Analysis of Connectivity Reliability of Multi-level Rail Transit Network Considering Passengers' Bounded Rationality

Yubo Zhang, Changfeng Zhu, Bin Ma, Xin Wang, and Enhua Xu

Abstract—**In order to ensure the safe operation of multi-level rail transit composite network, a composite network evaluation model was constructed by using complex network theory. This model analyzed the connectivity reliability of the composite network in response to sudden site failures through the global network efficiency and network connectivity. Among them, the network connectivity index from the perspective of passengers was proposed, the prospect theory is introduced to describe the connectivity of the entire composite network, and the connectivity measured by the largest connected subgraph is compared and analyzed. Matlab2018a software was used to simulate the attack process of different strategies, and the differences of the unweighted and passenger flow weighted composite network in dealing with different attacks were compared and analyzed. The connectivity reliability of the composite network was analyzed by taking the actual data of a municipal railway-subway as an example. The research results showed that the unweighted composite network appeared more vulnerable. The composite network connectivity from the passenger's point of view was directly proportional to the tolerance coefficient. Under different strategy attacks, the network connectivity index from the passenger's point of view reflected better composite network connectivity reliability. This paper provided a certain reference for analyzing the resilience of multi-level rail transit composite network.**

Index Terms—**Multi-level Rail Transit, Composite Network, Tolerance Coefficient, Prospect Theory, Connectivity reliability**

I. Introduction

In recent years, with the rapid advancement of the urbanization process, various types of rail transit have developed rapidly, but there have been "urban diseases" such as long travel distances and separation of work and housing caused by unbalanced regional development and many other factors. As an important infrastructure for constructing a reasonable spatial layout of a metropolitan area, the development of rail transit directly affects the rationality of the development of the metropolitan area. Therefore, how to reasonably integrate different rail transit networks is a difficult problem at present. Exploring the safety of each rail transit network, analyzing the topological characteristics of the rail transit network, and quantitatively calculating the reliability of each station are of great significance to ensuring the stable operation of rail transit.

Many scholars had studied the network in various fields, and the emergency material distribution networks had been studied in [2][3]. The bus network had been studied in terms of service reliability such as headway reliability and running time reliability, but there was a lack of system-level research based on network topology in [4][5][6][7]. Therefore, many scholars had done a lot of research on the topology of rail transit network by using complex network theory. First, the method of measuring the degree of damage to the network after being attacked was proposed in [8], which created a precedent for subsequent research. [9][10][11][12][13][14][15] studied the characteristics and elasticity of subway networks. By proposing a structural evaluation method based on system elasticity, a topological structural elasticity evaluation model of regional rail transit network was constructed in [16]. The topological characteristics and robustness of the railway express freight network were studied in [17] by considering the line level. [18] studied the vulnerability of the subway network from the perspective of associated infrastructure. Based on the improved network efficiency formula, the anti-destruction degree of the network under the simulated attack of 10 typical cities was analyzed in [19]. The invulnerability of urban rail networks in six developed cities in the world was analyzed from the perspective of network science in [20]. The vulnerability of high-speed railway networks in different countries was compared and analyzed in [21].

In view of the importance of complex network nodes, many scholars studied the important nodes in the network in [22] [23] [24] [25] [26] [27] by establishing spatial weighted degree model, comprehensive evaluation model of degree and intermediate number and key node identification method combining economic value and demand. Personnel deployment was studied in traffic emergency networks in [28][29]. The above literatures had made contributions to the

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research on the characteristics of rail transit network topology and the importance of complex network nodes. However, the actual weighted networks were rarely considered in above literatures.

In view of the shortcomings of the above literature, a rail transit network model based on passenger flow weighting was constructed by analyzing the effects of passenger flow proportional coefficient and node efficiency on stations in [30][31]. By considering the influence of OD flow distribution in the transportation network, a method for cascading failure analysis of subway network based on weighted coupled image lattice model was proposed by taking Beijing subway as an example in [32]. Different from the weight of passenger flow in the above literature, the air transport weighted network and road transport weighted network model were constructed by taking the route frequency and the number of departure trains as the side weights in [33][34][35]. Unfortunately, all of the above literatures took single-level network model as the research object, which failed to reflect the interaction relationship and comprehensive diversity among networks.

In view of the shortcomings of the above-mentioned single-level network model, a multi-level composite network model was constructed by considering the subway-bus multi-mode and multi-level topology network, and the cascading failure process of the network under emergencies was simulated and analyzed in [36][37][38]. The quantitative determination rule and process method of coupled stations based on ArcGIS were proposed, and the subway-bus composite network model was constructed based on this in [39]. A vulnerability assessment model based on the loss of average travel time was established by considering the connecting effect of bus on passenger flow under the scenario of station failure in [40]. It was worth mentioning that [41] constructed a three-level superimposition urban multi-mode transportation network model of car network, ground bus network and rail transit network by considering passenger flow and running time. Further than the above literatures, the city agglomeration composite transport network model was constructed by superposition with the number of departures, number of trains, flight frequency and flight frequency as the weight of the edge of road transport network, rail transport network, air transport network and waterway transport network in [42]. Although the complexity of multi-level networks was considered in the above literatures, traditional evaluation indexes such as maximum connected subgraph and global network efficiency were adopted. However, [43] used the number of tolerable paths between stations to measure the connectivity reliability between stations and the connectivity of the entire network, providing a new idea for measuring the connectivity reliability of the network.

The above literatures studied the reliability of network connectivity under random and deliberate attacks by constructing unweighted networks and weighted composite networks in various ways. However, there was almost no research on connectivity reliability of multi-level rail transit composite network in existing literatures. The existing literatures rarely considered the weighted composite network model, and did not consider the edge weight of the network from the actual situation. In the existing literatures, the measurement of network connectivity reliability was mainly based on the traditional network efficiency and maximum connectivity subgraph, while the measurement of composite network connectivity reliability was not carried out from the perspective of passengers.

