Finite-time and Fixed-time Synchronization for Inertial Memristive Neural Networks with Time-varying Delay and Linear Coupling

Jie Gao, Yuming Chen

Abstract—**This paper studies the finite-time and fixed-time synchronization for the coupled inertial memristive neural networks (CIMNNs) with time-varying delays. First, with the help of the differential inclusions theory and the interval matrix theory, the CIMNNs with state-dependent parameters which may be mismatched are transformed into an uncertain interval parametric system. Then, two novel controllers are designed to deal with the mismatched parameters and the time-varying delay. Moreover, according to the interval matrix theory, two sufficient conditions are attained which make the CIMNNs synchronize with the target trajectory in the fixed time. Finally, one numerical simulation example is given to demonstrate the effectiveness of the theoretical results.**

Index Terms—**Fixed-time synchronization, Inertial memristive neural networks, Interval matrix theory, Switching control.**

I. Introduction

SINCE the memristor is introduced to reflect the relation between magnetic flux and electric charge in [1], it has attracted many attentions due to its memory function, similar to the human brain [2]–[4]. Especially, if the resistors are displaced by the memristors in the neural networks, the traditional neural networks become the memristive neural networks (MNNs) with state-dependent parameters that could imitate the human brain more perfectly [5], [6]. Therefore, there are many efforts to study about the dynamical behaviors of the MNNs, such as stability, dissipativity, synchronization and periodicity [7]–[9]. Moreover, the achievements of theoretical research about MNNs could promote the applications such as associative memories [2], neural learning circuits [10], new classes of artificial neural systems [4].

In addition, most of the existing literatures focus on the first-order MNNs and a few about the second-order MNNs with an inertial term which is called inertial memristive neural networks (IMNNs). In [11], the inertial term was firstly introduced into Hopfield neural networks by Babcock and Westervelt who studied the generated dynamic behaviors from then. It was proved that the inertia in neural networks was a favorable tool which could generate the chaotic behaviour. Moreover, the authors in [12], [13] use the inertia to create the bifurcation behaviors including limit cycles, homoclinic and heteroclinic orbits and so on. Thus, it is essential to study the dynamical behaviors of the inertial memristive neural networks. In [14], the dissipativity of the inertial uncertain NNs was studied by using of the matrix measure. And the synchronization of the inertial BAM NNs was investigated by means of the matrix measure in [15]. It is in [16] studied the exponential stability for periodic solutions of inertial C-G type BAM NNs. Li and Zheng in [17] gave some criteria to reach synchronization for the coupled inertial memristive neural networks (CIMNNs) by themselves. In [18], the exponential stability for the inertial memristive neural networks was investigated with impulses and time-varying delays.

It is noted that synchronization in many practical situations must be attained in a finite time. Thus, the control accuracy becomes very important and the convergence time should be limited stringently. It is more essential to study the finite time synchronization which means that all the dynamical behaviors reach the same behavior in the finite time, but it is more chanllenging. In [19], finite-time synchronization problem of MNNs is studied by constructing a novel switching controller. The settling time obtained in [19] is estimated dependently on the initial values. Nevertheless, if the initial values are not given beforehand, it turns to be difficult to calculate the settling time. Then, it is necessary to consider the fixed-time synchronization which is irrelevant to the initial value. In [20], the fixed-time cluster synchronization problem is discussed for complex networks by designing a pinning controller. Next, the fixed-time pinning-controlled synchronization is investigated for the coupled neural networks with delays and discontinuous activations in [21]. However, there are only few studies about the fixed-time synchronization for the coupled inertial memristive neural networks (CIMNNs) with time-varying delays by using the interval matrix theory.

Motivated by the aforementioned discussions, this paper studies the finite-time and fixed-time synchronization of the CIMNNs with delays by using of the interval matrix method. Our main contributions of the study are outlined as: (1) by using of the differential inclusions theory and the interval matrix theory, the second-order CIMNNs with state-dependent parameters which may be mismatched are transformed into a first-order uncertain interval parametric system; (2) two discontinuous controllers with the sign function are constructed to obtain the finite-time and fixed-time synchronization, respectively; (3) two sufficient conditions are achieved with the help of the norms of the interval matrices, and need not utilize directly the the maximum absolute values of the memristive synaptic weights, which could lead to the less conservativeness.

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Jie Gao is an associate professor of the School of Science, Southwest Petroleum University, Chengdu 610500, China. e-mail: gaojiejr@foxmail.com (Corresponding Author)

Yuming Chen is a postgraduate student of the School of Science, Southwest Petroleum University, Chengdu 610500, China. e-mail: 786600316@qq.com.

The remainder of the paper is organized as follows. In Section 2, the mathematical model of the CIMNNs is introduced. In addition, some necessary preliminaries are provided. In Section 3, two discontinuous controllers are constructed to achieve the finite-time and fixed-time synchronization, respectively. Some sufficient conditions are obtained with the help of the upper bound norms of the interval matrices in this section. Section 4 gives one numerical simulation to show the effectiveness of the proposed controllers. Finally, the conclusions and future research are given in Section 5.

Notation: Throughout this article, the notations are standard. Denote $\mathbb{R}^n$ as the n-dimensional Euclidean space. $\mathbb{R}^{n\times n}$ represents the set of $n\times n$ matrices. $\|\cdot\|$ is the 2-norm which is defined by $\|\varphi\| = (\sum_{i=1}^{n}\varphi_i^2)^{1/2}$. $\mathrm{sign}(\cdot)$ denotes the sign function. $\mathrm{sign}(\omega) = [\mathrm{sign}(\omega_1), \mathrm{sign}(\omega_2), ..., \mathrm{sign}(\omega_n)]^T$ with $\omega \in \mathbb{R}^n$. The superscript T represents the matrix transposition. $\mathrm{diag}(d_1, d_2, ..., d_n)$ stands for a diagonal matrix of n dimensions.

