increment is increased for the traditional forward time centered space (FTCS) techniques. The Saulyev finite difference techniques, on the other hand, can still be used. The Saulyev finite difference technique, on the other hand, can handle numerical solutions in almost every scenario as shown in Table V, because the stability conditions are not constrained. As a result, the Saulyev finite difference technique can also be useful for computing a practical conceptual design of shoreline evolution when the number of grids is increased.

VII. Conclusion

The model includes a non-dimensional shoreline evolution model as well as a groin structure model. Physical parameters can now be manipulated by the model. The initial condition setting technique and boundary condition approaches are also presented where groin structural effects are involved. The traditional forward time centered space (FTCS) technique and the unconditionally stable Saulyev finite difference strategies are employed to approximate the incremental model every year. The numerical models available give a realistic simulation for predicting long-term coastal evolution. Because the stability requirements are not limited, the Saulyev finite difference Techniques can handle numerical solutions in practically any circumstance. Since the number of grids has increased, the Saulyev finite difference technique can be highly beneficial for computing a practical conceptual design of shoreline evolution. The simulation described here can be used to forecast the success of a groin system on a local beach.

References

[1] P. Phosri, and N. Pochai, “Numerical Computation of a Water-Quality Model with Advection-Diffusion-Reaction Equation Using an Upwind Implicit Scheme,” Thai Journal of Mathematics, vol. 19, no. 1, pp187-196, 2021

[2] A. Vongkok, and N. Pochai, “Numerical Models of Nitrogen Compound Measurements in a Stream with Removal Mechanism Using Saulyev Technique with Cubic Spline Interpolation,” Journal of Interdisciplinary Mathematics, vol. 22, no. 8, pp1235-1275, 2019

[3] P. Othata, and N. Pochai, “A Mathematical Model of Salinity Control in a River with an Effect of Internal Waves Using Two Explicit Finite Difference Methods,” Engineering Letters., vol. 29, no. 2, pp689-696, 2021

[4] P. Othata, and N.Pochai, “Irrigation Water Management Strategies for Salinity Control in the Chao Phraya River Using Sualyev Finite Difference Method with Lagrange Interpolation Technique,” Engineering Letters, vol. 29, no. 2, pp332-338, 2021

[5] W. Timpitak, and N. Pochai, “Numerical Simulations to a One-Dimensional Groundwater Pollution Measurement Model Through Heterogeneous Soil,” IAENG International Journal of Applied Mathematics, vol. 50, no. 3, pp558-565, 2020

[6] P. Phosri, and N. Pochai, “Explicit Finite Difference Techniques for a One-Dimensional Water Pollutant Dispersion Model in a Stream,” International Journal of

Simulation: Systems, Science and Technology, vol. 21, no.3, pp1.1-1.7, 2020
[7] N. Camcoon, and N. Pochai, "A Non-Dimensional Mathematical Model of Salinity Measurement in the Chao Phraya River Using a New Fourth Order Finite Difference Method with the Saulyev Technique," Engineering Letters, vol. 28, no. 4, pp978-984, 2020
[8] W. Kraychang, and N. Pochai, "A Simple Numerical Model for Water Quality Assessment with Constant Absorption Around Nok Phrao Island of Trang River," Engineering Letters, vol. 28, no. 3, pp912-922, 2020
[9] A. Wibowo, and S.H. Arbain, and N. Z. Abidin, "Combined Multiple Eural Networks and Genetic Algorithm with Missing Data Treatment: Case Study of Water Level Forecasting in Dungun River - Malaysia," IAENG International Journal of Computer Science, vol. 45, no. 2, pp246-254, 2018
[10] L. Dongping and Y. Yingchun and W. Jin, "Water Quality Assessment Based on Probabilistic Echo State Networks," IAENG International Journal of Computer Science, vol. 44, no. 3, pp270-276, 2017
[11] Y. Obnosov, and A. Kacimov, "Fluid Dynamics of Nonaqueous Phase Contaminants in Groundwater: Analytical Solutions and Analog with Zhukovsky's Trochoid," in Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2012, WCE 2012, 4-6 July, 2012, London, U.K., pp44-47
[12] US Army Corps of Engineers, "Shore Protection Planning and Design," Coastal Engineering Research Center, vol. 3, no. 4, 1966
[13] W. T. Bakker, "The Dynamics of Coast with a Groin System," Proceeding of 11th Coastal Engineering Conference 1968, September, 1968, London, U.K., pp492-517
[14] H. Hanson, M. Larson, and N.C. Kraus, "Analytical Solution of the One-line Model for Shoreline Change," US Army Corps of Engineer Waterways Experiment Station, CERC, 1987
[15] W. T. Bakker, and T. Edelman, "The Coastline of River Deltas," in Proc. 9th Conf. Coastal Engineering, Lisbon, 1964, pp199-218
[16] W. Grijimr, "Theoretical Form of Shoreline," in Proc. 7th Conf. Coastal Engineering, Hague, 1960, pp. 197-202.
[17] M.F. Ahmad, Subiyanto, R. Yaacob and M. Mamat, "Coastline Changes in Vicinity of Runway Platform of Sultan Mahmud Airport, Kuala Terengganu: Comparative Analysis of One-line Model Versus Satellite Data," Journal of Applied Sciences, vol.14, no.19, pp2234-2245, 2014
[18] W. Grijimr, "Theoretical Form of Shoreline," in Proc. 9th Conf. Coastal Engineering, Lisbon, 1964, pp219-235
[19] P. Unyapoti, and N. Pochai, "A One-dimensional Mathematical Model of Long-term Shoreline Evolution with Groin System Using an Unconditionally Stable Explicit Finite Difference Method," International Journal of Simulation: Systems, Science and Technology, vol. 21, no. 3, pp2.1-2.6, 2020
[20] P. Unyapoti, and N. Pochai, "A Shoreline Evolution Model with a Twin Groins Structure Using Unconditionally Stable Explicit Finite Difference Techniques," Engineering Letters, vol. 29, no. 1, pp288-296, 2021
[21] P. Unyapoti, and N. Pochai, "A Shoreline Evolution Model with a Groin Structure under Non-Uniform Breaking Wave Crest Impact," Computation, Computation 2021, vol. 9, no. 4, 2021
[22] M. M. Subiyanto, M. F. Ahmad, and M. L. Husain, "Comparison of Numerical Method for Forward and Backward Time Centered Space for Long-Term Simulation of Shoreline Evolution," Applied Mathematical Sciences, vol. 7, no. 104, pp5165-5173, 2013
[23] B. L. Mahute, and M. Soldate, "Mathematical Modeling of Shoreline Evolution," US Army Corps of Engineer Waterways Experiment Station, CERC, 1977
[24] G.D. Smith, "Transformation to Non-Dimensional Form, Numerical Solution of Partial Differential Equation: Finite Difference Methods," Oxford University, New York, pp11-12, 1985
[25] A. R. Mitchell, "Computational Methods in Partial Differential Equations," John Wiley & Sons Ltd., London, 1969
[26] L. X. Hoan, "Some Result of Comparison Between Numerical and Analytical Solutions of the One-Line Model for Shoreline Change," Vietnam Journal of Mechanics, vol. 28, no.2, pp94-102, 2006

S. Manilam is a Researcher Assistant of Centre of Excellence in Mathematics, MHESI, Bangkok 10400, Thailand.

N. Pochai is a Researcher of Centre of Excellence in Mathematics, MHESI, Bangkok 10400, Thailand.