Tracking Control of the Six-rotor UAV with Input Dead-zone and Saturation

Wangyue Di, Xin Tong, and Zhigang Li

Abstract—**A trajectory tracking controller is designed for a six-rotor Unmanned Aerial Vehicles (UAV) with input dead-zone and saturation. The six-rotor UAV system is divided into two subsystems by using the double closed-loop control strategy. The outer loop control is used to control the position subsystem, and the inner loop control is used to control the attitude subsystem. The dead-zone and saturation are non-smooth. This nonlinear property is approximated by a non-affine smooth function and transformed from non-affine to affine by using the mean-value theorem. Applied with backstepping method and the Nussbaum gain method, the tracking controller of the six-rotor UAV system is designed, which effectively solves the influence of input dead-zone and saturation on system instability, realizes tracking control, and ensures the stability of the system. Finally, the effectiveness of the proposed method is verified by the MATLAB simulation example.**

Index Terms—**the six-rotor UAV, input dead-zone, saturation, the Nussbaum gain method**

I. Introduction

The control system of the six-rotor UAV has the characteristics of strong coupling, underactuation and nonlinearity. Compared with the Quadrotor UAV (QUAV), the six-rotor UAV not only enhances the load capacity, but also has better stability and robustness, it has been widely used in civil, military and other fields [1]-[4]. Backstepping method has been a commonly used effective control method [5]-[9], it has been widely adopted in the control research of multi rotors UAV. By utilizing backstepping method and sliding mode control scheme to design controller, [10] ensured Bank-To-Turn (BTT) UAV accurate flight the stability of the system. [11] considered QUAV with input actuator failure and uncertain disturbance, the proposed robust fault-tolerant controller finally realized the fast tracking. [12] applied sliding mode control to the position and attitude control of four rotors, which improved the dynamic response to the system. Considering the QUAV with actuator failure, [13] designed controller by prescribed performance method and guaranteed the predetermined performance indicators. From the above literature, the actuator of UAV needs to consider the nonlinear characteristics, such as dead-zone, saturation. These nonlinear characteristics will reduce the flight control performance and lead to the instability of the system. In response to this problem, [14] proposed an adaptive tracking control strategy combined with the backstepping method to deal with the problem of the input dead-zone of the multi-input and multi-output system. [15] adopted the dynamic surface and the disturbance observer technique to solve distributed fault-tolerant cooperative control. [16] designed a robust control strategy to realize the distributed finite-time control of formations under the influence of saturation constraints on the input of multi-UAV attitude subsystems.

The problems considered in the above literature mainly focus on the case that the input of QUAV has dead-zone or saturation. Aiming at the tracking control problem that the six-rotor UAV has both dead-zone and saturation, this paper adopts the double closed-loop control strategy, combined with backstepping method and Nussbaum gain method to design the controller to realize the tracking of the desired trajectory of the six-rotor UAV, ensure the boundedness of all signals in the system, effectively solve the influence of the input with dead-zone and saturation on the instability of the six-rotor UAV system, and ensure the stability of the system.

II. Description of the Problem and Preliminaries

In order to facilitate the establishment of the mathematical model of the six-rotor UAV, it is assumed that the flight speed of the UAV is very low and it is not affected by the air resistance during flight, and the following assumptions are made:

1) It is assumed that the six-rotor UAV is a rigid body. The whole fuselage structure is symmetrical, and the structure does not undergo minor deformation during flight.

2) It is assumed that the mass distribution of the six-rotor UAV is uniform and its center coincides with its center of mass.

In order to simplify the design process of the controller, the dual closed-loop represents two subsystems. The outer loop is the position subsystem and the inner loop is the attitude subsystem. The controllers of the position subsystem and the attitude subsystem are designed respectively. Finally, the controller achieves the control goal. Position subsystem (1) and attitude subsystem (2), which are expressed as

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follows[17]:

$$\begin{cases} \ddot{x} = \dfrac{u_1}{m} H(u_x) \\ \ddot{y} = \dfrac{u_1}{m} H(u_y) \\ \ddot{z} = \dfrac{\cos\phi\cos\theta}{m} H(u_1) - g \\ u_x = \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ u_y = \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \end{cases}, \tag{1}$$

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\dfrac{J_y - J_z}{J_x} + \dfrac{a}{J_x} H(u_2) - \dfrac{J_r}{J_x}\dot{\theta}\Omega_r \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\dfrac{J_z - J_x}{J_y} + \dfrac{a}{J_y} H(u_3) - \dfrac{J_r}{J_y}\dot{\phi}\Omega_r \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\dfrac{J_x - J_y}{J_z} + \dfrac{1}{J_z} H(u_4) \end{cases}, \tag{2}$$

where x, y, z denote position of the six-rotor UAV; ϕ, θ, ψ represent the roll angle, pitch angle, and yaw angle, respectively; u_1, u_2, u_3, u_4 denote control input; m denotes the mass; g is the acceleration of gravity; a denotes the distance between the center of mass and the propeller; J_x, J_y, J_z are the moment of inertia; J_r is the total moment of inertia of the motor rotor and propeller rotating around the rotating shaft; $\Omega_r = \Omega^2_2 - \Omega^2_1 + \Omega^2_4 - \Omega^2_3 + \Omega^2_6 - \Omega^2_5$ is the sum of the rotational speeds of the six rotors.