In view of the above shortcomings, this paper will use the Space-L method to construct a multi-level rail transit composite network model, and compare and analyze the connection reliability of the unweighted composite network and the passenger flow weighted composite network to deal with random attacks and deliberate attacks. From the perspective of passengers, the prospect theory is introduced to describe the evaluation indexes of network connectivity reliability, so as to analyze the composite network connectivity reliability under different evaluation indexes. A comparative analysis of network connectivity from the perspective of passengers and traditional network connectivity measured by maximum connectivity will be carried out.

The rest of this paper is summarized as follows: Section II constructs multi-level rail transit composite network model. The bounded rational behavior analysis of passenger travel is in Section III. The composite network connectivity reliability measure index is in Section IV. The example analysis of the basic attributes of the composite network in a city in Section V. The connectivity reliability analysis of the composite network is in Section VI. Finally, the conclusion of this paper is in Section VII.

II. Composite Network Model Construction

A. Model Assumptions

Hypothesis 1: The difference in train direction, departure frequency, and traffic routes between the two stations are not considered.

Hypothesis 2: The shortest travel time of passengers shall be the minimum travel time of all passengers.

B. Unauthorized Network Model Construction

Based on the physical structure of the actual network, the stations in the multi-level rail transit network are abstracted as the nodes of the composite network, and the sections between stations are mapped to the edges between nodes. The Space-L method is used to establish a fully connected topology model composed of stations and lines in a multi-level rail transit network.

Based on the above description, the multi-level rail transit unweighted composite network model is constructed as shown in (1). Among them, the composite network includes the national railway main line sub-network, the inter-city railway sub-network, the municipal railway sub-network and the urban rail transit sub-network. The sites and lines of the four-level network are merged by means of superposition.

$$G_1 = (V, E) \tag{1}$$

Where, V represents the network node set in the unweighted composite network G_1, whose set is $V=\{v_1,v_2,\ldots,v_i,\ldots,v_j,\ldots,v_N\}$. N represents the total number of network nodes. E represents the network edge set in the unweighted composite network G_1, whose set is $E=\{e_1,e_2,\ldots,e_M\}$. M represents the number of road segments between pairs of stations that each sub-network line travels.

C. Passenger Flow Weighted Network Model Construction

The multi-level rail transit weighted composite network model is constructed in (2).

$$G_2 = (V, E, W) \tag{2}$$

Among them, W represents the passenger flow weight set of the network connection, and its set is $W=\{W_{s,t}|s,t\in V\}$. The OD passenger flow matrix between the stations of each line is allocated according to the shortest path. The weight matrix of composite network is obtained by counting the passenger flow of each side of the network.

III. Analysis of Bounded Rational Behavior of Passenger Travel

In the actual travel process, passengers are often bounded rationality. The tolerance coefficient was used to determine which connectivity paths passengers would use in [39]. The passenger's tolerance coefficient for travel time is proposed in the paper, and the passenger's tolerance coefficient for travel is defined as a reference point. When the ratio of the actual travel time of the passenger to the shortest travel time is not less than the travel tolerance coefficient, the passenger is satisfied from the node, that is, the passenger is profitable. On the contrary, the passenger will lose from this node, and the passenger is more sensitive to the loss than the profit. Therefore, the overall tolerance value function of passengers traveling from station v_i is as follows.

$$H(v_i) = \begin{cases} (\Delta A_i)^{\alpha}, \Delta A_i \geq 0 \\ -\lambda(-\Delta A_i)^{\beta}, \Delta A_i < 0 \end{cases} \tag{3}$$

Where, α and β are the sensitivity coefficients of gain and loss respectively, $0 < \alpha, \beta \leq 1$. In general, $\alpha=\beta=0.88$. λ is the loss avoidance coefficient, and $\lambda \geq 1$. In general, $\lambda=2.25$. ΔA_i represents the deviation value of the actual relative travel time of passengers compared to the reference point, which can be expressed by (4).

$$\Delta A_i = \sum_{j=1}^{N'} t_{ij} - A \tag{4}$$

Where, N' is the set of sites without node v_i. t_{ij} represents the actual relative travel time of passengers starting from node v_i, and $t_{ij} = t_{ij}^g / t_{ij}^{short}$. t_{ij}^g represents the travel time of passengers on the g path from node v_i to node v_j. t_{ij}^{short} represents the minimum travel time from node v_i to node v_j. A represents the travel endurance coefficient of passengers, which can be set according to specific conditions.

The overall tolerance value function curve of passengers travelling from station v_i is shown in Fig. 1.

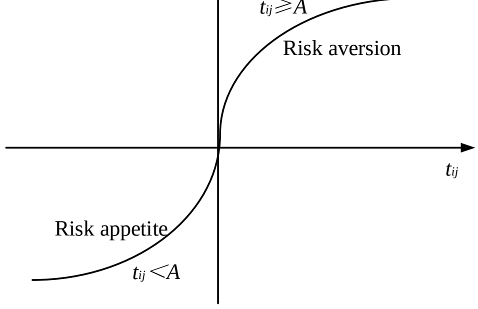

Fig.1 Value function graph

The weight function of passengers' overall tolerance to travel from station v_i is shown in (5). The weight function reflects that decision-makers tend to attach importance to low probability events while ignoring high probability events.

$$\omega(p) = \begin{cases} \dfrac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma}}, \Delta A_i \geq 0 \\ \dfrac{p^{\delta}}{[p^{\delta} + (1-p)^{\delta}]^{1/\delta}}, \Delta A_i < 0 \end{cases} \tag{5}$$

Where, $\omega(p)$ is the perceived probability of passengers choosing a certain route from the node. p is the actual probability that passengers choose a certain route to travel from the node. γ is the perceived probability coefficient of earnings. δ is the probability coefficient of loss perception. In general, γ=0.61, δ=0.69. The values of γ and δ determine the curvature of the weight function. The weight function curve is shown in Fig. 2. Passengers generally have the characteristics of overestimating small probabilities and underestimating large probabilities in the decision-making process.