II. Preliminaries

In this section, on the basis of the previous works [22], [23], an array of inertial memristive neural networks (IMNNs) with linear coupling and time-varying delays is given by the following equations:

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &= -D\frac{dx_i(t)}{dt} - Cx_i(t) + A(x_i(t))f(x_i(t)) \\ &\quad + B(x_i(t))f(x_i(t-\tau(t))) - h \\ &\quad \sum_{j=1}^{N} l_{ij}\Gamma(\frac{dx_j(t)}{dt} + x_j(t)), i = 1,2,...,N\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), ..., x_{in}(t))^T \in \mathbb{R}^n$ is the state sector of the ith neuron; the second derivative of $x_i(t)$ denotes the inertial term; $D = \mathrm{diag}(d_1, d_2, ..., d_n)$ and $C = \mathrm{diag}(c_1, c_2, ..., c_n)$ are positive definite constant matrices; the neuron feedback functions are given as

$$\begin{aligned}f(x_i(t)) &= (f_1(x_{i1}(t)), f_2(x_{i2}(t)), ..., f_n(x_{in}(t)))^T \\ f(x_i(t-\tau(t))) &= (f_1(x_{i1}(t-\tau(t))), f_2(x_{i2}(t-\tau(t))), \\ &\quad ..., f_n(x_{in}(t-\tau(t))))^T,\end{aligned} \tag{2}$$

where $\tau(t)$ is the time-varying delay with the upper bound τ, that is, $0 \le \tau(t) \le \tau$; $A(x_i(t)) = [a_{kj}(x_{ij}(t))]_{n\times n}$ and $B(x_i(t)) = [b_{kj}(x_{ij}(t))]_{n\times n}$. By using of the current-voltage characteristics and the property of the memristor, $a_{kj}(x_{ij}(t))$ and $b_{kj}(x_{ij}(t))$ are considered as follows:

$$a_{kj}(x_{ij}(t)) = \begin{cases} \hat{a}_{kj}, & |x_{ij}(t)| \le \Lambda_j, \\ \check{a}_{kj}, & |x_{ij}(t)| > \Lambda_j, \end{cases} \tag{3}$$

$$b_{kj}(x_{ij}(t)) = \begin{cases} \hat{b}_{kj}, & |x_{ij}(t)| \le \Lambda_j, \\ \check{b}_{kj}, & |x_{ij}(t)| > \Lambda_j, \end{cases} \tag{4}$$

where all the switching jumps Λ_j are non-negative, $\hat{a}_{kj}$, $\check{a}_{kj}$ and $\hat{b}_{kj}$, $\check{b}_{kj}$ are all known constant numbers; $h > 0$ is the network coupling strength and $\Gamma = \mathrm{diag}(\zeta_1, ..., \zeta_n)$ is the inner coupling matrix with assuming $\Gamma = I_n$; $M = (m_{ij})_{N\times N}$ is defined as the configuration matrix which denotes the network topology with $m_{ij} > 0, i \ne j$ if there exists a link from the jth node to the ith node and $m_{ij} = 0$ otherwise, and $m_{ii} = 0$; then, the corresponding Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as $l_{ij} = -m_{ij}, i \ne j$ and $l_{ii} = \sum_{j=1}^{N} m_{ij}$.

The target trajectory $s(t)$ of this paper satisfies the following equation:

$$\begin{aligned}\frac{d^2s(t)}{dt^2} &= -D\frac{ds(t)}{dt} - Cs(t) + A(s(t))f(s(t)) \\ &\quad + B(s(t))f(s(t-\tau(t))),\end{aligned} \tag{5}$$

where $s(t) \in \mathbb{R}^n$ denotes the state vector of the isolate memtistive neural network.

The initial values of the system (1) and (5) are given by $x_i(\varsigma) = \varphi_i(\varsigma), s(\varsigma) = \psi_i(\varsigma) \in \mathbb{C}^1([-\tau, 0]; \mathbb{R}^n)$.

Obviously, the right side of (1) and (5) are discontinuous with the switching state-dependent parameters, which differs from the traditional inertial neural networks studied in [24]. Therefore, the solutions of (1) and (5) are considered under the Filippov's sense. Thus, we will provide the definition of Filippov solution.

Definition 2.1: [25] For the differential system $\frac{dx}{dt} = f(t,x)$, where the function $f(t,x)$ is discontinuous on x. The set-valued map of $f(t,x)$ is considered as :

$$F(t,x) = \bigcap_{\delta>0}\bigcap_{\mu(E)=0} \overline{\mathrm{co}}[f(B(x,\delta)\backslash E)], \tag{6}$$

where $B(x,\delta) = \{y : \|y - x\| \le \delta\}$ represents the ball of center x and radius δ; and $\mu(E)$ denotes the Lebesgue measure of set E; $\overline{\mathrm{co}}[E]$ is the closure of the convex hull of some set E. A Filippov solution of $\frac{dx}{dt} = f(t,x)$ with initial condition $x(0) = x_0$ is absolutely continuous on any subinterval $t \in [t_1, t_2]$ of $[0, T]$, which conforms to $x(0) = x_0$ and the differential inclusion:

$$\frac{dx}{dt} \in F(t,x), \text{ for a.a. } t \in [0,T].$$

According to the differential inclusion theory [25] and Definition 1, the coupled IMNNs (1) can be transformed into the interval parametric systems as follows:

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &\in -D\frac{dx_i(t)}{dt} - Cx_i(t) + \overline{\mathrm{co}}[\underline{A}, \overline{A}]f(x_i(t)) \\ &\quad + \overline{\mathrm{co}}[\underline{B}, \overline{B}]f(x_i(t-\tau(t))) \\ &\quad - h\sum_{j=1}^{N} l_{ij}(\frac{dx_j(t)}{dt} + x_j(t)),\end{aligned} \tag{7}$$

where $\underline{A} = (\underline{a}_{kj})_{n\times n}$, $\overline{A} = (\bar{a}_{kj})_{n\times n}$, $\underline{B} = (\underline{b}_{kj})_{n\times n}$, $\overline{B} = (\bar{b}_{kj})_{n\times n}$ with $\bar{a}_{kj} = \max\{\hat{a}_{kj}, \check{a}_{kj}\}$, $\underline{a}_{kj} = \min\{\hat{a}_{kj}, \check{a}_{kj}\}$, $\bar{b}_{kj} = \max\{\hat{b}_{kj}, \check{b}_{kj}\}$, $\underline{b}_{kj} = \min\{\hat{b}_{kj}, \check{b}_{kj}\}$, $\overline{\mathrm{co}}[u,v]$ is the closure of the convex hull which is generated by the two different real numbers u and v. Then, according to the measurable selection theorem, one has $\widetilde{A}(x_i(t)) = (a_{kj}(t))_{n\times n} \in \overline{\mathrm{co}}[\underline{A}, \overline{A}]$ and $\widetilde{B}(x_i(t)) = (b_{kj}(t))_{n\times n} \in \overline{\mathrm{co}}[\underline{B}, \overline{B}]$ with $\underline{a}_{kj} \le a_{kj}(t) \le \bar{a}_{kj}$ and $\underline{b}_{kj} \le b_{kj}(t) \le \bar{b}_{kj}$ such that

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &= -D\frac{dx_i(t)}{dt} - Cx_i(t) + \widetilde{A}(x_i(t))f(x_i(t)) \\ &\quad + \widetilde{B}(x_i(t))f(x_i(t-\tau(t))) \\ &\quad - h\sum_{j=1}^{N} l_{ij}(\frac{dx_j(t)}{dt} + x_j(t)),\end{aligned} \tag{8}$$

The following aim of this paper is to make the coupled IMNNs (1) synchronize with the target trajectory (5) in the finite time or in the fixed time. However, it is difficult that all the nodes in the network could synchronize with the target trajectory without any control input. Then, it is necessary to design appropriate controllers to reach the synchronization.

Now, through an appropriate variable substitution: $w_i(t) = \frac{dx_i(t)}{dt} + x_i(t)$, the second-order system (7) could be changed into the two controlled first-order equations as follows:

$$\begin{cases} \dfrac{dx_i(t)}{dt} &= -x_i(t) + w_i(t) + u_{i1}(t), \\ \dfrac{dw_i(t)}{dt} &= -\Phi x_i(t) - \Psi w_i(t) + \widetilde{A}(x_i(t))f(x_i(t)) \\ &\quad +\widetilde{B}(x_i(t))f(x_i(t-\tau(t))) \\ &\quad -h\sum_{j=1}^{N} l_{ij}w_j(t) + u_{i2}(t), \end{cases} \tag{9}$$

where $\Phi = I_n + C - D$ and $\Psi = D - I_n$, $u_{i1}(t)$ and $u_{i2}(t)$ are the control inputs.

According to the interval uncertainty theory, the coupled IMNNs (7) could be transformed into the following form:

$$\begin{cases} \dfrac{dx_i(t)}{dt} &= -x_i(t) + w_i(t) + u_{i1}(t), \\ \dfrac{dw_i(t)}{dt} &= -\Phi x_i(t) - \Psi w_i(t) + (A_0 + E_A\Delta_A(t)F_A) \\ &\quad f(x_i(t)) + (B_0 + E_B\Delta_B(t)F_B)f(x_i(t-\tau(t))) \\ &\quad -h\sum_{j=1}^{N} l_{ij}w_j(t) + u_{i2}(t), \end{cases} \tag{10}$$

where $A_0 = \frac{1}{2}(\overline{A}+\underline{A})$, $B_0 = \frac{1}{2}(\overline{B}+\underline{B})$, $A_1 = (\alpha_{ij})_{n\times n} = \frac{1}{2}(\overline{A}-\underline{A})$, $B_1 = (\beta_{ij})_{n\times n} = \frac{1}{2}(\overline{B}-\underline{B})$, $\Delta_A(t), \Delta_B(t) \in \Delta$ with

$$\Delta = \{\mathrm{diag}(\delta_{ij}) \in \mathbb{R}^{n^2\times n^2} : -1 \le \delta_{ij} \le 1, i,j = 1,...,n\},$$

$$E_A = (\sqrt{\alpha_{11}}e_1,,...,\sqrt{\alpha_{1n}}e_1,...,\sqrt{\alpha_{n1}}e_n,,...,\sqrt{\alpha_{nn}}e_n)_{n\times n^2},$$

$$F_A = (\sqrt{\alpha_{11}}e_1,,...,\sqrt{\alpha_{1n}}e_n,...,\sqrt{\alpha_{n1}}e_1,,...,\sqrt{\alpha_{nn}}e_n)^T_{n^2\times n},$$

$$E_B = (\sqrt{\beta_{11}}e_1,,...,\sqrt{\beta_{1n}}e_1,...,\sqrt{\beta_{n1}}e_n,,...,\sqrt{\beta_{nn}}e_n)_{n\times n^2},$$

$$F_B = (\sqrt{\beta_{11}}e_1,,...,\sqrt{\beta_{1n}}e_n,...,\sqrt{\beta_{n1}}e_1,,...,\sqrt{\beta_{nn}}e_n)^T_{n^2\times n},$$

where $e_i \in \mathbb{R}^n$ denotes the n-dimensional column vector with the ith element being 1 and the others being 0.

Similarly, the target trajectory could be written as follows:

$$\begin{cases} \dfrac{ds(t)}{dt} &= -s(t) + r(t), \\ \dfrac{dr(t)}{dt} &= -\Phi s(t) - \Psi r(t) + (A_0 + E_A\Delta_A^1(t)F_A)f(s(t)) \\ &\quad +(B_0 + E_B\Delta_B^1(t)F_B)f(s(t-\tau(t))), \end{cases} \tag{11}$$

where $\Delta_A^1(t), \Delta_B^1(t) \in \Delta$.

Remark 1: The coupled IMNNs (1) could be controlled to converge to the the target trajectory (5) as long as the system (10) could reach synchronization with (11). However, in the process of achieving synchronization, $\Delta_A(t)$ and $\Delta_A^1(t)$ may not be equal, so are $\Delta_B(t)$ and $\Delta_B^1(t)$.

