Vibration Suppression of Flexible Joints Space Robot based on Neural Network

Lijian JIANG, Wenhui ZHANG, Jinmiao SHEN, Yangfan YE, Shuhua ZHOU

Abstract—**A vibration suppression control method based on neural network for space robot is proposed. At the same time, considering the uncertain factors such as model inaccuracy and joint flexibility, the dynamic model of space robot is established. Based on singular perturbation theory, it is decomposed into fast subsystem and slow subsystem models for control respectively. Using the nonlinear approximation characteristics of neural network, a radial basis function neural network (RBFNN) controller is designed to approximate the unknown nonlinear system. In order to obtain the best hidden layer Gaussian map, the Taylor linearization method is used to linearize the nonlinear Gaussian function of RBFNN, and the on-line adaptive adjustment rates of parameters including network weight, Gaussian function center and width are designed to improve the approximation accuracy of neural network. A robust controller is designed to suppress the external disturbances. A velocity differential feedback controller (VDFC) is designed to suppress the elastic vibration of flexible joints. Experimental results show the effectiveness of the proposed scheme.**

Index Terms—**Flexible joints, space robot, neural network; robust control, uniformly ultimately bounded.**

I. INTRODUCTION

With the development of space exploration, higher performance and precision of space robot manipulators are required[1]-[2]. Space robot can not only undertake maintenance, installation and other tasks, but also replace astronauts to perform dangerous tasks, which undoubtedly puts forward higher precision requirements for the trajectory tracking performance of space robot[3]-[4]. Space robot manipulators is a strongly coupled and nonlinear multi-input multi-output(MIMO) complex system[5]-[6]. In recent years, international scholars' researches on space robot manipulators mainly stay on the premise of rigid model hypothesis, and have produced many theoretical research results [7].

In fact, the space robot belongs to a typical flexible multi-body system. In particular, the actuating joints of the manipulators are non-rigid coupling with elastic vibration. These elastic sources are mainly due to the mechanical components such as hole and shaft coordination and harmonic reducer. The flexible link of the mechanical arm can absorb the external impact energy and reduce the hard damage, but the damping delay of the flexible link will cause the joint rotation angle and the motor rotor rotation angle cannot be synchronized, the accumulated dislocation error is obviously not conducive to the high-precision operation of the space robot manipulators[8]-[9].Moreover, in the undraped space environment, the vibration brought by the flexible link is more likely to be excited and attenuated slowly, or even to cause vibration diffusion, which obviously will seriously affect the real-time high-precision stability control of the space robot[10].

At present, international scholars have proposed many control schemes for MIMO(Multiple input multiple output) nonlinear systems[11]-[12]. Nanos[13] established a dynamic equation for space manipulators with flexible joints, and proposed a static feedback linearization control method. Under the condition of ensuring the trajectory tracking control accuracy, the calculation amount of this method is lower than that of the traditional linearization calculation, which has higher real-time performance. Ulrich[14] proposed a model reference adaptive control method based on singular perturbation theory for space robot with flexible joints, which has good reference value. Yang et al.[15] conducted dynamic modeling for flexible articular space manipulators, and conducted in-depth research on the impact of flexible joint vibration on the system, which provided references for subsequent studies. Liang J et al.[16]

Manuscript received January 28, 2022; revised September 13, 2022. This work was supported by the National Natural Science Foundation of China (61772247), Key projects of Natural Science Foundation of Zhejiang Province(LZ21F020003),the National Natural Science Foundation of Zhejiang Province (LY20E050002, LY18F030001) and Nanjing Xiaozhuang College Talent Fund (2020 NXY14). *(Corresponding author:Wenhui Zhang.)*

Lijian Jiang is a lecturer in College of Engineering, Lishui University, Lishui 323000, P. R. China (e-mail: 75918222@qq.com).

Wenhui Zhang is a professor in College of Electronic Engineering, Nanjing Xiaozhuang University, Nanjing 211171, P. R. China and with State Key Lab. for Novel Software Technology，Nanjing University, P.R. China (Corresponding author, e-mail: hit_zwh@126.com).

Jinmiao Shen is a postgraduate student in College of Mechanical Engineering, Zhejiang Sci-Tech University, Hangzhou 310000, P. R. China (e-mail: Zwh2022hit@126.com).

Yangfan YE is an engineer in Zhejiang Engineering Geophysical Survey and Design Institute Co., Ltd, Hangzhou 310000, P. R. China (e-mail: wclsxy@163.com).

Shuhua Zhou is a professor in School of Automotive Engineering, Zhejiang Technical Institute of Economics, Hangzhou, Zhejiang 310000, P. R. China (e-mail: 715685317@qq.com).

proposed a non-singular fuzzy terminal sliding mode control method for space manipulators with flexible joints. By using fuzzy adaptive control of sliding mode controller gain, the sliding mode vibration was reduced, and a good control effect was obtained at the same time. Chen Z Y et al.[17] studied the anti-disturbance motion control of uncertain elastic base and elastic joint space robot, as well as synchronous suppression of vibration of base and joint, and proposed an improved adaptive robust anti-interference control method, which provided a new idea for synchronous suppression of matrix elasticity and joint elasticity. Liu X et al.[18] designed controller based on RBFNN for an unknown model flexible joint robot, and applied it to single link flexible joint robot, the simulation results verified its reliability. Cui L L et al.[19] proposed a trajectory planning method, which was solved by particle swarm optimization algorithm to suppress the residual vibration generated at the end. Li WP et al. [20] proposed a hybrid controller that combines input shaping technology with ADRC (adaptive disturbance rejection controller) to reduce external disturbances or self-generated vibrations. Numerical simulation results show that the method is feasible.

