Analysis of the Capillary Rise of a Sedimentary Soil in a Subtropical Region

Jair de Jesus Arrieta Baldovino, Ronaldo Luis dos Santos Izzo, and Carlos Millan-Paramo

Abstract— **Very little research has been done on the capillary rise in soil for agriculture and geotechnical engineering. The capillary rate rise of water in fine granular soil is one of the major challenges for rising experiments in vertical tubes, as the time required for the water to reach the maximum height of capillary rise can vary from 100 to 400 days. The present research aimed to validate a correlation curve for the capillary rise of a silt-clay soil of the Guabirotuba Geological Formation (Paraná, Brazil) as a function of the time of water rise. The control variables of the study were: compaction, water content, porosity, hydraulic conductivity, soil column height, and rate of capillary rise. Results were gathered by comparing the behavior of capillary rise using the analytical solutions developed by Terzaghi and Lu and Likos. On analysis of the results, it was concluded that the equation proposed by Terzaghi was the most suitable for the silt-clay soil of this study.**

Index Terms— **Capillary Rise, Unsaturated Hydraulic, Porosity, Time, Agricultural Soils.**

I. INTRODUCTION

THE soils store water and makes it available to plants depending on the soil water content and suction. Water deficit decreases plant growth, leaf size, and photosynthesis is also affected because of direct effects on enzymatic processes, electrolytes transport, and chlorophyll content [1]. Also, the capillary phenomenon and the suction influence the soil mechanic behavior. So, it is essential in engineering, agriculture, and biology to study water by capillary ascension with which plants can count in their growth.

Liquid flow that is driven by the capillary effect is the main transport mechanism in the water-air system in which soil matrix solids are continually eroded by capillary rise from a lower elevation to a higher elevation [2,3]. Capillary rise phenomena lead to an increase in the degree of saturation of the soil. Thus, the increase in saturation decreases the resistance of the soil and alters the elastic modulus. These changes produce changes in the stress-strain response of the soil under external loads in transport infrastructure works, mainly the base and sub-base of pavements. Therefore, surface pathology in pavements is closely related to capillary erosion.

Capillarity is a consequence of the surface tension between the liquid film and the capillary tube wall. The height reached by the liquid depends on the liquid surface tension and the capillary tube radius. This phenomenon occurs in several circumstances: in the movement of water through the soil pores, especially in fine granular soils, and is essential for the circulation of sap by plant stems, for example ([4]–[8]).

Several studies have been conducted to demonstrate, understand, and analyze the phenomenon of capillarity in soils. For instance, Lane et al. [9] used a capillarimeter and an open tube to analyze capillary rise. Natural sandy gravel was mixed in desired portions to create eight soil classes, representing a wide grain size and distribution range.

Urbanization results in changes to both surface and groundwater flows. In arid regions, the typical impact of urbanization is increased soil moisture and regional or localized rise in the groundwater table. Because of the vast distance and area covered by pavements, regional groundwater flows must be considered, and seasonal impacts on the groundwater table must be understood. Localized mounding, however, can also be important to pavement performance.

Liu et al. [6] developed an approximation for capillary rise using only four parameters that apply to various soil types: the contact angle, the air entry height, porosity, and saturated hydraulic conductivity. Terzaghi [1] proposed an analytical solution demonstrating the capillary rise of any soil. Based on the solution developed by Terzaghi [1], Lu and Likos [2] also proposed an analytical solution, but unlike Terzaghi, they considered the permeability coefficient as nonlinear. The relatively recent emergence of unsaturated soil mechanics theory and associated constitutive models makes the application of knowledge about the degree and extent of wetting more practical. If unsaturated soil testing on subgrade materials is to be performed, the tests should be conducted at a water content associated with the wettest unsaturated soil condition since this corresponds to the lowest shear strength and highest compressibility

Based on the analytical solutions developed by Terzaghi and by Lu and Likos, the objective of the present study was to analyze the capillary rise of a silt-clay soil of the Guabirotuba Geological Formation, to verify which curve model best represents a capillary rise of this soil type.

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A. Terzaghi's analytical solution

Terzaghi [1] calculated the rate of capillary rise based on Darcy's law and as a function of the height of one column and the saturated hydraulic conductivity of the soil. Figure 1 conducts the conceptual model for the capillary rise in soils, defining the phenomenon as a direct relation between suction and degree of saturation; the capillary rise is directly related to the characteristic curve of soil suction.

