On the Creeping Flow in Prismatic Pipes

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***Abstract*— In the present paper we study a uniform unsteady incompressible creeping fluid flow in the finite pipes with the polygonal cross-section. Our purpose is to define the velocity of the flow for the certain pressure. The velocity components satisfy the Stokes linear system of equations (STS) with the appropriate initial-boundary conditions. The STS represents a linearized Navier-Stokes equations (NSE) for the small Reynolds number. In our work STS is studied for the specific pressure. We used the conformal mapping method and the Poisson formula and reduced the Stokes system to the system of Fredholm integral equations. By means of the stepwise approximation method the unique solution of this system is obtained. Several cases of the fluid flow in the pipes having regular polygons as a cross-section are considered.**

***Index Terms*—Conformal mapping, Creeping flow, Prismatic pipe, Stepwise approximation**

I. Introduction

Our surroundings are full of various pipes. They are important part of our life. Pipes are everywhere: artificial pipes (water pipes, oil pipes etc.) as well, as biological pipes (human capillary network for example). They are composed of many different materials (steel, plastic, etc.). Within the pipes, the pressure can be controlled: by pumps in artificial pipes, by the heart in the human body. Regulating pressure is necessary to prevent turbulence [1], [2], [4], [6], [8], [9], [17], [24], [26], [27], [33], [35], [39], [40], [42], [47]. As a rule, oil and gas pipes are cylindrical and has the circular cross-section [1], [2], [4], [9], [13], [26], [27], [39], [40]. Pipes with the polygonal cross-section are widely used in technological processes and medicine [1], [2], [5], [7], [9], [12], [13], [17], [24], [26], [27], [34]- [36], [46]-[48]. Besides, nanotubes have polygonal cross-section. For example, a single-walled carbon nanotubes (CNT) can have as the cross-section regular polygons with six, 10 or 20 angles (the vortices of the polygons are the carbon atoms) [5],[7], [12], [26], [27], [34], [36], [46], [48]. One can find the images of CNT at the web-page:
www.bing.com/videos/search?q=Carbon+nanotubes.

The thinnest (freestanding) single-walled CNT is about (3,3), (4,3), or (5,1) nm in diameter [5], [7], [34], [36], [46]. For the fluid flow in nanotubes there is no friction of the classical kind, but there exists the “quantum friction” [5], [7], [34], [36], [46], [48]. In our paper we suppose, that the creeping flow in the nanotubes is possible.

The flows in pipes attract the attention of many scientists. STS was first solved by Stokes (1845), for the circular pipe flow [41]. Boussinesq (1868), obtained the solutions of NSE for the pipes of rectangular, triangular and elliptic cross-sections with a constant pressure gradient [4]. Proudman (1914), derived the solutions for the flow in a pipe with the right-angled isosceles triangular cross-section [40]. Berker (1963), gave the solutions for the pipe flows with the cross-section of lemniscates, cardioids and limacon [2].

S.Tsangaris, D. Kondaxakis and, N. Vlachakis (2007) derived the exact solutions for the flow in porous circular pipes [43]. The solutions of the axi-symmetric STS for the fluid flow over the ellipsoidal bodies in the infinite channel are obtained in [17]. The non-smooth solutions of the NSE for the flow in the rectangular infinite prism are given in [22], [23].

Hence, it is important to define the velocity of the fluid flow in prismatic pipes not only experimentally, but also analytically. Those analytical solutions enable to predict the velocity of fluids for the different pressure without experiment.

II. Problem formulation

Let us consider the prismatic area D_z in Cartesian co-ordinates (x, y, z) $D_z = \{D_0 \times [0, l\_0]\}; 0 \leq z \leq l\_0;$ $l\_0 > 0$; , where l_0 is the certain constant. By S we denote the boundary of D_z and by D_0 the transversal cross-section of D_z, D_0 is the simply connected region of the plane xOy bounded by S_0 (S_0 is the polygon with vertices a_i, and the internal angles $\pi\alpha_i$, $i = 1, \cdots, n$). In D_z we study incompressible creeping fluid flow with the velocity $\vec{V}(V_x, V_y, V_z)$.