To study the control problem of space robot with joint flexibility, not only the trajectory tracking problem of rigid body motion should be considered, but also the vibration suppression problem caused by joint flexibility should be dealt with[21]-[25]. No root space robot system at the same time, the floating base substrate coupling effect of mechanical arm not only exist, but also exist in the process of fuel consumption and parameters such as liquid sloshing, grab and release target load uncertainty problem these increased the difficulty of controller design, caused a lot of traditional control strategy is difficult to apply[26]-[29].

On the basis of the above research, taking into account the uncertain factors such as model inaccuracy caused by joint flexibility and measurement level constraints, the main contributions of this paper are as follows:

1) Considering the uncertain factors such as model inaccuracy and joint flexibility, the dynamic model of space robot is established, it is decomposed into fast and slow subsystem models based on singular perturbation theory. Using the nonlinear approximation characteristics of neural network, RBFNN controller is designed to approximate the unknown nonlinear model, so there is no need for an accurate dynamic model.

2) The traditional neural network control only designs the weight adaptive learning rate to realize on-line real-time adjustment. Different from the traditional control method, in order to obtain the best hidden layer Gaussian map, the Taylor linearization method is used to linearize the nonlinear Gaussian function of RBFNN, and the on-line adaptive adjustment rate of parameters including network weight, Gaussian function center and width is designed to improve the approximation accuracy.

3) A robust controller is designed to suppress the external interference and improve the control accuracy. VDFC is designed to suppress the vibration. The proposed algorithm is simple, fast and real-time.

II. Dynamic Singularly Perturbed Modeling of Space Robot System with Flexible -joint

Based on Newton's second equation, the dynamic equation of a free floating space robot with flexible joints can be derived as follows[30]:

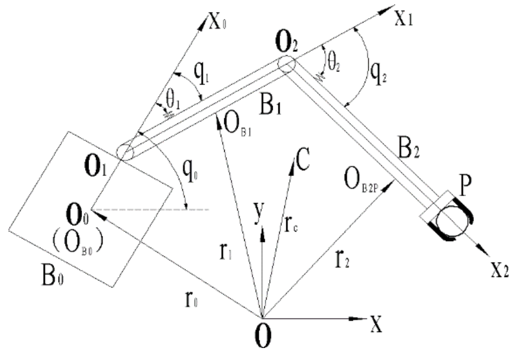

Fig. 1. Space robot with flexible joints model

$$J_\theta \ddot{\theta} + K(\theta - q) = \tau \tag{1}$$

$$D(q_0, q)\ddot{q} + N(q_0, q, \dot{q}, \dot{\theta}) - K(\theta - q) = 0 \tag{2}$$

Where $\theta = [\theta_1 \quad \theta_2]^{\mathbf{T}}$ are the rotor angles, $q = [q_1 \quad q_2]^{\mathbf{T}}$ are the angles of the arm joints, q_0 is matrix attitude angle of the carrier; $J_\theta = \mathrm{diag}(J_{\theta 1}, J_{\theta 2}) \in R^{2\times 2}$ is the diagonal positive definite inertia matrix of the motor; $D(q_0, q) \in R^{2\times 2}$ is the symmetric positive definite inertia matrix of the boom; $N(q_0, q, \dot{q}, \dot{\theta}) \in R^2$ are arrays containing coriolis force and centrifugal forces; $K = \mathrm{diag}(k_1, k_2)$ is the joint stiffness coefficient constant matrix; $\tau = [\tau_1 \quad \tau_2]^{\mathbf{T}}$ is the motor output torque array.

Considering that the space manipulator with flexible joints is a rigid flexible coupling nonlinear system, the singular perturbation theory is used to decompose the system into a slow subsystem representing the rigid characteristics and a fast subsystem representing the flexible characteristics, and then the sub-controllers of each subsystem are designed respectively for the two subsystem models:

$$\tau = \tau_s + \tau_f \tag{3}$$

Where τ is the master controller, τ_s is the slow subsystem controller, and τ_f is the fast subsystem controller.

Defining $K_1=\varepsilon^2 K$, ε is a minimal normal number. The system "fast" variable is $z=K(\theta-q)$, Substitute it into the system dynamics equations (10) and (11), and get:

$$D(q_0,q)\ddot{q}+N(q_0,q,\dot{q},\dot{\theta})=z \tag{4}$$

$$\varepsilon^2 J_\theta \ddot{z}+K_1 z=K_1(\tau-J_\theta \ddot{q}) \tag{5}$$

For fast subsystems, defining $K_f=\dfrac{K_2}{\varepsilon}$, K_2 is positive definite diagonal matrix. VDFC is designed：

$$\tau_f=K_f(\dot{q}-\dot{\theta}) \tag{6}$$

Substituting equation (6) into equation (5) to obtain the second-order differential dynamics equation of the fast subsystem:

$$\varepsilon^2 J_\theta \ddot{z}+\varepsilon K_2 \dot{z}+K_1 z=K_1(\tau_s-J_\theta \ddot{q}) \tag{7}$$

When $\varepsilon\to 0$,take the limit value $K=\dfrac{K_1}{\varepsilon^2}$,then $K\to\infty$,So the joint is equivalent to a rigid joint. Then $\theta=q$, $\ddot{\theta}=\ddot{q}$,Substitute into Eq. (1) and Eq. (2).The corresponding equivalent stiffness model can be obtained, which is also the dynamic equation of the slow subsystem:

$$M(q_0,q)\ddot{q}+C_s(q_0,q,\dot{q})=\tau_s \tag{8}$$

Where $M(q_0,q)=D(q_0,q)+J_\theta$, $C_s(q_0,q,\dot{q})$ is the new vector of $N(q_0,q,\dot{q},\dot{\theta})$ in $\dot{\theta}=\dot{q}$.

