Gaussian Mixture CBMeMBer Filter for Multi-Target Tracking with Non-Gaussian Noise

Xiangyu He, Yi Wang, and Shuqiang Yang

Abstract—**This paper presents a Gaussian mixture implementation method of the standard cardinality-balanced multi-target multi-Bernoulli (CBMeMBer) filter for addressing the multi-target tracking (MTT) problems with non-Gaussian model noises. With non-Gaussian model noises approximated by the weighted sum of Gaussian distributions, the closed-form recursions to the CBMeMBer filter are derived by using the Gaussian mixture expressions. The advantage of the proposed algorithm over the sequential Monte Carlo CBMeMBer (SMC-CBMeMBer) filter is that it can track multiple targets with similar tracking accuracies and significant reductions in computing time. The effectiveness of the proposed algorithm has been verified with the comparison results of numerical simulations.**

Index Terms—**Gaussian mixture, filtering, multi-Bernoulli, multi-target tracking, random finite set**

I. Introduction

THE research content of multi-target tracking (MTT) problem is to develop the approaches that can estimate the satisfactory tracking results by using uncertain measurements [1]. In MTT problems, the measurements received from an imperfect sensor are usually affected by clutter and noise [2]. Therefore, how to effectively track multiple targets under complex situations is a challenging research issue in both MTT theories and applications.

Most traditional MTT approaches [3]-[5] require the data association technique to determine the associations between sensor measurements and targets. However, the associations between measurements and targets are computationally intensive in most practical applications. The random finite set (RFS) theory [6], which is presented in recent years, has been successfully used to address different MTT problems. The probability hypothesis density (PHD) filter [7] and the cardinalized PHD (CPHD) [8] filter are two effective Bayesian filter algorithms to MTT based on RFS, which can effectively solve the combinatorial problems resulting from data association in traditional MTT approaches and have gained the attention of a lot of researchers [9]-[16]. Unfortunately, due to the existence of multiple integrals in their recursions, The PHD and CPHD filters are generally intractable in computing. Consequently, two implementation methods, which are the Gaussian mixture PHD (GM-PHD) filter for linear Gaussian models and the sequential Monte Carlo PHD (SMC-PHD) filter [17]-[19], respectively, have been developed to solve such difficulties. In comparison with the SMC-PHD filter, the GM-PHD filter is much more efficient and reliable in estimating target states, but its performance decreases dramatically for addressing the problems related to nearby and crossing target tracking. To overcome the drawback of missing targets that arises from tracking nearby and crossing targets in the GM-PHD filter, the competitive GM-PHD (CGM-PHD) filter, the penalized GMPHD (PGM-PHD) filter, the collaborative penalized GM-PHD (CPGM-PHD) tracker, the radiation intensity GM-PHD (RIGM-PHD) filter, and the refined GM-PHD (RGM-PHD) tracker have been developed in the research papers published recently [20]-[24]. In addition, in the field of MTT, the algorithms for tracking multiple extended targets are another important research topic. For the purpose of tracking multiple extended targets effectively in the RFS theory framework, the extended target PHD (ET-PHD) filter [25] and its implementation methods such as the extended target Gaussian mixture PHD (ET-GM-PHD) filter [26], the extended target particle PHD (ET-P-PHD) filter [27], the extended target box-particle PHD (ET-Box-PHD) filter [28], and the ellipse extended target box particle PHD (EET-BP-PHD) filter [29] have been developed successively.

Besides the PHD filter and CPHD filter, another well-known RFS-based MTT algorithm is the multi-target multi-Bernoulli (MeMBer) filter proposed recently [1]. The MeMBer filter makes use of the multi-Bernoulli RFS in its Bayesian recursions and a multi-Bernoulli parameter set to represent the multi-Bernoulli posterior densities at each time step, this makes its target state extraction reliable and convenient. However, a target number over-estimation problem occurs along with it. To remedy such a disadvantage, the cardinality-balanced MeMBer (CBMeMBer) filter which offers a nice solution by modifying the formulas for calculating the measurement-updated multi-target density was developed in [30]. Subsequently, the labeled multi-Bernoulli (LMB) filter has been developed as an improved approximation of the MeMBer filter [31]. By introducing the labeled RFS, the LMB filter overcomes the limit of high signal to noise ratio that the MeMBer filter requires. Moreover, a forward-backward MeMBer smoother and its sequential Monte Carlo implementation were developed for improving the tracking performances of the

Manuscript received January 12, 2022; revised June 13, 2022. This work was supported in part by the Key Scientific Research Program of Higher Education of Henan Province of China under Grant 21A510009, and the Key Scientific and Technological Project of Henan Province of China under Grants 212102210523 and 222102210301.

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CBMeMBer filter [32]. The existing implementations of the CBMeMBer filter mainly include sequential Monte Carlo CBMeMBer (SMC-CBMeMBer) and Gaussian mixture CBMeMBer (GM-CBMeMBer) filters [30]. The SMC-CBMeMBer filter is a generic implementation method with no limit on tracking model and noise, but it is potentially computationally inefficient since that a large number of target samples are used to approximate the posterior multi-target densities for reaching favourable results and a resampling scheme should be adopted to reduce the effect of degeneracy. By taking the predicted state estimations and measurement likelihoods into consideration, a high-speed algorithm was proposed to improve the computing efficiency of the SMC-CBMeMBer filter [33]. Improving the computational efficiency of data processing algorithms is an important research issue because it is of great significance to do so in practical applications. So far, there have been many published research papers regarding the issue [34]-[38]. The GM-CBMeMBer filter is a computationally efficient closed-form solution with applications constrained to linear Gaussian models. With linearization and the unscented transforms, the GM-CBMeMBer filter can be extended to accommodate nonlinear Gaussian tracking models; nevertheless, these extensions are still not adequate for handling non-Gaussian MTT problems.