In his study, Terzaghi made two assumptions: that Darcy's law for saturated soil is also applicable to unsaturated soil and that the hydraulic gradient (i), responsible for capillary ascension, can be described as follows (Equation 1):

$$i=\frac{h_c-z}{z} \tag{1}$$

Where h_c is the ultimate height of capillary rise, and z is the distance upwards of water above the water table. Applying Darcy's law in Equation 1, the function of saturation velocity that is derived and can be expressed as follows (Equation 2):

$$\eta\frac{dz}{dt}=k_s\frac{(h_c-z)}{z} \tag{2}$$

Where η is soil porosity, dt and d_z are the time and height differences, respectively, and k_s is the permeability coefficient of the saturated soil.

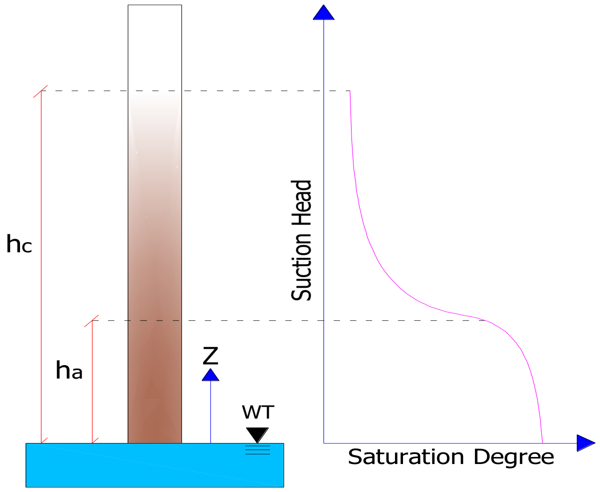


Fig. 1. The conceptual model for the capillary rise in soils.

Considering the boundary condition z equal to zero, when T is also zero, the solution of Equation 2 results in:

$$t=\frac{\eta h_c}{k_s}\left(\ln\frac{h_c}{h_c-z}-\frac{z}{h_c}\right) \tag{3}$$

B. Lu and Likos' analytical solution

Lu and Likos [2] developed a solution for the rate of capillary rise based on the equation put forward by Terzaghi [1]. The authors considered the permeability coefficient to be nonlinear from the point where the soil ceases to be saturated and enters the wetting front. The nonlinear permeability coefficient (k) was described by Gardner [10] as a function dependent on k_s, the suction height (h), and the rate of decrease in hydraulic conductivity with decreasing suction head (α) (Equation 4).

$$k=k_S\exp(-\alpha h) \tag{4}$$

The parameter α is proportional to pore size distribution. It is defined as the inverse of the saturation height (h_a), or air-entry head ($\alpha = 1/h_a$) and is between 1 cm^{-1} and 0.001 cm^{-1}. Considering Equations 3 and 4, the equation of the capillary rise defined by Lu and Likos is (Equation 5):

$$\frac{dz}{dt}=\frac{k_S}{\eta}\exp(-\alpha h)\frac{h_C}{h_C-z} \tag{5}$$

The solution of Equation 5 is proposed to determine the rate of capillary rise (Equation 6).

If linearity is considered in Equation 6, then *m* will be zero, and the equation reduces to Equation 3. However, if nonlinearity is considered, *m* equals 10 for a wide range of soils.

$$t=\frac{\eta}{k_S}\sum_{j=0}^{m=\infty}\frac{\alpha^j}{j!}\left({h_C}^{j+1}\ln\frac{h_C}{h_C-z}-\sum_{S=0}^{j}\frac{{h_C}^{S}{z}^{j+1-S}}{j+1-S}\right) \tag{6}$$

II. MATERIALS AND METHODOLOGY

A. Soil

Samples were collected from the third horizon of the Guabirotuba Formation, located in Curitiba, Brazil. The geographical coordinates are 25°26´31.2” S and 49°21´9.3” O. The soil presents a yellow appearance, has a clayish composition, and constitutes the leading lithological group of the Formation, varying in depth from 1 to 5 meters [11].

Experiments to determine the behavior of the refined grains were conducted according to Atterberg limit test D4318 [12]. The experiment to determine the relative density of the grains was performed according to D854 [13]

B. Capillary rise experiment

After collection, the soil was placed in an oven at a constant temperature (110 ºC) for 24 hours. The dry soil was compacted in a transparent acrylic cylindrical tube in 6 continuous layers of 30 cm each, and then the tube was placed on a tray with distilled water at constant height and temperature for the duration of the experiment (Figure 2). The exact mass and porosity of the compacted soil were calculated before starting the experiment.