III. Design of Robust Controller for Slow Subsystem base on Neural Network

A vibration suppression controller based on adaptive neural network is designed. The specific design idea is shown in the Fig.2 below:

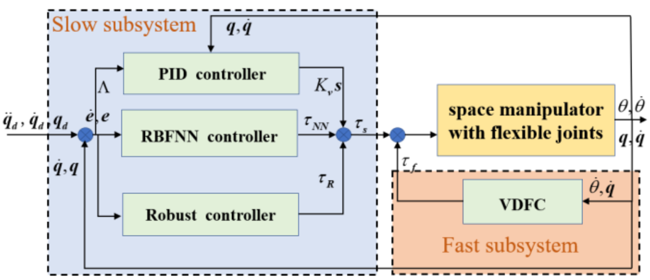

Fig. 2. Technical route of control system

In order to better design the control algorithm of the slow subsystem, Eq.(8) is rewritten into the following quasi-linear form. This quasi-linear treatment $C_s(q_0,q,\dot{q})$ only changes the expression form of the formula and does not generate any model precision loss:

$$M(q_0,q)\ddot{q}+C(q_0,q,\dot{q})\dot{q}=\tau_s \tag{9}$$

Eq.(9) has the following properties[30]:

Property 1： $C(q_0,q,\dot{q})\in R^2$,the form is not unique, $x^{\mathrm{T}}[\dot{M}(q_0,q)\ddot{q}-2C(q_0,q,\dot{q})]x=0$, $\forall x\in R^2$.

Property 2： For $M(q_0,q)$,there is a normal number λ_1 and λ_2 making it bounded : $\lambda_1\|x_1\|^2\le x_1^{\mathrm{T}}M(q_0,q)x_1\le\lambda_2\|x_1\|^2$, $\forall x_1\in R^2$.

Property 3 : There is a normal number λ_3 , $\|C(q_0,q,\dot{q})\|\le\lambda_3\|\dot{q}\|$.

Let $q_d=[q_{1d}\quad q_{2d}]^{\mathrm{T}}$ be the expected position vector of the joints of the slow subsystem, and e be the position tracking error of the subsystem.

$$e=q_d-q \tag{10}$$

Let s be the filter error sliding surface of subsystem:

$$s=\dot{e}+\Lambda e \tag{11}$$

Where $\Lambda=\Lambda^T>0$ is positive definite matrix, set q_r as the subsystem reference trajectory:

$$\dot{q}_r=\dot{q}_d+\Lambda e \tag{12}$$

In Eq. (12) and vertical equation (18), the closed-loop error equation of the subsystem can be obtained as follows:

$$M\dot{s}+Cs+\tau_s=M\ddot{q}_r+C\dot{q}_r \tag{13}$$

When the model parameters are known, K_v is defined as a positive definite matrix, and the following control law is designed to ensure the global asymptotic stability of the subsystem.

$$\tau_s=M(q_0,q)\ddot{q}_r+C(q_0,q,\dot{q})\dot{q}_r+K_v s \tag{14}$$

Proof: construct the following Lyapunov function:

$$V=\frac{1}{2}s^{\mathrm{T}}Ms$$

In combination with Eq. (13) and Eq. (14)：

$$\dot{V}=-s^{\mathrm{T}}K_v s\le 0$$

Then the control system is stable. However, in practical engineering, the parameters of the grasping load P at the end of the mechanical arm are uncertain, so it is difficult to obtain the accurate model of the subsystem, and extract the uncertain model f of the system, then

$$f=M(q_0,q)\ddot{q}_r+C(q_0,q,\dot{q})\dot{q}_r \tag{15}$$

Substituting Eq. (15) into Eq. (14), then the control law equation is:

$$\tau_s=f+K_v s \tag{16}$$

Therefore, if the uncertain part f can be accurately compensated, the control system will be stable.

In this paper, radial basis neural network (RBFNN) with universal approximation characteristics is selected as a compensator to realize approximation compensation for uncertain models f . W is the network weight, hidden layer

function $\varphi = \varphi(x, c, \sigma)$ is the Gaussian function, x is its input, c is its central parameter, and σ is its width parameter.
Suppose the optimal approximation of the neural network in the ideal case is τ_{NN}^*, and the approximation error is ε, then:

$$\begin{cases} f = \tau_{NN}^* + \varepsilon \\ \tau_{NN}^* = w^{*T}\varphi^* \end{cases} \tag{17}$$

Where w^* 、 c^* and σ^* are their respective optimal parameters, $\varphi^* = \varphi(x, c^*, \sigma^*)$ .The controller of the slow subsystem under the optimal approximation of the neural network is designed as

$$\tau_s = \tau_{NN}^* + \varepsilon + K_v s \tag{18}$$

The above is the controller design under ideal conditions, but the neural network is difficult to obtain its optimal approximation in practice. Let τ_{NN} be the actual approximate output of the neural network to f ; at the same time, in order to eliminate the influence of uncertainties, a robust controller is designed, and the control law Eq.(18) is modified as

$$\tau_s = \tau_{NN} + K_v s + \tau_R \tag{19}$$

$$\tau_{NN} = w^{\mathrm{T}}\varphi(x, c, \sigma) \tag{20}$$

RBFNN has the following properties: [31]:
Assumption： In an ideal situation, the optimal weight and network parameters w^*, c^*, σ^* and the approximation error ε are bounded, that is, $\| w^* \| \le w_M$, $\| c^* \| \le c_M$, $\| \sigma^* \| \le \sigma_M$, $\| \varepsilon \| \le \varepsilon_M$.
The optimal matrix Ξ^* and the estimation matrix Ξ to define the weight and hidden layer Gaussian parameters are:

$$\Xi^* = \begin{bmatrix} w^* & 0 & 0 \\ 0 & c^* & 0 \\ 0 & 0 & \sigma^* \end{bmatrix}$$

$$\Xi = \begin{bmatrix} w & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

According to assumption 1, $\| \Xi^* \| \le \Xi_M$ is established, Ξ_M is a positive real number, and $\| w^* \| \le \Xi_M$, $\| c^* \| \le \Xi_M$, $\| \sigma^* \| \le \Xi_M$.
Let the neural network approximation error of the system uncertainty model f be $\Delta\tau_{NN}$ .Defining $\tilde{w} = w^* - w$, $\tilde{\varphi} = \varphi^* - \varphi$, Based on Eq. (17) and Eq. (20), we can obtain:

$$\begin{aligned} \Delta\tau_{NN} &= \tau_{NN}^* + \varepsilon - \tau_{NN} \\ &= w^{*\mathrm{T}}\varphi^* - w^{\mathrm{T}}\varphi + \varepsilon \\ &= w^{*\mathrm{T}}\tilde{\varphi} + \tilde{w}^{\mathrm{T}}\varphi + \varepsilon \end{aligned} \tag{21}$$

Considering the generalization of neural network, when the center c and width C σ of hidden layer Gaussian function $\varphi(x, c, \sigma)$ are constant, its mapping relation is fixed. In order to obtain the best mapping, this paper plans to use Taylor linearization technology to design an adaptive neural network with adjustable hidden layer parameters to improve the control accuracy.
Defining $\frac{\partial \varphi_j}{\partial c}$ and $\frac{\partial \varphi_j}{\partial \sigma}$ as：

$$[\frac{\partial \varphi_j}{\partial c}]^T = \begin{bmatrix} \underbrace{0 \ \cdots \ 0}_{(j-1)\times m} & \frac{\partial \varphi_j}{\partial c_{j1}} & \cdots & \frac{\partial \varphi_j}{\partial c_{jm}} & \underbrace{0 \ \cdots \ 0}_{(N-j)\times m} \end{bmatrix}$$

$$[\frac{\partial \varphi_j}{\partial \sigma}]^T = \begin{bmatrix} \underbrace{0 \ \cdots \ 0}_{(j-1)\times m} & \frac{\partial \varphi_j}{\partial \sigma_{j1}} & \cdots & \frac{\partial \varphi_j}{\partial \sigma_{jm}} & \underbrace{0 \ \cdots \ 0}_{(N-j)\times m} \end{bmatrix}$$

$\tilde{\varphi}$ is expanded by Taylor series :

$$\tilde{\varphi} = H^{\mathrm{T}}\tilde{c} + Q^{\mathrm{T}}\tilde{\sigma} + o(\cdot) \tag{22}$$

Where $\tilde{\varphi} = [\tilde{\varphi}_1, \cdots, \tilde{\varphi}_N]^T$, $\tilde{c} = c^* - c$, $\tilde{\sigma} = \sigma^* - \sigma$, $H = [\frac{\partial \varphi_1}{\partial c} \cdots \frac{\partial \varphi_N}{\partial c}]_{c=c^*}$, $Q = [\frac{\partial \varphi_1}{\partial \sigma} \cdots \frac{\partial \varphi_N}{\partial \sigma}]_{\sigma=\sigma^*}$, $o(\cdot)$ is the higher order vector.
From Eq. (21) to Eq. (22):

$$\begin{cases} \Delta\tau_{NN} = w^{\mathrm{T}}(H^{\mathrm{T}}\tilde{c} + Q^{\mathrm{T}}\tilde{\sigma}) + \tilde{w}^{\mathrm{T}}\varphi + \rho \\ \rho = \tilde{w}^{\mathrm{T}}\tilde{\varphi} + w^{\mathrm{T}}o(\cdot) + \varepsilon \end{cases} \tag{23}$$

Since the output value of Gaussian function φ is not greater than 1, then $\| \tilde{\varphi} \| \le 1$, so $\| \tilde{w}^T\tilde{\varphi} \| \le \| \tilde{w}^T \| \cdot \| \tilde{\varphi} \| \le \| \tilde{w} \| \le \| \tilde{\Xi} \|$.

$$\begin{aligned} \left\| s^{\mathrm{T}}\rho \right\| &\le \left\| s^{\mathrm{T}} \right\| \left\| \rho \right\| = \left\| s^{\mathrm{T}} \right\| \left\| \tilde{w}^{\mathrm{T}}\tilde{\varphi} + w^{\mathrm{T}}o(\cdot) + \varepsilon \right\| \\ &\le \left\| s \right\| \left\| \tilde{\Xi} \right\| + \left\| s \right\| \left\| w \right\| \left\| o(\cdot) \right\| + \left\| s \right\| \left\| \varepsilon_M \right\| \end{aligned} \tag{24}$$

The robust controller τ_R is designed as:

$$\tau_R = \gamma \frac{s}{\| s \|} \left\| w \right\| \tag{25}$$

Where γ is a normal number and $\gamma \ge \left\| o(\cdot) \right\|$.

The adaptive learning law of network weights and parameters is：

$$\dot{\hat{w}} = \lambda K_1 \| s \| w - K_1 \varphi s^{\mathrm{T}} \tag{26}$$

$$\dot{\hat{c}} = \lambda K_2 \| s \| c - K_2 H w s \tag{27}$$

$$\dot{\hat{\sigma}} = \lambda K_3 \| s \| \sigma - K_3 Q w s \tag{28}$$

Where K_1, K_2 and c K_3 are normal diagonal matrices; λ is the normal number.