In many signal processing applications, Gaussian mixture models have been widely used for approximating non-Gaussian densities [39]-[42]. Inspired by the mechanism that Gaussian mixtures can approximate any given density, a new analytic implementation to the CBMeMBer filter for addressing the non-Gaussian MTT problems is proposed in this paper. While for the sake of clarity, only the linear tracking models are considered. Under the premises that the non-Gaussian tracking model noises are approximated by Gaussian mixture distributions, the state transition density and the measurement likelihood are defined firstly; then, the closed-form recursions to the CBMeMBer filter, in which the mean and variance can be updated by Kalman equations, are derived by using the standard conclusions for Gaussian distributions. The numerical simulation indicates that the proposed algorithm is a satisfactory solution for solving the MTT problems with non-Gaussian tracking models compared with the SMC-CBMeMBer filter.

The rest parts of this paper are organized as follows. Section Ⅱ defines the non-Gaussian tracking models and provides brief reviews of the standard CBMeMBer filter. Section Ⅲ elaborates the proposed algorithm. In Section Ⅳ, the numerical simulation results used to compare the performance of our algorithm with that of the SMC-CBMeMBer filter are studied. Some meaningful conclusions are drawn in the final Section Ⅴ.

II. Backgrounds

A. Tracking Model

In general, the linear tracking models which are used in the CBMeMBer filter can be written as

$$x_k = F_{k-1}x_{k-1} + q_{k-1}\,, \tag{1}$$

$$z_k = H_k x_k + r_k\,, \tag{2}$$

where k is the time index. x_k and z_k are the target state vector and the measurement vector, respectively. F_{k-1} denotes the state transition matrix and H_k denotes the observation matrix. q_{k-1} and r_k denote the process noise and the measurement noise, respectively.

Let $p(q_{k-1})$ and $p(r_k)$ denote the probability densities of q_{k-1} and r_k, respectively. As shown in [39], any density can be described as a Gaussian mixture expression. Therefore, for the linear non-Gaussian tracking models, the probability densities of q_{k-1} and r_k can be described as the expressions approximated by Gaussian functions. Suppose that $p(q_{k-1})$ and $p(r_k)$ are comprised of Gaussian mixtures of the form

$$p(q_{k-1}) = \sum_{i=1}^{N_{q,k-1}} w_{q,k-1}^{(i)} N(q_{k-1}; m_{q,k-1}^{(i)}, Q_{q,k-1}^{(i)})\,, \tag{3}$$

$$p(r_k) = \sum_{i=1}^{N_{r,k}} w_{r,k}^{(i)} N(r_k; m_{r,k}^{(i)}, R_{r,k}^{(i)})\,, \tag{4}$$

where $N(x;m,P)$ denotes a Gaussian density with mean m and covariance P. $N_{q,k-1}$, $w_{q,k-1}^{(i)}$, $m_{q,k-1}^{(i)}$, and $Q_{q,k-1}^{(i)}$ are the parameters that describe the probability density of the process noise q_{k-1}; similarly, $N_{r,k}$, $w_{r,k}^{(i)}$, $m_{r,k}^{(i)}$, and $Q_{r,k}^{(i)}$ are the parameters that describe the probability density of the measurement noise r_k; $\sum_{i=1}^{N_{q,k-1}} w_{q,k-1}^{(i)} = \sum_{i=1}^{N_{r,k}} w_{r,k}^{(i)} = 1$.

Then, according to (1)-(4), the state transition density $f_{k|k-1}(x_k \mid x_{k-1})$ and the measurement likelihood $g_k(z_k \mid x_k)$ can be rewritten as

$$f_{k|k-1}(x_k \mid x_{k-1}) = \sum_{i=1}^{N_{q,k-1}} w_{q,k-1}^{(i)} N(x_k; F_{k-1}x_{k-1} + m_{q,k-1}^{(i)}, Q_{q,k-1}^{(i)})\,, \tag{5}$$

$$g_k(z_k \mid x_k) = \sum_{i=1}^{N_{r,k}} w_{r,k}^{(i)} N(z_k; H_k x_k + m_{r,k}^{(i)}, R_{r,k}^{(i)})\,. \tag{6}$$

As described above, by using Gaussian mixture expressions to approximate the non-Gaussian model noises, the non-Gaussian tracking models can be described as banks of Gaussian noise models. This approach for approximating non-Gaussian noise largely simplifies the application of the non-Gaussian tracking models.

B. CBMeMBer Filter

The CBMeMBer filter is a recursive MTT algorithm, it makes use of the prediction and update equations to calculate the multi-target density at each time step. For clarity, the recursion equations of the CBMeMBer filter are briefly described below.

At time step $k-1$, suppose that the posterior multi-target density π_{k-1} can be represented by a multi-Bernoulli parameter set of the form

$$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}}\,, \tag{7}$$

where $r_{k-1}^{(i)}$ denotes the existence probability and $p_{k-1}^{(i)}$ denotes the state probability density function of the ith hypothesized track, respectively. M_{k-1} is the number of hypothesized tracks.

