Reliability Bayesian Analysis in Multicomponent Stress–Strength for Generalized Inverted Exponential Using Upper Record Data

Amal S. Hassan, Doaa M. Ismail, Heba F. Nagy

Abstract— **Stress–strength (SS) models are particularly relevant in the literature on reliability and engineering applications. In this study, we focus on multicomponent SS (MSS) reliability estimation, say $\Re_{c,t}$, which contains (*t*) independent and identically distributed (iid) random strength elements $Y_1, Y_2, ..Y_t$. Assume that *c* of *t* components of the system survive common random stress *X*, where ($1 \le c \le t$), and both the stress and strength variables follow the generalized inverted exponential distribution (GIED). The maximum likelihood and Bayesian approaches of estimating $\Re_{c,t}$ are considered via upper record values (URVs) for different sample sizes (*d*, *f*). The behavior of estimates is evaluated using a simulation study based on absolute bias (ABs) and mean squared error (MSEs) values in classical estimating. The Bayesian and credible interval estimators of $\Re_{c,t}$ are created using gamma prior distributions via squared error (SE) and linear exponential loss functions (LINEX). The Markov Chain Monte Carlo (MCMCO) method is used because there are no specific techniques for computing the proposed Bayesian estimates (BEs). Estimated risks (ERs) and average lengths (ALs) are also applied in a simulation analysis to test the precision of estimates, followed by an example of real datasets to validate results. Through simulation experiments, we discovered that accuracy measures decrease with an increase in the number of records.**

Index Terms— **Bayesian inference, Generalized inverted exponential model, Stress–strength model, Upper record values.**

I. INTRODUCTION

When the complete arrangements of perceptions are urgent to gather or a few perceptions have been stopped throughout an investigation, the benefit of record regard hypothesis appears. Many real-world situations, such as weather, sports, economics, and life testing, require the study of record values. In industry, various components break down under load. As an example, a wooden board cracks when enough load is exerted on it; additionally, under the strain of time, a battery will expire. However, the collapse pressure or failure point differs, even between identical items. Consequently, measurements can be performed consecutively in such experiments, with only values less (or greater) than all preceding values being recorded. Consequently, the number of measurements made is significantly less than the total sample size.

If $Z_1, Z_2, ...$ is an infinite series of independent and identically distributed (iid) random variables with a probability density function (pdf) $h(z)$ and a cumulative distribution function (cdf) $H(z)$, then observation Z_i is an upper record value (URV) if its value is greater than all preceding observations, i.e., $Z_i > Z_j$ for every $i < j$.

Suppose $\underline{u} = (u_1, u_2, ..., u_d)^T_{1\times d}$ correspond to the first (*d*) URVs from cdf $H(.;\theta)$ and pdf $h(.;\theta)$, then the joint distribution of the first (*d*) URVs (see [1]) is defined by

$$f(\underline{u};\theta) = f(u_d)\prod_{i=1}^{d-1}\frac{f(u_i;\theta)}{1-F(u_i;\theta)}, -\infty < u_1 < u_2 < ... < u_d < \infty. \quad (1)$$

The stress–strength (SS) model is one of the most basic tools in reliability testing. The SS structure $\Re = P(X < Y)$ gives the likelihood of failure when stress *X* exceeds strength *Y*. That is, it continues to function provided stress does not exceed strength. This concept was first introduced in [2] and then refined in [3]. Many researchers have addressed inferences using various methodologies and distributional assumptions (see, e.g., [4–9]).

The basic idea of $\Re = P(X < Y)$ can be changed to make a system with two or more components. Reliability was first developed in a multicomponent SS (MSS) model in [10] assuming that *c* of *t* components of the system survive a common random stress *X*, where ($1 \le c \le t$).

The MSS has several viable applications in many fields, such as logistical and military systems, communications, and industrial systems. For instance, in a vehicle with a V-8 motor, it might be conceivable to drive it if only four of the cylinders are gunfire. Thus, it can be denoted as a 4 out of 8: G system. Assume that a system with *t* similar components works if *c* or more of the components work together, where $1 \le c \le t$. The system is put under stress *X* in its working environment, which is a random variable with cdf $F(x)$. If

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the component strengths or minimum pressures required to cause manufacturing failure are iid random variables with cdf $G(y)$, the c-out-of-t system reliability, denoted by $\Re_{c,t}$, which was developed in [10], is given by

$$R_{c,t} = p\left[at\ least\ c\ of\ the\ (Y_1,Y_2,...,Y_t)\ exceed\ X\ \right] = \sum_{i=c}^{t}\binom{t}{i}\int_{-\infty}^{\infty}[1-G(y)]^{i}G(y)^{t-i}\,dF(y). \quad (2)$$

Other examples include an electrical power plant that produces the correct quantity of energy only if at least six of eight generating items are operational; a region's energy requirement is met only if six of eight wind roses are operational at all times. Many scholars have studied reliability estimation in MSS models based on complete samples for various SS distributions and sampling strategies, including exponentiated Pareto [11], exponentiated Weibull [12], proportional-reversed hazard rate model [13], generalized Pareto [14], inverse Chen [15], Chen [16], power Lindley [17], Topp-Leone [18], and inverted exponentiated Rayleigh [19], based on complete samples. Moreover, there are only a few works that are based on record values [20–22].

In this study, we focus on an MSS model that has t iid strength components and common stress. We assume that the strength and stress variables Y and X have a common scale parameter and follow a generalized inverted exponential distribution (GIED). URVs are used to discuss the estimation of MSS systems using maximum likelihood (ML) and Bayesian frameworks. We iteratively obtain the estimator of $\Re_{c,t}$ via the ML technique. In the Bayesian approach, the Markov Chain Monte Carlo (MCMC) method is used to derive Bayesian estimators of $\Re_{c,t}$ and Bayesian credible intervals. Finally, genuine datasets are provided for demonstration purposes with the analysis of the monthly concentration of sulfur dioxide in Long Beach, California. We study the validity of the MSS model for two datasets and show that GIED fits well for both datasets.

The remainder of this article is organized as follows. Section II introduces the expression for $\Re_{c,t}$ and its ML estimator. Section III considers Bayesian estimators for $\Re_{c,t}$ and the MCMC method for independent gamma priors within squared error (SE) and linear exponential (LINEX) loss functions as well as the highest posterior density credible intervals. In Section IV, real-world datasets are used to demonstrate theoretical outcomes. In Section V, we summarize the results and conclude this study.

II. MODEL DESCRIPTION & CLASSICAL ESTIMATION OF $\Re_{c,t}$

In this section, we derive the $\Re_{c,t}$ formulation for the MSS system for GIED. The ML estimator is derived from a URV. A numerical analysis is also presented.

A. Expression of $\Re_{c,t}$

The exponential distribution (ED) is a well-known lifetime model in reliability and survival analysis. In the literature, there are several types of EDs and their generalizations. Reference [23] proposed the GIED, a generalization of the inverted ED capable of modeling multiple failure rate forms. Several researchers have discussed the GIED's studies and applications (see, e.g., [24–28]). The GIED's cdf is given by

$$F(x;\alpha,\lambda) = 1-\left(1-e^{-\lambda/x}\right)^{\alpha} \qquad ;x>0,\alpha>0,\lambda>0, \quad (3)$$

where α and λ are the shape and scale parameters, respectively. The associated pdf is given by

$$f(x;\alpha,\lambda) = \frac{\alpha\lambda}{x^2}\left(1-e^{-\lambda/x}\right)^{\alpha-1} e^{-\lambda/x} \ ;x>0,\alpha>0,\lambda>0. \quad (4)$$

Let $F(x)$ be the cdf of the stress variable X and assume that the t strength components of an MSS system are iid with the common cdf $G(y)$. We also assume that X ~ GIED (α,λ) and Y ~ GIED (β,λ), hence the MSS model's reliability can be calculated as follows:

$$\begin{aligned}\Re_{c,t} &= \sum_{i=c}^{t}\binom{t}{i}\int_{0}^{\infty}[1-e^{-(\lambda/y)}]^{\alpha i+\beta-1}[1-(1-e^{-(\lambda/y)})^{\alpha}]^{t-i}\frac{\beta\lambda}{y^2}e^{-(\lambda/y)}dy \\ &= \rho\sum_{i=c}^{t}\binom{t}{i}\int_{0}^{1}z^{i+\rho-1}[1-z]^{t-i}dz \\ &= \rho\sum_{i=c}^{t}\binom{t}{i}B(\rho+i,t-i+1),\end{aligned} \quad (5)$$

where $\rho=\dfrac{\beta}{\alpha}$ and $B(.,.)$ is the beta function. Notably, Expression (5) is a function of α and β.