Now, define the synchronization errors as follows: $\Xi = (\varepsilon_1^T(t),...,\varepsilon_N^T(t))^T$ and $\Omega = (\varpi_1^T(t),...,\varpi_N^T(t))^T$ with $\varepsilon_i(t) = x_i(t) - s(t)$, and $\varpi_i(t) = w_i(t) - r(t)$. Then, the error systems could be rewritten as follows:

$$\begin{cases} \dfrac{d\varepsilon_i(t)}{dt} &= -\varepsilon_i(t) + \varpi_i(t) + u_{i1}(t), \\ \dfrac{d\varpi_i(t)}{dt} &= -\Phi\varepsilon_i(t) - \Psi\varpi_i(t) + (A_0 + E_A\Delta_A(t)F_A) \\ &\quad f(\varepsilon_i(t)) + (B_0 + E_B\Delta_B(t)F_B)f(\varepsilon_i(t-\tau(t))) \\ &\quad +\Pi(t) - h\sum_{j=1}^{N} l_{ij}(\varpi_j(t) - \varpi_i(t)) + u_{i2}(t). \end{cases} \tag{12}$$

where $f(\varepsilon_i(t)) = f(x_i(t)) - f(s(t))$, $f(\varepsilon_i(t-\tau(t))) = f(x_i(t-\tau(t))) - f(s(t-\tau(t)))$, $\Pi(t) = E_A(\Delta_A(t) - \Delta_A^1(t))F_Af(s(t)) + E_B(\Delta_B(t) - \Delta_B^1(t))F_Bf(s(t-\tau(t)))$.

The neuron activation functions $f_i(\cdot)$ satisfies the following assumptions.

Assumption1 : There are positive constants M_i such that $|f_i(z)| \le M_i$ for all $z \in \mathbb{R}^n, i = 1,...,N$.

Assumption2 : There are positive constants l_i such that $0 \le \frac{f_i(x)-f_i(y)}{x-y} \le l_i$, $\forall x,y \in \mathbb{R}^n$, $i = 1,...,N$.

Before giving our main results, some necessary lemmas are introduced in the following.

Lemma 2.2: [26] Suppose Assumption 1 holds. Then, there at least exists a local solution $x(t)$ of system (1) with initial condition $\varphi(s) = (\varphi_1(s),...,\varphi_n(s))^T \in \mathbb{R}((-\tau,0],\mathbb{R}^n)$, and the local solution $x(t)$ could be extended to $[0,+\infty)$ in the Filippov's sense.

Lemma 2.3: [27], [28] Suppose the function $V(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ is C-regular, where $x(t) : [0,+\infty) \to \mathbb{R}^n$ is absolutely continuous on any compact interval $[0,+\infty)$. If there is a continuous function $\gamma : (0,+\infty) \to \mathbb{R}$, with $\gamma(\sigma) > 0$ for all $\sigma \in (0,+\infty)$, such that

$$\dot{V}(t) \le -\gamma(V(t)), \quad \text{and} \quad \int_0^{V(0)} \frac{1}{\gamma(\sigma)}d\sigma = T < +\infty.$$

Then, we have $V(t) = 0$ for $t \ge T$. If $\gamma(\sigma) = K\sigma^\mu$, $0 < \mu < 1$ and $K > 0$, then the settling time is estimated by

$$T = \frac{V^{1-\mu}(0)}{K(1-\mu)}.$$

Lemma 2.4: [29] Let $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$ be a continuous radically unbounded function. If the following two conditions hold: (i) : $V(x) = 0 \Leftrightarrow x = 0$;
(ii) : There exist some constants $a,b > 0$, $0 < p < 1$ and $q > 1$ such that the solution $e(t)$ of the error system (12) satisfies $\dot{V}(e(t)) \le -aV^p(e(t)) - bV^q(e(t))$.

Then, the origin of the error system (12) is fixed-time stable, and the settling time is estimated by

$$T \le T_{\max} \triangleq \frac{1}{a(1-p)} + \frac{1}{b(q-1)}. \tag{13}$$

Lemma 2.5: [30] If all the constants $a_1,a_2,...,a_n$ are positive numbers and $0 < p \le 1, q > 1$, one has

$$(\sum_{i=1}^{n} a_i^p) \ge (\sum_{i=1}^{n} a_i)^p, \quad (\sum_{i=1}^{n} a_i^q) \ge n^{1-q}(\sum_{i=1}^{n} a_i)^q.$$

III. Main Results

In this section, two controllers will be designed to solve the finite-time synchronization and fixed-time synchronization problem of IMNNs in Section 3-A and Section 3-B, respectively.

A. Finite-time synchronization of IMNNs

In this subsection, a controller is designed to achieve the synchronization in finite time as follows:

$$\begin{cases} u_{i1}(t) = -\xi\,\mathrm{sign}(\varepsilon_i(t)) - \theta_{i1}\varepsilon_i(t), \\ u_{i2}(t) = -(\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1 + \eta)\mathrm{sign}(\varpi_i(t)) - h\gamma_i\varpi_i(t), \end{cases} \tag{14}$$

where $\xi,\eta,h > 0$, $\theta_{i1} > 0$, $\theta_{i2} > 0$ and $\Upsilon = \mathrm{diag}(\gamma_1,...,\gamma_n)$ with $\gamma_i \ge 0$ denotes the pinning control matrix. $\mathrm{sign}(\varepsilon_i(t)) =$

$(\mathrm{sgn}(\varepsilon_{i1}(t)),...,\mathrm{sgn}(\varepsilon_{in}(t)))^T$, sgn($\dot{}$) denotes the sign function with

$$\mathrm{sgn}(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0.\end{cases} \quad (15)$$

Theorem 1 will give the sufficient conditions to reach the synchronization under the controller (15) in the finite time.

