A DC Motor Speed Control System with Disturbance Rejection and Noise Reduction

Pyung Soo Kim and Su Yeol Kim

Abstract—**In this paper, a direct current (DC) motor speed control system with proportional-integral-derivative (PID) controller and Kalman filter is presented with consideration of disturbance, system variation, and feedback sensor noise and verified through computer simulations. Initially, a mathematical model for a DC motor system is introduced, after which the performance degradation due to disturbance and system variation in the basic open-loop control is revealed. To resolve this problem, a PID controller based feedback system is designed for the DC motor speed control system. Third, to improve the performance degradation due to feedback sensor noise that may occur during the feedback process, the Kalman filter is applied for the DC motor speed control system. Ultimately, it is verified that the designed DC motor speed control system with PID controller and Kalman filter not only satisfies all performance criteria but also has the ability to reject disturbance, cope with system variation and reduce feedback sensor noise.**

Index Terms—**PID Controller, Kalman Filter, DC Motor Speed Control, Disturbance, Noise.**

I. INTRODUCTION

DIRECT current (DC) motor is an electrical machine that converts electrical energy into mechanical energy. The DC motor has been the most commonly used as an actuator in control systems due to their features such cost-efficiency, ease of use, high performance, longevity and quiet operation. Moreover, they directly provide rotary motion and can provide transitional motion when coupled with wheels or drums and cables. Thus, the DC motor has been widely used in industrial applications, robot manipulators and home appliances, because of their high reliability, flexibility and low cost, where speed and position control of motor are required[1]-[3].

The greatest advantage of the DC motor is speed control which stands for intentional change of the rotational speed to a desired value required for performing the specific task. Thus, controlling the rotational speed of the DC motor has been an important issue and researched for quite a long time. Basically, the rotational speed is often measured and controlled in the DC motor speed control system.

Even if the DC motor speed control system works accurately in normal situations, it may undergo unpredictable uncertainties such as disturbance, system variation, and feedback sensor noise. These uncertainties, whether large or small, must be addressed because they adversely affect the performance of the DC motor speed control system. A feedback control can be considered for disturbance rejection and system variation. For example, the feedback control with the proportional-integral-derivative (PID) controller takes action to force the plant variable back toward the desired output whenever disturbance and system variation on the plant cause a deviation[4]-[7]. The estimation filter can be considered for noise reduction. For example, the well-known Kalman filter adjusts the currently measured sensor value by considering the past sensor data to reduce noise in the measured value[8]-[12].

This paper designs a DC motor speed control system with PID controller and Kalman filter for disturbance rejection, system variation handling and noise reduction under unpredictable uncertainties. Each detailed design process is verified through computer simulations. The performance degradation due to disturbance and system variation in the basic open-loop control is shown. A PID controller based feedback system is designed for the DC motor speed control system to resolve this problem. To improve the performance degradation due to feedback sensor noise that may occur during the feedback process, the Kalman filter is applied for the DC motor speed control system. It is verified that the designed DC motor speed control system with PID controller and Kalman filter not only satisfies all performance conditions but also has the ability to reject disturbance, cope with system variation and reduce noise.

This paper has the following structure. In Section II, the problem statement of DC motor speed control system is described. In Section III, the PID controller based feedback control is designed for disturbance rejection and system variation handling. In Section IV, the Kalman filtering is designed for noise reduction. Then, concluding remarks are given in Section V.

II. PROBLEM STATEMENT OF DC MOTOR SPEED CONTROL SYSTEM

The DC motor has been used as a common actuator in control systems. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide transitional motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in Fig. 1.

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this paper, it is assumed that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current by a constant factor. From Fig. 1, the following dynamic equations can be derived using

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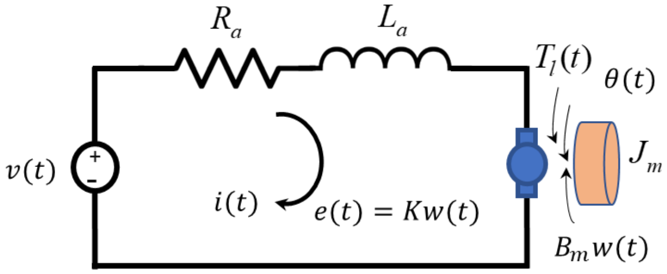

Fig. 1: Electric equivalent circuit of DC motor system

Newton's 2nd law and Kirchhoff's voltage law:

$$\begin{aligned} J_m\dot{w}(t) + B_m w(t) &= Ki(t), \\ L_a\dot{i}(t) + R_a i(t) &= v(t) - Kw(t). \end{aligned} \quad (1)$$

In addition, the transfer function of the DC motor system can be represented by.

$$\begin{aligned} G(s) &= \frac{W(s)}{V(s)} \\ &= \frac{K}{(J_m s + B_m)(L_a s + R_a) + K^2}[\frac{rad/s}{V}]. \end{aligned} \quad (2)$$

As shown in (1) and (2), the DC motor system consists of many kinds of variables and parameters, which are defined in Table I. Values of physical parameters used for computer simulations are set as shown in Table II.

TABLE I: Variables and parameters of DC motor system

Variables	
$w(t)$	rotational speed $[rad/s]$
$i(t)$	armature current $[A]$
$v(t)$	armature voltage $[V]$
Parameters	
J_m	moment of inertia of the rotor $[kg \cdot m^2]$
B_m	motor viscous friction constant $[N \cdot m/(rad/s)]$
K (K_e)	electromotive force constant $[V/(rad/s)]$
K (K_t)	motor torque constant $[N \cdot m/A]$
R_a	electric resistance $[\Omega]$
L_a	motor torque constant $[H]$

TABLE II: Values of physical parameters

Parameters	Values
J_m	0.01
B_m	0.1
K	0.01
R_a	1
L_a	0.5

From now on, the ultimate goal is to come up with some performance criteria that the DC motor speed control system should achieve. The performance specifications for the DC motor speed control system are shown in Table III. The desired rotational speed, denoted by w_d, is $1rad/s$. The control system should be able to achieve that speed in less than 2 seconds. A 5% overshoot and 1% steady-state error on the rotational speed are sufficient.