B. ML ESTIMATOR of $\Re_{c,t}$

When the existing data are drawn from a URV, we find the ML estimator of $\Re_{c,t}$. To acquire the ML estimator of $\Re_{c,t}$, we must first obtain the ML estimators of α and β. Now, let $\underline{u}=(u_1,u_2,...,u_d)$, and $\underline{v}=(v_1,v_2,...,v_f)$ be the observed URVs from GIED with parameters (α,λ) and (β,λ), respectively, where λ is known. The likelihood function of the observed samples of records $\underline{u}$ is obtained by substituting (3) and (4) into (1) as follows:

$$\ell_1(\underline{u}\mid\alpha) = \alpha^d\lambda^d(1-e^{-\lambda/u_d})^{\alpha}\prod_{i=1}^{d}e^{-\lambda/u_i}\left\{u_i^{2}(1-e^{-\lambda/u_i})\right\}^{-1}. \quad (6)$$

Similarly, the likelihood function of the obtained samples of records $\underline{v}=(v_1,v_2,...,v_f)$ is given by

$$\ell_2(\underline{v}\mid\beta) = \beta^f\lambda^f(1-e^{-\lambda/v_f})^{\beta}\prod_{j=1}^{f}e^{-\lambda/v_j}\left\{v_j^{2}(1-e^{-\lambda/v_j})\right\}^{-1}. \quad (7)$$

Consequently, the joint likelihood function of the obtained record values $\underline{u}$ and $\underline{v}$ denoted by $\ell(\underline{u},\underline{v}\mid\alpha,\beta)$ is given by

$$\ell(\underline{u},\underline{v}\mid\alpha,\beta) = \alpha^d\beta^f\lambda^{d+f}\ (1-e^{-\lambda/v_f})^{\beta}\ (1-e^{-\lambda/u_d})^{\alpha} \prod_{i=1}^{d}e^{-\lambda/u_i}\left\{u_i^{2}(1-e^{-\lambda/u_i})\right\}^{-1}\prod_{j=1}^{f}e^{-\lambda/v_j}\left\{v_j^{2}(1-e^{-\lambda/v_j})\right\}^{-1}. \quad (8)$$

The joint log-likelihood function, say ℓ^*, is obtained as follows:

$$\ell^* = d\ln(\alpha) + f\ln(\beta) + (d+f)\ln(\lambda) + \beta\ln(1-e^{-\lambda/v_f}) + \alpha\ln(1-e^{-\lambda/u_d}) - \sum_{i=1}^{d}\left[(\lambda/u_i) + 2\ln(u_i) + \ln(1-e^{-\lambda/u_i})\right] - \sum_{j=1}^{f}\left[(\lambda/v_j) + 2\ln v_j + \ln(1-e^{-\lambda/v_j})\right]. \quad (9)$$

Considering that λ is a known scale parameter, the partial derivatives of ℓ^* with respect to α and β are obtained, respectively, as follows:

$$\frac{\partial\ell^*}{\partial\alpha} = \ln\left(1-e^{-\lambda/u_d}\right) + \frac{d}{\alpha}, \quad \frac{\partial\ell^*}{\partial\beta} = \ln\left(1-e^{-\lambda/v_f}\right) + \frac{f}{\beta}. \quad (10)$$

Then, the ML estimators of α and β, represented by $\hat{\alpha}$ and $\hat{\beta}$, are obtained by, respectively, setting $\partial\ell^*/\partial\alpha$ and $\partial\ell^*/\partial\beta$ to be zero. In this study, $\hat{\alpha}$ and $\hat{\beta}$ are obtained as follows:

$$\hat{\alpha} = \frac{-d}{\ln\left(1-e^{-\lambda/u_d}\right)}, \hat{\beta} = \frac{-f}{\ln\left(1-e^{-\lambda/v_f}\right)}. \quad (11)$$

Therefore, based on the invariance property, the ML estimator of $\Re_{c,t}$ is derived by inserting $\hat{\alpha}$ and $\hat{\beta}$ in (5) as follows:

$$\hat{\Re}_{c,t} = \frac{\hat{\beta}}{\hat{\alpha}}\sum_{i=c}^{t}\binom{t}{i}B\left(\frac{\hat{\beta}}{\hat{\alpha}}+i, t-i+1\right) \quad (12)$$

C. NUMERICAL STUDY

Now, we conduct a comprehensive mathematical analysis of the ML estimate (MLE) for the strength and stress random variables at various parameter values and numbers of records. The most common accuracy assessments are the absolute biases (ABs) and mean squared errors (MSEs). The numerical study is conducted as follows.

- Create URV samples on the basis of the provided parameter values.

TABLE I
SELECTED VALUES of (α,β) and THE CORRESPONDING $\Re_{1,3}$ and $\Re_{2,4}$

(α,β)	$\Re_{1,3}$	$\Re_{2,4}$	(α,β)	$\Re_{1,3}$	$\Re_{2,4}$
(2.5,1)	0.475	0.332	**(0.5,0.5)**	0.75	0.6
(1.5,3)	0.9	0.8	**(0.7,0.5)**	0.653	0.495
(2,3)	0.848	0.723	**(0.3,1.5)**	0.982	0.952
(3,3)	0.75	0.6	**(0.6,1.7)**	0.944	0.875
(1,2)	0.9	0.8	**(0.8,2)**	0.931	0.851
(1.75,1)	0.584	0.428	**(0.4,1.2)**	0.95	0.886
(2,2)	0.75	0.6	**(1.2,0.3)**	0.344	0.228
(2,1.5)	0.668	0.51	**(1.1,0.9)**	0.693	0.537
(0.1,0.5)	0.982	0.952	**(2.5,0.6)**	0.333	0.22
(0.3,0.5)	0.869	0.752	**(2,0.4)**	0.29	0.188

- The URVs of MSS variables for (*d*, *f*) are selected to be (5, 5), (10, 5), (5, 10), (10, 10), (10, 15), (15, 10), and (15, 15).
- After obtaining the MLEs of α and β from (11) and replacing $\hat{\alpha}$ and $\hat{\beta}$ in (12), the MLE of $\Re_{c,t}$ is obtained.
- The ABs and MSEs of $\Re_{c,t}$ estimates are calculated with 5,000 repetitions.
- Numerical results are described in Tables II–VI and graphed in Figs. 1–6.
- The MSEs of $\hat{\Re}_{c,t}$ for $(c,t)=(2,4)$ are larger than those for $(c,t)=(1,3)$ with various values of (α,β) (Tables III–VI).
- At different actual values of (α,β), the MSEs of $\hat{\Re}_{c,t}$ for $(c,t)=(1,3)$ and (2, 4) reduce as the number of identical records (*i.e* $d=f$) increases (see Figs. 1 and 5).
- Fig. 2 illustrates that the MSEs of $\hat{\Re}_{c,t}$ at $(c,t)=(1,3)$ are less than the MSEs of $\hat{\Re}_{c,t}$ at $(c,t)=(2,4)$ for (α,β) = (2.5, 1).
- The MSEs of $\hat{\Re}_{c,t}$ at $(c,t)=(2,4)$ for different values of (α,β) decrease as the number of records increases (Fig. 3).
- The MSEs of $\hat{\Re}_{c,t}$ at $\alpha<\beta$ are less than those of $\hat{\Re}_{c,t}$ at $\alpha>\beta$ for selected values of (c,t) and at $d=f$.
- For $\alpha\ge\beta$, the MSEs of $\hat{\Re}_{c,t}$ for $d<f$ are less than those of $\hat{\Re}_{c,t}$ for $d>f$ for both values of (c,t) (Tables II and III).
- Figs. 4 and 6 demonstrate that the ABs of $\hat{\Re}_{c,t}$ for selected values of (α,β) decrease as the similar number of records increases.
- For $\alpha,\beta<1$, except at $(\alpha,\beta)=(0.1,0.5)$, the MSEs and ABs of $\hat{\Re}_{c,t}$ for $d>f$ are less than those of $\hat{\Re}_{c,t}$ for $d<f$ at $(c,t)=(1,3)$ (Table IV).
- For $\alpha<1,\beta>1$, the MSEs of $\hat{\Re}_{c,t}$ for $d<f$ are smaller than those of $\hat{\Re}_{c,t}$ for $d>f$ for both values of (c,t) (Table V).
- For $\alpha<1,\beta>1$, the MSEs of $\hat{\Re}_{c,t}$ for $(\alpha,\beta)=(0.3,1.5)$ have the least values for $d=f$ at $(c,t)=(2,4)$, (see (Fig. 7).
- For $\alpha>1,\beta<1$, the MSEs of $\hat{\Re}_{c,t}$ for $(\alpha,\beta)=(2,0.4)$ have the least values (c,t) or both values of $d\ne f$ (Fig. 8).

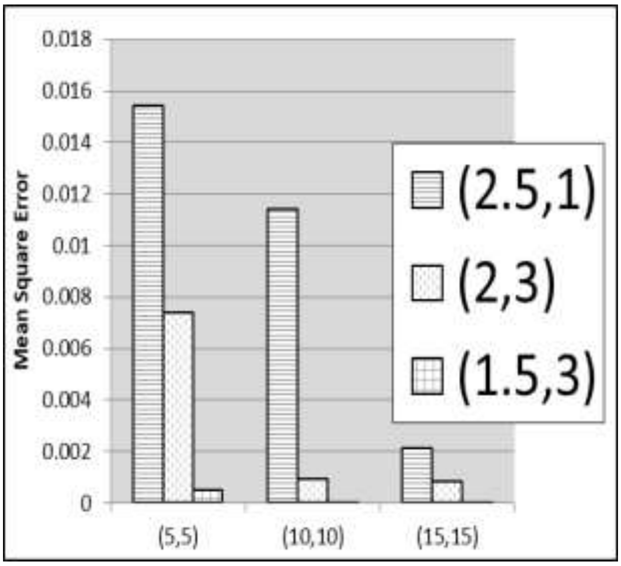

Fig. 1. MSEs of $\hat{\Re}_{c,t}$ for different (α,β) values at (c, t)=(1.3), $d = f$