At time step k, let $\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$ denote the parameter set of the multi-Bernoulli RFS of birth targets. Then, the predicted multi-target density $\pi_{k|k-1}$ is also a multi-Bernoulli and given by

$$\pi_{k|k-1} = \{(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}} \cup \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}\}_{i=1}^{M_{\Gamma,k}}, \quad (8)$$

where

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} \int p_{S,k} p_{k-1}^{(i)}(x_{k-1}) dx_{k-1}, \quad (9)$$

$$p_{P,k|k-1}^{(i)}(x_k) = \frac{\int p_{S,k} f_{k|k-1}(x_k \mid x_{k-1}) p_{k-1}^{(i)}(x_{k-1}) dx_{k-1}}{\int p_{S,k} p_{k-1}^{(i)}(x_{k-1}) dx_{k-1}}, \quad (10)$$

where $p_{S,k}$ is the target survival probability and $f_{k|k-1}(x_k \mid x_{k-1})$ denotes the single target transition density.

At time step k, if the predicted multi-target density $\pi_{k|k-1}$ is

$$\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}}. \quad (11)$$

Then, the updated multi-target density π_k at time step k can be approximated by a multi-Bernoulli as

$$\pi_k \approx \{(r_{L,k}^{(i)}, p_{L,k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z_k), p_{U,k}(\cdot; z_k))\}_{z_k \in Z_k}, \quad (12)$$

where

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - \chi_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)} \chi_{k|k-1}^{(i)}}, \quad (13)$$

$$p_{L,k}^{(i)}(x_k) = \frac{1 - p_{D,k}}{1 - \chi_{k|k-1}^{(i)}} p_{k|k-1}^{(i)}(x_k), \quad (14)$$

$$r_{U,k}(z_k) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1 - r_{k|k-1}^{(i)}) \psi_k^{(i)}(z_k)}{(1 - r_{k|k-1}^{(i)} \chi_{k|k-1}^{(i)})^2}}{\kappa_k(z_k) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \psi_k^{(i)}(z_k)}{1 - r_{k|k-1}^{(i)} \chi_{k|k-1}^{(i)}}}, \quad (15)$$

$$p_{U,k}(x_k; z_k) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} p_D p_{k|k-1}^{(i)}(x_k) g_k(z_k \mid x_k)}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} \psi_k^{(i)}(z_k)}, \quad (16)$$

$$\chi_{k|k-1}^{(i)} = \int p_{D,k} p_{k|k-1}^{(i)}(x_k) dx_k, \quad (17)$$

$$\psi_k^{(i)}(z_k) = \int p_{D,k} p_{k|k-1}^{(i)}(x_k) g_k(z_k \mid x_k) dx_k, \quad (18)$$

where $p_{D,k}$ is the detection probability, Z_k is the measurement set received at time step k, $g_k(\cdot \mid x)$ denotes the single-target measurement likelihood, and $\kappa_k(z_k)$ is the intensity of clutter.

III. The Proposed Algorithm

In this section, a closed-form solution to the CBMeMBer filter with non-Gaussian noises is elaborated. The analytic implementation for the CBMeMBer filter with non-Gaussian model noises is derived by using Gaussian mixtures in a similar way to the GM-CBMeMBer filter, and the detailed formulation of the analytic implementation method is described as follows.

A. Prediction

At time step $k-1$, suppose that the posterior multi-target density π_{k-1} is a multi-Bernoulli and its form is the same as (7), where $p_{k-1}^{(i)}(x_{k-1})$ can be expressed as

$$p_{k-1}^{(i)}(x_{k-1}) = \sum_{j=1}^{L_{k-1}^{(i)}} w_{k-1}^{(i,j)} N(x_{k-1}; m_{k-1}^{(i,j)}, P_{k-1}^{(i,j)}). \quad (19)$$

Then, the form of the predicted multi-target density $\pi_{k|k-1}$ at time step k is the same as (8). Suppose that the probability density of the ith birth track is $p_{\Gamma,k}^{(i)}(x_k) = \sum_{j=1}^{L_{\Gamma,k}^{(i)}} w_{\Gamma,k}^{(i,j)} N(x_k; m_{\Gamma,k}^{(i,j)}, P_{\Gamma,k}^{(i,j)})$, substituting (5) and (19) into (9) and (10), the parameters for the surviving targets in $\pi_{k|k-1}(x_k)$ can be computed as follows:

$$r_{P,k|k-1}^{(i)} = p_{S,k} r_{k-1}^{(i)}, \quad (20)$$

$$p_{P,k|k-1}^{(i)}(x_k) = \sum_{j=1}^{L_{k-1}^{(i)}} \sum_{l=1}^{N_{q,k-1}} w_{P,k|k-1}^{(i,j,l)} N(x_k; m_{P,k|k-1}^{(i,j,l)}, P_{P,k|k-1}^{(i,j,l)}), \quad (21)$$

$$w_{P,k|k-1}^{(i,j,l)} = w_{k-1}^{(i,j)} w_{q,k-1}^{(l)}, \quad (22)$$

$$m_{P,k|k-1}^{(i,j,l)} = F_{k-1} m_{k-1}^{(i,j)} + m_{q,k-1}^{(l)}, \quad (23)$$

$$P_{P,k|k-1}^{(i,j,l)} = F_{k-1} P_{k-1}^{(i,j)} F_{k-1}^T + Q_{q,k-1}^{(l)}. \quad (24)$$

B. Update

Suppose that at time step k, the form of the multi-Bernoulli posterior density $\pi_{k|k-1}(x_k)$ is the same as (11), where $p_{k|k-1}^{(i)}(x_k)$ can be expressed as

$$p_{k|k-1}^{(i)}(x_k) = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} N(x_k; m_{k|k-1}^{(i,j)}, P_{k|k-1}^{(i,j)}). \quad (25)$$