TABLE III: Performance specifications

Index	Criteria
Desired rotational speed	$1\ rad/s$
Settling time	$< 2sec$
Overshoot	$< 5\%$
Steady-state error	$< 1\%$

III. PID CONTROLLER BASED FEEDBACK CONTROL FOR DISTURBANCE REJECTION

A. Limitations of Open-loop Control

An open-loop control is easy and conceptually simple. For the open-loop control to achieve desired output, a static control input must be set using a couple of ways. The first way is to find a suitable control input corresponding to the desired output by increasing it from a small value. In other words, the open-loop system is tuned to make the actual output go the desired output through trial and error. The second way is to apply the final value theorem of Laplace transform to the open-loop transfer function (2). In this paper, the second way is adopted.

In the DC motor speed control system with desired rotational speed $w_d = 1rad/s$, if the voltage source is assumed as $v(t) = \alpha V$, the following is obtained from the final value theorem:

$$\begin{aligned} \lim_{t\to\infty} w(t) &= \lim_{s\to\infty} sW(s) \\ &= \left\{ \frac{s\alpha K}{s\left[(J_m s + B_m)(L_a s + R_a) + K^2\right]} \right\}_{s=0} \\ &= w_d[rad/s]. \end{aligned} \quad (3)$$

Then, $\alpha \approx 10$ is obtained. Thus, the open-loop response of the system to a step input of $10V$ is simulated. From Fig. 2, it is shown that when $v(t) = 10V$ is applied to the DC motor system, a maximum speed of $1rad/s$ can be only achieved. In addition, the open-loop control exhibits no overshoot or oscillations. So far, the open-loop control seems to work perfectly. However, it takes the motor $2.07sec$ to reach its steady-state speed, which this does not satisfy $2sec$ settling time criterion.

In addition, there can be a disturbance issue. The DC motor speed control system is now operating with the static voltage source $10V$. However, in real situations, there can be unknown disturbance which is often in the form of unpredicted variations on the system that cause the actual rotational speed to rise or fall sharply and unpredictably. To address this, it is assumed in this paper that the disturbance affects the voltage source and can drop the voltage. So, to see the effect of the disturbance in simulation, the disturbance is assumed to act at $5sec$ and simulation time is extended to $8sec$. As shown in Fig. 3, the actual rotational speed drops significantly. This means the open-loop control system fails if there is an unpredicted disturbance acting on the DC motor speed control system.

Moreover, there can be a robustness issue due to a system variation. Until now, the DC motor speed control system is now operating with the normal values of physical parameters as shown in Table I. However, even if the DC motor speed control system is represented in the mathematical model (1) accurately on a long time scale, it may undergo unpredictable

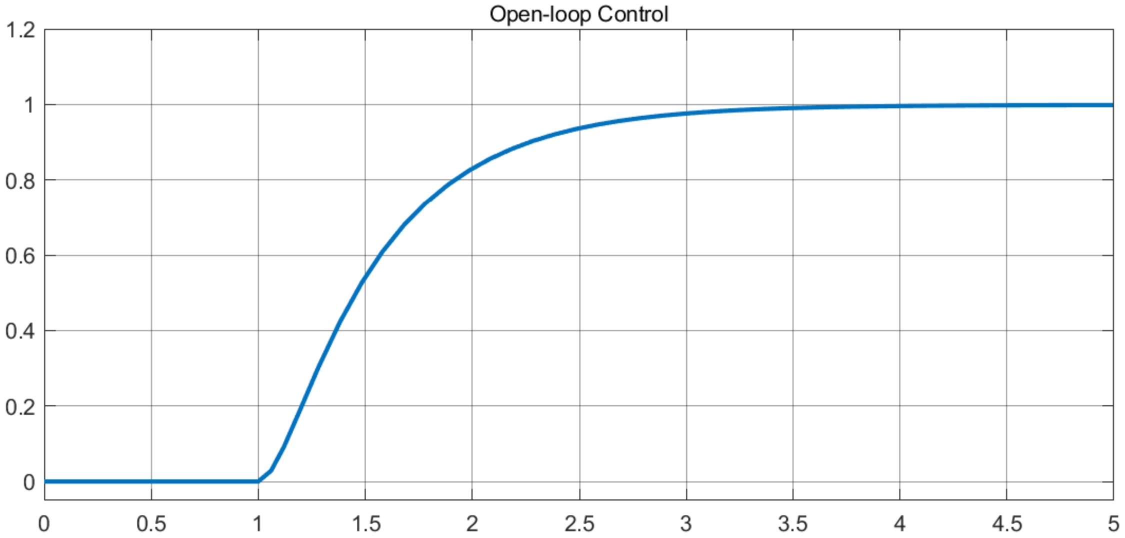

Fig. 2: Result for open-loop control

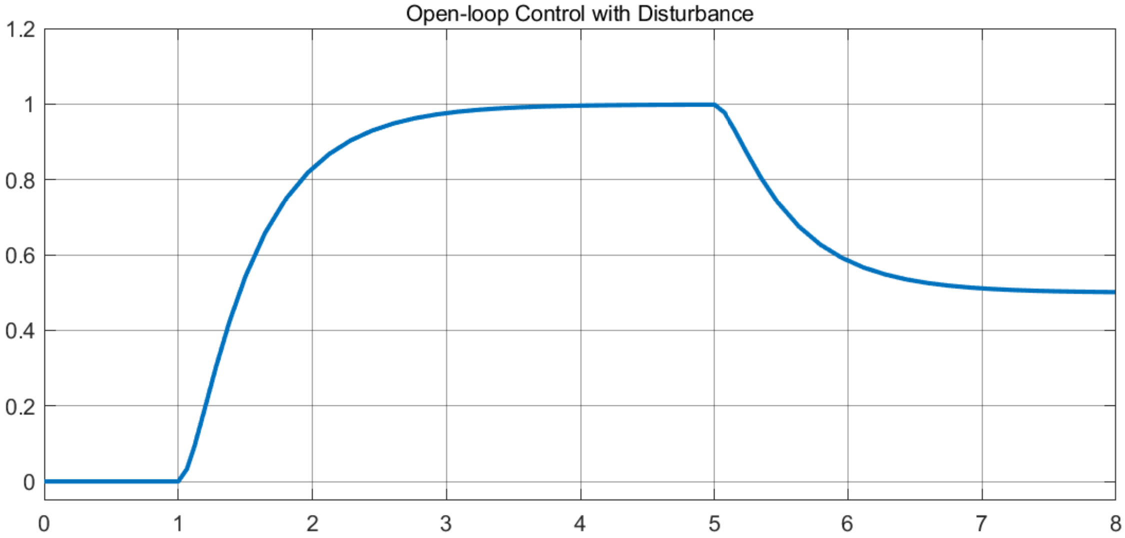

Fig. 3: Result for open-loop control with disturbance

TABLE IV: System variations

Parameters	Normal	System variation	
		Case 1	Case 2
B_m	0.1	0.105	0.11
R_a	1	1.05	1.1