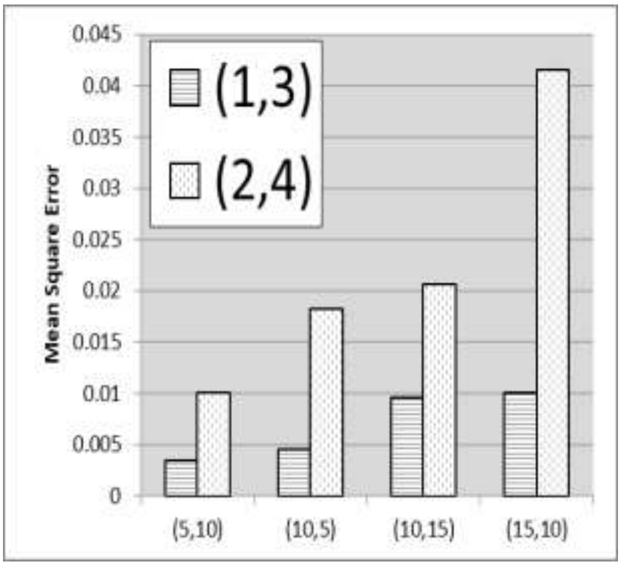

Fig. 2. MSEs of $\hat{\Re}_{c,t}$ for $(\alpha,\beta)=(2.5,1)$ at $(c,t)=(1,3)$ $(c,t)=(2,4)$ and $d \neq f$

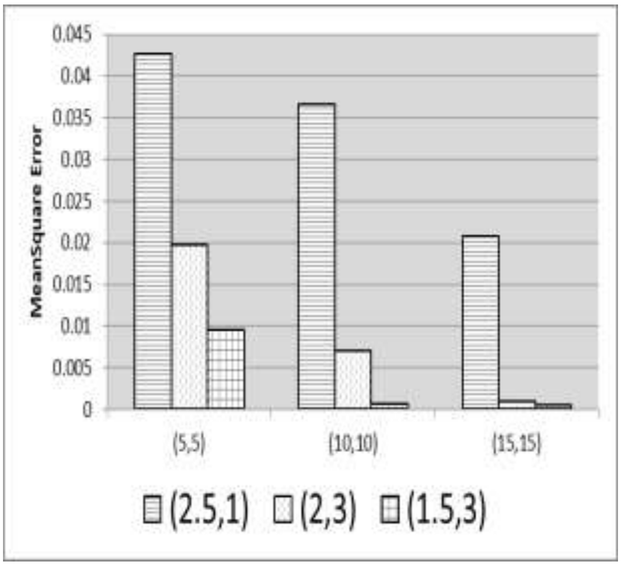

Fig. 3. MSEs of $\hat{\Re}_{c,t}$ for different (α,β) values at $(c,t)=(2,4)$ and $d = f$

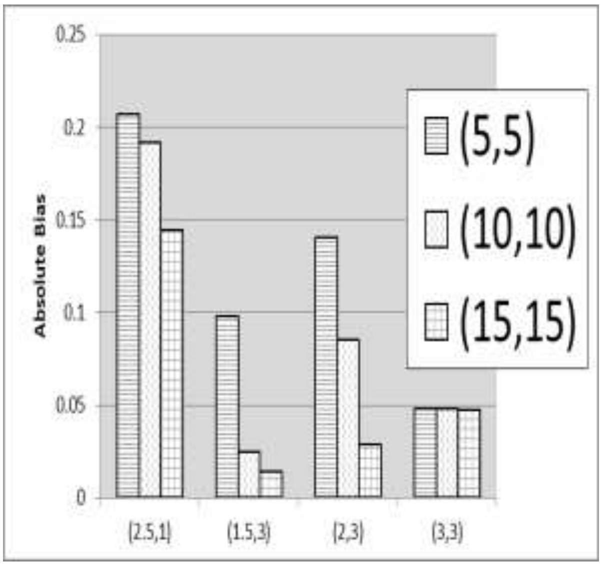

Fig. 4. ABs of $\hat{\Re}_{c,t}$ for different (α,β) values $(c,t)=(2,4)$ and $d = f$

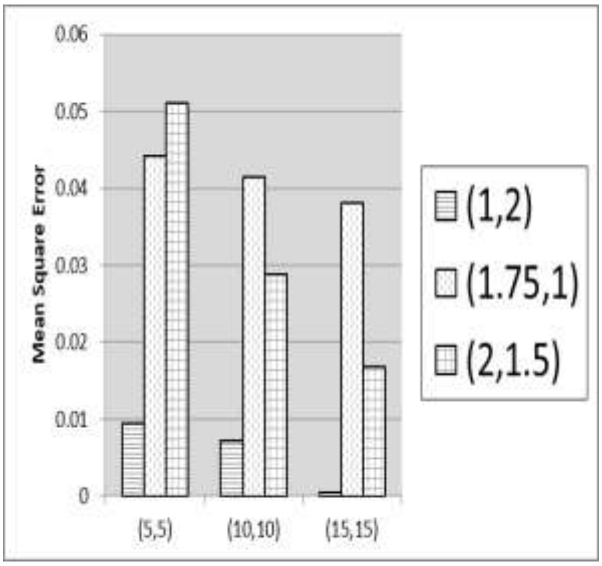

Fig. 5. MSEs of $\hat{\Re}_{c,t}$ for different (α,β) values at $(c,t)=(2,4)$ and $d = f$

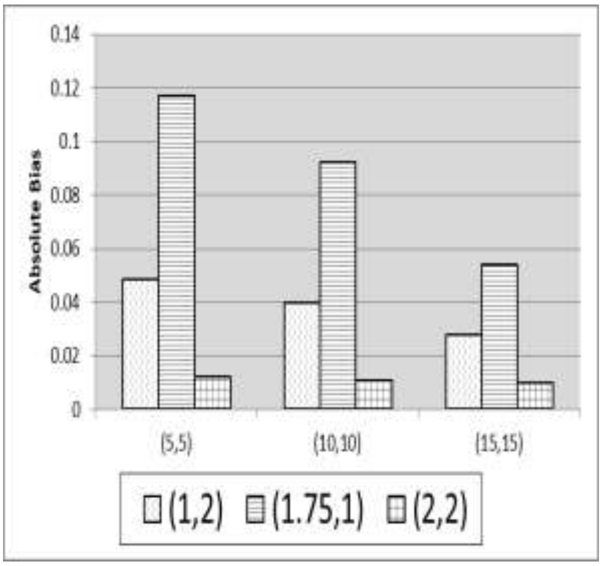

Fig. 6. ABs of $\hat{\Re}_{c,t}$ for different (α,β) values at $(c,t)=(1,3)$ and $d = f$

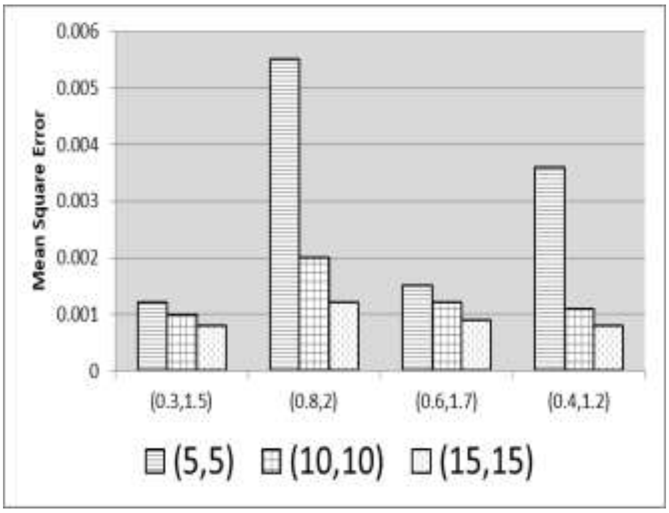

Fig. 7. MSEs of $\hat{\Re}_{c,t}$ for different (α,β) values at $(c,t)=(2,4)$ and $d=f$

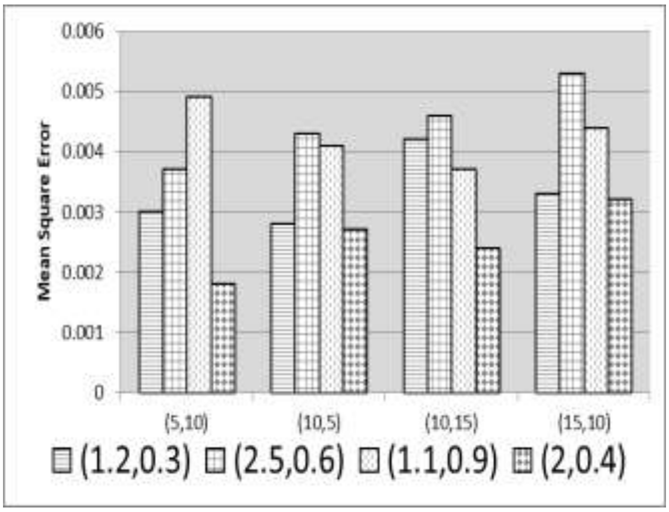

Fig. 8. MSEs of $\hat{\Re}_{c,t}$ for different (α,β) values at $(c,t)=(1,3)$ and $d\neq f$