IV. Stability Analyses of Control System

Theorem： For the dynamic model of space robot Eq. (9), adopted the controller Eq. (19), neural network controller Eq. (20), robust controller Eq. (25) and adaptive learning rates Eq. (26)～Eq. (28), it can ensure that the control system is uniformly ultimately bounded (UUB).

$$V=\frac{1}{2}s^{\mathrm{T}}Ms+\frac{1}{2}tr(\tilde{w}^{\mathrm{T}}K_1^{-1}\tilde{w})+\frac{1}{2}tr(\tilde{c}^{\mathrm{T}}K_2^{-1}\tilde{c})+\frac{1}{2}tr(\tilde{\sigma}^{\mathrm{T}}K_3^{-1}\tilde{\sigma}) \quad (29)$$

Differentiating both sides:

$$\dot{V}=s^{\mathrm{T}}M\dot{s}+\frac{1}{2}s^{\mathrm{T}}\dot{M}s+tr(\tilde{w}^{\mathrm{T}}K_1^{-1}\dot{\tilde{w}})+tr(\tilde{c}^{\mathrm{T}}K_2^{-1}\dot{\tilde{c}})+tr(\tilde{\sigma}^{\mathrm{T}}K_3^{-1}\dot{\tilde{\sigma}}) \quad (30)$$

In combination with Eq. (13) and Eq. (16), the following equation can be obtained:

$$M\dot{s}=f-\tau_s-Cs \quad (31)$$

By substituting Eq. (17), Eq. (18) and Eq. (21) into the above equation, the closed-loop system error equation can be obtained as:

$$M\dot{s}=\Delta\tau_{NN}-K_v s-\tau_R-Cs \quad (32)$$

Substituting the above equation into Eq. (30) to get:

$$\dot{V}=s^{\mathrm{T}}(\Delta\tau_{NN}-K_v s-\tau_R-Cs)+\frac{1}{2}s^{\mathrm{T}}\dot{M}s+tr(\tilde{w}^{\mathrm{T}}K_1^{-1}\dot{\tilde{w}})+tr(\tilde{c}^{\mathrm{T}}K_2^{-1}\dot{\tilde{c}})+tr(\tilde{\sigma}^{\mathrm{T}}K_3^{-1}\dot{\tilde{\sigma}}) \quad (33)$$

Substituting Eq. (23) and Eq. (26) ~ Eq. (28) into the above equation:

$$\begin{aligned}\dot{V}=&-s^{\mathrm{T}}K_v s+s^{\mathrm{T}}w^{\mathrm{T}}(H^{\mathrm{T}}\tilde{c}+Q^{\mathrm{T}}\tilde{\sigma})+s^{\mathrm{T}}\tilde{w}^{\mathrm{T}}\hat{\varphi}+s^{\mathrm{T}}\rho\\&-s^{\mathrm{T}}\tau_R+tr[\tilde{w}^{\mathrm{T}}(\lambda\|s\|w-\varphi s^{\mathrm{T}})]\\&+tr[\tilde{c}^{\mathrm{T}}(\lambda\|s\|c-Hws)]\\&+tr[\tilde{\sigma}^{\mathrm{T}}(\lambda\|s\|\sigma-Qws)]\end{aligned} \quad (34)$$

Substituting equation $\Xi=\Xi^*-\tilde{\Xi}$ and Eq. (24) into the above formula

$$\begin{aligned}\dot{V}\le&-s^{\mathrm{T}}K_v s+\lambda\|s\|tr[\tilde{\Xi}^{\mathrm{T}}(\Xi^*-\tilde{\Xi})]+\|s\|\|\tilde{\Xi}\|\\&+\|s\|\|w\|\|o(\cdot)\|+\|s\|\|\varepsilon_M\|-s^{\mathrm{T}}\tau_R\end{aligned} \quad (35)$$

Because $tr[\tilde{\Xi}^{\mathrm{T}}(\Xi^*-\tilde{\Xi})]\le\|\tilde{\Xi}\|\|\Xi^*\|-\|\tilde{\Xi}\|^2$.

$$\begin{aligned}\dot{V}\le&-s^{\mathrm{T}}K_v s+\lambda\|s\|(\|\tilde{\Xi}\|\|\Xi^*\|-\|\tilde{\Xi}\|^2)+\|s\|\|\tilde{\Xi}\|\\&+\|s\|\|w\|\|o(\cdot)\|+\|s\|\|\varepsilon_M\|-\gamma\|w\|\|s\|\\\le&-\|s\|[K_{v\min}\|s\|+\lambda(\|\tilde{\Xi}\|-\alpha)^2-\beta]\end{aligned} \quad (36)$$

Where $K_{v\min}$ is the minimum singular value of K_v, $\alpha=\frac{\Xi_M}{2}+\frac{1}{2\lambda}$, $\beta=\varepsilon_M+\lambda\alpha^2$.

When

$$\|s\|>\frac{\beta}{K_{v\min}} \text{ 或 } \tilde{\Xi}>\alpha+\sqrt{\frac{\beta}{\lambda}} \quad (37)$$

Then

$$\dot{V}\le 0$$

Therefore, $\dot{V}$ is negative outside the closed set .According to the extended Lyapunov theory, all signals s and $\tilde{\Xi}$ of the subsystem are uniformly ultimately bounded (UUB).

V. Simulation Examples and Analysis

The simulation experiment of the free floating space robot arm of the flexible joint in fig.1,The length of the mechanical arm B_i along the x_i shaft is 4.5m, O_1 O_0 distance 0.5m, O_1 and O_{B1} are 2m, the distance of O_2 and O_{B2P} is 2.5m.