According to the dynamic model (1), the position subsystem has one input and three outputs, which is an underactuated subsystem. So u_x, u_y are defined as two virtual control inputs. According to the dynamic model, there is a strongly coupling relationship between translation motion and attitude motion. The input of the inner loop controller is determined by the outer loop controller, so u_x, u_y are used to provide the expected values of roll angle and pitch angle of the inner loop. Specific expectations are given later.

$H(u)$ is the input affected by dead-zone saturation and can be represented as [18]:

$$H(u) = \begin{cases} w_N, u(t) \le b_{m1} \\ t_m(u), b_{m1} < u(t) \le b_{m0} \\ 0, b_{m0} < u(t) \le b_{n1} \\ t_n(u), b_{n1} < u(t) \le b_{n0} \\ w_P, u(t) > b_{n0} \end{cases}, \tag{3}$$

where $u(t)$ denotes the plant inputs affected by dead-zone saturation nonlinearity; $t_m(u)$ and $t_n(u)$ are unknown smooth nonlinear functions; w_N and w_P are the unknown saturation values; $b_{m1} < b_{m0} < 0$ and $0 < b_{n1} < b_{n0}$ are unknown input nonlinearity parameters. The following Lemmas and Assumptions can be introduced.

Assumption 1 [19]: There are unknown positive parameters $b_{ml}, b_{mv}, b_{nl}, b_{nv}$, such that:

$$0 < b_{ml} < t'_m(u) < b_{mv} < \infty, \forall u \in [b_{m1}, b_{m0}], \tag{4}$$

$$0 < b_{nl} < t'_n(u) < b_{nv} < \infty, \forall u \in [b_{n1}, b_{n0}]. \tag{5}$$

Lemma 1 [19]: To approximate the dead-zone saturation function $H(u)$, a smooth function $D(u)$ is defined as:

$$\begin{aligned} D(u) = &\frac{-w_N}{2}\tanh s_1\left(u - \frac{b_{m0}}{s_1} + \iota_l\right) - \frac{w_N}{2}\tanh\left(b_{m0} - \iota_l s_1\right) \\ &+ \frac{w_P}{2}\tanh s_2\left(u - \frac{b_{n0}}{s_2} - \iota_r\right) + \frac{w_P}{2}\tanh\left(b_{n0} + \iota_r s_2\right), \end{aligned} \tag{6}$$

where s_1, s_2, ι_l, and ι_r are positive parameters, which can be changed the value to reduce the error. $H(u)$ and $D(u)$ are depicted in Fig 1. $H(u)$ can be rewritten as:

$$H(u) = P(u) + D(u), \tag{7}$$

where $P(u) = H(u) - D(u)$ is the error. Based on the mean-value theorem, there exist a constant $\mu \in (u_0, u)$, such that $D(u) = D(u_0) + \dot{D}(u_\mu)(u - u_0)$, let $u_0 = 0$, such that $D(u) = \dot{D}(u_\mu)u$. Then, one can obtain:

$$H(u) = \dot{D}(u_\mu)u + P(u). \tag{8}$$

Assumption 2: There exists an unknown positive constant P, such that:

$$P(u) \le P. \tag{9}$$

Assumption 3 [20]: The smooth function $\dot{D}(u_\mu)$ is non-decreasing. There is positive constant d, such that:

$$0 < d < \dot{D}(u_\mu). \tag{10}$$

Assumption 4: The reference trajectory x_r, y_r, z_r, ψ_r, and its first derivative are continuous and bounded.

Assumption 5: The Euler angles ϕ, θ, ψ of the six-rotor UAV satisfy $\phi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\psi \in (-\pi, \pi)$.

III. Design of Controller

In this section, a tracking controller is designed for the six-rotor UAV with input dead-zone and saturation. The six-rotor UAV has four inputs and six degrees of freedom outputs. It is a underactuated, strongly coupled and nonlinear system. Therefore, this paper adopts the double closed-loop control strategy that the attitude subsystem is the inner loop and the position subsystem is the outer loop, so as to design the controller. Achieve the control objectives: 1) all signals in the closed-loop system are bounded. 2) The six-rotor UAV system can realize the trajectory tracking control.