III. BAYESIAN ESTIMATORS OF $\Re_{c,t}$

In this section, we seek the Bayesian estimator of $\Re_{c,t}$ under the assumption that the random variables α and β have gamma prior distributions with the following pdfs:

$$\pi_1(\alpha)\propto\alpha^{a-1}e^{-b\alpha},\ \pi_2(\beta)\propto\beta^{c-1}e^{-k\beta}$$

where *a*, *b*, *c*, and *k* are assumed to be known. Assuming the independence of parameters, the joint prior distribution of parameters α and β is as follows:

$$\pi(\alpha,\beta)=\alpha^{a-1}\beta^{c-1}e^{-(b\alpha+k\beta)}.$$

Given the observed sample, the joint density function of parameters is calculated as follows:

$$\pi(\alpha,\beta\,|\,\underline{u},\underline{v})=\ell(\underline{u},\underline{v}\,|\,\alpha,\beta)\pi(\alpha,\beta)=$$
$$\alpha^{d+a-1}\beta^{f+c-1}\lambda^{d+f}\,e^{-(b\alpha+k\beta)}(1-e^{-\lambda/v_f})^{\beta}\,(1-e^{-\lambda/u_d})^{\alpha} \quad (13)$$
$$\prod_{i=1}^{d}e^{-\lambda/u_i}\left\{u_i^{\,2}(1-e^{-\lambda/u_i})\right\}^{-1}\prod_{j=1}^{f}e^{-\lambda/v_j}\left\{v_j^{\,2}(1-e^{-\lambda/v_j})\right\}^{-1}.$$

Thus, the posterior pdf of α and β can be expressed as

$$\pi^*(\alpha,\beta\,|\,\underline{u},\underline{v})=\frac{\ell(\underline{u},\underline{v}\,|\,\alpha,\beta)\pi(\alpha,\beta)}{\int_0^\infty\int_0^\infty\ell(\underline{u},\underline{v}\,|\,\alpha,\beta)\pi(\alpha,\beta)d\alpha d\beta}. \quad (14)$$

The Bayesian estimator of $\Re_{c,t}$, denoted by $\tilde{\Re}_{c,t}$, is its posterior mean, which is produced by assuming the SE loss function.

$$\tilde{\Re}_{c,t}=E(\Re_{c,t}\,|\,\underline{u},\underline{v})=\int_0^\infty\int_0^\infty\Re_{c,t}\,\pi^*(\alpha,\beta\,|\,\underline{u},\underline{v})d\alpha d\beta. \quad (15)$$

Furthermore, the Bayesian estimator of $\Re_{c,t}$ for the LINEX loss function, denoted by $\dddot{\Re}_{c,t}$ is as follows:

$$\dddot{\Re}_{c,t}=\frac{-1}{w}\log E(e^{-w\Re_{c,t}})$$
$$=\frac{-1}{w}\log\left(\int_0^\infty\int_0^\infty e^{-w\Re_{c,t}}\,\pi^*(\alpha,\beta\,|\,\underline{u},\underline{v})d\alpha d\beta\right),\qquad w\neq 0. \quad (16)$$

An explicit expression for the Bayesian estimator of $\Re_{c,t}$ is difficult to realize because the posterior density function $\pi^*(\alpha,\beta\,|\,\underline{u},\underline{v})$ has a composite form. Consequently, we estimate these integrations using the MCMC algorithm within the SE and LINEX loss functions, and the Metropolis-Hastings (M–H) technique will be used to calculate Bayes estimates (BEs) and credible interval widths.

A. MCMCO TECHNIQUE

The MCMC simulation is used to investigate the behavior of $\Re_{c,t}$'s MSS. Gamma priors are used to create BEs in the SE and LINEX loss functions. $\Re_{c,t}$'s BE accuracy is measured using the ABs, estimated risks (ERs), and average lengths (ALs). The different choices of URV are (*d*, *f*) = (5, 5) (5, 10), (10, 5), (10, 10), (10, 15), (15, 10), and (15, 15).

The true values for $(c,t)=(1,3)$ are 0.75, 0.85, 0.9, and 0.67 and for $(c,t)=(2,4)$ are 0.6, 0.72, 0.8, and 0.51, The following are five possible sets of hyper-parameter values to be considered: Prior I: $(a,b),(c,k)=(4,1),(4,1)$

Prior II: $(a,b),(c,k)=(3,2),(2,4)$

Prior III: $(a,b),(c,k)=(6,1),(4,1.5)$

Prior IV: $(a,b),(c,k)=(2,2),(3,3)$

Prior V: $(a,b),(c,k)=(4,4),(2,2)$.

Additionally, we take $(w=-2,2)$. The results are based on 5,000 replications in total. In the Bayesian literature, the M–H process is a common subcategory of the MCMC technique for modeling deviations from the posterior density and providing good predicted results. The key issue with using the MCMC is simulating samples from the posterior density and then using the M–H technique to generate the BEs of $\Re_{c,t}$ inside SE and LINEX loss functions. Using acceptance/rejection criteria, it converges to the target distribution. According to [29], the M–H algorithm works as follows.

a) Specify the number of samples *N* and a beginning parameter value $\Re_{c,t}^{0}$.

b) For *i* = 2 to N, set $\Re_{c,t}=\Re_{c,t}^{i-1}$.

c) Generate *u* from $\text{uniform}(0,1)$.

d) From the proposal density, select a candidate parameter $\Re_{c,t}^{*}$.

e) If $u \le \dfrac{\pi(\theta^*)\, g(\theta \mid \theta^*)}{\pi(\theta)\, g(\theta^* \mid \theta)}$, then set $\Re^i_{c,t} = \Re^*_{c,t}$; otherwise, set $\Re^i_{c,t} = \Re_{c,t}$.

f) Go back to stage (b) and repeat the preceding steps N times with $i = i + 1$.

B. SIMULATED RESULTS

The simulated conclusions are documented in Tables VII–XI and described using Figs. 9–12; therefore, we draw the following conclusions:

- At true value $\Re_{c,t} = 0.6$, the ER of $\ddot{\Re}_{c,t}$ at $w = 2$ is the least for a distinct number of records, except at $(d, f) = (10,15)$, via prior I (Fig. 9).
- At true value $\Re_{c,t} = 0.75$, the ER of $\ddot{\Re}_{c,t}$ at $w = -2$ is the least for the number of records (5, 5), (5, 10), and (10, 15), whereas the ER of $\ddot{\Re}_{c,t}$ at $w = 2$ is the least for (10, 5), (10, 10), (15, 10), and (15, 15) via prior I (Fig. 10).
- The ER of $\ddot{\Re}_{c,t}$ within the LINEX loss function and the ER of $\tilde{\Re}_{c,t}$ within the SE loss function decrease as the number of records increases (Fig. 11).
- Compared with the analogous ER of $\tilde{\Re}_{c,t}$ within the SE loss function, the ER of $\ddot{\Re}_{c,t}$ within the LINEX loss function has the least values in all circumstances (Figs. 10 and 11).
- For both values of (c,t), the ABs of $\tilde{\Re}_{c,t}$ via the SE loss function and $\ddot{\Re}_{c,t}$ under the LINEX loss function decrease as the number of records increases (Tables VII–IX).
- The AL of $\tilde{\Re}_{c,t}$ under SE loss function has the least value for $(c,t) = (1,3)$ and $d \ne f$ via LINEX $w = 2, -2$ loss function using prior III. Additionally, the AL decreases as the number of records increases (Fig. 12).
- The ER of $\ddot{\Re}_{c,t}$ at LINEX ($w = 2$) is the least for all the number of records for $(c,t) = (1,3)$ via prior IV (Fig. 13).
- At true value $\Re_{c,t} = 0.25$, the ER of $\ddot{\Re}_{c,t}$ at LINEX ($w = -2$) is the least for a similar number of records using prior V (Table XI).
- The ER of $\ddot{\Re}_{c,t}$ at LINEX ($w = 2$) is the least for $(c,t) = (1,3)$ and $d = f$ via prior IV (Table X).
- The ER of $\ddot{\Re}_{c,t}$ at LINEX ($w = 2$) is the least for $(c,t) = (2,4)$ and $d > f$ via prior III (Table IX).

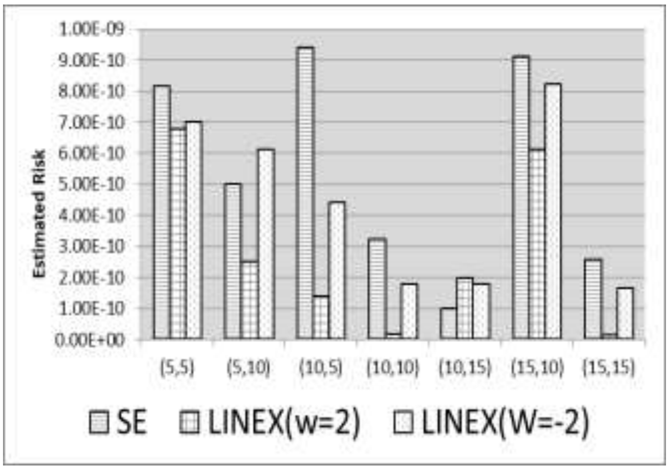

Fig. 9. ER of $\tilde{\Re}_{c,t}$ and $\ddot{\Re}_{c,t}$ at $(c,t) = (2,4)$ for prior I

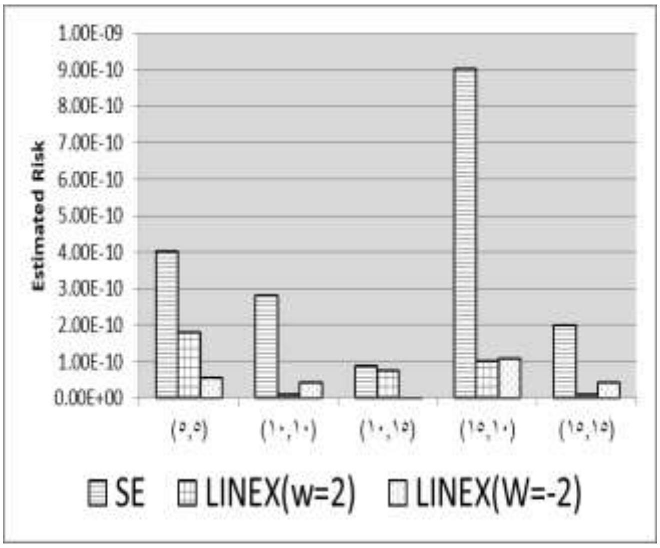

Fig. 10. ER of $\tilde{\Re}_{c,t}$ and $\ddot{\Re}_{c,t}$ at $(c,t) = (1,3)$ for prior I