Then, the updated multi-target density $\pi_k(x_k)$ at time step k is the same as (12). Substituting (6) and (25) into (13)-(18), the equations to calculate the parameters in $\pi_k(x_k)$ are derived as follows:

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - p_{D,k}}{1 - p_{D,k} r_{k|k-1}^{(i)}}, \quad (26)$$

$$p_{L,k}^{(i)}(x_k) = p_{k|k-1}^{(i)}(x_k), \quad (27)$$

$$r_{U,k}(z_k) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1 - r_{k|k-1}^{(i)}) \psi_k^{(i)}(z_k)}{(1 - p_{D,k} r_{k|k-1}^{(i)})^2}}{\kappa_k(z_k) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \psi_k^{(i)}(z_k)}{1 - p_{D,k} r_{k|k-1}^{(i)}}}, \quad (28)$$

$$\psi_k^{(i)}(z_k) = p_{D,k} \sum_{j=1}^{J_{k|k-1}^{(i)}} \sum_{l=1}^{N_{r,k}} w_{r,k}^{(l)} w_{k|k-1}^{(i,j)} \times N(z_k; H_k m_{k|k-1}^{(i,j)} + m_{r,k}^{(l)}, R_{r,k}^{(l)} + H_k P_{k|k-1}^{(i,j)} H_k^T), \quad (29)$$

$$p_{U,k}(x_k; z_k) = [\sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{J_{k|k-1}^{(i)}} \sum_{l=1}^{N_{r,k}} w_{U,k}^{(i,j,l)}(z_k)]^{-1} \times \sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{J_{k|k-1}^{(i)}} \sum_{l=1}^{N_{r,k}} w_{U,k}^{(i,j,l)}(z_k) \times N(x_k; m_{U,k}^{(i,j,l)}(z_k), P_{U,k}^{(i,j,l)}), \quad (30)$$

$$w_{U,k}^{(i,j,l)}(z_k) = \frac{p_{D,k} r_{k|k-1}^{(i)} w_{r,k}^{(l)} w_{k|k-1}^{(i,j)}}{1 - r_{k|k-1}^{(i)}} \times N(z_k; H_k m_{k|k-1}^{(i,j)} + m_{r,k}^{(l)}, R_{r,k}^{(l)} + H_k P_{k|k-1}^{(i,j)} H_k^T), \quad (31)$$

$$m_{U,k}^{(i,j,l)}(z_k) = m_{k|k-1}^{(i,j)} + K_{U,k}^{(i,j,l)}(z_k - H_k m_{k|k-1}^{(i,j)} - m_{r,k}^{(l)}), \tag{32}$$

$$P_{U,k}^{(i,j,l)} = [I - K_{U,k}^{(i,j,l)} H_k] P_{k|k-1}^{(i,j)}, \tag{33}$$

$$K_{U,k}^{(i,j,l)} = P_{k|k-1}^{(i,j)} H_k^T [R_{r,k}^{(l)} + H_k P_{k|k-1}^{(i,j)} H_k^T]^{-1}. \tag{34}$$

Similar to the GM-CBMeMBer filtering process, As time goes on, the number of the Gaussian functions representing the multi-target densities is growing without bound, and this phenomenon can cause high computational cost. Therefore, the pruning and merging procedures aimed at reducing the number of Gaussian functions also need to be implemented after update computing in the proposed algorithm. Furthermore, by means of standard approximation methods, the derived recursions to the CBMeMBer filter for linear non-Gaussian tracking models can be extended to accommodate nonlinear non-Gaussian tracking models.

C. Tracking Result Extraction

At time step k, the updated multi-Bernoulli parameter set $\pi_k = \{(r_k^{(i)}, p_k^{(i)})\}_{i=1}^{M_k}$ is used to extract the tracking results which involves the estimated number of targets and corresponding target state estimations.

The target state estimations are obtained by computing the means of the Gaussian functions which represent the multi-target posterior densities of the hypothesized tracks with existence probabilities greater than 0.5. The number of targets at each time step is estimated by summing up the updated existence probabilities. At time step k, let $\hat{N}_k$ denote the estimated number of targets, then the equation for computing $\hat{N}_k$ can be written as

$$\hat{N}_k = \mathrm{round}\left(\sum_{i=1}^{M_k} r_k^{(i)}\right) \tag{35}$$

where $\mathrm{round}(\cdot)$ represents the rounding operation.

IV. Simulation Results

For comparison, a two-dimensional linear non-Gaussian scene with different parameters is used to perform the designed simulation experiments. The environment for performing the simulation experiments was: AMD A8-6600K APU with Radeon HD(tm) Graphics 3.9 GHz, 4 GB DDR3 1600 Memory, Windows 7, and MATLAB R2012a. The sample interval is $\Delta = 1$ s. At time step k, the target state vector $x_k = [p_{x,k}, v_{x,k}, p_{y,k}, v_{y,k}]^T$ is defined as the positions and velocities of a moving target, while the target generated measurement is the target position information affected by noise. In the state transition density given in (5) and the measurement likelihood given in (6), the parameters are set as follows:

$$F_{k-1} = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_{k-1} = \begin{bmatrix} \Delta^2/2 & \Delta & 0 & 0 \\ 0 & 0 & \Delta^2/2 & \Delta \end{bmatrix}^T,$$

$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $N_{q,k-1} = N_{r,k} = 2$, $w_{q,k-1}^{(1)} = 0.6$, $w_{q,k-1}^{(2)} = 0.4$, $w_{r,k}^{(1)} = 0.8$, $w_{r,k}^{(2)} = 0.2$; $m_{q,k-1}^{(1)} = m_{r,k}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $m_{q,k-1}^{(2)} = \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix}$, $m_{r,k}^{(2)} = \begin{bmatrix} 2.8 \\ 2.8 \end{bmatrix}$; $Q_{q,k-1}^{(1)} = R_{r,k}^{(1)} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$, $Q_{q,k-1}^{(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R_{r,k}^{(2)} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$.