changes, such as jumps in frequency, phase, and velocity. These changes are called system variations and effect typically occur over a short time horizon. It is necessary to see how the DC motor speed control system will respond to these system variations. To investigate this, the open-loop control system with system variations as shown in Table IV is simulated to see how system variations affect the response. As shown in Fig. 4, it is observed that the DC motor's speed settles to different values. This means that open-loop control system cannot deal with system variations, and it needs calibration each time.

Therefore, it is needed to design a feedback control which speeds up the response significantly and rejects the unpredicted disturbance's affect, deals with system variations without negatively affecting the other dynamic performance metrics.

B. PID Controller Based Feedback Control

1) Overview of PID Control: The PID controller is widely employed because it is very understandable and because it is quite effective. One attraction of the PID controller is that all engineers understand conceptually differentiation and integration, so they can implement the control system even without a deep understanding of control theory. Further, even though the compensator is simple, it is quite sophisticated in that it captures the history of the system through integration and anticipates the future behavior of the system through differentiation. The output $u(t)$ of a PID controller, which is equal to the control input, i.e. the voltage source $v(t)$, to

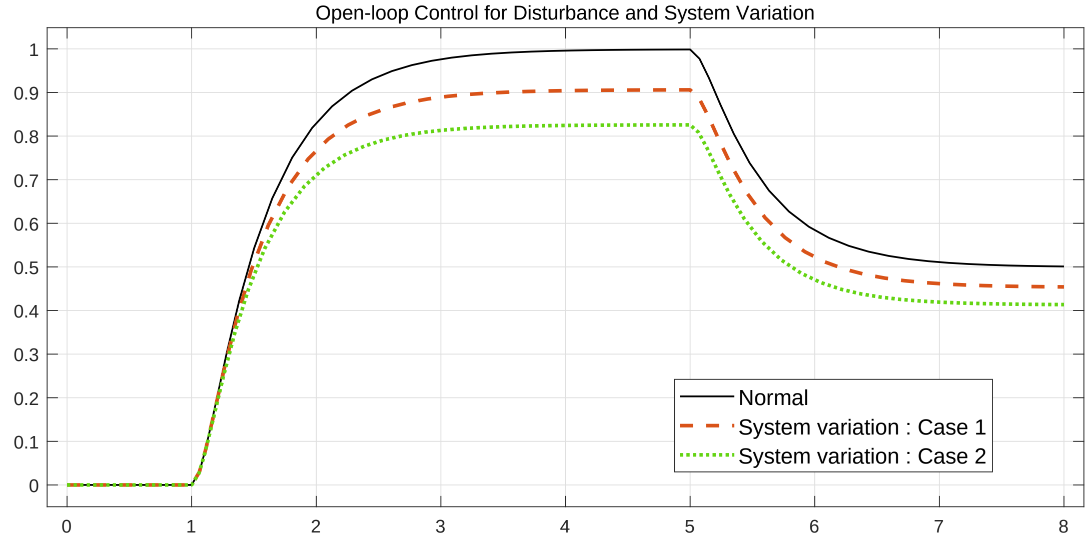

Fig. 4: Result for open-loop control with disturbance and variations

the plant, i.e. the DC motor system, is calculated in the time domain from the feedback error as follows:

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_p \frac{de(t)}{dt}. \tag{4}$$

The variable $e(t)$ represents the error, the difference between the desired output w_D and the actual output $w(t)$. This error $e(t)$ is fed to the PID controller, and the controller computes both the derivative and the integral of this error signal with respect to time. The control input $u(t)$ to the plant is equal to the proportional gain K_p times the magnitude of the error plus the integral gain K_i times the integral of the error plus the derivative gain K_d times the derivative of the error. This control input $u(t)$ is fed to the plant and the new output is obtained. The new output is then fed back and compared to the reference to find the new error $e(t)$. The controller takes this new error signal and computes an update of the control input. This process continues while the controller is in effect.

2) P Controller Design: First, a simple proportional controller is applied. As shown in Fig. 5, applying the proportional controller with a gain of $K_p = 50$, that is, $C(s) = 50$ shows that the steady-state error is too large. Then, as shown in Fig. 6, increasing the proportional gain $K_p = 100$, that is, $C(s) = 100$ reduces the steady-state error while increasing overshoot. Therefore, it appears that not all of the performance criteria can be met with a simple proportional controller.

A little experimentation verifies that a proportional controller is insufficient for meeting the given performance specifications. Thus, derivative and/or integral terms must be added to the controller.

3) PID Controller Design: Adding an integral term will eliminate the steady-state error to a step reference and a derivative term will often reduce the overshoot. Thus, a PID controller with $K_i = 1$ and $K_d = 1$ is applied while maintaining $K_p = 100$. However, as shown in Fig. 7, applying the PID controller with small K_i and K_d shows that both the steady-state error and the overshoot are still too large. Thus, the gain tuning is required to obtain better performance. The long tail on the step response is due to the fact that the integral gain is small and, therefore, it takes a long time for the integral action to build up and eliminate the steady-state error. This process can be sped up by increasing the value of K_i. Hence, K_i is changed to 200 while maintaining $K_p = 100$ and $K_d = 1$. As expected, Fig. 8 indicates that the steady-state error is now eliminated much more quickly than before. It also becomes possible to reject the disturbance. However, the large K_i has greatly increased the overshoot. Thus, K_d is increased to 10 in an attempt to reduce the overshoot. As expected, Fig. 9 indicates that the increased K_d reduced the resulting overshoot. It also becomes possible to reject the disturbance.

Now, if a PID controller is applied with $K_p = 100$, $K_i = 200$, and $K_d = 10$, all of performance criteria is satisfied. Therefore, as opposed to open-loop control, it is shown that the feedback control with the PID controller speeds up the response significantly and compensates for the unpredicted disturbance.