A. Position Subsystem Controller Design

The position control of the outer-loop includes horizontal direction x, y, height subsystem z direction control. In order to facilitate mathematical processing, (1) can be written in the following matrix form:

$$\begin{cases} \dot{d}_1 = d_2 \\ \dot{d}_2 = B_1 \circ H_1 + F_1 \end{cases}, \tag{11}$$

where $d_1 = [z, x, y]^T$, $B_1 = \left[\frac{\cos\phi\cos\theta}{m}, \frac{u_1}{m}, \frac{u_1}{m}\right]^T$,

$F_1=[-g,0,0]^T, H(U_1)=[u_1,u_x,u_y]^T$. $\circ$ represents the Hadamard product, i.e., the matrices $A=(a_{mn})$ and $B=(b_{mn})$, if have the same dimension, then have $(A\circ B)_{mn}=a_{mn}\times b_{mn}$.

Step 1:

The tracking errors are defined as:

$$Z_1=d_1-d_r, \tag{12}$$

$$Z_2=d_2-X, \tag{13}$$

where $d_1=[z_r,x_r,y_r]^T$ is the desired trajectory, $X=[\alpha_{z,1},\alpha_{x,1},\alpha_{y,1}]^T$ is input of the virtual controller.

According to (12) and (13), ones have:

$$\dot{Z}_1=\dot{d}_1-\dot{d}_r=d_2-\dot{d}_r=Z_2+X-\dot{d}_r. \tag{14}$$

Choose the Lyapunov function:

$$V_{p1}=\frac{1}{2}Z_1^TZ_1. \tag{15}$$

Then, using (14) yields:

$$\dot{V}_{p1}=Z_1^T\left(Z_2+X-\dot{d}_r\right). \tag{16}$$

The virtual control rate is generated by:

$$X=-K_1\circ Z_1+\dot{d}_r, \tag{17}$$

where $K_1=[k_{1,1},k_{1,2},k_{1,3}]^T$ is designed parameter.

Then, substituting (17) into (16) gives:

$$\dot{V}_{p1}=-(K_1\circ Z_1)^TZ_1+Z_1^TZ_2. \tag{18}$$

Step 2:

With (8), (11), and (13) gives:

$$\dot{Z}_2=\dot{d}_2-\dot{X}=B_1\circ\dot{D}_1\circ U_1+B_1\circ P_1+F_1-\dot{X}, \tag{19}$$

where

$\dot{D}_1=\left[\dot{D}(u_{\mu,1}),\dot{D}(u_{\mu,x}),\dot{D}(u_{\mu,y})\right]^T, U_1=[u_1,u_x,u_y]^T$,

$P_1=\left[P(u_1),P(u_x),P(u_y)\right]^T$.

Defining the Lyapunov function as:

$$V_{p2}=V_{p1}+\frac{1}{2}Z_2^TZ_2. \tag{20}$$

By invoking (18) and (19), the derivative of V_{p2} is:

$$\begin{aligned}\dot{V}_{p2}&=-(K_1\circ Z_1)^TZ_1+Z_2^TZ_1\\&+Z_2^T\left(B_1\circ\dot{D}_1\circ U_1+B_1\circ P_1+F_1-\dot{X}\right).\end{aligned} \tag{21}$$

Based on Young's inequality and Assumption 2, ones have:

$$Z_2^T(B_1\circ P_1)\le\frac{Z_2^T(B_1\circ B_1\circ Z_2)}{2}+\frac{\|P_1\|^2}{2}. \tag{22}$$

Then, substituting (22) into (21) gives:

$$\begin{aligned}\dot{V}_{p2}&\le-(K_1\circ Z_1)^TZ_1+Z_2^TZ_1\\&+Z_2^T\left(B_1\circ\dot{D}_1\circ U_1-\dot{X}+\frac{Z_2\circ B_1\circ B_1}{2}+F_1\right)+\frac{\|P_1\|^2}{2}.\end{aligned} \tag{23}$$

Design control inputs U_1 for the position subsystem:

$$U_1=B_2\circ N_1\circ\bar{U}_1, \tag{24}$$

$$\bar{U}_1=Z_1-\dot{X}+\frac{Z_2\circ B_1\circ B_1}{2}+K_2\circ Z_2+F_1, \tag{25}$$

$$\dot{o}_1=Z_2\circ\bar{U}_1, \tag{26}$$

where $N_1=\left[N(o_z),N(o_x),N(o_y)\right]^T, o_1=[o_z,o_x,o_y]^T$, $B_2=\left[\frac{m}{\cos\phi\cos\theta},\frac{m}{u_1},\frac{m}{u_1}\right]^T, K_2=[k_{2,1},k_{2,2},k_{2,3}]^T$ is design parameters.