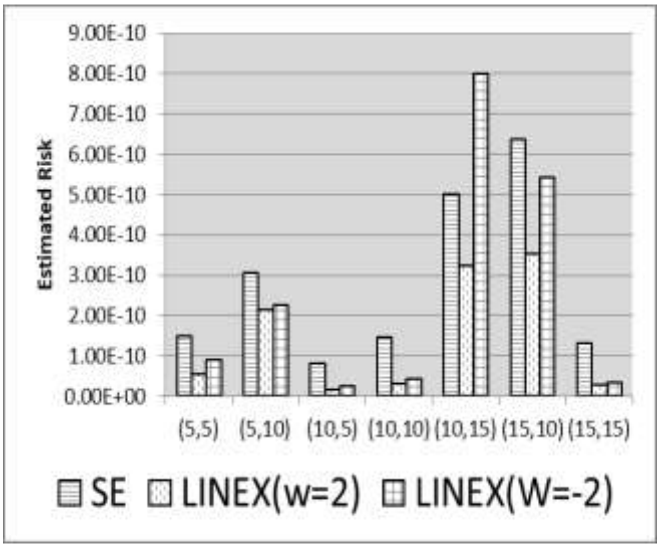

Fig. 11. ER of $\tilde{\Re}_{c,t}$ and $\ddot{\Re}_{c,t}$ at $(c,t) = (1,3)$ for prior II

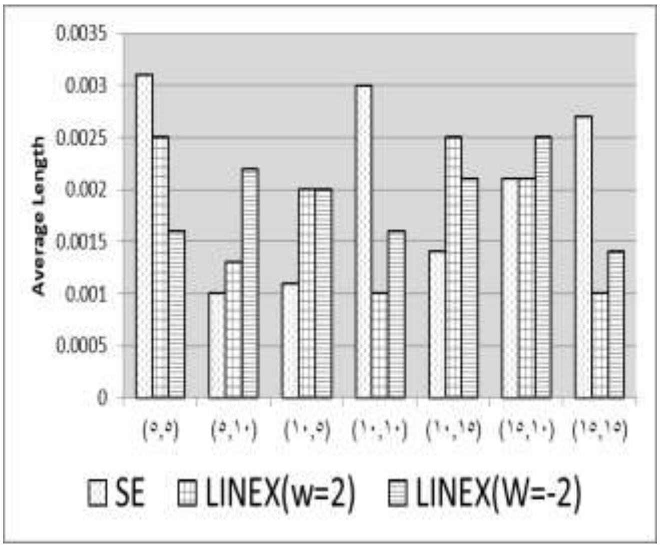

Fig. 12. AL of $\tilde{\Re}_{c,t}$ and $\ddot{\Re}_{c,t}$ at $(c,t) = (1,3)$ and $(\alpha, \beta) = (1.5, 3)$ for Prior III

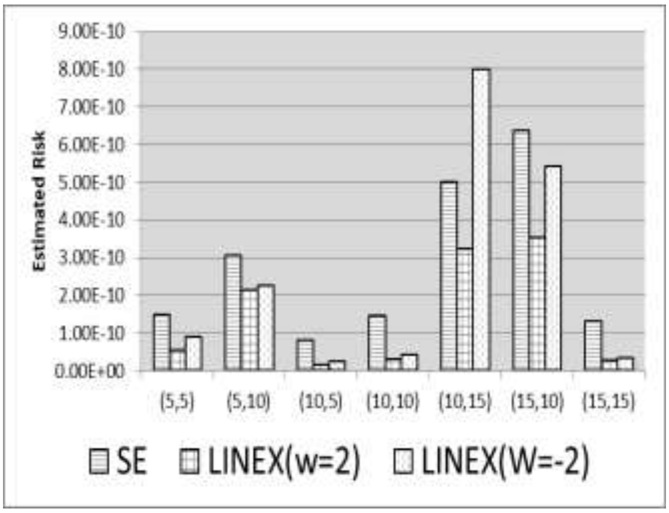

Fig. 13. ER of $\tilde{\Re}_{c,t}$ and $\ddot{\Re}_{c,t}$ at $(c,t)=(1,3)$ for prior IV

IV. ACUTAL DATA IMPLEMENTATION

In this section, we use two real datasets for demonstration purposes. The two datasets were used in [30]. The data represent the concentration of sulfur dioxide in March (Data Group I) and August (Data Group II), in Long Beach, California, from 1956 to 1974. The two datasets are fitted separately with the GIED using the Kolmogorov–Smirnov goodness-of-fit test; the results are illustrated in Table XII and Figs. 14 and 15.

TABLE XII
K-S TEST and CORESSPONDING P-VALUES for GROUPS I and II

	K-S	PV
Data Group I	0.187	0.488
Data Group II	0.258	0.141

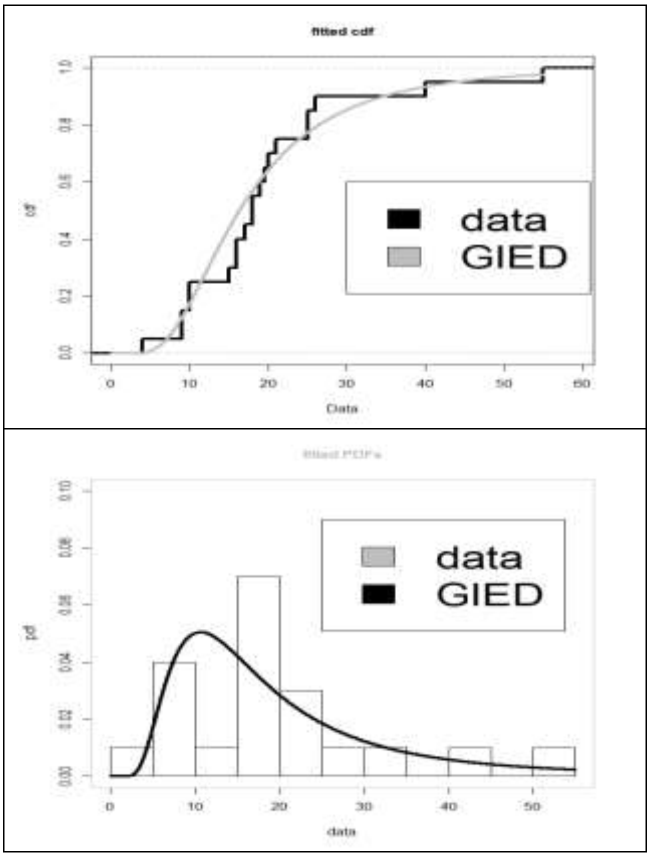

Fig. 14. Characteristics and limitations of K-S test for data of group I

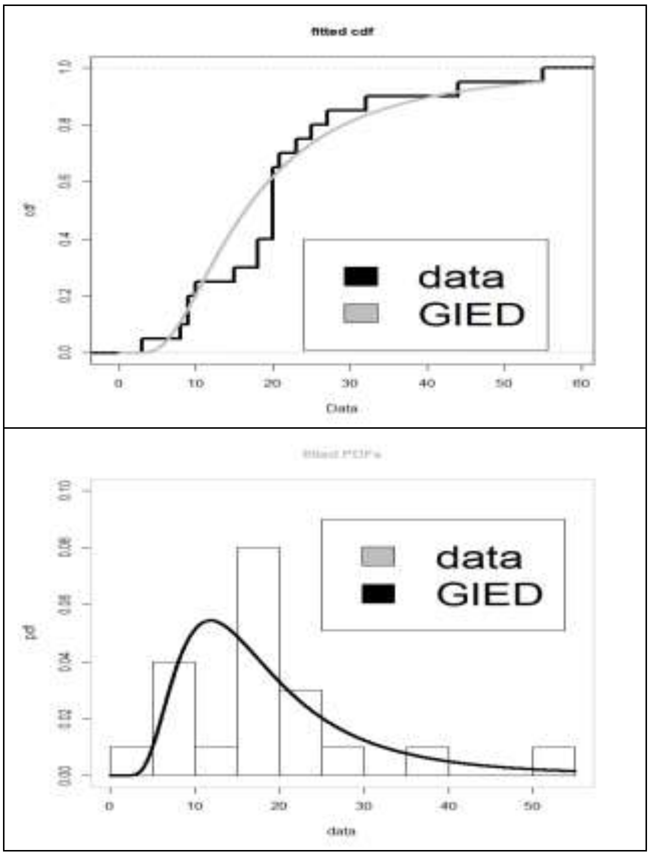

Fig. 15. Characteristics and limitations of K-S test for data of group II

According to Data Groups I and II, the URVs are obtained as follows: u = 21, 25, 26, 40, 55, v = 44, 55.

We calculate the estimates of $\Re_{c,t}$ using the ML and Bayesian approaches. Using the above URVs, the MLEs of $\Re_{c,t}$ for $(c,t)=(1,3)$ are calculated (Table XIII).

To analyze the data from the Bayesian approach, we do not have any prior information on $\Re_{c,t}$, so we compute various BEs using the noninformative prior. We follow the suggestions considered in [31], which are almost similar to Jeffreys priors. Under the same prior distributions, we compute BEs of $\Re_{c,t}$ under SE and LINEX loss functions (Table XIII).