The reason that open-loop control cannot speed up the response significantly as well as compensate for the disturbance is that it only provides a static voltage source to the DC motor. On the other hand, the feedback control changes the voltage source to the DC motor dynamically instead of keeping it constant. Especially, the feedback control sees that the error is growing when there is disturbance. And it increases the voltage source to the DC motor, which in turn increases the rotational speed of the DC motor. And in this way, the error is pulled back to zero.

In addition, as shown in Fig. 10, the feedback control with the PID controller can deal with system variations of Table IV. In Fig. 10, the enlarged view near the overshoot section is also included to compare feedback controls for system variations.

IV. KALMAN FILTERING FOR NOISE REDUCTION

A. Performance Degradation due to Feedback Sensor Noises

In order that the feedback control in the DC motor speed control system can adjust the error, a feedback sensor is required to measure output i.e. the DC motor's actual

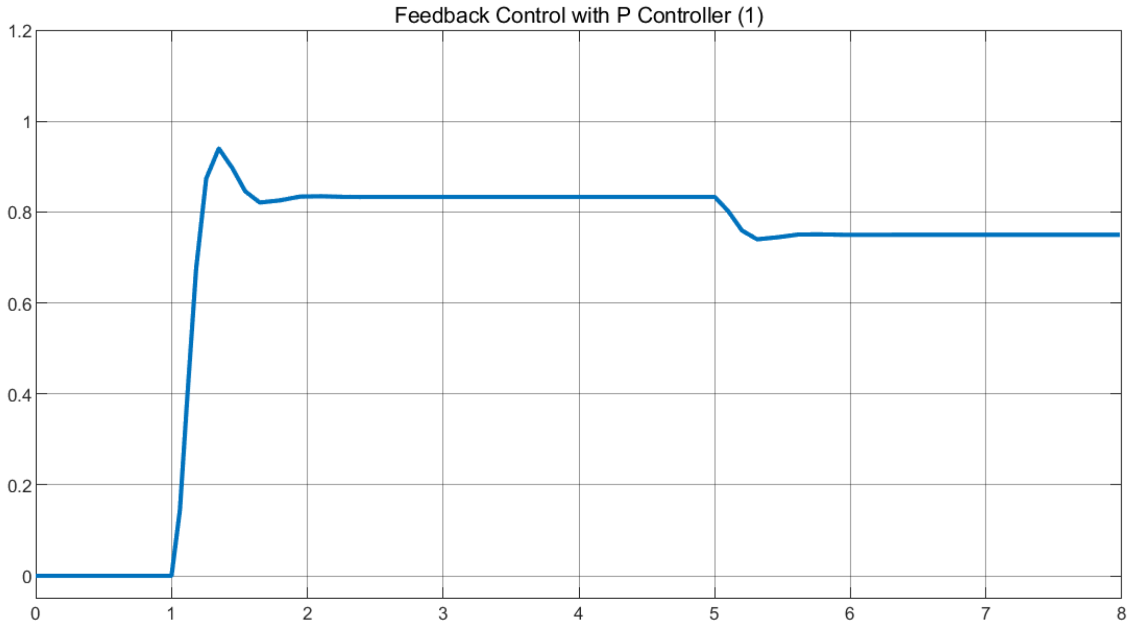

Fig. 5: Result for feedback control with P controller (Case 1)

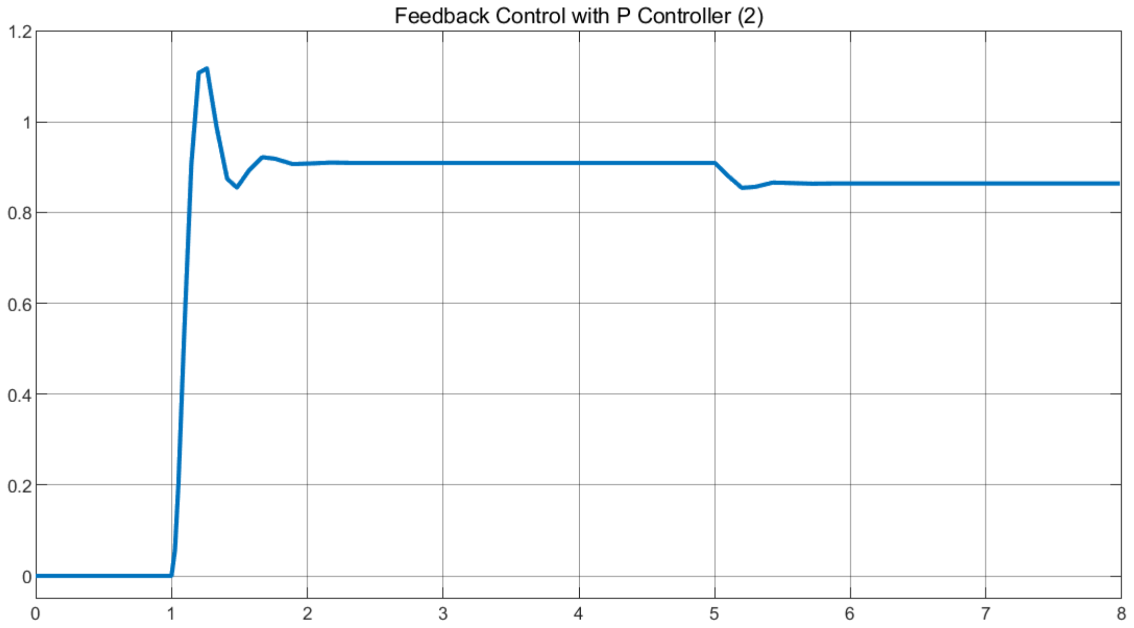

Fig. 6: Result for feedback control with P controller (Case 2)

rotational speed. Unfortunately, the feedback sensor can be often noisy. The noise coming from a sensor is thermal noise arising from thermal motions of charges within the sensor. Another low-level source of noise is shot noise related to the fact that charge is quantized. The feedback sensor noise is random variations of sensor output unrelated to variations in sensor input. Therefore, when the feedback sensor measures output imperfectly due to the noise, the control accuracy is affected in the conventional feedback control system. A couple of noise signals with noise variances $\sigma = 0.0001$ and $\sigma = 0.00001$ are considered for simulations. As shown in Fig. 11 and 12, the actual rotational speed of the DC motor speed control system is very noisy for both noise signals. Especially, the larger the noise variance, the worse the performance. The measured output i.e. the actual rotational speed should be corrected. Thus, the filtering should be applied to get the noise reduction of DC motor's rotational speed.