For the small Reynolds number the velocity components satisfy the Stokes system of equations [1], [2], [4], [6], [8], [9], [20], [21], [24], [25]-[27], [29]- [31], [33], [35], [37], [39], [42], [44], [47]

$$\frac{\partial V_x}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} = F_x + \nu \Delta V_x, \quad (1)$$

$$\frac{\partial V_y}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial y} = F_y + \nu \Delta V_y, \quad (2)$$

$$\frac{\partial V_z}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial z} = F_z + \nu \Delta V_z, \quad (3)$$

where t is a time, $\vec{F}(F_x, F_y, F_z)$ is the body force per unit mass, P is the pressure, $\rho$is the density, ν is the viscosity of the fluid, Δ is the Laplacian operator.

We consider the system (1), (2), (3) with the equation of continuity

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0, \quad (4)$$

and the initial-boundary conditions

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$$V_x|s = V_y|s = V_z|s = 0 \quad, \tag{5}$$

$$V_x(x,y,z,0) = V_x^0(x,y,z),$$
$$V_y(x,y,z,0) = V_y^0(x,y,z), \tag{6}$$
$$V_z(x,y,z,0) = V_z^0(x,y,z),$$

where $V_x^0(x,y,z), V_y^0(x,y,z), V_z^0(x,y,z)$, are some double-differentiable functions.

It is well known, that the pressure P satisfies the Helmholtz equation [1], [2], [4], [6], [8], [9], [20], [21], [24]-[27], [29]-[31], [33], [35], [37], [39], [42], [44], [47]

$$\Delta P = \rho div\vec{F}. \tag{7}$$

We admit, that the pressure is regulated and can be represented in the form

$$P(x,y,z,t) = a\,exp(-\alpha t)(l-z)P_0(x,y), \tag{8}$$

P_0 is the double differentiable function in D_0, a and $\alpha > 0$ are some given constants.

Besides, we suppose

$$F_x = exp(-\alpha t)(l-z)F_x^0(x,y),$$
$$F_y = exp(-\alpha t)(l-z)F_y^0(x,y), \tag{9}$$
$$F_z = exp(-\alpha t)F_z^0(x,y),$$

where $F_x^0(x,y), F_y^0(x,y), F_z^0(x,y)$ are given continuous functions in D_0, l > l_0.

We seek the solutions of the system (1), (2), (3) in the form

$$V_x = exp(-\alpha t)(l-z)V_x^0(x,y),$$
$$V_y = exp(-\alpha t)(l-z)V_y^0(x,y), \tag{10}$$
$$V_z = exp(-\alpha t)V_z^0(x,y),$$

where the functions $V_x^0(x,y), V_y^0(x,y), V_z^0(x,y)$ are unknown.

According to (8), (9), (10) the system (1), (2), (3), (4) becomes

$$\Delta V_x^0 + \frac{\alpha}{\nu}V_x^0 = \frac{1}{\rho\nu}\frac{\partial P_0}{\partial x} - \frac{1}{\nu}F_x^0, \tag{11}$$
$$\Delta V_y^0 + \frac{\alpha}{\nu}V_y^0 = \frac{1}{\rho\nu}\frac{\partial P_0}{\partial y} - \frac{1}{\nu}F_y^0, \tag{12}$$
$$\Delta V_z^0 + \frac{\alpha}{\nu}V_z^0 = -\frac{a}{\rho\nu}P_0(x,y) - \frac{1}{\nu}F_z^0, \tag{13}$$
$$\frac{\partial V_x^0}{\partial x} + \frac{\partial V_y^0}{\partial y} = 0, \tag{14}$$

By (8), (9), (10) the initial-boundary conditions (5), (6) become

$$V_x^0|S_0 = V_y^0|S_0 = V_z^0|S_0 = 0\,, \tag{15}$$
$$V_x(x,y,z,0) = (l-z)\,V_x^0(x,y),$$
$$V_y(x,y,z,0) = (l-z)\,V_y^0(x,y),$$
$$V_z(x,y,z,0) = V_z^0(x,y).$$

The equation (7) becomes

$$\frac{\partial^2 P_0}{\partial x^2} + \frac{\partial^2 P_0}{\partial y^2} = \rho\,(\frac{\partial F_y^0}{\partial x} + \frac{\partial F_x^0}{\partial y}).$$

We have to solve the following problem

PROBLEM A. In the area D_0 find the double-differentiable functions V_x^0, V_y^0, V_z^0 satisfying the system (11), (12), (13), (14) and the boundary conditions (15).