TABLE XIII
BAYES and MLEs ESTIMATE of $\Re_{c,t}$ for the ACUTAL DATA

	(c,t)=(1,3)
ML estimate of $\Re_{c,t}$	0.7522
BEs of $\Re_{c,t}$ under SE and LINEX loss functions	
SE	0.8214
LINEX (w =2)	0.8208
LINEX (w =−2)	0.8221

IV. CONCLUSION

In this study, we focus on analyzing multicomponent SS reliability using record data when both the strength and stress variables are GIED with different shape parameters. The reliability of MSS is examined using ML and Bayesian processes of estimation. Samples are taken from strength and stress distributions, and their measures are given in units of URVs. To assess the precision of the various estimations, we use MCMC techniques. According to the simulation study, the MSE and average bias for two selections of (c,t)

decrease as the number of records increases, which confirms the MLE's consistency characteristic of $\Re_{c,t}$. Concerning the MCMC method, compared with the analogous ER of $\tilde{\Re}_{c,t}$ within SE loss, we infer that the ER of $\ddot{\Re}_{c,t}$ within LINEX loss holds the least values in most cases. Finally, the implementation of actual data demonstrates that the reliability estimates for our model approximate one, demonstrating its use in reality.

TABLE II

NUMERICAL RESULTS of $\hat{\Re}_{c,t}$ for DIFFERENT VALUES of α and β

$(\alpha,\beta)=(2.5,1)$					$(\alpha,\beta)=(1.5,3)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0154	0.1242			(5,5)	0.0005	0.0235
		(5,10)	0.0035	0.0952			(5,10)	0.0024	0.0111
		(10,5)	0.0045	0.0673			(10,5)	0.0068	0.0829
(1,3)	**0.47**	(10,10)	0.0114	0.0893	**(1,3)**	**0.9**	(10,10)	0.0001	0.0109
		(10,15)	0.0096	0.0980			(10,15)	0.0011	0.0338
		(15,10)	0.0101	0.1006			(15,10)	0.0005	0.0242
		(15,15)	0.0021	0.0291			(15,15)	0.0001	0.0111
		(5,5)	0.0426	0.2066			(5,5)	0.0095	0.0976
		(5,10)	0.0100	0.1003			(5,10)	0.0084	0.0121
		(10,5)	0.0182	0.0911			(10,5)	0.0513	0.2265
(2,4)	**0.33**	(10,10)	0.0366	0.1915	**(2,4)**	**0.8**	(10,10)	0.0006	0.0247
		(10,15)	0.0206	0.1436			(10,15)	0.0016	0.0407
		(15,10)	0.0415	0.2038			(15,10)	0.0008	0.0297
		(15,15)	0.0208	0.1444			(15,15)	0.0005	0.0143
$(\alpha,\beta)=(2,3)$					$(\alpha,\beta)=(3,3)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0074	0.0858			(5,5)	0.0026	0.0201
		(5,10)	0.0009	0.0311			(5,10)	0.0048	0.0695
		(10,5)	0.0008	0.0291			(10,5)	0.0076	0.0877
(1,3)	**0.85**	(10,10)	0.0009	0.0804	**(1,3)**	**0.75**	(10,10)	0.0002	0.0160
		(10,15)	0.0060	0.0254			(10,15)	0.0010	0.0326
		(15,10)	0.0049	0.0439			(15,10)	0.0012	0.0170
		(15,15)	0.0008	0.0288			(15,15)	0.0001	0.0155
		(5,5)	0.0197	0.1402			(5,5)	0.0066	0.0480
		(5,10)	0.0064	0.0804			(5,10)	0.0432	0.2097
		(10,5)	0.0008	0.0291			(10,5)	0.0437	0.1171
(2,4)	**0.72**	(10,10)	0.0070	0.0850	**(2,4)**	**0.6**	(10,10)	0.0022	0.0478
		(10,15)	0.0063	0.0795			(10,15)	0.0031	0.0561
		(15,10)	0.0049	0.0439			(15,10)	0.0047	0.0685
		(15,15)	0.0009	0.0289			(15,15)	0.0020	0.0471

TABLE III

NUMERICAL RESULTS of $\hat{\Re}_{c,t}$ for DIFFERENT VALUES of α and β

$(\alpha,\beta)=(1,2)$					$(\alpha,\beta)=(1.75,1)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0058	0.0485			(5,5)	0.0211	0.1171
		(5,10)	0.0004	0.0311			(5,10)	0.0071	0.0875
		(10,5)	0.0004	0.0200			(10,5)	0.0077	0.0721
(1,3)	**0.9**	(10,10)	0.0005	0.0399	**(1,3)**	**0.58**	(10,10)	0.0210	0.0921
		(10,15)	0.0042	0.0217			(10,15)	0.0088	0.0924
		(15,10)	0.0032	0.0210			(15,10)	0.0200	0.0977
		(15,15)	0.0004	0.0276			(15,15)	0.0016	0.0541
		(5,5)	0.0094	0.0921			(5,5)	0.0441	0.1244
		(5,10)	0.0080	0.0844			(5,10)	0.0318	0.1018
		(10,5)	0.0071	0.0345			(10,5)	0.0329	0.1000
(2,4)	**0.8**	(10,10)	0.0072	0.0712	**(2,4)**	**0.43**	(10,10)	0.0415	0.1143
		(10,15)	0.0061	0.0813			(10,15)	0.0392	0.1032
		(15,10)	0.0054	0.0310			(15,10)	0.0402	0.2100
		(15,15)	0.0005	0.0300			(15,15)	0.0381	0.1111
$(\alpha,\beta)=(2,2)$					$(\alpha,\beta)=(2,1.5)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0041	0.0119			(5,5)	0.0222	0.1422
		(5,10)	0.0044	0.0218			(5,10)	0.0050	0.1000
		(10,5)	0.0058	0.0257			(10,5)	0.0055	0.0952
(1,3)	**0.75**	(10,10)	0.0002	0.0106	**(1,3)**	**0.67**	(10,10)	0.0187	0.0976
		(10,15)	0.0038	0.0213			(10,15)	0.0044	0.0988
		(15,10)	0.0038	0.0155			(15,10)	0.0062	0.0990
		(15,15)	0.0001	0.0100			(15,15)	0.0131	0.0871
		(5,5)	0.0064	0.0298			(5,5)	0.0511	0.2311
(2,4)	**0.6**	(5,10)	0.0085	0.0317	**(2,4)**	**0.51**	(5,10)	0.0312	0.1481
		(10,5)	0.0097	0.0419			(10,5)	0.0399	0.0987
		(10,10)	0.0039	0.0223			(10,10)	0.0288	0.0989

$(\alpha,\beta)=(1,2)$					$(\alpha,\beta)=(1.75,1)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(10,15)	0.0051	0.0244			(10,15)	0.0207	0.0989
		(15,10)	0.0066	0.0301			(15,10)	0.0279	0.0911
		(15,15)	0.0031	0.0115			(15,15)	0.0167	0.0885

TABLE IV

NUMERICAL RESULTS of $\hat{\Re}_{c,t}$ for DIFFERENT VALUES of α and β

$(\alpha,\beta)=(0.1,0.5)$					$(\alpha,\beta)=(0.3,0.5)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0091	0.0954			(5,5)	0.0038	0.0621
		(5,10)	0.0080	0.0896			(5,10)	0.0029	0.0547
		(10,5)	0.0021	0.0459			(10,5)	0.0018	0.0430
(1,3)	**0.98**	(10,10)	0.0004	0.0219	**(1,3)**	**0.87**	(10,10)	0.0001	0.0135
		(10,15)	0.0071	0.0841			(10,15)	0.0041	0.0713
		(15,10)	0.0052	0.0712			(15,10)	0.0037	0.0630
		(15,15)	0.0002	0.0101			(15,15)	0.0001	0.0133
		(5,5)	0.0092	0.0960			(5,5)	0.0095	0.0975
		(5,10)	0.0103	0.1016			(5,10)	0.0031	0.0560
		(10,5)	0.0097	0.0989			(10,5)	0.0055	0.0747
(2,4)	**0.95**	(10,10)	0.0049	0.0703	**(2,4)**	**0.75**	(10,10)	0.0028	0.0530
		(10,15)	0.0087	0.0863			(10,15)	0.0040	0.0712
		(15,10)	0.0091	0.0899			(15,10)	0.0056	0.0746
		(15,15)	0.0026	0.0315			(15,15)	0.0011	0.0256
$(\alpha,\beta)=(0.5,0.5)$					$(\alpha,\beta)=(0.7,0.5)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0014	0.0385			(5,5)	0.0031	0.0562
		(5,10)	0.0026	0.0409			(5,10)	0.0038	0.0618
		(10,5)	0.0015	0.0315			(10,5)	0.0007	0.0276
(1,3)	**0.75**	(10,10)	0.0004	0.0222	**(1,3)**	**0.65**	(10,10)	0.0015	0.0397
		(10,15)	0.0033	0.0610			(10,15)	0.0035	0.0635
		(15,10)	0.0023	0.0511			(15,10)	0.0025	0.0501
		(15,15)	0.0003	0.0211			(15,15)	0.0009	0.0311
		(5,5)	0.0053	0.0734			(5,5)	0.0040	0.0632
		(5,10)	0.0072	0.0849			(5,10)	0.0042	0.0655
		(10,5)	0.0026	0.0453			(10,5)	0.0011	0.0345
(2,4)	**0.6**	(10,10)	0.0050	0.0711	**(2,4)**	**0.5**	(10,10)	0.0036	0.0600
		(10,15)	0.0047	0.0701			(10,15)	0.0049	0.0713
		(15,10)	0.0061	0.0791			(15,10)	0.0032	0.0340
		(15,15)	0.0028	0.0517			(15,15)	0.0030	0.0317