B. *Kalman Filtering*

The Kalman filter is known to be the best linear unbiased estimator for linear systems with Gaussian process and measurement noise. The Kalman filter has been a standard choice and a beautiful reference for the state estimation. The Kalman filter's closed-form recursive equations have turned it into arguably the most popular and widely used estimator, with applications ranging from the aerospace and aircraft industries to seismology and weather forecasting[8]-[12]. To

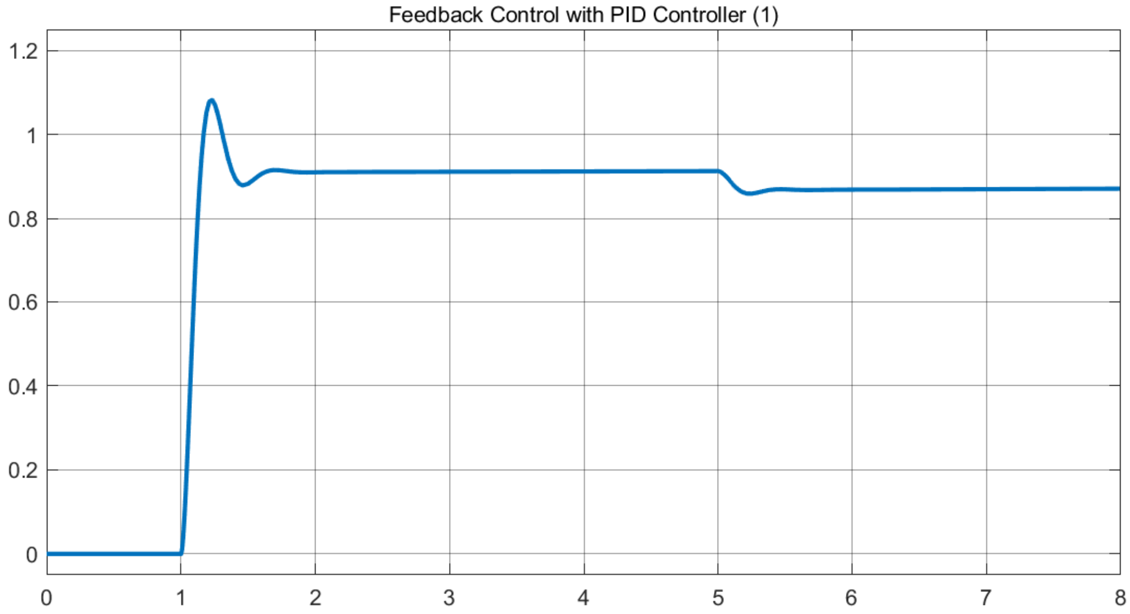

Fig. 7: Result for feedback control with PID controller (Case 1)

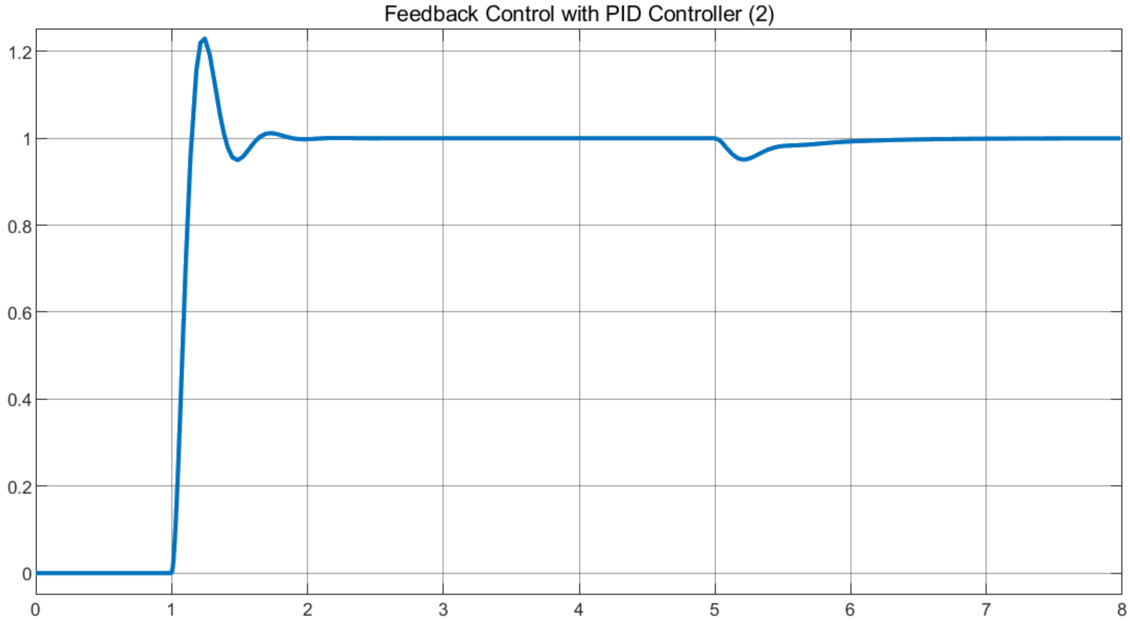

Fig. 8: Result for feedback control with PID controller (Case 2)

apply the state estimation filtering, the state-space realization is required for the DC motor system. The state-space approach is a generalized time domain method for modeling, analyzing and designing a wide range of control systems and is particularly well suited to digital computational technique. The dynamic equation (1) of the DC motor system can be represented in the continuous-time state-space model as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{5}$$

with variables and matrices

$$\begin{aligned} x(t) &\triangleq \begin{bmatrix} w(t) \\ i(t) \end{bmatrix}, \; A = \begin{bmatrix} -B_m/J_m & K/J_m \\ -K/L_a & -R_a/L_a \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \end{aligned}$$

where $x(t)$ is state variable with rotational speed $w(t)$ and armature current $i(t)$, $y(t)$ is output variable with rotational speed $w(t)$, $u(t)$ is control input variable with voltage source $v(t)$.

