High-Gain-Observer-Based Output Feedback Adaptive Controller Design with Command Filter and Event-Triggered Strategy

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Abstract--**This article considers the tracking control problem in nonlinear systems, and thus designs a novel high-gain-observer based adaptive tracking controller. For optimal control problems in the case of parameter perturbations in nonlinear systems, the adaptive control method which designs readily adjustable control laws and adaptive parameters is adopted in backstepping. To address the problem of containing immeasurable states in physical systems, a high gain observer is designed to obtain unknown states. The high gain parameters can better regulate the observed performance of the system. Meanwhile, by introducing the fuzzy logic system (FLS), the unknown construction in the system is estimated. In addition, an event triggering strategy with a fixed threshold is included to conserve network resources. Furthermore, the command filter is used in the controller design to solve "over-complication" and "excess parameterization" problems in the backstepping. Through the Lyapunov function and the recursive deduction of the command filter, the signals in the close loop system (CLS) are all ultimately bounded while the tracking error can converge to a small area. Ultimately, the effectiveness of the design method is demonstrated by simulations.**

Index Terms—**adaptive control; backstepping; high gain observer; event triggering strategy; command filter**

I. Introduction

It is known that nonlinear systems play an important role in describing real physical systems. Nowadays, the tracking control of nonlinear systems [1-3] receives increasing attention. In this context, many scholars have designed different controllers to solve this problem. Among the methods, the design of controllers based on backstepping method [4] is gradually applied more widely. It derives the control laws of the system in a step-by-step approach. For the problem of systems with parameters to be designed, an adaptive control method based on backstepping [5-9] is proposed. This method can design adaptive parameters upon derivation, while the control law is always updated to meet the requirements of system control. Due to the immeasurable states and uncertain structures in the system, the high gain observer and the fuzzy logic system (FLS) [10-12] are adopted in the controller. With this method, the complexity caused by the unknown systems is solved. Moreover, the repeated partial differential of the control laws will occur when we deduce the control laws by using backstepping. If the order of the system is higher, it will cause great trouble to derive the theory. For this reason, the command filter [13-16] is developed to simplify the process of deriving control laws. In most cases, the controller does not need to provide the control input at all times. Frequent updates of input may lead to wasted communication resources. Thus, the event triggering strategy [17-20] containing a fixed threshold are brought into the design process. It can save communication resources to improve system performance. The fixed-time output feedback controller designed by Hou et al. [15] utilizes command filter to reduce the complexity of the derivation, but does not consider the problem of wasted communication resources due to continuous input. Pang et al. [4] designed a suspension system controller which can achieve better tracking performance. However, it is not suitable when the states are immeasurable. It can be known that no articles in the current literature works aiming for the design of nonlinear system controllers with unknowns and obtaining the simplification of the design complexity and the optimization of the system performance simultaneously. Therefore, a novel high-gain-observer based output feedback controller which can save system resources is designed. Finally, all bounded signals can be uniformly contained in the CLS and the controller can achieve good tracking performance.

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II. Problem description and preparation

To better solve the problem, we consider the output feedback nonlinear system in the following form:

$$\begin{aligned} \dot{x}_k &= f_k(\overline{x}_k) + x_{k+1} \quad (1 \le k \le n-1,), \\ \dot{x}_n &= f_n(\overline{x}_n) + u(t), \\ \mathrm{y} &= \mathrm{x}_1, \end{aligned} \tag{1}$$

In the system, $\overline{x}_k = [x_1,...,x_k]^T (k = 1,2,...,n)$ is the system state vector. $y = x_1$ and $u(t)$ represent the output and input of the nonlinear system respectively, where $u(t)$ satisfies $u(t) = 0$ when $t \le 0$. Besides, $f_k(\cdot)$ (k=1,2,...,n) denotes the unknown nonlinear functions that exists universally in real systems and it has $f_k(0) = 0$.

For the system (1), we propose the following control objectives:

- Under the condition that the system states $x_2, x_3, ... x_n$ are immeasurable, design an adaptive controller with backstepping method to ensure the output y can track the reference signal y_r. Meanwhile, the tracking error can converge to a small area which can be described ultimately.
- The signals in the CLS are all bounded.
- When system communication resources are limited, the controller can use an event-triggered strategy to adjust input changes, ultimately reducing wasted resources and improving system performance.
- In the process of designing the control laws, the complexity of the design can be simplified by using the command filter design method.

For a better derivation, we have the following lemma and assumption:

Lemma 1 [17]**:** For random $\bar{p} \in R$ and $\Gamma > 0$, there is the following inequality:

$$|\bar{p}| - \bar{p}\tanh(\frac{\bar{p}}{\Gamma}) \leq 0.2785\Gamma. \tag{2}$$

Assumption 1: Suppose that the setting signal y_r and its derivative $y_r^{(k)}$ are continuous and bounded.

III. Design of High Gain Observer

Fuzzy logic system is a kind of system with fuzzy logic and fuzzy concept. The fuzzy logic system has the following four components: fuzzy inference engine, knowledge base, fuzzifier , and defuzzier. The knowledge base has the following reference laws:

R^l : X_1 represents H_1^k, X_2 represents H_2^k, and,..., X_n represents H_n^k, then $X = [X_1, ..., X_n]^T$ is the input of the FLS. Y stands for P^k, which is the output of the FLS. B stands for the number of reference laws. Furthermore, P^k and $H_1^k, H_2^k, ..., H_n{}^k$ both refer to fuzzy sets of the system. To better give the mathematical definition of the fuzzy logic system, we express FLS as:

$$Y(X) = \frac{\sum_{k=1}^{\mathrm{B}} \bar{Y}_\Gamma \prod_{i=1}^{n} \wp H_i^k(X_i)}{\sum_{k=1}^{\mathrm{B}} \left[\prod_{i=1}^{n} \wp H_i^k(X_i)\right]}, \tag{3}$$

where $\bar{Y}_\Gamma = \max_{y \in R} \mu P^k(y)$, and fuzzy membership functions are represented by $\mu H_i^k(X_i)$ and $\mu P^k(y)$. Fuzzy membership functions are determined by the fuzzy sets.