TABLE V

NUMERICAL RESULTS of $\hat{\Re}_{c,t}$ for DIFFERENT VALUES of α and β

$(\alpha,\beta)=(0.3,1.5)$					$(\alpha,\beta)=(0.8,2)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0009	0.0306			(5,5)	0.0029	0.0543
		(5,10)	0.0001	0.0112			(5,10)	0.0015	0.0399
		(10,5)	0.0009	0.0308			(10,5)	0.0026	0.0510
(1,3)	**0.98**	(10,10)	0.0008	0.0297	**(1,3)**	**0.93**	(10,10)	0.0010	0.0326
		(10,15)	0.0002	0.0213			(10,15)	0.0025	0.0498
		(15,10)	0.0005	0.0271			(15,10)	0.0031	0.0583
		(15,15)	0.0004	0.0172			(15,15)	0.0009	0.0293
		(5,5)	0.0012	0.0354			(5,5)	0.0055	0.0747
		(5,10)	0.0004	0.0200			(5,10)	0.0036	0.0601
		(10,5)	0.0035	0.0598			(10,5)	0.0037	0.0612
(2,4)	**0.95**	(10,10)	0.0010	0.0317	**(2,4)**	**0.85**	(10,10)	0.0020	0.0417
		(10,15)	0.0011	0.0341			(10,15)	0.0029	0.0487
		(15,10)	0.0017	0.0349			(15,10)	0.0034	0.0571
		(15,15)	0.0008	0.0298			(15,15)	0.0012	0.0345
$(\alpha,\beta)=(0.6,1.7)$					$(\alpha,\beta)=(0.4,1.2)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0011	0.0344			(5,5)	0.0014	0.0376
		(5,10)	0.0023	0.0488			(5,10)	0.0011	0.0337
		(10,5)	0.0038	0.0617			(10,5)	0.0016	0.0403
(1,3)	**0.94**	(10,10)	0.0008	0.0299	**(1,3)**	**0.95**	(10,10)	0.0003	0.0180
		(10,15)	0.0020	0.0476			(10,15)	0.0018	0.0347
		(15,10)	0.0031	0.0412			(15,10)	0.0021	0.0377
		(15,15)	0.0004	0.0201			(15,15)	0.0001	0.0112
		(5,5)	0.0015	0.0399			(5,5)	0.0036	0.0605
(2,4)	**0.87**	(5,10)	0.0026	0.0513	**(2,4)**	**0.89**	(5,10)	0.0023	0.0487
		(10,5)	0.0046	0.0684			(10,5)	0.0030	0.0548

$(\alpha,\beta)=(0.3,1.5)$					$(\alpha,\beta)=(0.8,2)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(10,10)	0.0012	0.0349			(10,10)	0.0011	0.0313
		(10,15)	0.0029	0.0439			(10,15)	0.0029	0.0481
		(15,10)	0.0040	0.0611			(15,10)	0.0051	0.0621
		(15,15)	0.0009	0.0215			(15,15)	0.0008	0.0199

TABLE VI

NUMERICAL RESULTS of $\hat{\Re}_{c,t}$ for DIFFERENT VALUES of α and β

$(\alpha,\beta)=(1.2,0.3)$					$(\alpha,\beta)=(2.5,0.6)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0043	0.0657			(5,5)	0.0015	0.0387
		(5,10)	0.0030	0.0553			(5,10)	0.0037	0.0609
		(10,5)	0.0028	0.0532			(10,5)	0.0043	0.0657
(1,3)	**0.34**	(10,10)	0.0012	0.0351	**(1,3)**	**0.33**	(10,10)	0.0012	0.0357
		(10,15)	0.0042	0.0651			(10,15)	0.0046	0.0623
		(15,10)	0.0033	0.0521			(15,10)	0.0053	0.0647
		(15,15)	0.0009	0.0183			(15,15)	0.0010	0.0301
		(5,5)	0.0054	0.0736			(5,5)	0.0026	0.0511
		(5,10)	0.0038	0.0621			(5,10)	0.0051	0.0720
		(10,5)	0.0040	0.0635			(10,5)	0.0046	0.0683
(2,4)	**0.23**	(10,10)	0.0017	0.0414	**(2,4)**	**0.22**	(10,10)	0.0018	0.0435
		(10,15)	0.0063	0.0743			(10,15)	0.0063	0.0752
		(15,10)	0.0052	0.0633			(15,10)	0.0072	0.0629
		(15,15)	0.0013	0.0231			(15,15)	0.0014	0.0256

$(\alpha,\beta)=(1.1,0.9)$					$(\alpha,\beta)=(2,0.4)$				
(c, t)	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**	**(c, t)**	**Real $\Re_{c,t}$**	**(d, f)**	**MSE**	**AB**
		(5,5)	0.0039	0.0627			(5,5)	0.0021	0.0462
		(5,10)	0.0049	0.0704			(5,10)	0.0018	0.0426
		(10,5)	0.0041	0.0645			(10,5)	0.0027	0.0520
(1,3)	**0.69**	(10,10)	0.0035	0.0597	**(1,3)**	**0.29**	(10,10)	0.0012	0.0354
		(10,15)	0.0037	0.0617			(10,15)	0.0024	0.0215
		(15,10)	0.0044	0.0573			(15,10)	0.0032	0.0341
		(15,15)	0.0025	0.0252			(15,15)	0.0009	0.0178
		(5,5)	0.0047	0.0689			(5,5)	0.0037	0.0614
		(5,10)	0.0070	0.0837			(5,10)	0.0023	0.0485
		(10,5)	0.0073	0.0859			(10,5)	0.0028	0.0528
(2,4)	**0.54**	(10,10)	0.0044	0.0670	**(2,4)**	**0.19**	(10,10)	0.0017	0.0462
		(10,15)	0.0051	0.0623			(10,15)	0.0026	0.0246
		(15,10)	0.0082	0.0823			(15,10)	0.0042	0.0352
		(15,15)	0.0032	0.0314			(15,15)	0.0010	0.0201

TABLE VII

MEASURES of ACCURACY of $\Re_{c,t}$ ESTIMATES for PRIOR I

	(c,t)=(1, 3)					**(c,t)=(2, 4)**			
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
SE	0.75	(5,5)	0.0001	4.01E-10	0.0018	0.6	0.0001	8.17E-10	0.0020
LINEX (w =2)			0.0009	1.80E-10	0.0021		0.0018	6.77E-10	0.0046
LINEX (w=−2)			0.0003	5.66E-11	0.0014		0.0003	7.01E-10	0.0021
SE		(5,10)	0.0015	4.84E-10	0.0024		0.0015	5.00E-10	0.0026
LINEX (w =2)			0.0010	2.28E-10	0.0018		0.0011	2.51E-10	0.0021
LINEX (w=−2)			0.0008	1.00E-11	0.0015		0.0009	6.11E-10	0.0016
SE		(10,5)	0.0001	8.13E-10	0.0018		0.0003	9.41E-10	0.0019
LINEX (w =2)			0.0016	1.11E-10	0.0032		0.0019	1.38E-10	0.0030
LINEX (w=−2)			0.0012	4.01E-10	0.0015		0.0031	4.43E-10	0.0014
SE		(10,10)	0.0001	2.80E-10	0.0018		0.0001	3.22E-10	0.0019
LINEX (w =2)			0.0002	1.04E-11	0.0018		0.0002	1.70E-11	0.0017
LINEX (w=−2)			0.0003	4.33E-11	0.0013		0.0002	6.01E-10	0.0020
SE		(10,15)	0.0002	8.66E-11	0.0017		0.0003	9.87E-11	0.0013
LINEX (w =2)			0.0006	7.60E-11	0.0013		0.0009	1.97E-10	0.0017
LINEX (w=−2)			0.0001	3.93E-12	0.0015		0.0009	1.80E-10	0.0019
SE		(15,10)	0.0006	9.02E-10	0.0015		0.0007	9.12E-10	0.0013
LINEX (w =2)			0.0007	9.98E-11	0.0010		0.0009	6.11E-10	0.0017
LINEX (w=−2)			0.0007	1.06E-10	0.0014		0.0008	8.21E-10	0.0016
SE		(15,15)	0.0001	2.00E-10	0.0016		0.0001	2.59E-10	0.0016
LINEX (w =2)			0.0001	1.02E-11	0.0013		0.0001	1.56E-11	0.0014
LINEX (w=−2)			0.0002	4.19E-11	0.0013		0.0001	4.02E-10	0.0020