Readings of the height of capillary rise were taken periodically, more frequently in the beginning, and then at greater intervals. The experiment was finalized when the water reached a height of 180 cm.

The soil was extracted, and a sample was collected each 10cm to determine the water content.

C. Permeability test

The permeability test was performed according to D2434-68 [14]. A soil sample was placed in a core permeability tube with the same specific dry mass and soil porosity as in the capillary rise tube to obtain the permeability of the soil in which capillary rise was being studied. At the end of the experiment, the permeability coefficient was calculated.

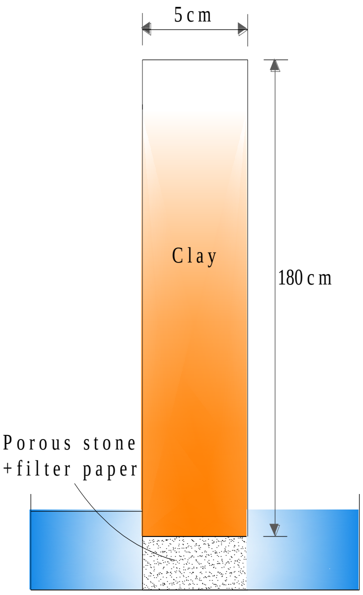


Fig. 2. Schematic Drawing of the Capillary Rise Experiment.

D. Analytical solutions

The curves obtained in the experiment will be compared with those of the analytical solution of Terzaghi (Equation 3) and with the analytical solution of Lu and Likos (Equation 6).

III. RESULTS AND DISCUSSIONS

The soil used in this study was silt-clayey (Figure 3), with a liquid limit of 43.56% and a plastic limit of 12.93%. The specific gravity of the grains was 2.71.

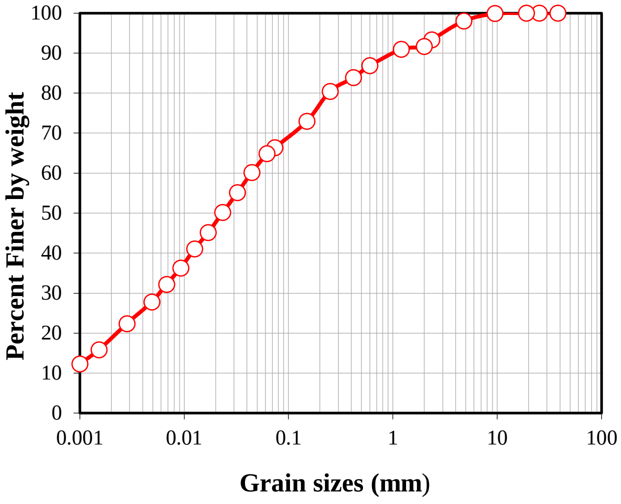


Fig. 3. Grain size distribution curve.

The maximum dry unit weight, obtained by the compaction test (Standard Proctor), resulted in 15.6 kN/m³, at an optimum water content of 24% (Figure 4).

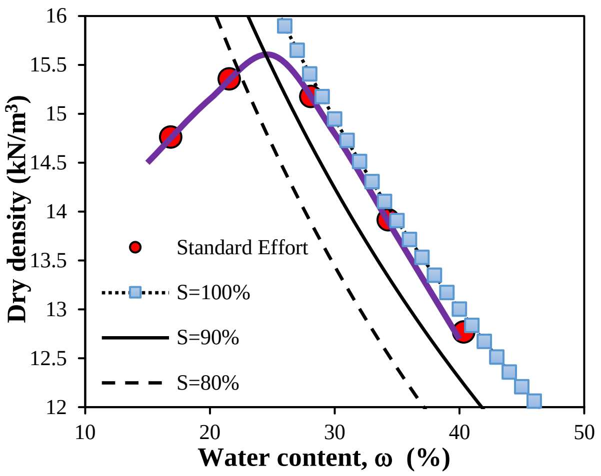


Fig. 4. Standard Proctor Compaction Curve.

The permeability coefficient (K_s) from the soil, which has a porosity (h) of 0.607, the same as the soil compacted in the tube, was $2.39\text{x}10^{-5}$ cm/s.