For the state-space model (5), the Kalman filter provides an optimal state estimate $\hat{x}(t)$ for the system state $x(t)$ as

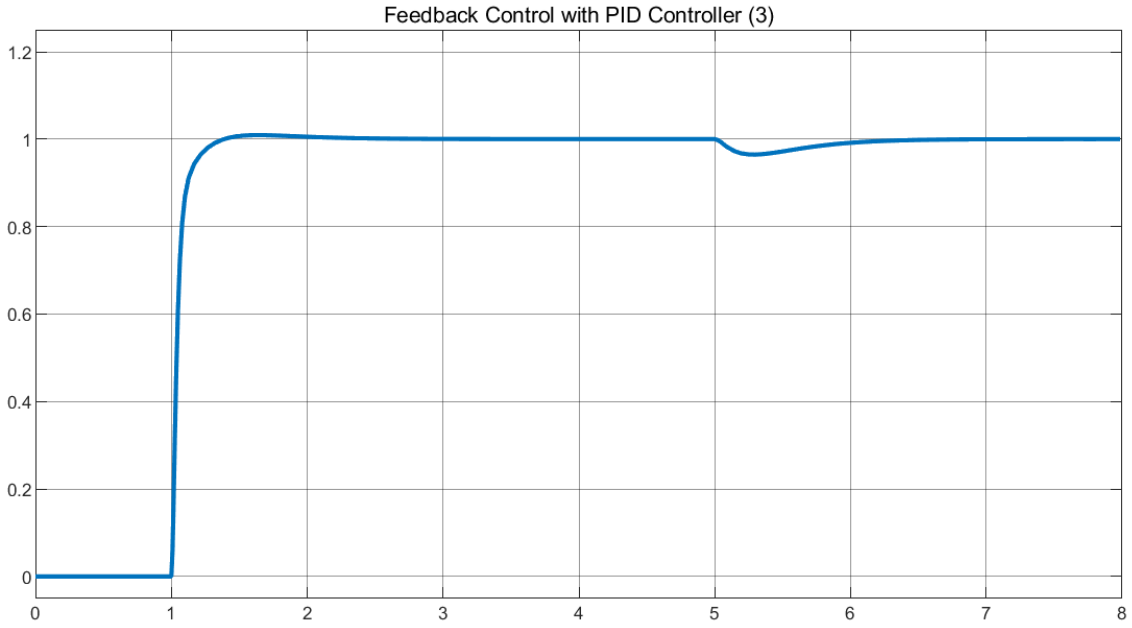

Fig. 9: Result for feedback control with PID controller (Case 3)

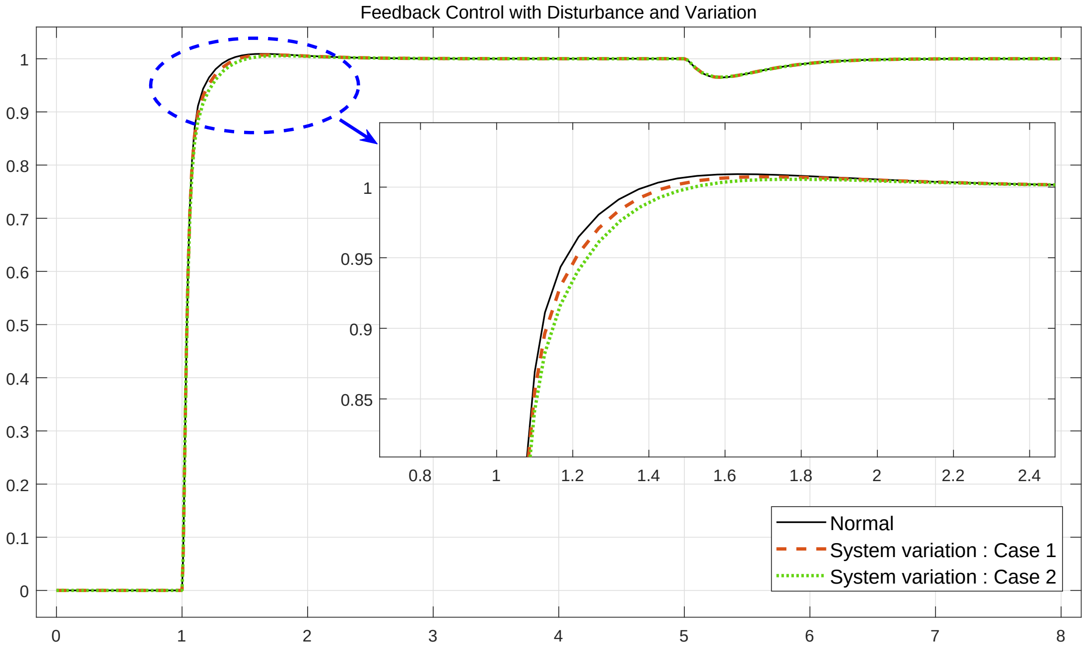

Fig. 10: Result for feedback control with disturbance and variations

follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t)+P(t)C^TR^{-1}\Big[y(t)-C\hat{x}(t)\Big]+Bu(t),\\ P(t) &= AP(t)+P(t)A^T+GQG^T\\ &\quad -P(t)C^TR^{-1}CP(t),\end{aligned}$$

with the initial state $\hat{x}(t_0) = \bar{x}(t_0)$. $P(t)$ is the error covariance of the estimate $\hat{x}(t)$ with initial value $P(t_0)$. Q and R are useful design parameters for the Kalman filter. These parameters can make the tradeoff between the noise reduction and the tracking speed of the state estimation. In this paper, how to properly set these parameters will not be discussed.

The filtered rotational speed $\hat{w}(t)$ is fed to the computation of error $e(t) = w_d - \hat{w}(t)$. Then, this error $e(t)$ is fed to the PID controller to compute the voltage source. Finally, it is verified from Fig. 13 that the DC motor speed control system with both PID controller and Kalman filter is sufficient for meeting the given performance criteria and compensates for the unpredicted disturbance as well as the feedback sensor noise. Fig. 14 shows the ultimate block diagram for DC motor speed control system. As shown in Fig. 15, this block diagram is implemented with the commercial software *MathWorks Simulink* which is the *MathWorks MATLAB*-based graphical programming environment for modeling, simulating and analyzing multidomain dynamical systems.