TABLE VIII

MEASURES of ACCUARCY of $\Re_{c,t}$ ESTIMATES for PRIOR II

		(c,t)=(1, 3)					(c,t)=(2, 4)		
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
SE	0.85	(5,5)	0.0004	4.58E-10	0.0023	0.72	0.0004	6.24E-10	0.0025
LINEX (w =2)			0.0001	4.31E-10	0.0015		0.0003	5.70E-10	0.0017
LINEX (w=−2)			0.0007	3.96E-10	0.0014		0.0011	5.85E-10	0.0019
SE		(5,10)	0.0003	2.88E-11	0.0001		0.0024	1.23E-09	0.0023
LINEX (w =2)			0.0002	9.27E-12	0.0028		0.0002	1.33E-11	0.0030
LINEX (w=−2)			0.0003	2.80E-11	0.0001		0.0004	8.82E-10	0.0016
SE		(10,5)	0.0004	4.96E-10	0.0027		0.0003	9.05E-10	0.0029
LINEX (w =2)			0.0002	3.21E-10	0.0012		0.0003	2.36E-09	0.0012
LINEX (w=−2)			0.0000	9.08E-18	0.0012		0.0006	9.27E-11	0.0018
SE		(10,10)	0.0002	3.48E-10	0.0012		0.0003	3.88E-10	0.0012
LINEX (w =2)			0.0001	2.81E-10	0.0012		0.0001	4.32E-10	0.0014
LINEX (w=−2)			0.0004	3.17E-10	0.0001		0.0010	5.20E-10	0.0015
SE		(10,15)	0.0005	4.11E-10	0.0023		0.0006	7.77E-10	0.0022
LINEX (w =2)			0.0002	2.58E-10	0.0019		0.0004	3.40E-10	0.0020
LINEX (w=−2)			0.0001	1.61E-10	0.0011		0.0001	3.34E-10	0.0017
SE		(15,10)	0.0002	1.27E-11	0.0002		0.0003	3.15E-10	0.0012
LINEX (w =2)			0.0015	4.64E-10	0.0006		0.0016	4.76E-10	0.0008
LINEX (w=−2)			0.0024	2.59E-10	0.0031		0.0030	4.23E-10	0.0034
SE		(15,15)	0.0002	1.33E-10	0.0008		0.0001	1.78E-10	0.0010
LINEX (w =2)			0.0001	1.45E-10	0.0011		0.0000	4.21E-10	0.0011
LINEX (w=−2)			0.0001	3.00E-10	0.0000		0.0001	4.55E-10	0.0013

TABLE IX

MEASURES of ACCURACY of $\Re_{c,t}$ ESTIMATES for PRIOR III

		(c,t)=(1, 3)					(c,t)=(2, 4)		
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
SE	0.9	(5,5)	0.0004	3.52E-11	0.0031	0.8	0.0004	2.70E-10	0.0042
LINEX (w =2)			0.0023	3.18E-11	0.0025		0.0030	1.26E-10	0.0028
LINEX (w=−2)			0.0004	4.15E-11	0.0016		0.0005	5.41E-11	0.0018
SE		(5,10)	0.0003	4.03E-10	0.0010		0.0008	5.52E-10	0.0016
LINEX (w =2)			0.0001	2.80E-12	0.0013		0.0009	1.69E-10	0.0018
LINEX (w=−2)			0.0010	2.25E-10	0.0022		0.0012	6.85E-10	0.0024
SE		(10,5)	0.0002	1.00E-11	0.0011		0.0011	4.11E-10	0.0017
LINEX (w =2)			0.0007	9.94E-11	0.0020		0.0009	9.95E-11	0.0017
LINEX (w=−2)			0.0009	1.44E-11	0.0020		0.0008	1.31E-10	0.0021
SE		(10,10)	0.0002	3.45E-11	0.0030		0.0004	2.56E-10	0.0035
LINEX (w =2)			0.0005	3.00E-11	0.0010		0.0007	8.89E-11	0.0023
LINEX (w=−2)			0.0004	4.10E-11	0.0016		0.0004	4.29E-11	0.0017
SE		(10,15)	0.0003	2.24E-11	0.0014		0.0015	5.00E-10	0.0031
LINEX (w =2)			0.0014	3.96E-10	0.0025		0.0014	4.66E-10	0.0025
LINEX (w=−2)			0.0010	2.00E-10	0.0021		0.0001	2.48E-10	0.0021
SE		(15,10)	0.0012	6.16E-10	0.0021		0.0011	7.02E-10	0.0010
LINEX (w =2)			0.0004	3.22E-11	0.0021		0.0015	5.11E-11	0.0025
LINEX (w=−2)			0.0016	5.13E-10	0.0025		0.0017	7.58E-10	0.0026
SE		(15,15)	0.0001	2.00E-11	0.0027		0.0003	1.41E-10	0.0030
LINEX (w =2)			0.0003	2.61E-11	0.0010		0.0002	4.15E-11	0.0017
LINEX (w=−2)			0.0000	3.75E-13	0.0014		0.0002	1.06E-11	0.0016

TABLE X

MEASURES of ACCURACY of $\Re_{c,t}$ ESTIMATES for PRIOR IV

		(c,t)=(1, 3)					(c,t)=(2, 4)		
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
SE	0.67	(5,5)	0.0008	1.49E-10	0.0015	0.51	0.0008	2.33E-10	0.0017
LINEX (w =2)			0.0005	5.30E-11	0.0021		0.0014	9.26E-10	0.0022
LINEX (w=−2)			0.0006	9.02E-11	0.0034		0.0007	9.90E-11	0.0036
SE		(5,10)	0.0005	3.05E-10	0.0010		0.0008	7.62E-10	0.0014
LINEX (w =2)			0.0001	2.14E-10	0.0013		0.0009	6.29E-10	0.0018
LINEX (w=−2)			0.0003	2.25E-10	0.0022		0.0011	6.91E-10	0.0022
SE		(10,5)	0.0002	8.03E-11	0.0001		0.0005	4.81E-10	0.0017
LINEX (w =2)			0.0007	1.72E-11	0.0018		0.0009	4.95E-11	0.0019
LINEX (w=−2)			0.0009	2.54E-11	0.0016		0.0009	1.41E-10	0.0020
SE		(10,10)	0.0006	1.45E-10	0.0011		0.0007	2.02E-10	0.0013
LINEX (w =2)			0.0004	3.00E-11	0.0015		0.0010	8.82E-11	0.0021
LINEX (w=−2)			0.0004	4.10E-11	0.0016		0.0004	4.27E-11	0.0019
SE		(10,15)	0.0004	5.01E-10	0.0014		0.0015	5.55E-10	0.0031
LINEX (w =2)			0.0013	3.22E-10	0.0025		0.0014	4.96E-10	0.0025

		(c,t)=(1, 3)					**(c,t)=(2, 4)**		
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
LINEX (w=−2)			0.0011	8.00E-10	0.0021		0.0001	9.48E-10	0.0021
SE			0.0008	6.36E-10	0.0016		0.0011	7.42E-10	0.0010
LINEX (w =2)		(15,10)	0.0004	3.52E-10	0.0014		0.0015	5.11E-10	0.0019
LINEX (w=−2)			0.0012	5.43E-10	0.0012		0.0017	7.23E-10	0.0020
SE			0.0004	1.31E-10	0.0009		0.0006	1.46E-10	0.0011
LINEX (w =2)		(15,15)	0.0002	2.64E-11	0.0014		0.0003	5.15E-11	0.0016
LINEX (w=−2)			0.0001	3.25E-11	0.0008		0.0002	4.16E-11	0.0007

TABLE XI

MEASURES of ACCURACY of $\Re_{c,t}$ ESTIMATES for PRIOR V

		(c,t)=(1, 3)					**(c,t)=(2, 4)**		
Loss function	**Real $\Re_{c,t}$**	**(d,f)**	**AB**	**ER**	**AL**	**Real $\Re_{c,t}$**	**AB**	**ER**	**AL**
SE	0.25		0.0012	3.22E-10	0.0030	0.23	0.0007	1.00E-10	0.0024
LINEX (w =2)		(5,5)	0.0011	2.55E-10	0.0025		0.0006	9.29E-11	0.0017
LINEX (w=−2)			0.0003	1.31E-11	0.0030		0.0007	1.08E-10	0.0025
SE			0.0006	5.03E-10	0.0010		0.0008	5.11E-10	0.0011
LINEX (w =2)		(5,10)	0.0001	2.90E-12	0.0013		0.0005	2.69E-11	0.0015
LINEX (w=−2)			0.0002	2.35E-10	0.0022		0.0011	4.35E-10	0.0021
SE			0.0004	1.05E-11	0.0011		0.0008	1.31E-10	0.0016
LINEX (w =2)		(10,5)	0.0007	9.74E-11	0.0020		0.0008	1.94E-10	0.0017
LINEX (w=−2)			0.0005	1.14E-11	0.0020		0.0007	1.81E-10	0.0021
SE			0.0009	1.22E-11	0.0025		0.0007	1.01E-10	0.0024
LINEX (w =2)		(10,10)	0.0007	2.01E-11	0.0014		0.0005	7.89E-11	0.0016
LINEX (w=−2)			0.0005	1.01E-11	0.0017		0.0005	1.29E-11	0.0017
SE			0.0003	2.28E-11	0.0011		0.0005	2.04E-10	0.0030
LINEX (w =2)		(10,15)	0.0008	2.96E-10	0.0021		0.0010	5.46E-10	0.0021
LINEX (w=−2)			0.0009	2.08E-10	0.0021		0.00010	3.41E-10	0.0020
SE			0.0011	5.16E-10	0.0019		0.0011	8.72E-10	0.0010
LINEX (w =2)		(15,10)	0.0004	3.12E-11	0.0013		0.0008	5.61E-11	0.0024
LINEX (w=−2)			0.0012	1.13E-10	0.0010		0.0014	6.58E-10	0.0021
SE			0.0006	1.00E-11	0.0020		0.0007	2.41E-11	0.0022
LINEX (w =2)		(15,15)	0.0003	2.00E-11	0.0011		0.0003	4.35E-11	0.0016
LINEX (w=−2)			0.0000	2.75E-12	0.0014		0.0002	1.06E-11	0.0015

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