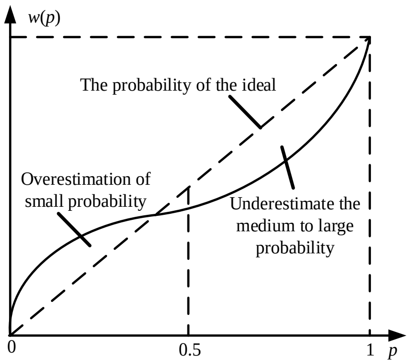

Fig.2 Weight function curve

IV. Composite Network Connectivity Reliability Measurement Index

The global efficiency of the composite network and the connectivity of the composite network are taken as the dynamic evaluation index of the overall connectivity reliability of the composite network. The static basic properties and specific dynamic evaluation indexes of the composite network are as follows.

A. Composite Network Static Basic Properties

A.1. Degree and Degree Distribution of Nodes

The degree k_i of an unweighted composite network node v_i is defined as the number of edges connected to the node. The greater the degree of a node, the more "important" the node is in a sense. The average value of the degree k_i of all nodes represents the average degree of the network, which is defined as $\langle k \rangle$, and its calculation is shown in (6). The degree distribution $P(k)$ represents the proportion of nodes whose degree value is exactly k in all nodes.

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i \tag{6}$$

The point strength S_i of the weighted composite network node v_i is defined as the sum of the edge weights associated with it, which is calculated as the following formula (7). The average point strength of the network is defined as the average value of S_i.

$$S_i = \sum_{j \in N_i} W_{ij} \tag{7}$$

Where, N_i represents the set of neighbor points of node v_i. W_{ij} represents the weight of the edge connecting nodes v_i and v_j.

A.2. Average Path Length

The distance d_{ij} between nodes v_i and v_j in the unweighted composite network is defined as the number of edges on the shortest path connecting these two nodes. The maximum value of the distance between any two nodes in the composite network is the diameter D of the network, which is calculated as the following (8).

$$D = \max_{1 \le i < j \le N} d_{ij} \tag{8}$$

The average value of d_{ij} is called the average path length L of the composite network, which is calculated as the following (9).

$$L = \frac{1}{C_N^2} \sum_{1 \le i < j \le N} d_{ij} \tag{9}$$

The shortest distance d_{ij}^w between two nodes v_i and v_j in the weighted composite network is defined as the weighted sum of edge weights on the shortest path connecting these two nodes. The average value of the shortest path length is called the average path length L^w of the weighted network, which is calculated as the following (10).

$$L^w = \frac{2}{N(N-1)} \sum_{i \ne j} d_{ij}^w \tag{10}$$

A.3. Clustering Coefficient

In the unweighted composite network, the ratio of the actual number of edges A_i between the k_i neighbor nodes of node v_i and the total possible number of edges is called the clustering coefficient C_i of node v_i, which is calculated as the following formula (11).

$$C_i = \frac{A_i}{C_{k_i}^2} \tag{11}$$

The clustering coefficient C of the entire network is the average value of the clustering coefficients C_i of all nodes v_i, which is calculated as the following formula (12).

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i \tag{12}$$

The clustering coefficient of the weighted composite network is defined in (13).

$$C_i^w = \frac{2}{k_i(k_i - 1)} \sum_{j,h \in V, j \ne h} (w'_{ij} w'_{jh} w'_{hi})^{\frac{1}{3}} \tag{13}$$

Where, $w'_{ij} = w_{ij} / \max\{w_{ij}\}$ is the normalized weight.

B. Global Efficiency of Composite Network

Based on the definition of the distance between adjacent nodes, the global efficiency EL and EL_w of the unweighted and weighted composite network are as follows (14) and (15) respectively.

$$EL = \frac{1}{N(N-1)} \sum_{i \ne j} \frac{1}{d_{ij}} \tag{14}$$

$$EL_w = \frac{1}{N(N-1)} \sum_{i \ne j} \frac{1}{d_{ij}^w} \tag{15}$$

C. Composite Network Connectivity

C.1. Maximum Connectivity of CompositeNetwork

The changes of complex network performance caused by the removal of network sites or edges are essentially related to the phenomenon of lattice seepage in [44]. For the general complex network, this phenomenon corresponds to the appearance of the maximum connected component of the network, and its disappearance is related to the vulnerability of the complex network. With the continuous advancement of the simulation attack process, the maximum connected component of the network will gradually become smaller, that is, the network operation coverage will gradually become smaller.

The overall connectivity of the composite network is an important index to measure its connectivity reliability when it is subjected to continuous attacks. The maximum connectivity measure can better describe the connectivity of each node in the network. The maximum connected subgraph is defined as the subgraph in which all nodes in the network are connected with the least number of edges. The ratio of the number of nodes in the maximum connected subgraph to the number of all nodes in the network is the maximum connectivity SL. Therefore, the following (16) represents the maximum connectivity of the composite network.

$$SL = \frac{n_s}{N} \tag{16}$$

Where, n_s represents the number of nodes in the maximum connected subgraph. The greater the maximum connectivity, the higher the connectivity between network nodes and the stronger the connectivity reliability of the network.

C.2. Network Connectivity from the Passenger Perspective

The higher the overall tolerance of passengers to travel time, the stronger the connectivity of the composite network. Therefore, the overall tolerance of passengers for travel time can be used to measure the connectivity of the composite network. The prospect theory is introduced to describe the passenger's overall tolerance of travel time.