Lemma 2 [21]: Define smooth functions $V(\cdot)$ and $o(\cdot)$, on $[0,t_f)$, $N(o)$ be any continuous function, if the following properties:

$$\lim_{r\to\infty}\sup\frac{1}{r}\int_0^r N(o)d_o=\infty, \tag{27}$$

$$\lim_{r\to\infty}\inf\frac{1}{r}\int_0^r \mathrm{N}(o)d_o=-\infty. \tag{28}$$

hold, then $N(o)$ is called a Nussbaum-type function. If the following property holds:

$$V(t)\le\sigma_0+e^{-\sigma_1 t}\int_0^t\left(f(\tau)N(o)+1\right)\dot{o}e^{\sigma_1\tau}d_\tau, \tag{29}$$

where $f(\tau)$ is a time-varying unknown function that takes values in the closed interval $L=[l,l^+]$ with $0\notin L$, σ_0 and σ_1 are positive constants, then it can be obtained that $\mathrm{V}(t),\mathrm{N}(o)$, and $\int_0^t f(\tau)N(o)\dot{o}d_\tau$ are bounded on $[0,t_f)$.

Applying Assumption 3 and Lemma 2, and noting that $\dot{D}_1>0$, similar to the processing method in [21]-[22], by combining the equalities (24)-(26), (23) can be rewritten as:

$$\begin{aligned}\dot{V}_{p2}&\le-(K_1\circ Z_1)^TZ_1+\frac{\|P_1\|^2}{2}\\&+Z_2^T\left(Z_1+\dot{D}_1\circ N_1\circ\bar{U}_1-\dot{X}+\frac{Z_2\circ B_1\circ B_1}{2}+F_1\right)\\&\le-(K_1\circ Z_1)^TZ_1+\frac{\|P_1\|^2}{2}+Z_2^TZ_1+Z_2^T\bar{U}_1-Z_2^T\bar{U}_1\\&+Z_2^T\left(\dot{D}_1\circ N_1\circ\bar{U}_1-\dot{X}+\frac{Z_2\circ B_1\circ B_1}{2}+F_1\right)\\&\le-(K_1\circ Z_1)^TZ_1-(K_2\circ Z_2)^TZ_2\\&+\dot{o}_1^T\left(\dot{D}_1\circ N_1+I_1\right)+\frac{\|P_1\|^2}{2}.\end{aligned} \tag{30}$$

where $I_1=[1,1,1]^T$.

In this paper, a double closed-loop control strategy is adopted, and the input in the horizontal direction u_x,u_y are used to inversely solve the desired roll angle and pitch angle as:

$$\phi_r=\arcsin\left(u_x\sin\psi-u_y\cos\psi\right), \tag{31}$$

$$\theta_r=\arcsin\left(\frac{u_x\cos\psi+u_y\sin\psi}{\cos\phi_r}\right). \tag{32}$$

B. Attitude Subsystem Controller Design

In this section, the controller of the attitude subsystem of the inner loop will be designed, which is similar to the derivation process of the position subsystem. The state equation of the attitude subsystem obtained from (2) is:

$$\begin{cases} \dot{h}_1 = h_2 \\ \dot{h}_2 = B_3 \circ H_2 + F_2 \end{cases}, \tag{33}$$

where $h_1 = [\phi, \theta, \psi]^T$, $H_2 = [H(u_2), H(u_3), H(u_4)]^T$,

$B_3 = \left[\frac{a}{J_x}, \frac{a}{J_y}, \frac{1}{J_z}\right]^T$,

$F_2 = \left[\dot{\theta}\dot{\psi}\frac{J_y - J_z}{J_x} - \frac{J_r}{J_x}\dot{\theta}\Omega_r, \dot{\phi}\dot{\psi}\frac{J_z - J_x}{J_y} - \frac{J_r}{J_y}\dot{\phi}\Omega_r,\right.$ $\left.\dot{\phi}\dot{\theta}\frac{J_x - J_y}{J_z}\right]^T$.

Step1:

The tracking errors are defined as:

$$Z_3 = h_1 - h_r, \tag{34}$$

$$Z_4 = h_2 - Q. \tag{35}$$

where $h_r = [\phi_r, \theta_r, \psi_r]^T$ is the desired trajectory, $Q = [\alpha_{\phi,1}, \alpha_{\theta,1}, \alpha_{\psi,1}]^T$ is input of the virtual controller.

According to (33) and (34), ones have:

$$\dot{Z}_3 = h_2 - \dot{h}_r = Z_4 + Q - \dot{h}_r. \tag{36}$$

Choose the Lyapunov function:

$$V_{a1} = \frac{1}{2} Z_3^T Z_3. \tag{37}$$

Then, using (36) yields:

$$\dot{V}_{a1} = Z_3^T \left(Z_4 + Q - \dot{h}_r\right). \tag{38}$$

The virtual control rate is generated by:

$$Q = -K_3 \circ Z_3 + \dot{h}_r, \tag{39}$$

where $K_3 = [k_{3,1}, k_{3,2}, k_{3,3}]^T$ is designed parameter.