The birth process is a multi-Bernoulli with density $\pi_{\Gamma,k} = \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{3}$, where $r_{\Gamma,k}^{(1)} = 0.02$, $r_{\Gamma,k}^{(2)} = r_{\Gamma,k}^{(3)} = 0.03$, and

$$p_{\Gamma,k}^{(i)}(x_k) = N(x_k; m_{\Gamma,k}^{(i)}, P_{\Gamma,k}^{(i)}), \tag{36}$$

where $m_{\Gamma,k}^{(1)} = [150,20,0,33]^T$, $m_{\Gamma,k}^{(2)} = [100,15,0,15]^T$, $m_{\Gamma,k}^{(3)} = [650,10,0,10]^T$, $P_{\Gamma,k}^{(1)} = P_{\Gamma,k}^{(2)} = P_{\Gamma,k}^{(3)} = \mathrm{diag}([2,1,2,1])$, and $\mathrm{diag}(\cdot)$ denotes the diagonal matrix.

In the simulation, the probabilities of target survival and detection are $p_{S,k} = 0.99$ and $p_{D,k} = 0.98$, respectively. At each time step in the SMC-CBMeMBer filter and proposed algorithm, updated tracks are pruned with a pruning threshold of $T_r = 10^{-3}$ and a maximum of $T_{\max} = 100$ tracks. For the SMC-CBMeMBer filter, a maximum of $L_{\max} = 600$ and a minimum of $L_{\min} = 200$ particles are imposed for each hypothesized track, and multinomial resampling method is adopted for the numerical simulations. In addition, after the update computing in the proposed algorithm, the Gaussian functions representing each updated track are pruned and merged by using a weight threshold of $T_w = 10^{-3}$, a merging threshold of $U = 4$, and a maximum of $J_{\max} = 100$ components. The clutter is modeled as a Poisson RFS with the rate $r = 12$ over the surveillance region.

In the simulation, the optimal subpattern assignment (OSPA) distance [43] is used to evaluate the target localization accuracy, At time step k, define $X = \{x_i\}_{i=1}^{|X|}$ as the target state estimation set and $Y = \{y_{\pi_i}\}_{i=1}^{|Y|}$ as the true target state set, then the equation for computing the OSPA distance between X and Y is defined by

$$d_{OSPA}(X,Y) = \left(\frac{1}{|X|}\left(\min_{\pi\in\Pi_{|X|}} \sum_{i=1}^{|Y|} d^{(c)}(x_i, y_{\pi_i})^p + c^p(\| X| - |Y\|)\right)\right)^{\frac{1}{p}}, \tag{37}$$

where $d^{(c)}(x_i, y_{\pi_i})$ represents the Euler distance between the true target state x_i and estimated target state y_{π_i}.

In additon, the mean number of targets estimation error (NTE) [18], [21] is used to evaluate the target number estimation accuracy, the equation for computing the NTE between X and Y is defined by

$$\mathrm{NTE}(X,Y) = E\{|Y| - |X|\} \tag{38}$$

In order to reach reliable results, 100 Monte Carlo (MC) trials are performed for each algorithm on the same target tracks but with independently generated clutter measurements and measurement noise. The parameters of the OSPA distance are set to $p = 2$ and $c = 50$ m in the simulations.

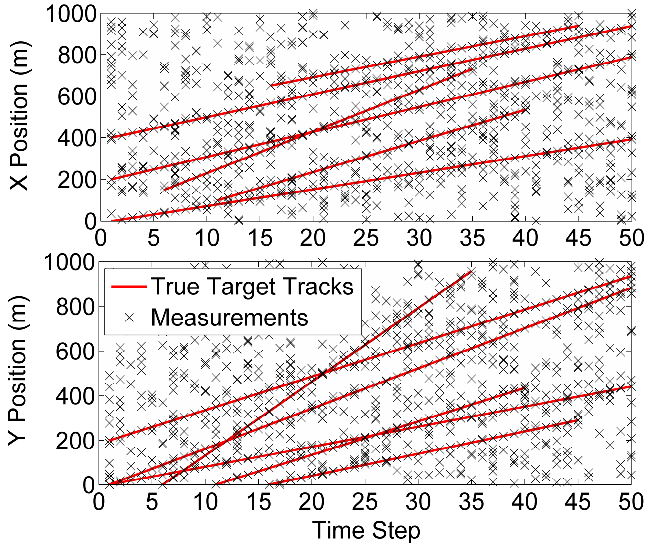

Fig. 1. True target tracks and measurements.

Fig. 1 plots the x and y positions of the true target tracks and measurements versus time step in one MC trail, where the true target tracks and the measurements are denoted by the red solid lines and the black multiplication signs, respectively.