Theorem 3.1: Suppose Assumption 1 and 2 are satisfied and assume there are two positive constants $\rho(A)$ and $\rho(B)$ such that $\|A_0+E_A\Delta_A(t)F_A\|\le\rho(A)$ and $\|B_0+E_B\Delta_B(t)F_B\|\le\rho(B)$. The IMNNs (10) could synchronize with the target trajectory (11) in a finite time with the help of the controller (15), if $\theta_{i1}\ge\frac{1}{2}\max_{1\le i\le n}(1+c_i-d_i)^2$, $\theta_{i2}\ge\rho(B)l_i$, $h\lambda_1(H)\ge\max_{1\le i\le n}(2-d_i+\frac{1}{2}\rho^2(A)l_i^2)$ and $\eta>2M\|A_1+B_1\|$. In addition, the settling time could be estimated by $T=2V^{\frac{1}{2}}(0)/\mu$ with $\mu=\min\{\xi,\eta-2M\|A_1+B_1\|\}$.

Proof Consider the following non-smooth Lyapunov function as:

$$V(t)=V_1(t)+V_2(t), \quad (16)$$

where $V_1(t)=\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^T(t)\varepsilon_i(t)$ and $V_2(t)=\frac{1}{2}\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)$.

Calculating the derivative of $V_1(t)$ along the trajectories of the error system (12), one can get

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{n}\varepsilon_i^T(t)\dot{\varepsilon}_i(t)=\sum_{i=1}^{n}\varepsilon_i^T(t)[-\varepsilon_i(t)+\varpi_i(t)+u_{i1}(t)]\\ &\le -\sum_{i=1}^{n}(\frac{1}{2}+\theta_{i1})\varepsilon_i^T(t)\varepsilon_i(t)+\sum_{i=1}^{n}\frac{1}{2}\varpi_i^T(t)\varpi_i(t)\\ &\quad -\xi\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1.\end{aligned} \quad (17)$$

Similarly, the derivative of $V_2(t)$ is

$$\begin{aligned}\dot{V}_2(t) &= -\sum_{i=1}^{n}\varpi_i^T(t)\Phi\varepsilon_i(t)-\sum_{i=1}^{n}\varpi_i^T(t)\Psi\varpi_i(t)\\ &\quad+\sum_{i=1}^{n}\varpi_i^T(t)(A_0+E_A\Delta_A(t)F_A)f(\varepsilon_i(t))\\ &\quad+\sum_{i=1}^{n}\varpi_i^T(t)(B_0+E_B\Delta_B(t)F_B)f(\varepsilon_i(t-\tau(t)))\\ &\quad+\sum_{i=1}^{n}\varpi_i^T(t)\Pi(t)-h\sum_{i=1}^{n}\varpi_i^T(t)(\sum_{j=1}^{N}l_{ij}\varpi_j(t)\\ &\quad+\gamma_i\varpi_i(t))-\sum_{i=1}^{n}(\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1\\ &\quad+\eta)\varpi_i^T(t)\mathrm{sign}(\varpi_i(t))\end{aligned} \quad (18)$$

By using the Hölder inequality, one can get

$$\sum_{i=1}^{n}-\varpi_i^T(t)\Phi\varepsilon_i(t)\le\sum_{i=1}^{n}\frac{1}{2}\varpi_i^T(t)\varpi_i(t)+\sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^T(t)\Phi^2\varepsilon_i(t). \quad (19)$$

Under Assumption 1 and 2, one can have

$$\begin{aligned}&\sum_{i=1}^{n}\varpi_i^T(t)(A_0+E_A\Delta_A(t)F_A)f(\varepsilon_i(t))\\ &\le\sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^T(t)\varepsilon_i(t)+\sum_{i=1}^{n}\frac{1}{2}\rho^2(A)l_i^2\varpi_i^T(t)\varpi_i(t)\end{aligned} \quad (20)$$

$$\begin{aligned}&\sum_{i=1}^{n}\varpi_i^T(t)(B_0+E_B\Delta_B(t)F_B)f(\varepsilon_i(t-\tau(t)))\\ &\le\sum_{i=1}^{n}\rho(B)l_i\|\varpi_i(t)\|_1\|\varepsilon_i(t-\tau(t))\|_1,\end{aligned} \quad (21)$$

and

$$\begin{aligned}\|\Pi(t)\| &= \|E_A(\Delta_A(t)-\Delta_A^1(t))F_Af(s(t))\\ &\quad+E_B(\Delta_B(t)-\Delta_B^1(t))F_Bf(s(t-\tau(t)))\|\\ &\le 2\|E_AF_A+E_BF_B\|M=2M\|A_1+B_1\|.\end{aligned} \quad (22)$$

Then, one can get that

$$\sum_{i=1}^{n}\varpi_i^T(t)\Pi(t)\le 2M\|A_1+B_1\|\sum_{i=1}^{n}\|\varpi_i(t)\|_1 \quad (23)$$

and

$$\begin{aligned}&\sum_{i=1}^{n}\varpi_i^T(t)(\sum_{j=1}^{N}l_{ij}\varpi_j(t)+\gamma_i\varpi_i(t))\\ &=\Omega^T(H\otimes I_N)\Omega\ge\lambda_1(H)\Omega^T\Omega\\ &=\lambda_1(H)\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)\end{aligned} \quad (24)$$

$$\begin{aligned}&\sum_{i=1}^{n}\varpi_i^T(t)(\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1+\eta)\mathrm{sign}(\varpi_i(t))\\ &=\sum_{i=1}^{n}(\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1\|\varpi_i(t)\|_1+\eta\|\varpi_i(t)\|_1)\end{aligned} \quad (25)$$

Substituting (19)-(25) into (18), one can obtain

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t)+\dot{V}_2(t)\\ &\le -\sum_{i=1}^{n}\theta_{i1}\varepsilon_i^T(t)\varepsilon_i(t)+\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)\\ &\quad-\xi\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1+\sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^T(t)\Phi^2\varepsilon_i(t)\\ &\quad-\sum_{i=1}^{n}\varpi_i^T(t)\Psi\varpi_i(t)+\sum_{i=1}^{n}\frac{1}{2}\rho^2(A)l_i^2\varpi_i^T(t)\varpi_i(t)\\ &\quad+2M\|A_1+B_1\|\sum_{i=1}^{n}\|\varpi_i(t)\|_1-h\lambda_{\min}(H)\\ &\quad\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)-\sum_{i=1}^{n}(\theta_{i2}-\rho(B)l_i)\|\varpi_i(t)\|_1\|\\ &\quad\varepsilon_i(t-\tau(t))\|_1-\eta\sum_{i=1}^{n}\|\varpi_i(t)\|_1.\end{aligned} \quad (26)$$