Then, substituting (39) into (38) gives:

$$\dot{V}_{a1} = -\left(K_3 \circ Z_3\right)^T Z_3 + Z_3^T Z_4. \tag{40}$$

Step 2:

With (8), (33), and (35) gives:

$$\dot{Z}_4 = \dot{h}_2 - \dot{Q} = B_3 \circ \dot{D}_2 \circ U_2 + F_2 + B_3 \circ P_2 - \dot{Q}, \tag{41}$$

where

$\dot{D}_2 = \left[\dot{D}(u_{\mu,2}), \dot{D}(u_{\mu,3}), \dot{D}(u_{\mu,4})\right]^T$, $U_2 = [u_2, u_3, u_4]^T$,

$P_2 = [P(u_2), P(u_3), P(u_4)]^T$.

Define the Lyapunov function as:

$$V_{a2} = V_{a1} + \frac{1}{2} Z_4^T Z_4. \tag{42}$$

By invoking (40) and (41), the derivative of V_{a2} is:

$$\begin{aligned} \dot{V}_{a2} &= -\left(K_3 \circ Z_3\right)^T Z_3 + Z_4^T Z_3 \\ &+ Z_4^T \left(B_3 \circ \dot{D}_2 \circ U_2 + F_2 + B_3 \circ P_2 - \dot{Q}\right). \end{aligned} \tag{43}$$

Based on Young's inequality and Assumption 2, ones have:

$$Z_4^T \left(B_3 \circ P_2\right) \le \frac{Z_4^T (Z_4 \circ B_3 \circ B_3)}{2} + \frac{\|P_2\|^2}{2}. \tag{44}$$

Then, substituting (44) into (43) gives:

$$\begin{aligned} \dot{V}_{a2} &\le -\left(K_3 \circ Z_3\right)^T Z_3 + Z_4^T Z_3 + \frac{\|P_2\|^2}{2} \\ &+ Z_4^T \left(B_3 \circ \dot{D}_2 \circ U_2 - \dot{Q} + \frac{Z_4 \circ B_3 \circ B_3}{2} + F_2\right). \end{aligned} \tag{45}$$

Design control inputs U_2 for the position subsystem:

$$U_2 = B_4 \circ N_2 \circ \bar{U}_2, \tag{46}$$

$$\bar{U}_2 = Z_3 - \dot{Q} + \frac{Z_4 \circ B_3 \circ B_3}{2} + K_4 \circ Z_4 + F_2, \tag{47}$$

$$\dot{o}_2 = Z_4 \circ \bar{U}_2, \tag{48}$$

where $N_2 = [N(o_\phi), N(o_\theta), N(o_\psi)]^T$, $o_2 = [o_\phi, o_\theta, o_\psi]^T$, $B_4 = \left[\frac{J_x}{a}, \frac{J_y}{a}, \frac{J_z}{a}\right]^T$. $K_4 = [k_{4,1}, k_{4,2}, k_{4,3}]^T$ is design parameters.

Applying Assumption 3, by combining the equalities (46)-(48), (45) can be rewritten as:

$$\begin{aligned} \dot{V}_{a2} &\le -\left(K_3 \circ Z_3\right)^T Z_3 + \frac{\|P_2\|^2}{2} + Z_4^T Z_3 \\ &+ Z_4^T \left(\dot{D}_2 \circ N_2 \circ \bar{U}_2 - \dot{Q} + \frac{Z_4 \circ B_3 \circ B_3}{2} + F_2\right) \\ &\le -\left(K_3 \circ Z_3\right)^T Z_3 + \frac{\|P_2\|^2}{2} + Z_2^T \bar{U}_2 - Z_2^T \bar{U}_2 \\ &+ Z_4^T \left(Z_3 + \dot{D}_2 \circ N_2 \circ \bar{U}_2 - \dot{Q} + \frac{Z_4 \circ B_3 \circ B_3}{2} + F_2\right) \\ &\le -\left(K_3 \circ Z_3\right)^T Z_3 - \left(K_4 \circ Z_4\right)^T Z_4 \\ &+ \dot{o}_2^T \left(\dot{D}_2 \circ N_2 + I_2\right) + \frac{\|P_2\|^2}{2}, \end{aligned} \tag{49}$$

where $I_2 = [1,1,1]^T$.