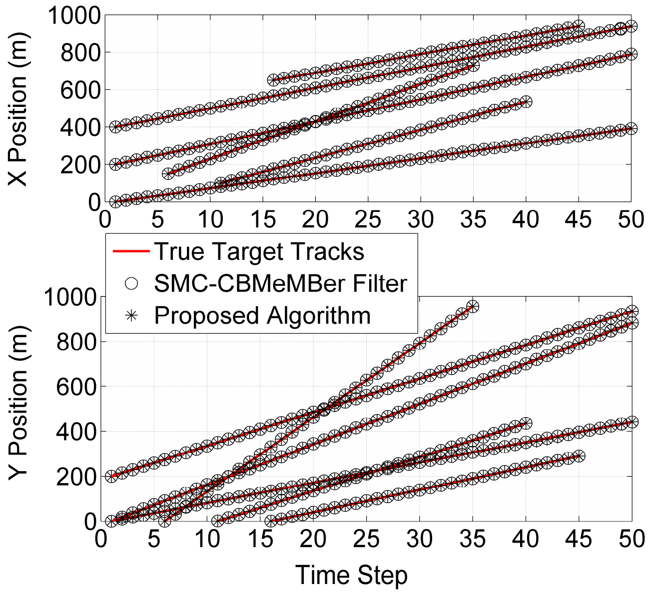

Fig. 2. True target tracks and position estimations.

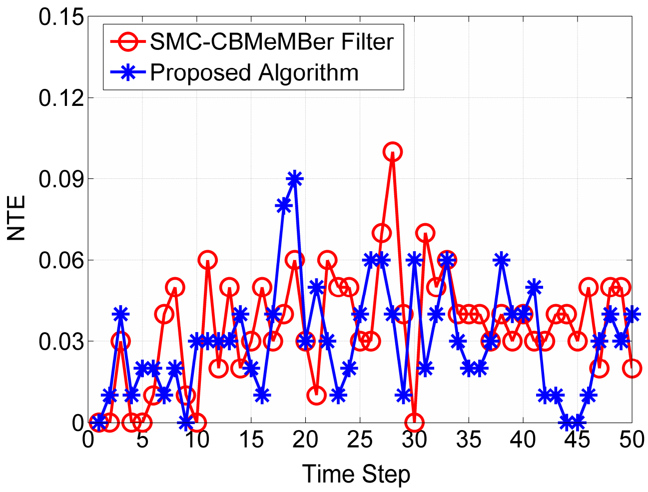

Fig. 3. NTEs for different algorithms.

Fig. 2 gives the results of the target position estimations for both algorithms superimposed on the true target tracks over 50 time steps. As seen from Fig. 2, the position estimations for both algorithms are close to the true target tracks during the whole period. It indicates that the proposed algorithm is able to track multiple targets well.

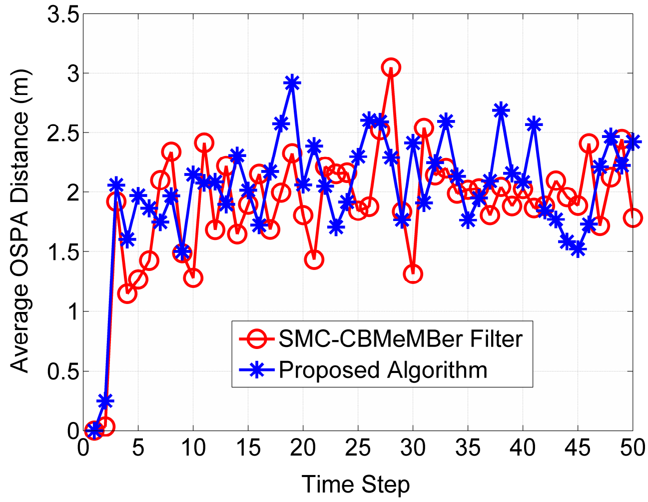

Fig. 4. Average OSPA distances for different algorithms.

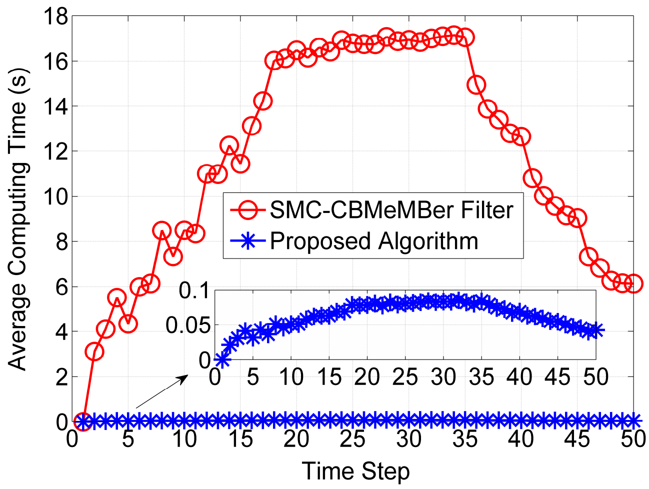

Fig. 5. Average computing time for different algorithms versus time step.

The NTEs, the OSPA distances and the computing time are used to analyze the tracking performances of different algorithms, with the results presented in Figs. 3, 4 and 5, respectively. As seen from Fig. 3, at each time step, the NTEs for both algorithms are very small, this means that the target number estimations computed by both algorithms are reliable and close to the true number of targets. Fig. 4 indicates that the proposed algorithm performs similarly to the SMC-CBMeMBer filter, i.e., the target localization accuracy of the proposed algorithm is almost the same as that of the SMC-CBMeMBer filter. However, it is observed from Fig. 5 that the proposed algorithm performs at a much faster computing speed than the SMC-CBMeMBer filter does throughout the entire process. The reason is that the SMC-CBMeMBer filter requires a significant amount of time to obtain the target track parameters by update computing, discard updated particles with low weights and reproduce updated particles with high weights after update computing.