Under the conditions in Theorem 1, one can get

$$\begin{aligned}\dot{V}(t) &\le -\xi\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1-(\eta-2M\|A_1+B_1\|)\sum_{i=1}^{n}\|\varpi_i(t)\|_1\\ &\le -\mu(\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1+\sum_{i=1}^{n}\|\varpi_i(t)\|_1)\le-\mu(V(t))^{\frac{1}{2}}\end{aligned} \quad (27)$$

where $\mu=\min\{\xi,\eta-2M\|A_1+B_1\|\}$.

Then, based on Lemma 2.3, we could estimate the settling time $T=2V^{\frac{1}{2}}(0)/\mu$. □

B. *Fixed-time synchronization of IMNNs*

This subsection investigates the fixed-time synchronization for IMNNs. The novel controllers $u_{i1}(t)$ and $u_{i2}(t)$ with the sign function $sign(\cdot)$ are designed as follows:

$$\begin{cases}u_{i1}(t) = -\xi\mathrm{sign}(\varepsilon_i(t))-\theta_{i1}\varepsilon_i(t)-\nu\mathrm{sig}^q(\varepsilon_i(t)),\\ u_{i2}(t) = -(\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1+\eta)\mathrm{sign}(\varpi_i(t))\\ \qquad\quad -\nu\mathrm{sig}^q(\varpi_i(t))-h\gamma_i\varpi_i(t),\end{cases} \quad (28)$$

where $\nu>0$ and $q>1$, the other parameters as in the controller (15).

Theorem 3.2: Suppose Assumption 1 and 2 are satisfied and assume there are two constants $\rho(A)>0$ and

$\rho(B)>0$ such that $\|A_0+E_A\Delta_A(t)F_A\|\leq\rho(A)$ and $\|B_0+E_B\Delta_B(t)F_B\|\leq\rho(B)$. Then, under the controller (28), the IMNNs (10) could synchronize with the target trajectory (11) in a finite time which is independent of the initial values, if $\theta_{i1}\geq\frac{1}{2}\max_{1\leq i\leq n}(1+c_i-d_i)^2$, $\theta_{i2}\geq\rho(B)l_i$, $h\lambda_1(H)\geq\max_{1\leq i\leq n}(2-d_i+\frac{1}{2}\rho^2(A)l_i^2)$ and $\eta>2M\|A_1+B_1\|$. Moreover, the settling time could be estimated by $T_{\max}=\frac{2}{\mu}+\frac{1}{\nu(q-1)}$ with $\mu=\min\{\xi,\eta-2M\|A_1+B_1\|\}$, $q>1$ and $\nu>0$.

Proof We also consider the same non-smooth Lyapunov function as the above as follows:

$$V(t)=V_1(t)+V_2(t), \tag{29}$$

where $V_1(t)=\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^T(t)\varepsilon_i(t)$ and $V_2(t)=\frac{1}{2}\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)$.

Then, under the controller (28), calculating the derivative of $V_1(t)$ and $V_2(t)$ along the trajectories of the error system (12), one can get

$$\begin{aligned}\dot V_1(t) &= \sum_{i=1}^{n}\varepsilon_i^T(t)\dot\varepsilon_i(t)=\sum_{i=1}^{n}\varepsilon_i^T(t)[-\varepsilon_i(t)+\varpi_i(t)+u_{i1}(t)]\\ &\leq -\sum_{i=1}^{n}(\frac{1}{2}+\theta_{i1})\varepsilon_i^T(t)\varepsilon_i(t)+\sum_{i=1}^{n}\frac{1}{2}\varpi_i^T(t)\varpi_i(t)\\ &\quad -\xi\textstyle\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1-\nu\displaystyle\sum_{i=1}^{n}\|\varepsilon_i(t)\|^{q+1}.\end{aligned} \tag{30}$$

Combining (19)-(25), one can get

$$\begin{aligned}\dot V_2(t) \leq\ & \sum_{i=1}^{n}\frac{1}{2}\varpi_i^T(t)\varpi_i(t)+\sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^T(t)\Phi^2\varepsilon_i(t)\\ &-\sum_{i=1}^{n}\varpi_i^T(t)\Psi\varpi_i(t)+\sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^T(t)\varepsilon_i(t)\\ &+\sum_{i=1}^{n}\frac{1}{2}\rho^2(A)l_i^2\varpi_i^T(t)\varpi_i(t)\\ &+\sum_{i=1}^{n}\rho(B)l_i\|\varpi_i(t)\|_1\|\varepsilon_i(t-\tau(t))\|_1\\ &+2M\|A_1+B_1\|\sum_{i=1}^{n}\|\varpi_i(t)\|_1\\ &-h\Omega^T(H\otimes I_N)\Omega-\textstyle\sum_{i=1}^{n}\theta_{i2}\|\varepsilon_i(t-\tau(t))\|_1\|\\ &\varpi_i(t)\|_1-\displaystyle\sum_{i=1}^{n}\eta\|\varpi_i(t)\|_1-\nu\sum_{i=1}^{n}\|\varpi_i(t)\|^{q+1}.\end{aligned} \tag{31}$$