The mass of the branch and the inertia of the rotation are: $m_0=100\mathrm{kg}$, $m_1=m_2=1.5\mathrm{kg}$, $J_0=20\mathrm{kg}\cdot\mathrm{m}^2$, $J_1=J_2=2.5\mathrm{kg}\cdot\mathrm{m}^2$.

The expected trajectory is:

$q_{1d}=0.5\pi[0.1t-0.5\sin(0.2\pi t)/\pi]$

$q_{2d}=0.5\pi[1-0.1t+0.5\sin(0.2\pi t)/\pi]$

Controller gain and parameter： $K=\mathrm{diag}(200,200)$； $J_\theta=\mathrm{diag}(0.5,0.5)$ ； $K_v=\mathrm{diag}(25,25)$ ； $\lambda=0.3,\gamma=45$； $\Lambda=\mathrm{diag}(5,5)$.

Initial state： $q(0)=\theta(0)=[0.2 \quad 1.4]^{\mathrm{T}}$

The initial weight of neural network is 0. The width and the center parameters are randomly selected in the input output domain, and the hidden layer nodes are 35.

a) When τ_f is turned on

In order to verify the effectiveness of the designed control system, the control system is experimentally studied when the controller of the rapid change subsystem τ_f is turned on. The experimental simulation results are shown in Fig.3- Fig.4. Fig.3 shows the situation that the joint angle position tracks the desired trajectory, and Fig.4 shows the situation that the motor rotor angle position tracks the joint angle position.

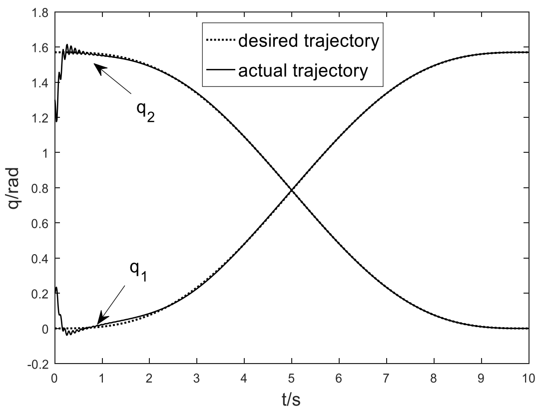

Fig.3. Joint angular motion trajectory of robot (τ_f is turned on)

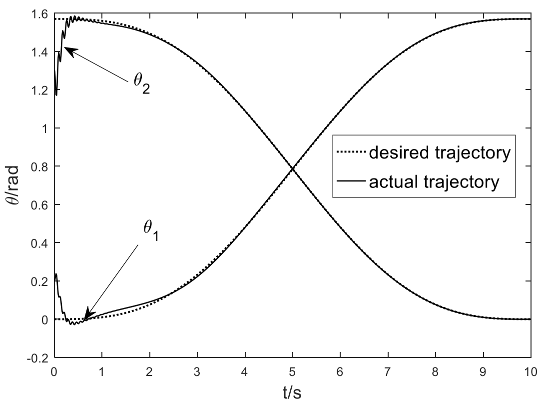

Fig.4. Motor rotor angle trajectory tracking (τ_f is turned on)

It can be seen from Fig.3 that even in the case of large initial error, the actual trajectory of joint angle can achieve rapid response, and the actual trajectory of joint angle can accurately track the desired trajectory in about 0.5s, and its overshoot is small. This shows that the designed controller is effective, which can ensure the actual trajectory to track the desired trajectory accurately and quickly. It can be seen from Fig.4 that even in the case of large initial error, the motor rotor angle can accurately track the joint angle in about 2s, and its overshoot is small. This shows that the designed VDFC can respond quickly and suppress the vibration caused by joint flexibility.

b) When τ_f is turned off

When the controller τ_f of the rapid change subsystem is turned off, that is, when the vibration suppression controller τ_f is not started, verify the ability of the robust controller based on neural network to suppress external interference and the high-precision control effect, and carry out experimental research on the control system. The experimental results are shown in Fig.5-Fig.6. Fig.5 shows the situation that the joint angle position tracks the desired trajectory, and Fig.6 shows the situation that the motor rotor angle position tracks the joint angle position.

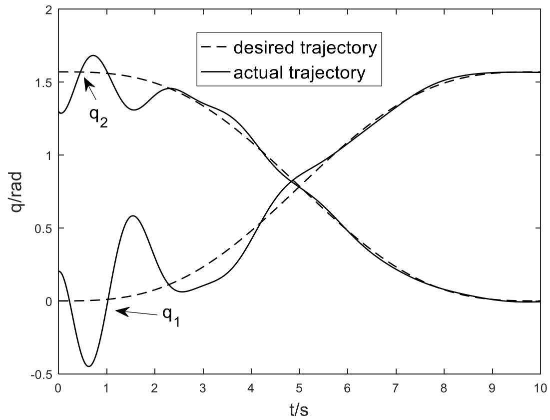

Fig.5. The joint angular motion trajectory (τ_f is turned off)

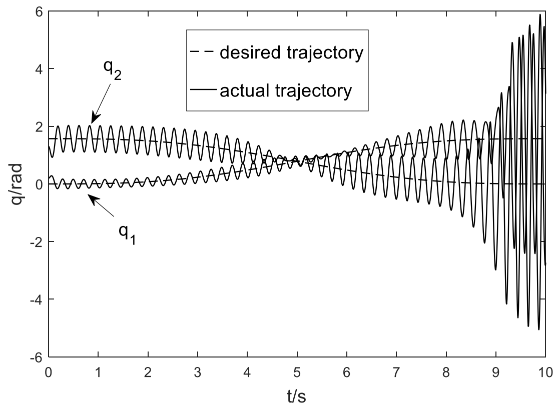

Fig.6. Angular trajectory of the joint(τ_f is turned off)