III. SOLUTION OF PROBLEM A.

We use the conformal mapping method and map the area D_0, at the rectangle D_1, of $w = \xi + i\eta$, plane, $D_1 = \{-a_0/2 \le \xi \le a_0/2; 0 \le \eta \le b_0\}$, $a_0; b_0$ are certain positive constants.

The conformal mapping of D_1, onto the area D_0, is of the form

$$f_1(w) = C_1\int_0^{z_0}\prod_{i=1}^{n}(t-b_i)^{\alpha_i-1}\,dt + C_2, \tag{16}$$
$$z_0 = snw,$$

$C_1,\ C_2,\ b_i; b_1 < b_2 < \ldots < b_n;\quad i = 1,\cdots,n;$ are certain constants, snw is the Jakobi sinus with the periods $2a_0; 2ib_0$ [14], [28], [32].

The mapping (16) is obtained by the combination of two mappings: first, we map the polygon D_0 onto the upper half plane by means of the Schvarz-Christoffel formula and then we map the upper half plane onto the rectangle D_1. $C_1,\ C_2,\ b_i,\quad i = 1,\cdots,n;$ are called the parameters of the conformal mapping, three of them can be chosen arbitrary and other parameters we can found by the formulas [10], [11], [14]- [16], [28], [32], [38], [45]

$$|a_i a_{i+1}| = \int_{sn(b_i)}^{sn(b_{i+1})}|f_1'(t)|\,dt,\ i = 1,\cdots,n; \tag{17}$$

For the calculation of C_1 we use the residue theory [10], [11], [14]-[16], [28], [32], [38], [45].

By means of the mapping $f_1(w)$ we can reduce the system (11), (12), (13), (14) to the following system in w= $\xi + i\eta$ plane

$$\Delta V_x^0 + \frac{\alpha}{\nu}|f_1'(w)|^2 V_x^0 = |f'_1(w)|^2\frac{1}{\rho\nu}\frac{\partial P_0}{\partial x}\frac{\partial x}{\partial \xi} - |f'_1(w)|^2\frac{1}{\nu}F_x^0, \tag{18}$$

$$\Delta V_y^0 + \frac{\alpha}{\nu}|f_1'(w)|^2 V_y^0 = |f'_1(w)|^2\frac{1}{\rho\nu}\frac{\partial P_0}{\partial y}\frac{\partial y}{\partial \eta} - |f_1'(w)|^2\frac{1}{\nu}F_y^0, \tag{19}$$

$$\Delta V_z^0 + \frac{\alpha}{\nu}|f'_1(w)|^2 V_z^0 = -|f'_1(w)|^2\frac{a}{\rho\nu}P_0(\xi,\eta) - |f_1'(w)|^2\frac{1}{\nu}F_z^0, \tag{20}$$

with the boundary conditions

$$V_x^0|S_1 = V_y^0|S_1 = V_z^0|S_1 = 0\,, \tag{21}$$

S_1 is the boundary of D_1.

By the equation of continuity (14) in our previous work we have proved that $\frac{\alpha}{2\pi\nu}$ can not be the eigenvalue of equations (18), (19), (20), and hence they have the unique solutions [20], [21], [24], [25].