For further evaluating the effectiveness of the proposed algorithm, 100 MC trials are performed for the proposed algorithm and the SMC-CBMeMBer filter with different clutter rates and detection probabilities. First, the detection probability $p_{D,k}$ ranges from 0.93 to 0.98, the clutter rate remains unchanged at r=12 and other parameters which are used in the MC trials are the same as that of the tracking scenario given above. The time averaged NTEs, time averaged OSPA distances and average computing time versus detection probability are shown in Figs. 6, 7 and 8, respectively. Second, the clutter rate r ranges from 1 to 30 under the condition of an unchanged detection probability $p_{D,k}$=0.98, and other parameters which are used in the MC trials are the same as that of the tracking scenario given above. The time averaged NTEs, time averaged OSPA distances and average computing time versus clutter rate are shown in Figs. 9, 10 and 11, respectively.

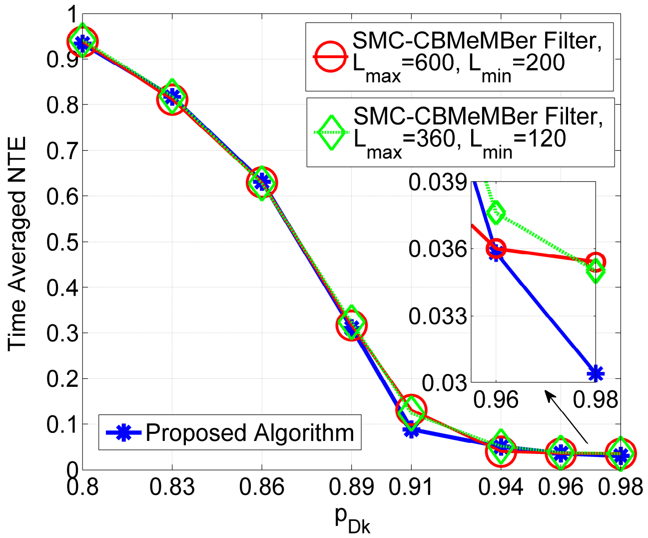

Fig. 6. Time averaged NTEs for different algorithms versus detection probability. The clutter rate r=12 is fixed.

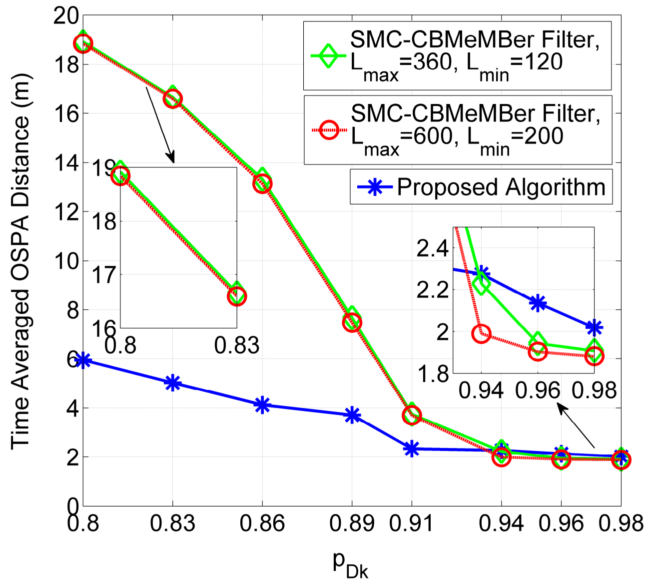

Fig. 7. Time averaged OSPA distances for different algorithms versus detection probability. The clutter rate r=12 is fixed.

In Figs. 6 and 7, it is obvious that all filtering algorithms used in simulation experiments are significantly affected by the value of detection probability. A low value of detection probability can lead to a unreliable tacking result, i.e., in low detection probability simulation experiments, both the time averaged OSPA distances and the time averaged NTEs of different algorithms are lower. However, the difference in the time averaged OSPA distances tends to be significant with a decrease in the detection probability. This indicates that the tracking accuracies of the proposed algorithm are better than those of the other two filtering algorithms in the case of low detection probabilities.

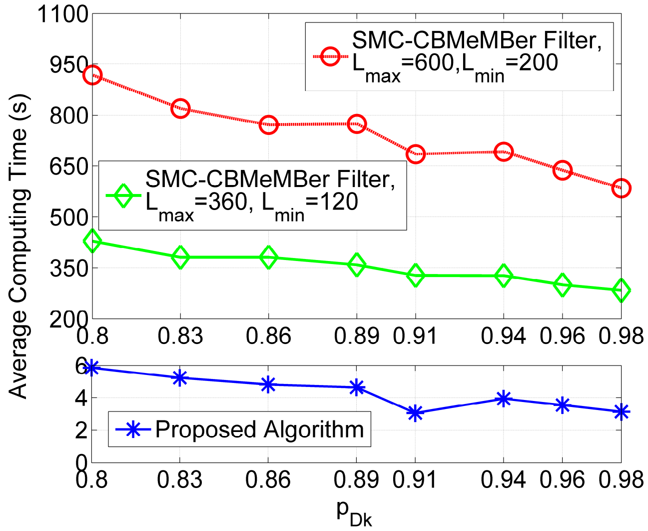

Fig. 8. Average computing time for different algorithms versus detection probability. The clutter rate r=12 is fixed.