It can be seen from Fig.5 that even when τ_f is closed, the actual trajectory of the joint angle accurately tracks the desired trajectory in about 6s, and the overshoot is large in the initial stage. This shows that the designed controller is effective. Even if the vibration suppression controller τ_f of the fast subsystem is turned off, the actual trajectory can still track the desired trajectory accurately. However, compared with Fig.3, it can be seen that the tracking time is significantly longer and the overshoot is larger, which shows that the controller τ_f is effective for suppressing elastic vibration. It can be seen from Fig.6 that the motor rotor angle cannot accurately track the joint angle when it is closed. This shows that the designed VDFC is effective in reducing the angular error caused by elastic vibration.

VI. CONCLUSIONS

In order to improve the control accuracy of space robot with flexible joints, an adaptive robust control method based on neural network is proposed. At the same time, considering the uncertain factors such as model inaccuracy and joint flexibility, the dynamic model of space robot is established. Based on singular perturbation theory, it is decomposed into fast and slow subsystem models for control respectively. Using Taylor linearization method, RBFNN controller is designed to approximate the unknown nonlinear model, and the online real-time adjustment of base width, center and weight in neural network is realized. A robust controller is designed to compensate the approximation error and suppress the external interference, which improves the control accuracy and robustness. A controller based on linear velocity feedback is designed to suppress the elastic vibration of flexible joints. Experimental results show the effectiveness of the proposed scheme.

REFERENCES

[1] V. M. Sukhanov, A. V. Silaev, and V. M. Glumov, “Dynamic equations of free-flying space robot for feedback control tasks,” *Automation and Remote Control*, vol. 76, no. 8, pp. 1446-1454, 2015.

[2] B. W. Zhang, X. Ye, and X. Ji, “RBF neural network adaptive control for space robots without speed feedback signal,” *Transactions of the Japan Society for Aeronautical and Space Sciences,* vol. 56, no. 6, pp. 317-322, 2013.

[3] X. Shao, G. Sun, C. Xue, and X. Li, “Nonsingular terminal sliding mode control for free-floating space manipulator with disturbance,” *Acta Astronautica,* vol. 181, pp. 396-404, 2021.

[4] L. Xie, and L. Chen, “Modeling, saturation robust fuzzy sliding control and vibration suppression of free-floating flexible-joint flexible-link space robot,” *Jixie Gongcheng Xuebao/Journal of Mechanical Engineering,* vol. 51, no. 1, pp. 76-82, 2015.

[5] H. Chen, C. Wang, B. Zhang, and D. Zhang, “Saturated tracking control for nonholonomic mobile robots with dynamic feedback,” *Transactions of the Institute of Measurement and Control,* vol. 35, no. 2, pp. 105-116, 2013.

[6] A. K. Muhammad, S. Okamoto, and J. H. Lee, “Comparisons of proportional and active-force controls on vibration of a flexible link manipulator using a piezoelectric actuator through calculations and experiments,” *Engineering Letters,* vol. 22, no. 3, pp. 134-141, 2014.

[7] Y. Tao, T. Wang, Y. Wang, L. Guo, H. Xiong, and D. Xu, “A new variable stiffness robot joint,” *Industrial Robot,* vol. 42, no. 4, pp. 371-378, 2015.

[8] Xiaoyu Qu, Xiaohua Li, and Xiaojie Cao, "Quantized Adaptive Bounded-H-infinity Tracking Control for a Class of Stochastic Nonaffine Nonlinear Systems," *IAENG International Journal of Applied Mathematics*, vol. 52, no.2, pp.298-307, 2022

[9] K.-H. Nam, B.-S. Kim, and J.-B. Song, “Compliant actuation of parallel-type variable stiffness actuator based on antagonistic actuation,” *Journal of Mechanical Science and Technology,* vol. 24, no. 11, pp. 2315-2321, 2010.

[10] Z. Wenhui, L. Hongsheng, Y. Xiaoping, H. Jiacai, and H. Mingying, “Adaptive robust control for free-floating space robot with unknown uncertainty based on neural network,” *International Journal of Advanced Robotic Systems,* vol. 15, no. 6, pp.1-11 ,2018.

[11] J. Liu, Q. Zhang, and Z. Luo, “Dynamical analysis of fuzzy cellular neural networks with periodic coefficients and time-varying delays,” *IAENG International Journal of Applied Mathematics,* vol. 46, no. 3, pp. 298-304, 2016.

[12] H. Xu, X. Ouyang, N. Zhao, and Y. Zhou, “Event-Triggered Adaptive Prescribed Performance Control of Uncertain Nonlinear Systems,” *IAENG International Journal of Computer Science,* vol. 48, no. 4, pp.977-985 ,2021.

[13] K. Nanos, and E. G. Papadopoulos, “On the dynamics and control of flexible joint space manipulators,”*Control Engineering Practice,* vol. 45, pp. 230-243, 2015.

[14] S. Ulrich, and J. Z. Sasiadek, “Trajectory tracking control of flexible-joint space manipulators,” *Canadian Aeronautics and Space Journal,* vol. 58, no. 1, pp. 47-59, 2012.

[15] Y. Yang, W. Xu, and Z. Mu, "Dynamic modeling and vibration properties study for flexible-joint space manipulators," *2014 IEEE International Conference on Robotics and Biomimetics, IEEE ROBIO 2014.* pp. 2443-2448, 2014.