We have now reduced the system (18), (19), (20), to the system of the Fredholm integral equations. Taking into the account the Poisson formula and the boundary conditions (21) the system (18), (19), (20), becomes [3], [24]

$$V_x^0 - \frac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f'_1(w)\right|^2 V_x^0\, dx_1 dy_1$$
$$= -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 \Psi_1^*\, dx_1 dy_1, \qquad (22)$$

$$V_y^0 - \frac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 V_y^0\, dx_1 dy_1$$
$$= -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 \Psi_2^*\, dx_1 dy_1, \qquad (23)$$

$$V_z^0 - \frac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 V_z^0\, dx_1 dy_1$$
$$= -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 \Psi_3^*\, dx_1 dy_1, \qquad (24)$$

where

$$\Psi_1^* = \frac{1}{\rho\nu}\frac{\partial P_0}{\partial\xi} - \frac{1}{\nu}F_x^0,$$
$$\Psi_2^* = \frac{1}{\rho\nu}\frac{\partial P_0}{\partial\eta} - \frac{1}{\nu}F_y^0,$$
$$\Psi_3^* = -\frac{a}{\rho\nu}P_0(\xi,\eta) - \frac{1}{\nu}F_z^0,$$
$$f_1'(w) = C_1 cnwdnw$$
$$X(snw-b_1)^{\alpha_1-1}(snw-b_2)^{\alpha_2-1}\cdots(snw-b_n)^{\alpha_n-1},$$

$G_0$is the Green function [3], [24], [28], [32]

$$G_0(w;w_0) = -\ln\left|\frac{snw-snw_0}{\bar{s}nw-snw_0}\right|, \qquad (25)$$

snw, cnw and dnw are the Jakobi functions [14], [28], [32].

By means of Banach's theorem we obtain [3], [24], [32]: If $\frac{\alpha}{2\pi\nu} < \frac{1}{M}$, where

$$\int_{D_1} |G_0(\xi,\eta,x_1,y_1)|\,\left|f_1'(w)\right|^2 dx_1 dy_1 \le M;\ (\xi,\eta)\in D_1,$$

then there exists the unique solutions of the system (22), (23), (24) and they are given by the formulas

$$V_x^0 = \lim_{n\to\infty} V_{xn};\ V_y^0 = \lim_{n\to\infty} V_{yn};\ V_z^0 = \lim_{n\to\infty} V_{zn}, \qquad (26)$$

where

$$V_{x0} = -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\left|f_1'(w)\right|^2 \Psi_1^* dx_1 dy_1, \qquad (27)$$

$$V_{xn} = V_{x0} + \tfrac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\left|f'_1(w)\right|^2 V_{x(n-1)} dx_1 dy_1,$$

$$V_{y0} = -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 \Psi_2^* dx_1 dy_1, \qquad (28)$$

$$V_{yn} = V_{y0} + \frac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\left|f_1'(w)\right|^2 V_{y(n-1)} dx_1 dy_1,$$

$$V_{z0} = -\tfrac{1}{2\pi}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\,\left|f_1'(w)\right|^2 \Psi_3^* dx_1 dy_1, \qquad (29)$$

$$V_{zn} = V_{z0} + \tfrac{\alpha}{2\pi\nu}\int_{D_1} G_0\,(\xi,\eta,x_1,y_1)\left|f_1'(w)\right|^2 V_{z(n-1)} dx_1 dy_1.$$

Having found the velocity components, we define the profile of the velocity by the formula

$$\left|\vec{V}\right| = exp(-\alpha t)\sqrt{(l-z)^2(V_x^0)^2 + (l-z)^2\left(V_y^0\right)^2 + (V_z^0)^2}.$$

REMARK 1. The formula (16) contains $n+3$ parameters C_1, C_2, b_i, $i=1,\cdots,n$; three of them can be chosen arbitrary and other parameters can be found by the formula (17). Generally, we can find those parameters numerically, as they are given in the implicit form [10], [11], [15], [18], [19], [24], [28], [45], but in case of regular polygons we can diminish the number of parameters by means of the Riemann–Schwarz symmetry principle [3], [10], [28], [32]. For example, all parameters can be defined for the triangle, rectangle, pentagon, hexagon and 8-gon (Octagon). For 7-gon (Heptagon), 10-gon (Decagon) only one parameter should be defined numerically, and for 9-gon (Nonagon)-2 parameters. Examples are given below.