Then, one has

$$\begin{aligned}\dot V(t) &\leq -\xi\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1-(\eta-2M\|A_1+B_1\|)\\ &\quad\sum_{i=1}^{n}\|\varpi_i(t)\|_1-\nu\sum_{i=1}^{n}(\|\varpi_i(t)\|^{q+1}-\nu\sum_{i=1}^{n}\|\varepsilon_i(t)\|^{q+1}\\ &\leq -\mu(\sum_{i=1}^{n}\|\varpi_i(t)\|_1+\sum_{i=1}^{n}\|\varepsilon_i(t)\|_1)\\ &\quad -\nu(\sum_{i=1}^{n}\|\varpi_i(t)\|^{q+1}+\sum_{i=1}^{n}\|\varepsilon_i(t)\|^{q+1})\\ &\leq -\mu(\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)+\sum_{i=1}^{n}\varepsilon_i^T(t)\varepsilon_i(t))^{\frac{1}{2}}\\ &\quad -\nu(\sum_{i=1}^{n}\varpi_i^T(t)\varpi_i(t)+\sum_{i=1}^{n}\varepsilon_i^T(t)\varepsilon_i(t))^{\frac{q+1}{2}}\\ &= -\mu V^{\frac{1}{2}}(t)-\nu V^{\frac{q+1}{2}}(t).\end{aligned} \tag{32}$$

Then, based on Lemma 2.4, the IMNNs (10) can synchronize with the target trajectory (11) in the fixed time and $T_{\max}=\frac{2}{\mu}+\frac{1}{\nu(q-1)}$. □

Remark 2: In [17], two sufficient synchronization criteria for IMNNs are obtained without controlling and the synchronization time is not limited. In this paper, two sufficient conditions for the finite-time and fixed-time synchronization of IMNNs are achieved by means of the signal function sign. Moreover, the larger the values of the parameters ξ,η,ν,q, the shorter the synchronization time.

IV. An illustrative example

This section will provide one simulation example to demonstrate the effectiveness of the theoretical results.

Example: Consider the coupled memristive inertial neural network system as follows:

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &= -D\frac{dx_i(t)}{dt}-Cx_i(t)+A(x_i(t))f(x_i(t))\\ &\quad +B(x_i(t))f(x_i(t-\tau(t)))-h\sum_{j=1}^{N}l_{ij}\\ &\quad \Gamma(\tfrac{dx_j(t)}{dt}+x_j(t)),\quad i=1,2,3,4\end{aligned} \tag{33}$$

and the target trajectory is

$$\begin{aligned}\frac{d^2s(t)}{dt^2} &= -D\frac{ds(t)}{dt}-Cs(t)+A(s(t))f(s(t))\\ &\quad +B(s(t))f(s(t-\tau(t))),\end{aligned} \tag{34}$$

where $D=\begin{pmatrix}1.8 & 0\\ 0 & 1.9\end{pmatrix}$, $C=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $\Gamma=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $f(x_i(t))=(\tanh(x_{i1}(t),\tanh(x_{i2}(t))^T$, $\tau(t)=\frac{e^t}{1+e^t}$, and the connection memristor-based weights matrices are listed as followed:

$$A(x_i(t))=\begin{bmatrix}a_{11}(x_{i1}) & a_{12}(x_{i2})\\ a_{21}(x_{i1}) & a_{22}(x_{i2})\end{bmatrix},$$

$$B(x_i(t))=\begin{bmatrix}b_{11}(x_{i1}) & b_{12}(x_{i2})\\ b_{21}(x_{i1}) & b_{22}(x_{i2})\end{bmatrix},$$

where the connection weights between memristors are given as:

$$a_{11}(x)=\begin{cases}1.7,|x|\leq 0.1,\\ 1.5,|x|>0.1,\end{cases}\quad a_{12}(x)=\begin{cases}1.8,|x|\leq 0.1,\\ 1,|x|>0.1,\end{cases} \tag{35}$$

$$a_{21}(x)=\begin{cases}1.2,|x|\leq 0.1,\\ 0.5,|x|>0.1,\end{cases}\quad a_{22}(x)=\begin{cases}0.8,|x|\leq 0.1,\\ 1,|x|>0.1,\end{cases} \tag{36}$$

$$b_{11}(x)=\begin{cases}-1.5,|x|\leq 0.1,\\ -1.2,|x|>0.1,\end{cases}\quad b_{12}(x)=\begin{cases}1.0,|x|\leq 0.1,\\ 0.8,|x|>0.1,\end{cases} \tag{37}$$

$$b_{21}(x)=\begin{cases}0.8,|x|\leq 0.1,\\ 1,|x|>0.1,\end{cases}\quad b_{22}(x)=\begin{cases}-1.4,|x|\leq 0.1,\\ -1.6,|x|>0.1.\end{cases} \tag{38}$$

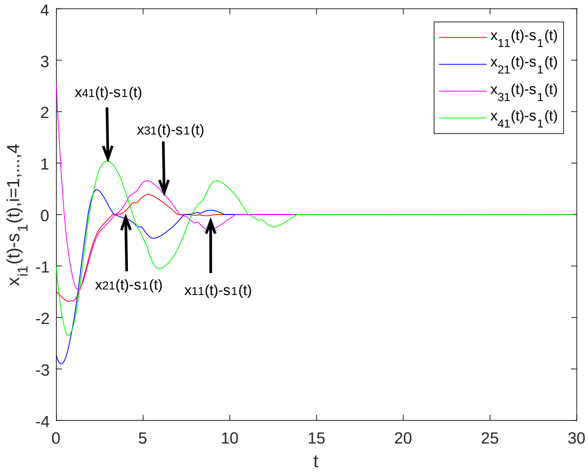
Then, we could calculate that there are $\rho(A)=2.8515$ and $\rho(B)=2.3548$ such that $\|A_0+E_A\Delta_A(t)F_A\|\leq\rho(A)$ and $\|B_0+E_B\Delta_B(t)F_B\|\leq\rho(B)$. And one can get $\|A_1+B_1\|=0.702$. Moreover, it is easy to verify that the activation functions $f_i(\cdot)$ satisfy Assumption 1 with $M_i=1,l_i=1$, $(i=1,2)$. The according Laplacian matrix is considered as follows:

$$L=\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\\ -1 & 0 & 2 & -1\\ 0 & -1 & -1 & 2\end{bmatrix}.$$