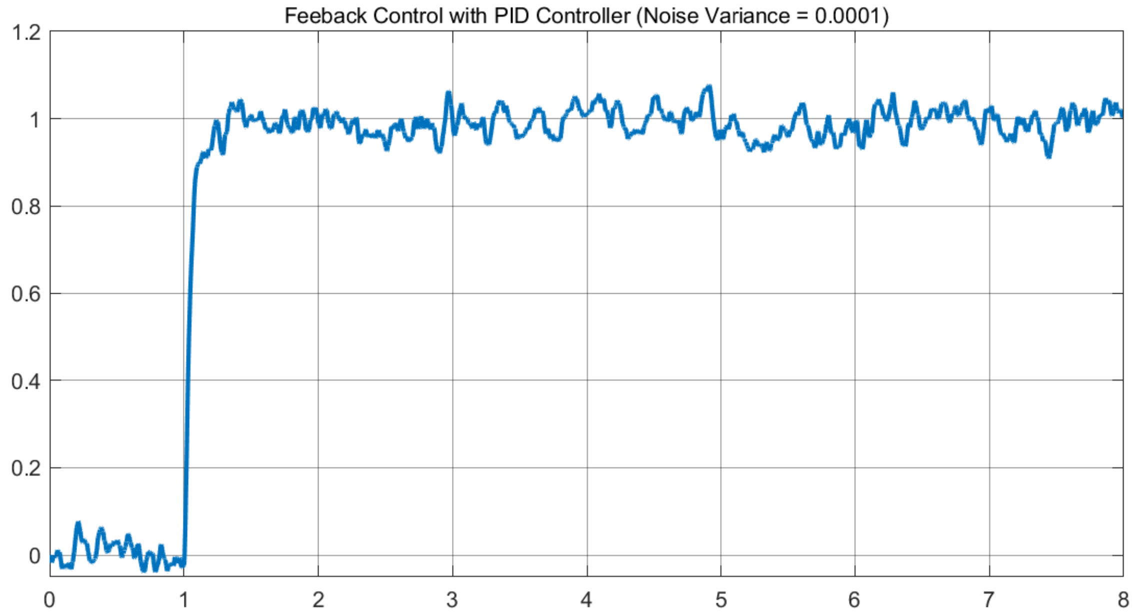

Fig. 11: Result for feedback control with PID controller ($\sigma = 0.0001$)

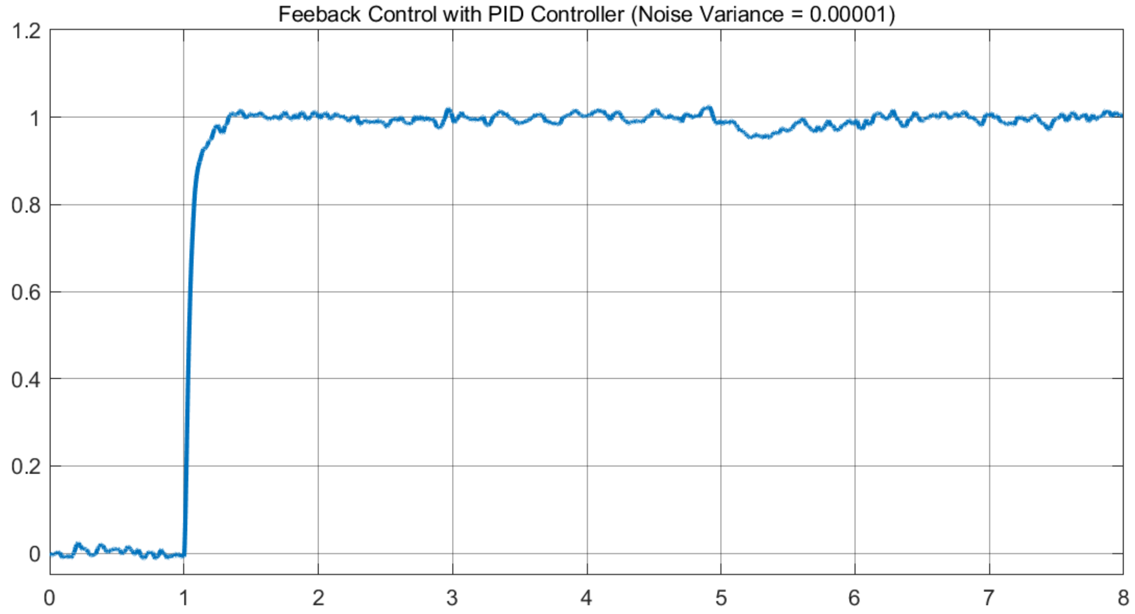

Fig. 12: Result for feedback control with PID controller ($\sigma = 0.00001$)

V. CONCLUSIONS

This paper has designed the DC motor speed control system with PID controller and Kalman filter for disturbance rejection and noise reduction when there are disturbance and feedback sensor noise. Each detailed design process has been verified through computer simulations. The performance degradation due to disturbance and system variation in the basic open-loop control has been shown. To resolve this problem, a PID controller based feedback system has been designed for the DC motor speed control system. To improve the performance degradation due to feedback sensor noise that may occur during the feedback process, the Kalman filter has been applied for the DC motor speed control system. It has been verified that the designed DC motor speed control system with PID controller and Kalman filter not only satisfies all performance criteria but also has the ability of disturbance rejection, system variation handling, and noise reduction.