According to the analysis of the bounded rationality of passenger travel in the second section, the overall tolerance function of passengers traveling from station v_i can be obtained as the comprehensive prospect value of prospect theory, as shown in the following (17).

$$f(v_i) = H(v_i) \times \omega(p) \tag{17}$$

The connectivity F_i of the station is characterized by the overall tolerance of passengers traveling from station v_i, which is the following (18). Thus, the composite network connectivity F from the perspective of passengers can be obtained as shown in (19). Where, M_f represents the normalization coefficient of connectivity.

$$F_i = \sum_{i=1}^{N} f(v_i) / M_f \tag{18}$$

$$F = \frac{\sum_{i=1}^{N} F_i}{N \times M_f} \tag{19}$$

V. An Example Analysis of Basic Attributes of Composite Network in a City

A. The Example Data

It is known that there are 2 lines and 27 stations in the municipal railway sub-network of a certain city. There are 14 lines and 282 stations in the subway sub-network. There are 16 lines and 309 stations in the municipal railway-subway composite network of the city. All stations are numbered according to the line sequence, and stations with two or more lines passing through are numbered only once. The topology mapping of a municipal railway-subway composite network is shown in Fig. 3.

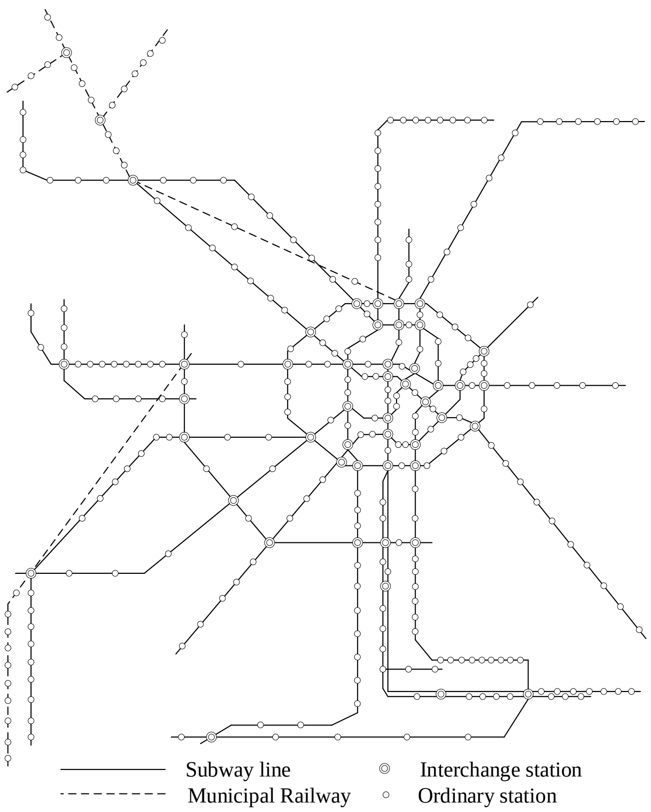

Fig.3 Topology mapping of a municipal railway-subway composite network

B. Analysis of Static Basic Properties of Composite Network

The calculation of the basic evaluation index of the composite network is shown in Table I.

TABLE I
BASIC EVALUATION INDEX OF COMPOSITE NETWORK

Index	Unweighted network	Weighted network
Number of nodes(N)	309	
Number of edges(M)	350	
The network diameter(D)	40	44
Average degree/Point strength($<k>/S$)	2.2654	3.4493
Average path length(L)	14.4383	20.1602
Clustering coefficient(C)	0.0016	0.00078
Global Efficiency of Composite Network(EL)	0.10	0.12

As can be seen from Table I, there are 350 connecting edges between stations in the city's composite network. If the cross-section passenger flow weighting is not considered, the average degree of each station is 2.2654, which means that each station has 2.2654 adjacent stations on average. It indicates that only one rail transit line passes through most of the stations in the composite network, and there are relatively few intersections between the lines. The average path length is 14.4383 and the network diameter is 40, that is, 14.438 stations are needed to reach any two stations in the composite network on average, and the longest connecting edge needs to pass 40 stations to reach it. It shows that the average line length of the composite network is relatively short, and the composite network has good accessibility. The clustering coefficient of the entire network is 0.0016, which indicates that the connections between the city's composite network stations are not close. The fitting relationship between node degree and cumulative probability is shown in Fig. 4 below.

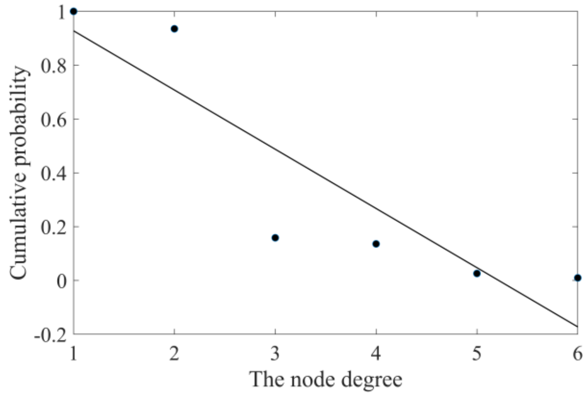

Fig.4 Fitting diagram of node degree and cumulative probability

It can be seen from Fig. 4 that the degree value has an approximately linear relationship with the cumulative probability. It shows that the composite network is a scale-free network, which has the characteristics of low degree of most nodes, but a small number of nodes with high degree. The degree distribution is shown in Table II.

TABLE II
DEGREE DISTRIBUTION OF UNWEIGHTED COMPOSITE NETWORK

k	1	2	3	4	5	6
$P(k)$	$\frac{20}{309}$	$\frac{240}{309}$	$\frac{7}{309}$	$\frac{34}{309}$	$\frac{5}{309}$	$\frac{3}{309}$

If the cross-section passenger flow weighting is considered, the average point strength of each station of the composite network is 3.4493, that is, the adjacent stations of each station have an average daily cross-section passenger flow of 34,493 people. It shows that the passenger flow balance of the composite network based on the cross-section passenger flow weight is relatively good, and there is no congestion. The average path length is 20.1602 and the network diameter is 44, that is, 20.1602 stations are needed to reach any two stations in the composite network on average in the case of passenger flow weighting, and the longest connecting edge needs to pass 44 stations to reach it. It shows that the average line length of the composite network is relatively long when the weight of passenger flow is considered, which also accords with the situation of large passenger flow that may occur in actual travel. The clustering coefficient of the entire network is 0.00078, which indicates that the connections between the city's composite network stations are not close.