IV. Stability analysis

Considering the system (1), (2) of the six-rotor UAV, under assumptions 1-5, the virtual control rates (17) and (39), the actual control rates (24) and (46). It is guaranteed that 1) all signals in the closed-loop system are bounded. 2) realizing the trajectory tracking control of the six-rotor UAV system.

Proof:

Select the Lyapunov function:

$$\begin{aligned} V &= V_{p1} + V_{p2} + V_{a1} + V_{a2} \\ &= \frac{1}{2} Z_1^T Z_1 + \frac{1}{2} Z_2^T Z_2 + \frac{1}{2} Z_3^T Z_3 + \frac{1}{2} Z_4^T Z_4. \end{aligned} \tag{50}$$

With (30) and (49) , ones obtain:

$$
\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{4}\left(K_i \circ Z_i\right)^T Z_i + \sum_{i=1}^{2}\frac{\left\|P_i\right\|^2}{2} \\
&+\sum_{i=1}^{2}\left(\dot{o}_i^T\left(\dot{D}_i \circ N_i + I_i\right)\right) \\
&\leq -aV + b + \sum_{i=1}^{2}\left(\dot{o}_i^T\left(\dot{D}_i \circ N_i + I_i\right)\right),
\end{aligned} \tag{51}
$$

where $a = \min 2\left\{k_{i,j}\right\}$, let $i = 1,2,3,4$, respectively. let $j = 1,2,3$, respectively.

From $\left[0,t\right]$ to integrate on (51) yields:

$$
\begin{aligned}
V\left(t\right) &\leq e^{-at}V\left(0\right) \\
&+\int_0^t e^{-a(t-s)}\left(\sum_{i=1}^{2}\left(\dot{o}_i^T\left(\dot{D}_i \circ N_i + I_i\right)\right)+b\right)d_s \\
&\leq e^{-at}V\left(0\right)+\frac{b}{a} \\
&+\int_0^t e^{-a(t-s)}\left(\sum_{i=1}^{2}\left(\dot{o}_i^T\left(\dot{D}_i \circ N_i + I_i\right)\right)\right)d_s
\end{aligned} \tag{52}
$$

According to Lemma 2, it can be inferred that o_i and $V\left(t\right)$ are bounded, by $V = V_{p1} + V_{p2} + V_{a1} + V_{a2}$, then all the signals in the system are bounded. We can achieve control objectives 1), 2).

V. SIMULATION

In this section, MATLAB is used to numerically simulate the tracking controller of the six-rotor UAV designed in this paper, and the altitude subsystem and attitude subsystem are simulated to verify the effectiveness of the control method. The achieved control objective is that the six-rotor UAV system inputs with dead-zone and saturation can track the desired trajectory. The desired altitude of the six-rotor UAV is set as the tracking curve $z_r = \sin(t)$, the initial state of the system $z_1(0) = 0$, $z_2(0) = 1.8$, the Nussbaum-type function is selected as $N\left(o_z\right) = o_z^{\,2}\cos(o_z)$, $o_z(0)=1.9$, the design parameters are $k_{1,1} = 0.8$, $k_{2,1} = 12$. The desired attitude angle is selected as $\eta_r = \phi_r = \theta_r = \psi_r = 0$, the initial state of the six-rotor UAV is given as $h_1\left(0\right) = \left[0.4, 0.2, 0.3\right]^T$, $h_2\left(0\right) = \left[0.7, 0.6, 0.9\right]^T$, the Nussbaum-type function is selected as $N\left(o_2\right) = o_2^{\,2}\cos(o_2)$, $o_2(0) = \left[1.3, 1.2, 1.1\right]^T$, the design parameters are chosen as $K_3 = \left[0.9, 1.1, 0.9\right]^T$, $K_4 = \left[1.6, 0.9, 0.6\right]^T$. The design parameters of the four inputs of the six-rotor UAV with dead-zone and saturation nonlinearity are shown in TABLE I. The function is expressed as $t_m\left(u\right) = \frac{w_N}{\left(b_{m1} - b_{m0}\right)}\left(u - b_{m0}\right)$, $t_n\left(u\right) = \frac{w_P}{\left(b_{n1} - b_{n0}\right)}\left(u - b_{n0}\right)$, The parameters of the six-rotor UAV simulation model are presented in TABLE II.

Using the above system parameters, the simulation is carried out on MATLAB, and the trajectory tracking control of the input with dead-zone saturation can be realized. Fig 2-7 stand for the simulation results.

Fig 2 shows the trajectory tracking of the altitude subsystem of the six-rotor UAV. From Fig 2, it can be seen that the UAV follows the desired trajectory in about 2.5 s, which shows better tracking properties. Fig 3 shows the control input with dead-zone and saturation for the altitude subsystem. It can be seen from the image that the input is bounded. From Fig 2 and Fig 3, we can see that in the high subsystem, all signals are bounded, and the output signal can track the desired reference signal. The designed tracking controller of the altitude subsystem is effective.