[16] J. Liang, and L. Chen, “Nonsingular fuzzy terminal sliding mode control and elastic vibration suppressing of a free-floating space robot with flexible joints based on trajectory tracking of the singular perturbation method,” *Zhendong yu Chongji/Journal of Vibration and Shock,* vol. 32, no. 23, pp. 6-12, 2013.

[17] Z.-Y. Chen, T.-T. Zhang, and Y.-S. Guo, “Adaptive Robust Anti-interference Control and Vibration Suppression for an Elastic-base Elastic-joint Space Robot,” *Zidonghua Xuebao/Acta Automatica Sinica,* vol. 44, no. 7, pp. 1271-1281, 2018.

[18] X. Liu, C. Yang, Z. Chen, M. Wang, and C.-Y. Su, “Neuro-adaptive observer based control of flexible joint robot,” *Neurocomputing,* vol. 275, pp. 73-82, 2018.

[19] L. Cui, H. Wang, and W. Chen, “Trajectory planning of a spatial flexible manipulator for vibration suppression,” *Robotics and Autonomous Systems,* vol. 123, pp. 103-116, 2020.

[20] W. P. Li, B. Luo, and H. Huang, “Active vibration control of Flexible Joint Manipulator using Input Shaping and Adaptive Parameter Auto Disturbance Rejection Controller,” *Journal of Sound and Vibration,* vol. 363, pp. 97-125, 2016.

[21] C.-H. Lee, and W.-C. Wang, “Robust adaptive position and force controller design of robot manipulator using fuzzy neural networks,” *Nonlinear Dynamics,* vol. 85, no. 1, pp. 343-354, 2016.

[22] M. W. Spong, “Modeling and Control of Elastic Joint Robots,” *Journal of Dynamic Systems, Measurement, and Control,* vol. 109, no. 4, pp. 310-318, 1987.

[23] W. Zhang, N. Qi, J. Ma, and A. Xiao, “Neural integrated control for a free-floating space robot with suddenly changing parameters,” *Science China Information Sciences,* vol. 54, no. 10, pp. 20-29, 2011.

[24] A. Flores-Abad, O. Ma, K. Pham, and S. Ulrich, “A review of space robotics technologies for on-orbit servicing,” *Progress in Aerospace Sciences,* vol. 68, pp. 1-26, 2014.

[25] K. Ibrahim, and A. B. Sharkawy, “A hybrid PID control scheme for flexible joint manipulators and a comparison with sliding mode control,” *Ain Shams Engineering Journal,* vol. 9, no. 4, pp. 3451-3457, 2018.

[26] P. Efimova, and D. Shymanchuk, “Dynamic Model of Space Robot Manipulator,” *Applied Mathematical Sciences,* vol. 9, pp. 4653-4659, 2015.

[27] H. Delavari, P. Lanusse, and J. Sabatier, “Fractional order controller design for a flexible link manipulator robot,” *Asian Journal of Control,* vol. 15, no. 3, pp. 783-795, 2013.

[28] H. Halalchi, E. Laroche, and G. I. Bara, “Flexible-link robot control using a Linear Parameter Varying systems methodology regular paper,” *International Journal of Advanced Robotic Systems,* vol. 11, no. 1, pp. 21-36, 2014.

[29] Y. Li, S. Tong, and T. Li, “Fuzzy adaptive dynamic surface control for a single-link flexible-joint robot,” *Nonlinear Dynamics,* vol. 70, no. 3, pp. 2035-2048, 2012.

[30] W. Zhang, J. Shen, X Ye, and S. Zhou, “Error Model-Oriented Vibration Suppression Control of Free-Floating Space Robot with Flexible Joints Based on Adaptive Neural Network,” *Engineering Applications of Artificial Intelligence,* vol.114,no.2,pp.105-128,2022.

[31] A. Zhu, and S. X. Yang, “A neural network approach to dynamic task assignment of multirobots,” *IEEE Transactions on Neural Networks,* vol. 17, no. 5, pp. 1278-1286, 2006.

Lijian Jiang received B.Eng. degree in mechanical design, manufacturing and automation from Zhejiang University in 2000, M.Eng. degree in control engineering from Wuhan University of Technology in 2012.

At present, he is lecturer of Lishui University. His research interests mainly include mechatronics and control system design, etc and so on.

Wenhui Zhang received B.Eng. degree in mechanical design, manufacturing and automation from Harbin Institute of Technology in 2004, M.Eng. degree in aircraft design from Harbin Institute of Technology in 2008 and Ph.D degree in Aerospace Science and technology from Harbin Institute of Technology in 2011.

At present, he is professor of Nanjing Xiaozhuang University. His research interests mainly include robot dynamics and mechatronics design and so on.

Jinmiao Shen received B.Eng. degree in Mechanical Design, Manufacturing and Automation from Jiangxi Agricultural University, Nanchang, China, in 2020.

At present, he is currently pursuing M.S. degree in School of Mechanical and Automatic Control, Zhejiang Sci-Tech University. His research interests include Robot Dynamics and nonlinear robust adaptive controls.

Ye Yangfan received M.S. degree in precision mechanical engineering from Zhejiang Shuren University in 2014.

At present, he is an engineer of Zhejiang Engineering Geophysical Exploration Institute Co., Ltd., mainly engaged in research on vibration testing.

Zhou Shuhua received Ph.D degree in engineering from Zhejiang University in 1992 and M.Eng. degree in engineering from Tongji University in 2003.

At present, he is a professor of College of Automotive Engineering, Zhejiang Technical Institute of Economics, mainly engaged in the research of mechanism design and performance analysis.