VI. Connectivity Reliability Analysis of Composite Network in a City

A. Attack Strategy

Random attack and deliberate attack are used to study the connectivity reliability of the composite network. The specific attack strategy is as follows.

(1) Attack the nodes in the composite network randomly, so that the nodes and corresponding connecting edges fail simultaneously.

(2) Attack the composite network nodes according to the ordering of the node degree (point strength ordering), that is, deliberately attacking the node, so that the failed node and the corresponding connection edge are invalidated at the same time.

B. Comparative Analysis of Global Efficiency of Composite Networks under Different Attack Strategies

Based on the OD swiping data provided by AFC system in a certain day, the global network efficiency of the municipal railway-subway composite network under the two attack strategies is analyzed.

Since random attacks have strong uncertainties, we take the average global efficiency of the composite network under 100 random attacks. The global efficiency change diagram of the unweighted composite network under the two attack strategies is shown in Fig. 5.

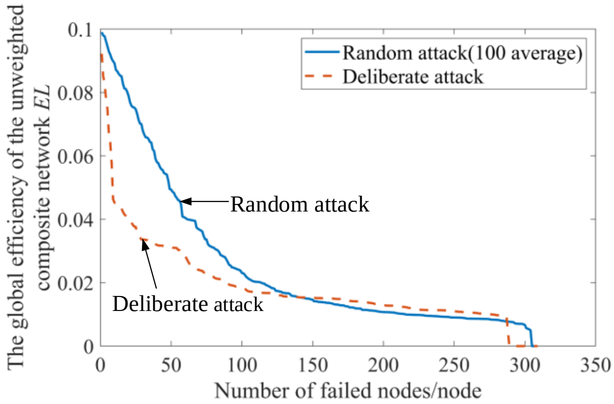

Fig.5 The global efficiency change diagram of the unweighted composite network

As can be seen from Fig. 5, for the unweighted composite network, the global efficiency of the composite network decreases with the increase of the number of failed nodes. The global efficiency of composite network decreases faster when deliberate attack is adopted. When the number of failed nodes is about 10, the global efficiency of the composite network has been reduced to half of that without failure. It shows that the degree of nodes in the unweighted network reflects the importance of nodes. When the number of failed nodes is less than 123 (40%), deliberate attack has a great impact on the global efficiency of the composite network, and then affects the connectivity reliability of the composite network. The reason is that the early stage of the deliberate attack makes the node with a large degree of failure, which is highly offensive to the network. When the number of failed nodes exceeds 123, the global efficiency of the composite network drops below 0.02, and the composite network is close to collapse. At this time, random attack has a great impact on the global efficiency of the composite network, and then affects the connectivity reliability of the composite network. The reason is that only the stations with the smallest degree are left after deliberate attack, while the random attack still retains a few important nodes with larger degrees.

The global efficiency change diagram of the weighted composite network under the two attack strategies is shown in Fig. 6 below.

As can be seen from Fig. 6, for the unweighted composite network, the global efficiency of the composite network decreases with the increase of the number of failed nodes. In the case of considering the weight of passenger flow, regardless of the number of failed nodes, deliberate attack has a greater impact on the global efficiency of the composite network, which in turn affects the connectivity reliability of the composite network. It shows that under the deliberate attack strategy, the point strength reflecting the sum of the associated weights of nodes has a greater impact on the passenger flow weighted composite network.

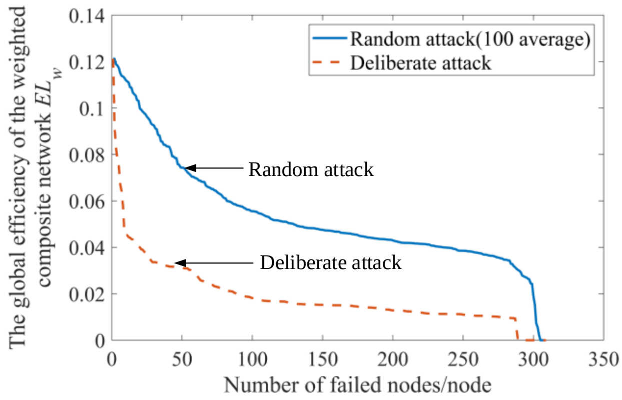

Fig.6 The global efficiency change diagram of the weighted composite network

By comparing Fig. 5 and Fig. 6, it can be seen that in the face of random attack, the composite network based on passenger flow weighting shows stronger connectivity reliability than the unweighted composite network. The initial value of the global efficiency of the composite network considering the weight of passenger flow is higher than the initial value of the global efficiency of the unweighted composite network. It shows that the passenger flow strengthens the communication between the stations and makes the whole composite network more closely connected. Under the two attack strategies, the global efficiency of the unweighted composite network decreases faster than the global efficiency of the weighted composite network. It shows that in actual operation, the ability of the composite network that considers the weight of passenger flow to respond to attack is stronger than that of an unweighted composite network that only considers the topology, that is, the unweighted composite network is more vulnerable.

C. Comparative Analysis of Connectivity of Composite Networks under Different Attack Strategies

C.1. Maximum Connectivity of Composite Networks under Different Attack Strategies

Based on the municipal railway-subway composite network structure shown in Fig. 3, the maximum connectivity changes of the composite network under the two strategies are analyzed as shown in Fig. 7.

Since random attacks have strong uncertainties, we take the maximum connectivity averages of the composite network under 100 random attacks.