TABLE I
INPUT DEAD-ZONE SATURATION PARAMETERS TABLE

	w_N	b_{m1}	b_{m0}	b_{n1}	b_{n0}	w_p
u_1	-22	-22	-0.5	-0.5	22	22
u_2	-8	-8	-0.42	-0.42	8	8
u_3	-8	-8	-0.42	-0.42	8	8
u_4	-4	-4	-0.1	-0.1	4	4

TABLE II
THE SIMULATION MODEL PARAMETERS TABLE

Parameter	m	J_x	J_y	J_z	J_r	a	g
Value	2	0.8	0.8	1.3	0.01	0.2	9.81
Units	Kg	Kg/m^2	Kg/m^2	Kg/m^2	Kg/m^2	m	m/s^2

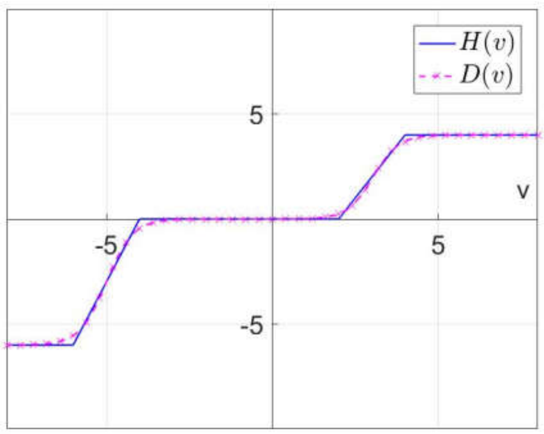

Fig 1. The curves for *H(u)* and *D(u)*

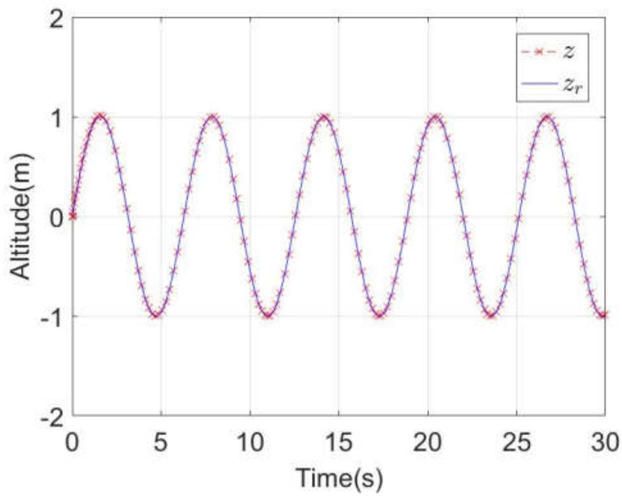

Fig 2. Trajectory tracking curve for altitude subsystem

The trajectory tracking of the attitude subsystem of the six-rotor UAV is shown in Fig 4, it can be seen that under the given initial value, all three attitude angles of the six-rotor UAV can finally follow the desired trajectory in about 8 s. The control input and dead-zone saturation input of the attitude subsystem are shown in Fig 5-7. Under the condition that the input has dead-zone and saturation, it can be seen that all signals are bounded.