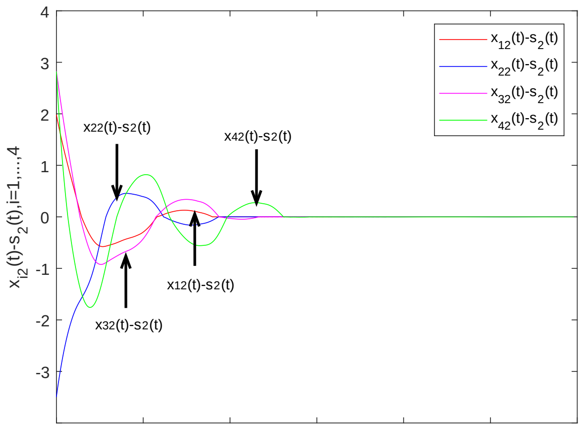
The IMNNs cannot synchronize with the target trajectory without control inputs from the initial values $x_1(t)=$

$(1.1, 0.2)^T$, $x_2(t) = (0.1, -6.1)^T$, $x_3(t) = (6.5, 1.2)^T$, $x_4(t) = (2.3, 2.1)^T$, $s(t) = (2.5, -2.1)^T$ $\forall t \in [-1, 0)$.

Pick $\xi = 2.5$, $\theta_{i1} = 0.5$, $\theta_{i2} = 2.5$, $\eta = 2.5$, $h = 12$ in the controller (33). Let $\gamma_1 = \gamma_2 = 1, \gamma_3 = \gamma_4 = 0$, that is, the first and second nodes are pinned. Then, we can calculate $\lambda_1(H) = 0.3820$. It can be easily checked that all the conditions in Theorem 3.1 are satisfied. Therefore, (33) could synchronize with the target trajectory (34) in a finite time under the controller (15). The time evolutions of the synchronization errors are shown in Fig.1 from the initial values $x_1(t) = (1.1, 0.2)^T$, $x_2(t) = (0.1, -6.1)^T$, $x_3(t) = (6.5, 1.2)^T$, $x_4(t) = (2.3, 2.1)^T$, $s(t) = (2.5, -2.1)^T$ $\forall t \in [-1, 0)$.



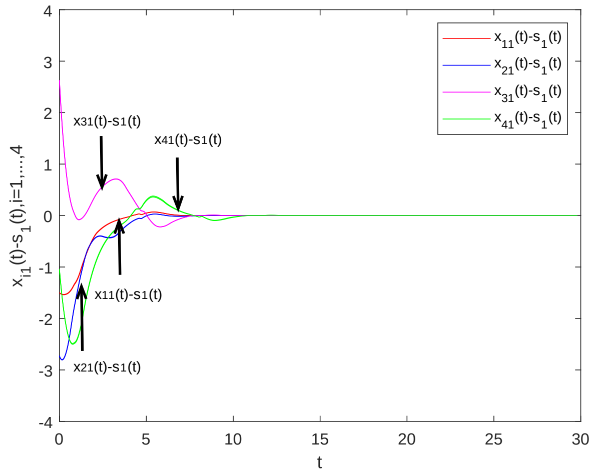
(a)



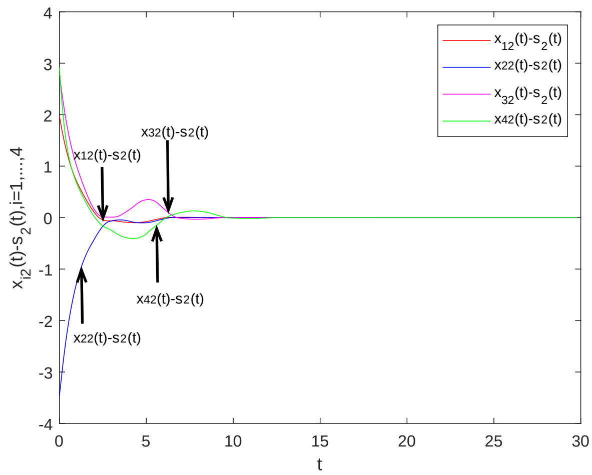
(b)

Fig. 1: The dynamics of the errors between (33) and (34) under the controller (15)

Similarly, under the controller (28), choose the parameters $\xi, \theta_{i1}, \theta_{i2}, \eta, h$ as in the controller (15). The pinned nodes and the network topology are the same as the above. Apparently, all the conditions in Theorem 3.2 are still held. Therefore, (33) could synchronize with the target trajectory (34) in a finite time which is independent of the initial values. The time evolutions of the errors are shown in Fig.2 from the initial values $x_1(t) = (1.1, 0.2)^T$, $x_2(t) = (0.1, -6.1)^T$, $x_3(t) = (6.5, 1.2)^T$, $x_4(t) = (2.3, 2.1)^T$, $s(t) = (2.5, -2.1)^T$ $\forall t \in [-1, 0)$.



(a)



(b)

Fig. 2: The dynamics of the errors between (33) and (34) under the controller (28)

V. Conclusion

This paper investigates the finite-time synchronization and fixed-time synchronization for the CIMNNs with time-varying delays. To deal with the mismatched state-dependent parameters, an uncertain interval parametric system is built with the help of the differential inclusions theory and the interval matrix theory. Then, we design two novel controllers with sign function and abtain two sufficient conditions which could make the CIMNNs synchronize with the target trajectory in the finite and fixed time. Finally, the effectiveness of the obtained results is illustrated by the numerical simulation example.

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Jie Gao received the B.S. degree in information and computing science from Xiangtan University in 2004, the M.S. degree in computational mathematics from Wuhan University in 2006, and the Ph.D. degree in applied mathematics from the University of Electronic Science and Technology in 2018. She is an associate professor in Southwest Petroleum University in Sichuan. Her research interests include synchronization of memristive neural networks and event-triggered control of multi-agent systems.

Yuming Chen received the B.S. degree in Materials Science and Engineering from Southwest Petroleum University in 2018. His research interests include synchronization of complex networks and event-triggered control of multi-agent systems.