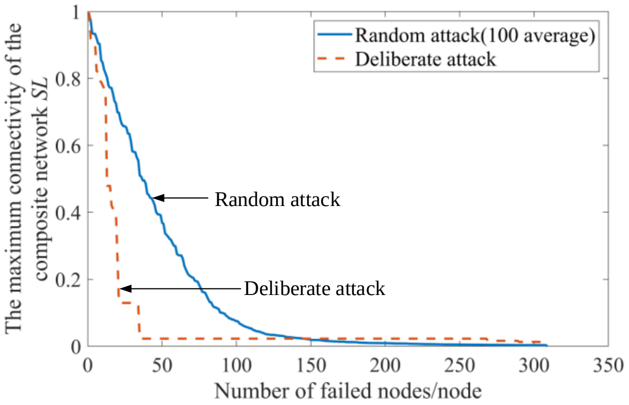

Fig.7 The maximum connectivity change diagram of the composite network

As can be seen from Fig. 7, the maximum connectivity of the composite network decreases with the increase of the number of failed nodes. The maximum connectivity of composite network decreases faster when deliberate attack is adopted. When the number of failed nodes is about 13, the maximum connectivity of the composite network has been reduced to half of that without failure. It shows that the failure of a node with a larger degree value will seriously affect the maximum connectivity of the composite network, and then affect the connectivity reliability of the composite network. When the number of failed nodes is 35 (11%), the maximum connectivity decreases to 0.023, and the composite network is close to collapse.

When the number of failed nodes is less than 123 (40%), deliberate attack has a great impact on the maximum connectivity of the composite network, and then affects the connectivity reliability of the composite network. The reason is that the early stage of the deliberate attack makes the node with a large degree of failure, which is highly offensive to the network. When the number of failed nodes exceeds 123, the maximum connectivity of the composite network drops below 0.02, and the composite network is close to collapse. At this time, random attack has a great impact on the maximum connectivity of the composite network, and then affects the connectivity reliability of the composite network. The reason is that only the stations with the smallest degree are left after deliberate attack, while the random attack still retains a few important nodes with larger degrees.

C.2. Connectivity from the Perspective of Passengers under Different Attack Strategies

According to the shortest travel time between any two stations under normal operation conditions and the actual travel time of passengers obtained according to AFC data, the comprehensive prospect value of passengers' overall tolerance to travel paths starting from a certain station under different tolerance coefficients is calculated, that is, the connectivity of each station. Thus, the connectivity of each station and the connectivity of the entire network under different tolerance coefficients are shown in Fig. 8 and Fig. 9. It is known that M_f=1000.

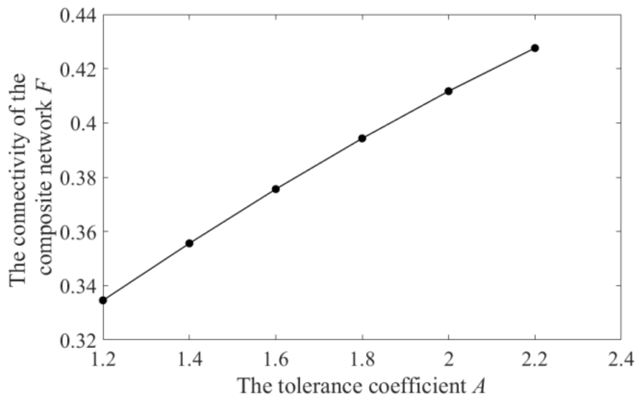

Fig.8 The composite network connectivity under different tolerance coefficients

As shown in Fig. 8, the connectivity of the composite network increases with the increase of the tolerance coefficient. The reason is that when the actual travel time that passengers can tolerate is greater than the shortest travel time, the greater the comprehensive prospect value of tolerability, the greater the connectivity of the composite network, and the stronger the connectivity reliability of the composite network. It shows that the connections between the stations are closer.

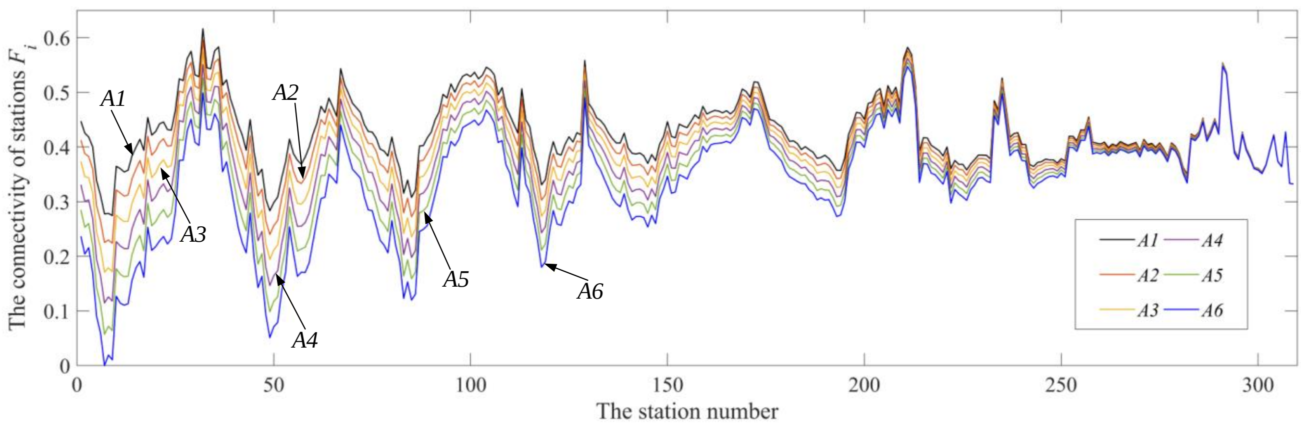

Fig.9 The connectivity of stations under different tolerance coefficients

In Fig. 9, *A1* indicates that the tolerance coefficient is 2.2, *A2* indicates that the tolerance coefficient is 2.0, *A3* indicates that the tolerance coefficient is 1.8, *A4* indicates that the tolerance coefficient is 1.6, *A5* indicates that the tolerance coefficient is 1.4, and *A6* indicates that the tolerance coefficient is 1.2.