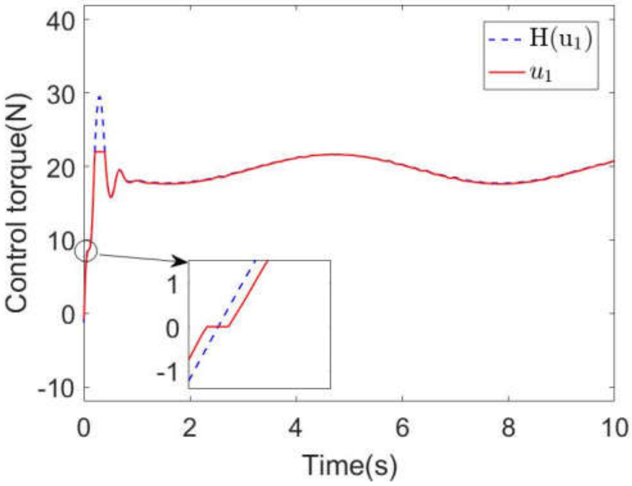

Fig 3. Altitude subsystem control input and dead-zone saturation input

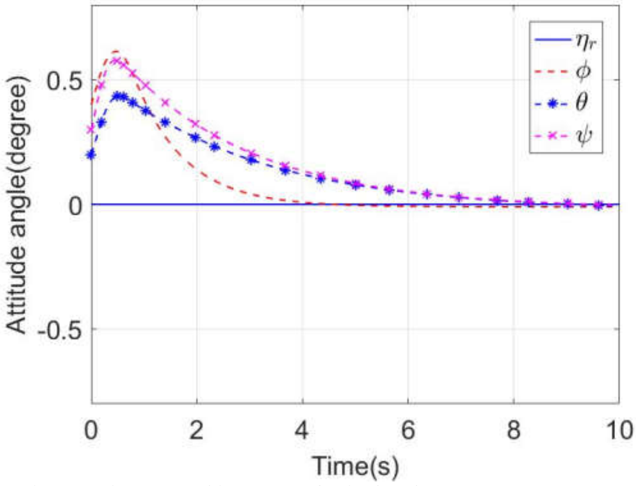

Fig 4. Trajectory tracking curve of attitude subsystem

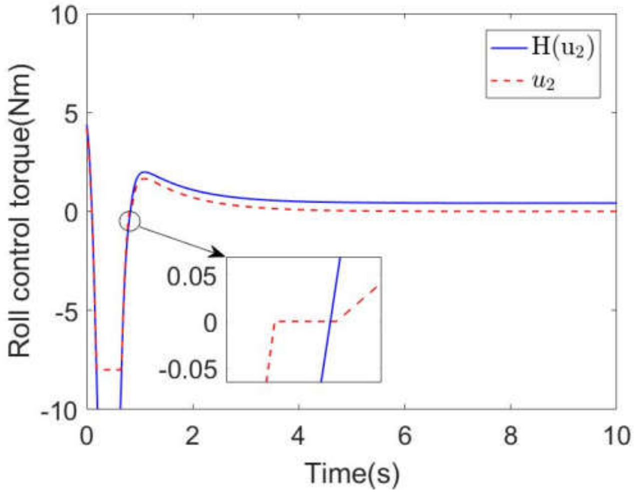

Fig 5. Roll subsystem control input and dead-zone saturation input

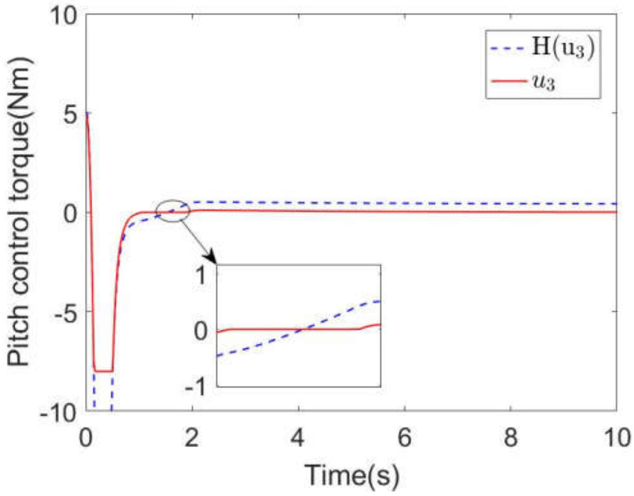

Fig 6. Pitch subsystem control input and dead-zone saturation input

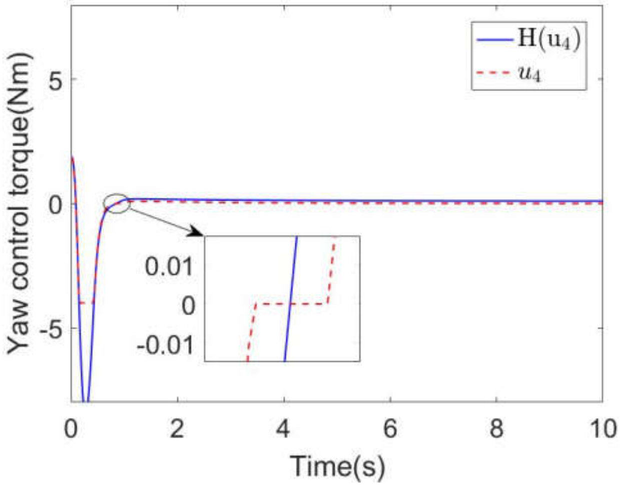

Fig 7. Yaw subsystem control input and dead-zone saturation input

VI. Conclusion

In this paper, the tracking control problem of six-rotor UAV with dead-zone saturation input is considered, this paper adopts a double closed-loop control strategy to design a trajectory tracking controller. By using approximation and mean-value theorem, the dead-zone saturation of nonlinear characteristics is transformed into a smooth function that is easy to handle, which solves the influence of the input dead-zone saturation on the system. Combining the Backstepping method and the Nussbaum gain method to realize the control of the QUAV. The simulation results verify that the proposed control scheme can make the six-rotor UAV system tracks the desired trajectory signal well, which proves the feasibility and effectiveness of the controller. On this basis, the following work designs a practical finite time tracking controller to realize the finite time tracking control of the six-rotor UAV.

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