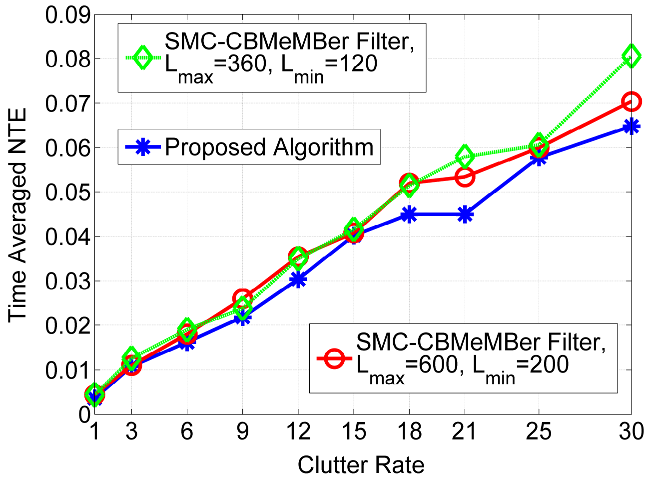

Fig. 9. Time averaged NTEs for different algorithms versus clutter rate. The detection probability $p_{D,k}$=0.98 is fixed.

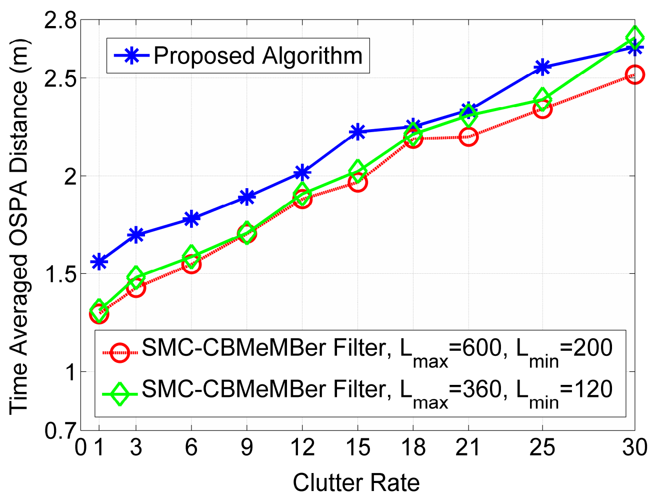

Fig. 10. Time averaged OSPA distances for different algorithms versus clutter rate. The detection probability $p_{D,k}$=0.98 is fixed.

From Figs. 6, 7 and 8, it can be also seen that the proposed algorithm can achieve satisfactory tracking results with a much smaller computational cost compared with the SMC-CBMeMBer filter in different detection probabilities.

Figs. 9 and 10 show the time averaged NTEs and time averaged OSPA distances under different clutter rates. As expected, the number of clutter measurements has an influence on the tracking performance, and the tracking accuracies of all filtering algorithms decrease with the increase of the clutter rate. However, the time averaged NTEs of the proposed algorithm tend to be lower than other filtering algorithms with an increase in the clutter rate. As seen from Fig. 11, the computing time of all filtering algorithms is growing as the clutter rate increases, but the average time that completing one MC trial requires of the proposed algorithm is much smaller than that of the SMC-CBMeMBer filter.

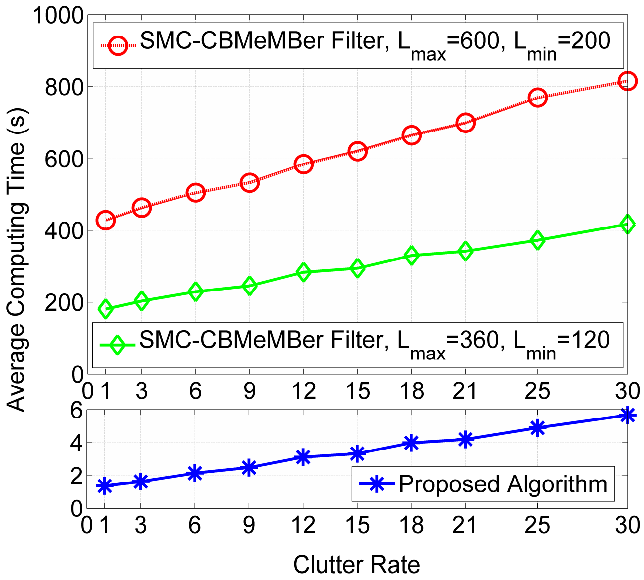

Fig. 11. Average computing time for different algorithms versus clutter rate. The detection probability $p_{D,k}$=0.98 is fixed.

In addition, from Figs. 6, 7, 8, 9, 10 and 11, it can be observed that the greater the number of particles is used in SMC-CBMeMBer filter the better the tracking accuracy will be achieved. However, the improved tracking accuracy is obtained at the expense of additional computational load. In a word, all these results presented in Figs. 6, 7, 8, 9, 10 and 11 confirm that the proposed algorithm can achieve a much higher processing rate compared with the SMC-CBMeMBer filter, and can also address the linear non-Gaussian tracking problems with satisfactory results.

V. Conclusion

Under the assumptions of linear non-Gaussian models, a Gaussian mixture implementation method of the CBMeMBer filter, which can be seen as a generalized extension of the GM-CBMeMBer filter, is derived in this paper. Based on the Gaussian mixture models of non-Gaussian model noises and the standard conclusions for Gaussian distributions, the formulation of the proposed filter is presented in detail. The proposed filter overcomes the drawbacks that are existed in both the GM-CBMeMBer filter and the SMC-CBMeMBer filter. The designed simulations demonstrate that the proposed algorithm can track a time-varying and unknown number of targets with much less computing time under complex environments than SMC-CBMeMBer filter.

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