It can be seen from Fig. 9 that the tolerance coefficient has a greater impact on station connectivity, and the greater the tolerance coefficient, the greater the connectivity of each station. Meanwhile, the stations with the strongest (weakest) connectivity are the strongest (weakest) under different tolerance coefficients. The connectivity of stations numbered 283-309 is less affected by the tolerance coefficient, because these nodes are municipal railway stations, and the actual travel time of passengers is not much different from the shortest travel time, so that the station connectivity is less affected by the tolerance coefficient.

The connectivity changes of the composite network under the two attack strategies are shown in Fig. 10.

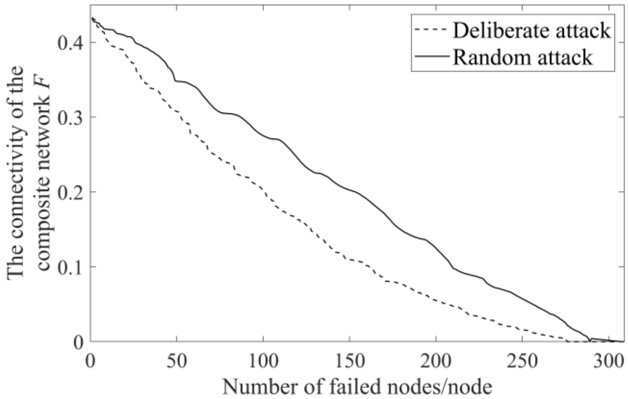

Fig.10 The connectivity change diagram of the composite network

It can be seen from Fig. 10 that the greater the number of failed nodes, the weaker the connectivity of the composite network. Fig. 10, Fig. 5, and Fig. 6 generally show the same evolution law, indicating that the connectivity of the composite network described from the perspective of passengers has a similar network connectivity reliability measurement function to the global efficiency of the composite network. However, under deliberate attack, the connectivity of the composite network from the passenger perspective decreases more slowly than the global efficiency of the network, and the greater the number of failed nodes required to cause the network to collapse. Solving the global efficiency of the network requires calculating the shortest distance between all stations, which leads to more time-consuming calculations. Therefore, it is reasonable to propose the connectivity from the passenger perspective as a measure of the connectivity reliability of the composite network in the paper.

By comparing Fig. 7 and Fig. 10, it can be seen that when dealing with the attack of the two strategies, the composite network connectivity of passenger perspective decreases at a slower rate than the maximum connectivity as the number of failed nodes increases. It shows that the connectivity measurement method from the passenger perspective proposed in this paper has better connectivity reliability than the maximum connectivity.

D. Sensitivity Analysis of the Prospect Theory

It shows the impact of changes in the gain sensitivity coefficient α and the loss sensitivity coefficient β on the connectivity of the composite network in Fig. 11.

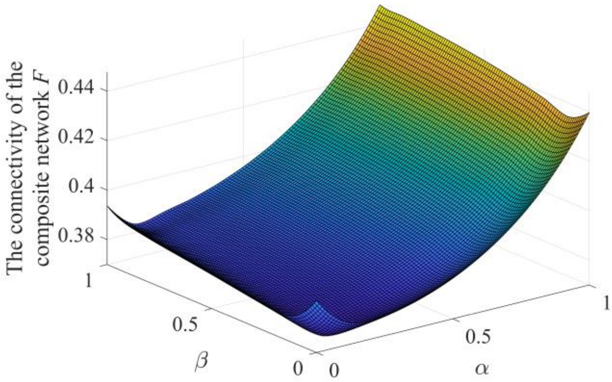

Fig.11 Graph of the effect of α and β on the connectivity of the composite network

It can be seen from Fig. 11 that when α remains unchanged, the connectivity of the composite network increases with the increase of β. When β is constant, with the increase of α, the connectivity of the composite network also increases, and the increase degree is greater. It shows that passengers are more sensitive to the loss of travel, thus reflecting the changes in the connectivity of the composite network.

It shows the impact of changes in the gain sensitivity coefficient α and the tolerance coefficient A on the connectivity of the composite network in Fig. 12.

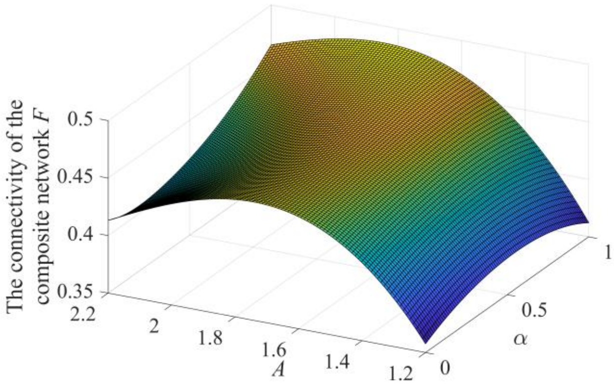

Fig.12 Graph of the effect of α and A on the connectivity of the composite network

It can be seen from Fig. 12 that when α is constant, the connectivity of the composite network increases first and then decreases with the increase of A. When A is 1.7, the composite network connectivity is maximum. When A is constant, with the increase of α, the connectivity of the composite network also increases, but the degree of increase is not obvious. It shows that under the influence of the gain sensitivity coefficient α, when the actual travel time of passengers is 1.7 times the shortest travel time, the overall connectivity of the composite network is the best.

It shows the impact of changes in the loss sensitivity coefficient β and the tolerance coefficient A on the connectivity of the composite network in Fig. 13.