Figure 5 displays the behavior of the capillary rise of the silt-clayey soil determined in the laboratory (experimental curve) compared to the prediction of capillary rise using the solution developed by Terzaghi (Equation 3).

On the first day, the capillary water reached 35 cm in height. On the tenth day, the capillary height was 0.5 h_c, 90 cm in height. The capillary rise time to reach 180 cm was 190 days.

Up to a height of approximately 20 cm, the experimental curve has an almost precise correlation to the Terzaghi solution. Between 20 and 40 cm (near the point of air-entry), this correlation is lost a little and then resumed from a height of 60cm with an almost precise correlation (97%).

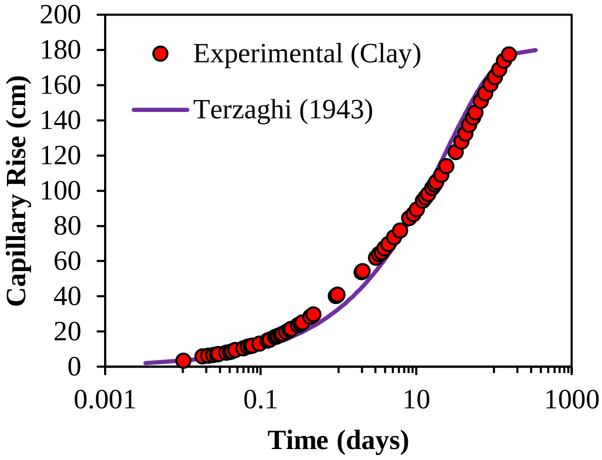
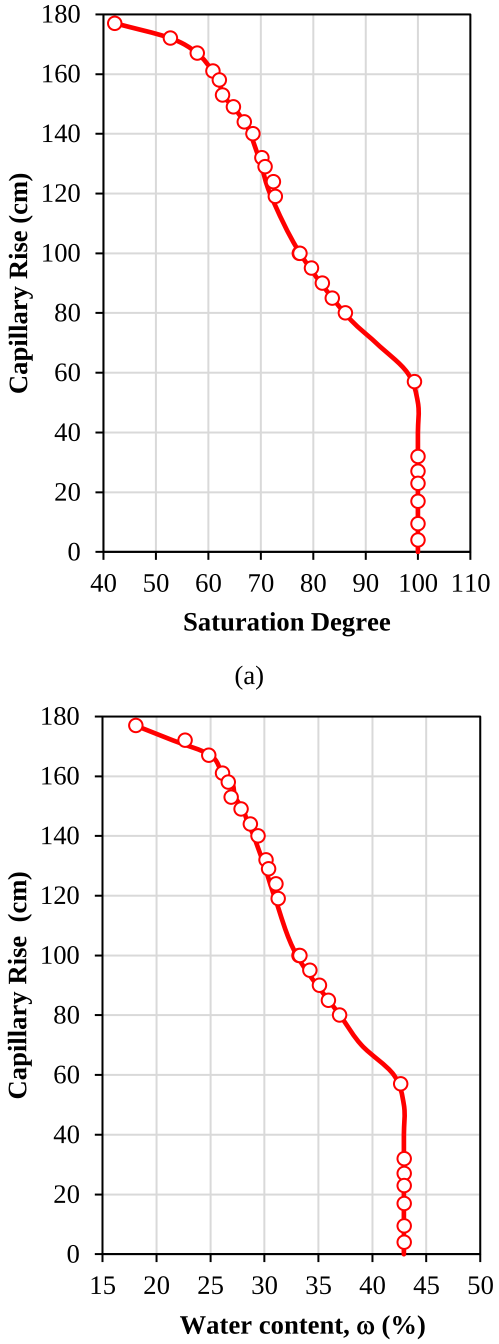


Figure 5. Comparison Between the Experimental Curve and the Terzaghi Solution.

The relationship between capillary rise height and soil saturation degree is shown in Figure 6a, and Figure 6b leads the relationship between capillary rise height and soil water content, both when the water reached height h_c.

Up to height h_a (60 cm), the soil reached saturation S = 100%. From this point, which is considered the point of air entry and where, theoretically, the coefficient of permeability k is saturated, the soil began to demonstrate unsaturated behavior, as described by Equation (4). At height h_c, the soil reached saturation S = 40%. At height $h_c - h_a$, in which $40\% \leq S \leq 100\%$, changes in soil suction were not considered for the analysis, as proposed by Lu and Likos.