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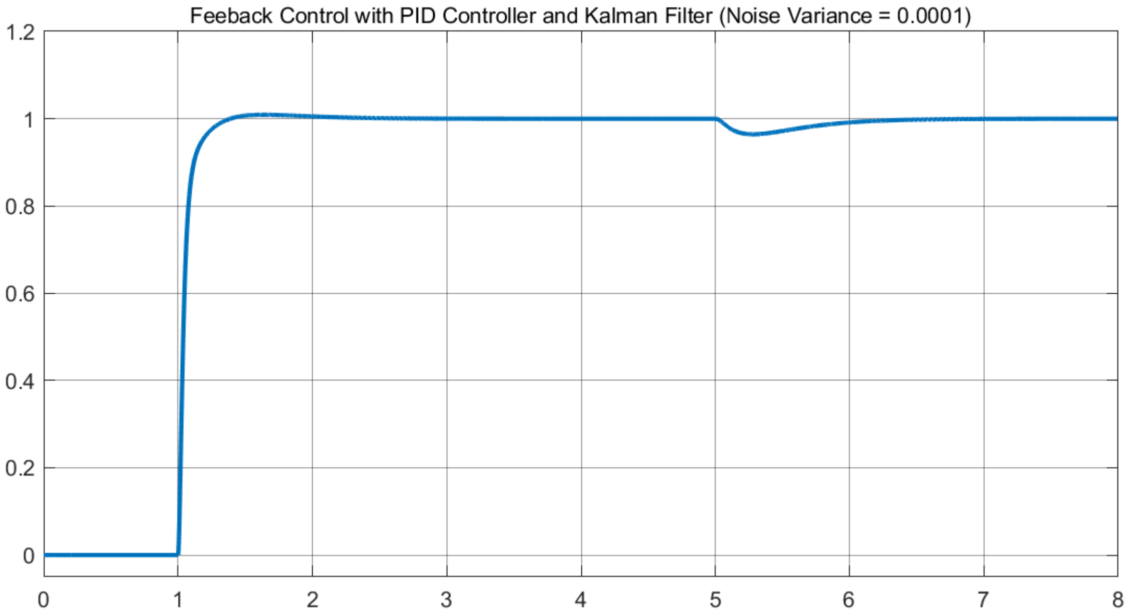

Fig. 13: Result for feedback control with PID controller and Kalman filter ($\sigma = 0.0001$)

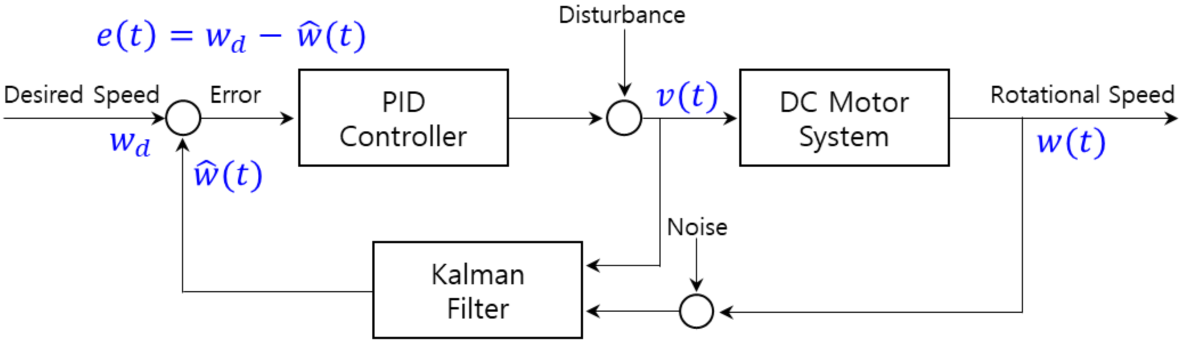

Fig. 14: Ultimate block diagram for DC motor speed control system

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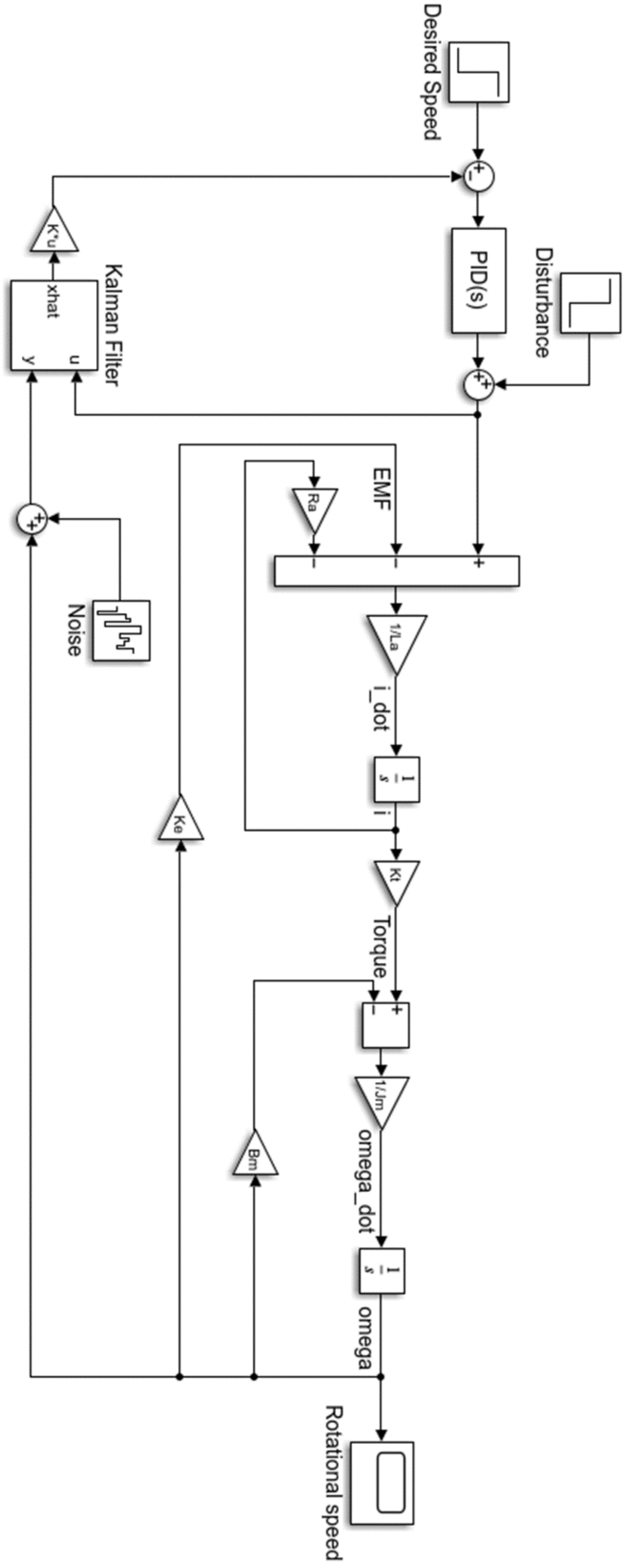

Fig. 15: *MathWorks Simulink* implementation for DC motor speed control system