1. For the regular pentagon with the vortices $a_1,\ a_2,\ a_3,\ a_4, a_5$, (Fig.1)

$$f_1(w) = C_1\int_0^{z_0}\left(t^2-b_1{}^2\right)^{-2/5}\left(t^2-b_2{}^2\right)^{-2/5} dt, \qquad (30)$$

$$a = C_1\cos\frac{3\pi}{10}\int_0^{\infty}\left(t^2-b_1{}^2\right)^{-2/5}\left(t^2-b_2{}^2\right)^{-2/5} dt.$$

2. For the regular hexagon with the vortices $a_1,\ a_2,\ a_3,\ a_4, a_5,\ a_6$ (Fig.2) [18], [19]

$$f_1(w) = C_1\int_0^{z_0}\left(t^2-b_1{}^2\right)^{-\frac{2}{3}}\left(t^2-b_2{}^2\right)^{-\frac{1}{3}} dt, \qquad (31)$$

$$a\sqrt{3} = 2C_1\int_0^{\infty}\left(t^2-b_1{}^2\right)^{-2/3}\left(t^2-b_2{}^2\right)^{-1/3} dt.$$

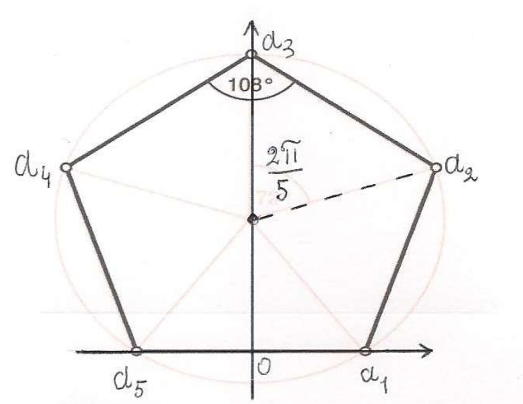

Fig.1. The regular Pentagon

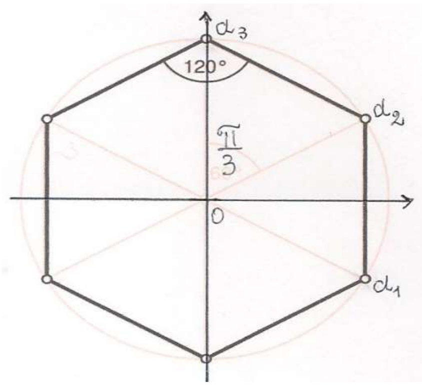

Fig.2. The regular Hexagon

3. For the regular Octagon with the vortices $a_i; i = 1,\ldots,8$; (Fig.4)

$$f_1(w) = C_1 \int_0^{z_0} \left(t^2 - {b_1}^2\right)^{-\frac{5}{8}} \left(t^2 - {b_2}^2\right)^{-\frac{1}{4}} dt \quad (32)$$

$$a = 2C_1 \cos\frac{3\pi}{8} \int_0^{\infty} \left(t^2 - {b_1}^2\right)^{-5/8} \left(t^2 - {b_2}^2\right)^{-1/4} dt.$$

4. For the regular Heptagon with the vortices $a_i; i = 1,\ldots,7$; (Fig.3)

$$f_1(w) = C_1 \int_0^{z_0} \left(t^2 - {b_1}^2\right)^{-\frac{2}{7}} \left(t^2 - {b_2}^2\right)^{-\frac{2}{7}} \left(t^2 - {b_3}^2\right)^{-\frac{2}{7}} dt, \quad (33)$$

$$a = C_1 \cos\frac{5\pi}{14} \int_0^{\infty} \left(t^2 - {b_1}^2\right)^{-2/7} \left(t^2 - {b_2}^2\right)^{-2/7} \left(t^2 - {b_3}^2\right)^{-2/7} dt.$$

5. For the regular Decagon with the vortices $a_i; i = 1,\ldots,10$; (Fig.5)

$$f_1(w) = C_1 \int_0^{z_0} \left(t^2 - {b_1}^2\right)^{-\frac{3}{5}} \left(t^2 - {b_2}^2\right)^{-\frac{1}{5}} \left(t^2 - {b_3}^2\right)^{-\frac{1}{5}} dt, \quad (34)$$

$$a = 2C_1 ctg\frac{2\pi}{5} \int_0^{\infty} \left(t^2 - {b_1}^2\right)^{-3/5} \left(t^2 - {b_2}^2\right)^{-1/5} \left(t^2 - {b_3}^2\right)^{-1/5} dt.$$