(a)

(b)

Fig. 6. a) Relationship between the height of the capillary rise and the degree of soil saturation and b) relationship with the water content of the soil.

The comparison between the experimental capillary rise curve and that of Lu and Likos is demonstrated in Figure 7. The parameter α to determine the theoretical rise curve of Lu and Likos was determined as:

$$\alpha = {}^{1}/_{h_a} \text{ and } h_c.\alpha = x \quad \text{so,}$$

$$x = {}^{h_c}/_{h_a} \rightarrow \text{ to } h_a = 60\text{ cm and } h_c = 180\text{ cm}$$

$$\text{where} \quad \alpha = 0{,}017\text{ cm}^{-1} \text{ and } h_c\alpha = 3$$

By comparing the Lu and Likos curve (Figure 7) with the experimental curve, it can be observed that the correlation of the experimental points to the Lu and Likos curve was not precise, unlike what was observed in the comparison between these experimental points and the Terzaghi curve (Figure 5).

A possible lack of homogeneity in the soil column introduced by the layered compaction process may have caused variation in the unsaturated hydraulic conductivity and, therefore, Equation 6 did not have a good fit.

The correlation of the experimental points of the capillary rise of the soil can be described by Equation 7 (t in days and h_c in cm), shown in Figure 8.

$$h_c = 183{,}53 - 44{,}72^{-0{,}27t} - 108{,}29^{-0{,}0175t} - 25{,}57^{-2{,}92t} \quad (R^2 = 1{,}00) \tag{7}$$

As there was no good correlation between the curve of Lu and Likos (Figure 7) and the experimental points, the relation αh_c was selected without using the experimental points. The values of 2, 1, and 0.36 were proposed for this relation. These were replaced in Equation 6, and new theoretical correlation curves were plotted (Figure 9).

Figure 9 proves that to the extent to which the value is chosen for the relation αh_c decreases, the correlation of the empirical curves in comparison to the experimental points improves since the height (z) and capillary rise time (t) decrease. It was, thus, confirmed that the best value for αh_c was 0.36.

Adopting the value of 0.36 for the relation αh_c and comparing the new theoretical curve with the Terzaghi curve, it was found that both have a very close correlation (Figure 10).

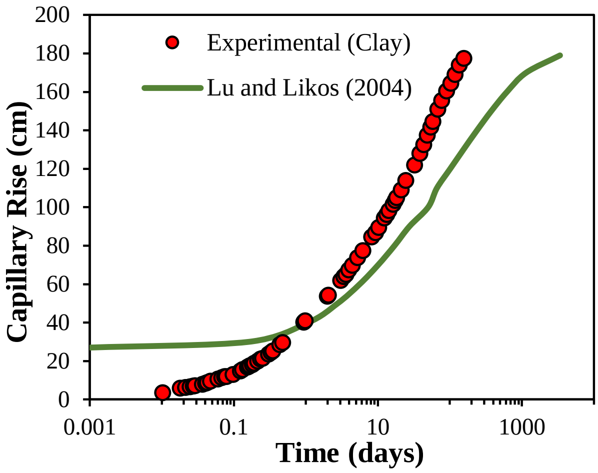


Fig. 7. Comparison Between the Lu and Likos (2004) Curve and the Experimental Curve.

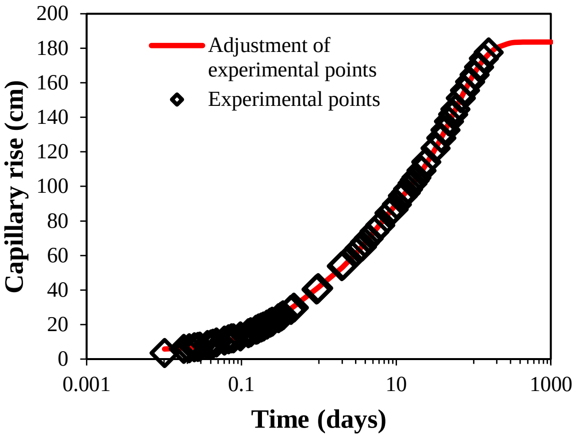


Fig. 8. Experimental Correlation curve.