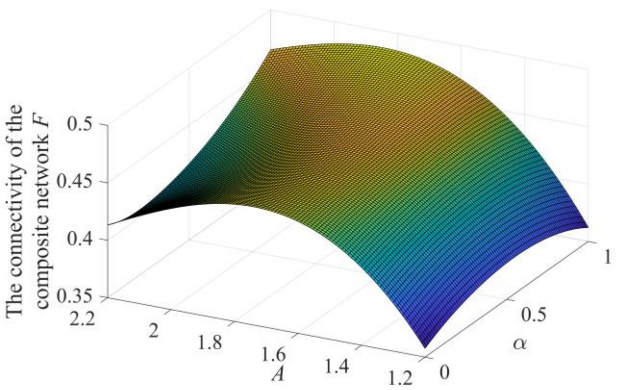

Fig.13 Graph of the effect of β and A on the connectivity of the composite network

It can be seen from Fig. 13 that when β is constant, the connectivity of the composite network increases first and then decreases with the increase of A. When A is 1.75, the composite network connectivity is maximum. When A is constant, the composite network connectivity is almost constant with the increase of β. It shows that under the influence of the loss sensitivity coefficient β, when the actual travel time of passengers is 1.75 times the shortest travel time, the overall connectivity of the composite network is the best.

In conclusion, the tolerance coefficient A plays a decisive role in the connectivity of the composite networks.

E. Identification of Important Stations in Composite Network

After attacking the nodes of the unweighted composite network and the passenger flow weighted composite network, the nodes that have a greater impact on the global network efficiency and network connectivity are listed. It shows the changes in the global efficiency index of the multi-level rail transit composite network node after being attacked in Table III.

It can be seen from Table III that the No. 16 station, the No. 32 station and the No. 47 station have a higher degree of importance in the multi-level rail transit unweighted composite network, and they are important transfer stations for the city's urban rail transit sub-network lines. The No. 36 station and the No. 113 station are the important stations connecting the urban rail transit sub-network and the municipal railway sub-network, and their failure has caused great damage to the entire composite network.

TABLE III
CHANGES IN THE EFFICIENCY MEASUREMENT INDEX OF THE ATTACKED NETWORK OF COMPOSITE NETWORK NODES

Numbers of nodes attacked in turn	*EL*	The rate of change of *EL* /%	*k*	*S*
16	0.09224	-	6	24.52
32	0.08513	7.7	6	7.79
47	0.08292	2.6	6	36.65
3	0.07805	5.87	5	14.29
13	0.07492	4.01	5	25.7
36	0.06726	10.23	5	3.44
90	0.06415	4.62	5	14.83
113	0.05779	9.92	5	7.86
4	0.04631	19.85	4	11.25

From the perspective of the global efficiency change rate of the network, the global efficiency change rate of the network is larger when all nodes with degrees of 5 and 6 fail. It means that the failure of a node with a larger degree is likely to make the entire composite network on the verge of collapse. Therefore, it is necessary to perform key maintenance on the sites listed in Table III. The point strength value of a station with a larger degree value is not the largest, indicating that the actual passenger flow peak does not appear at the transfer station with the largest degree value, which is consistent with the actual travel situation of passengers.

It shows the changes in the connectivity index of the multi-level rail transit composite network node after being attacked in Table IV.

TABLE IV
CHANGES IN THE CONNECTIVITY MEASUREMENT INDEX OF THE ATTACKED NETWORK OF COMPOSITE NETWORK NODES

Numbers of nodes attacked in turn	*SL*	The rate of change of *SL* /%	*F*	The rate of change of *F* /%	*k*
16	0.9968	-	0.4320	-	6
32	0.9191	7.79	0.4274	1.06	6
47	0.9159	0.35	0.4271	0.06	6
3	0.9061	1.06	0.4259	0.3	5
13	0.9029	0.36	0.4257	0.05	5
36	0.8220	8.96	0.4219	0.88	5
90	0.8188	0.39	0.4205	0.33	5
113	0.7929	3.16	0.4176	0.69	5
4	0.7896	0.41	0.4172	0.1	4

It can be seen from Table IV that for nodes with a larger degree value, the change rate of connectivity of the composite network and the maximum connectivity of the composite network from the perspective of passengers are non-monotonic and fluctuate. By comparing the change rate of SL and F when the nodes are attacked by deliberate attack in turn, it can be found that the change rate of F is smaller than that of SL with each additional deliberate attack. It is proved again that the passenger perspective connectivity measurement method proposed in this paper has better connectivity reliability than the maximum connectivity when evaluating the composite network response to attacks.

VII. CONCLUSION

By using complex network theory and introducing prospect theory, a multi-level rail transit composite network connectivity reliability evaluation model is constructed in the paper. Taking the actual data of the municipal railway-subway as an example, the connectivity reliability of multi-level rail transit composite network is analyzed. The following conclusions can be obtained.

(1) In order to reflect the characteristics of passenger travel, considering the bounded rationality of passenger travel, the tolerance coefficient is proposed to describe the relative travel time of passengers. It is reasonable and effective to introduce the prospect theory to measure the connectivity of the composite network, so as to construct a multi-level rail transit composite network connectivity reliability evaluation model.

(2) When dealing with different strategy attacks, the composite network that considers passenger flow weighting is more stable than the unweighted network that only considers the topology. That is, the unweighted composite network has weaker connectivity reliability than the passenger flow weighted network.

(3) The tolerance coefficient has a great influence on the connectivity of stations, and the connectivity of stations increases with the increase of the tolerance coefficient. However, when the tolerance coefficient exceeds a certain value, passengers will certainly be unable to bear too long travel time, and the connectivity of the station will be reduced.

(4) The connectivity of the composite network from the passenger perspective requires more failed nodes than the global efficiency of the network to collapse the composite network and consumes less computing time. When measuring connectivity, the composite network connectivity from the passenger perspective is better than the maximum connectivity. Therefore, the network connectivity from the perspective of passengers proposed in the paper can be used as the measurement index to measure the connectivity reliability of composite network in practical engineering research.

(5) Through the analysis of important sites identification, it is found that the failure of nodes with large degree value is easy to make the whole composite network on the verge of collapse. Therefore, important site maintenance is required.

The disadvantage is that this paper assumes the tolerance coefficient value, and the tolerance coefficient after actual investigation is more meaningful for analyzing the connectivity reliability of composite network. Future research may consider the actual investigation of passenger tolerance coefficient. At the same time, the focus of future research can be taken into account such factors as upstream and downstream routes and departure frequency, so as to make the research more practical.

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