6. For the regular Nonagon with the vortices $a_i; i = 1,\ldots,9$;

$$f_1(w) = C_1 \times \int_0^{z_0} \left(t^2 - {b_1}^2\right)^{-2/9} \left(t^2 - {b_2}^2\right)^{-2/9} \left(t^2 - {b_3}^2\right)^{-2/9} \left(t^2 - {b_4}^2\right)^{-2/9} dt,$$

$$f_1(w) = C_1 \cos\frac{7\pi}{18} \times \int_0^{\infty} \left(t^2 - {b_1}^2\right)^{-2/9} \left(t^2 - {b_2}^2\right)^{-2/9} \left(t^2 - {b_3}^2\right)^{-2/9} \left(t^2 - {b_4}^2\right)^{-2/9} dt. \quad (35)$$

In the formulas (30), (31), (32), (33), (34), (35) $|a_1 a_2| = a; z_0 = snw \ and \ b_1, b_2, b_3, b_4,$ are the real numbers $0 < b_1 < b_2 < b_3 < b_4$; b_1, b_2, can be chosen arbitrary and b_3, b_4 can be defined from the formula (17)

$$a = \int_{snb_2}^{snb_3} |f_1'(t)|\, dt = \int_{sn\ 3}^{sn\ 4} |f_1'(t)|\, dt.$$

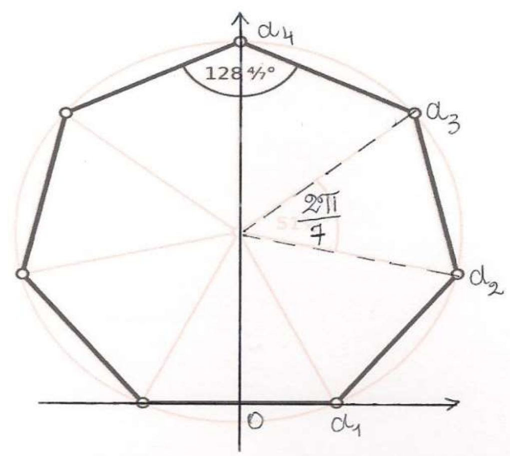

Fig.3. The regular Heptagon

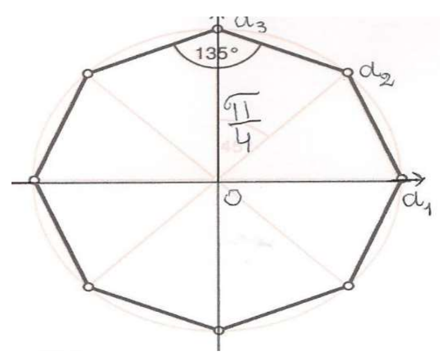

Fig.4. The regular Octagon

REMARK 2. Here we have studied the boundary value problem for the creeping flows in pipes. The free boundary problem for the creeping flows were studied in the previous works of the author [20], [21], [25].

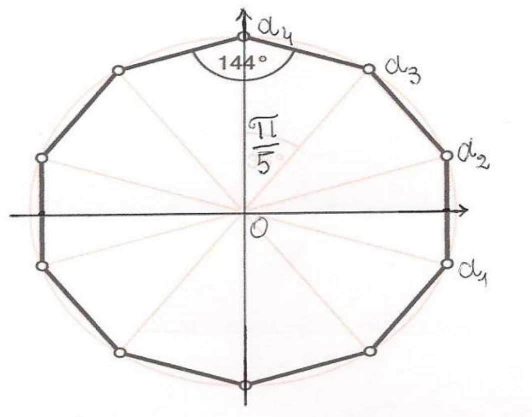

Fig.5. The regular Decagon

IV. Conclusion

If the pressure is given by the formula (8), where α is rather small, then the creeping flow in the prismatic pipes is possible and the velocity components of the flow are given in the explicit form by the formulas (26), (27), (28), (29). Those formulas represent analytic solutions.

In the forthcoming works we will give the solutions of Problem A. numerically.

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