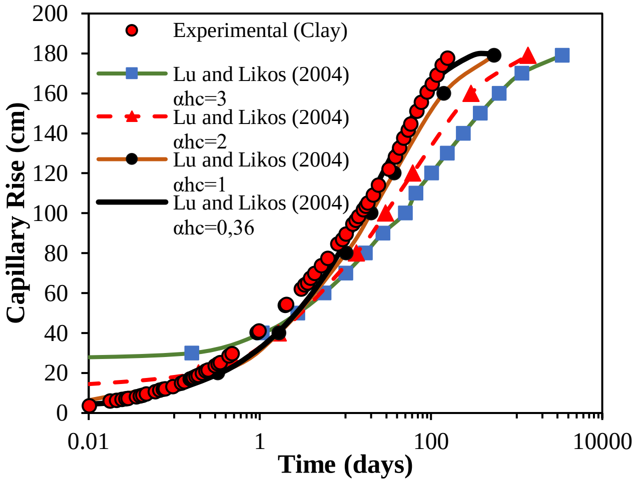


Fig. 9. Comparison between the Lu and Likos Curve, the Experimental Curve and the Theoretical Correlation Curves.

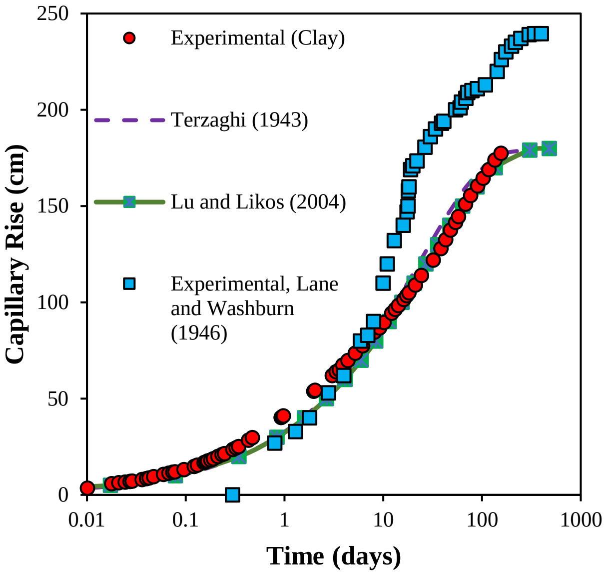


Fig. 10. Best fit for Lu and Likos Compared With the Correlation of Terzaghi's Equation and the Experimental Findings.

Analysis of Figure 10 reveals a deviation in the theoretical curves in the time between the heights of 100 and 180 cm. At height h_a, the behavior of the experimental points correlated better to Equation 6, when $\alpha h_c=0.36$, revealing that in this section of the soil column, the capillary water rise behavior was that of the unsaturated permeability coefficient. Figure 10 also proves the capillary rise of silt sand reported by Lane and Washburn (1946), where $k_s=6.2\text{x}10^{-5}$ cm/s, $\eta=0.40$, $h_c=239.6$ cm; and $h_a=175$ cm. It can be observed that this soil is less porous than the soil of the present research (0.40 > 0.607), with k_s being smaller and, h_c and h_a, therefore, more significant.

Moisture migration by capillary rise and its potential negative impact on infrastructure performance are well known to the engineering profession. For example, structural distress from moisture migration by capillary rise through concrete floors and slabs has been mitigated with moisture barriers and open-graded gravel placed beneath the slab [9]. Soils of silt, clay, and fine sand, for which capillary rise potential is significant, are susceptible to frost heave, which can damage pavements. In fact, the height of capillary rise, along with grain-size distribution and the soil-water characteristic curve can be found to correlate well with frost heave potential [7].

IV. CONCLUSION

Two analytical solutions were used to predict the rate of capillary rise of clayey soil. They were comparing the analytical solutions with the results obtained in the laboratory, it can be said that the solution proposed by Terzaghi [1] had a better fit, with porosity and the saturated permeability coefficient being the main control parameters, while in Lu and Likos' [2] solution, the parameter αh_c was a better fit.

The height of capillary rise as a function of time can be calculated using the specific gravity of the soil, the dry unit weight, the saturated hydraulic conductivity, and the air-entry head. All these variables are easy to calculate and determine in a laboratory.

An equation describing the capillary behavior of the studied soil was proposed as a function of time and a maximum height of capillary rise.

The use of two solutions can predict the rate of capillary rise of soil with the same characteristics as the studied soil. The solution of Lu and Likos tends to predict a longer time due to the unsaturated behavior of the soil column.

In addition, these solutions can be used to analyze geotechnical problems where capillary rise influences the behavior of structures, such as surface foundations and pavements, in the same way in the area of the soils in agriculture.

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