Thus, we have the following equations:

$$\varphi_k = \frac{\prod_{i=1}^{n} \wp H_i^k(X_i)}{\sum_{k=1}^{\mathrm{B}} \left[\prod_{i=1}^{n} \wp H_i^k(X_i)\right]}. \tag{4}$$

Then, we can express (3) as:

$$Y(\chi) = \hat{\theta}^T \varphi(X), \tag{5}$$

where $\varphi(X) = \left[\varphi_1(X), \varphi_2(X), ..., \varphi_{\mathrm{B}}(X)\right]^T$, and $\hat{\theta} = \left[\hat{\theta}_1, ..., \hat{\theta}_{\mathrm{B}}\right]^T = \left[\bar{Y}_1, ..., \bar{Y}_{\mathrm{B}}\right]^T$.

Lemma 2 [12]: For any real number ε, there is a continuous function $f(x)$ that satisfies the following equation:

$$\sup_{\chi \in R} \left| f(\chi) - \theta^T \varphi(\chi) \right| \leq \varepsilon. \tag{6}$$

According to the above lemma, the unknown functions can be replaced by the FLS described as below.

$$\hat{f}_i\left(\hat{\bar{x}}_i \mid \hat{\theta}_i\right) = \hat{\theta}_i^T \varphi_i\left(\hat{\bar{x}}_i\right), \tag{7}$$

We can define the optimum parameter θ_i^* in θ_i as follows:

$$\theta_i^* = \arg\min_{\theta_i \in \mathbb{C}_i} \left\{ \sup_{(\hat{\bar{x}}_i, \bar{x}_i) \in \mathbb{Q}_i} \left| \hat{f}_i\left(\hat{\bar{x}}_i \mid \theta_i\right) - f_i\left(\bar{x}_i\right) \right| \right\}, \tag{8}$$

where $\mathbb{C}_i$ is a closed mathematical set of θ_i. Likewise, $\mathbb{Q}_i$ is a closed mathematical set of $\hat{\bar{x}}_i$ and $\bar{x}_i$. Finally, we can express the approximation error as:

$$\varepsilon_i = f_i\left(\bar{x}_i\right) - \hat{f}_i\left(\hat{\bar{x}}_i \mid \theta_i^*\right) \; (|\varepsilon_i| \leq \varepsilon_i^*, \varepsilon_i^* > 0) \tag{9}$$

From (1), we define $x = \left[x_1, x_2, ... x_n\right]^T$. Therefore, the derivative of x is transformed as:

$$\dot{x} = A'x + W'y + \sum_{i=1}^{n} H_i f_i(\bar{x}_i) + Ru \tag{10}$$

where $A' = \begin{bmatrix} -d_1 l & & \\ -d_2 l^2 & I_{n-1} & \\ \vdots & & \\ -d_n l^n & 0 \; \cdots & 0 \end{bmatrix}$, $W' = \left[d_1 l \quad d_2 l^2 \quad \ldots \quad d_n l^n\right]^T$,

$H_i = \left[\underbrace{0, 0, ..., 1}_{i}, ..., 0\right]_{1 \times n}^T$, $R = [0, ..., 1]^T$.

Then, we construct the following high gain observer to estimate immeasurable states.

$$\begin{cases} \dot{\hat{x}}_1 = d_1 l e_1 + \hat{f}_1(\hat{\bar{x}}_1) + \hat{x}_2, \\ \dot{\hat{x}}_2 = d_2 l^2 e_1 + \hat{f}_2(\hat{\bar{x}}_2) + \hat{x}_3, \\ \vdots \\ \dot{\hat{x}}_n = d_n l^n e_1 + \hat{f}_n(\hat{\bar{x}}_n) + u. \end{cases} \tag{11}$$

where $e_1 = x_1 - \hat{x}_1$, l is a constant set to be greater than 1 in the later procedure and $d_i (i = 1, ..., n)$ are given parameters.

From (11), (10) is denoted as:

$$\dot{\hat{x}} = A'\hat{x} + W'y + \sum_{i=1}^{n} H_i \hat{f}_i\left(\hat{\bar{x}}_i \middle| \theta_i\right) + Ru, \tag{12}$$

We define $e = \left[e_1, ..., e_n\right]^T$ to represent the error of the the state observation. It has:

$$e = x - \hat{x}. \tag{13}$$

Then, we have:

$$\dot{e} = \dot{X} - \dot{\hat{X}} = A'e + \varepsilon + \sum_{i=1}^{n} H_i \tilde{\theta}_i^T \varphi_i\left(\hat{x}_i\right), \tag{14}$$

where $\tilde{\theta}_i = \theta_i^* - \theta_i (i = 1, ..., n)$ and $\varepsilon = \left[\varepsilon_{1,} \cdots \varepsilon_n\right]^T$.

Select the following coordinate transformation:

$$\rho_i = \frac{e_i}{l^{i-1}}, i = 1, 2, ..., n, \tag{15}$$

where $\rho = \left[\rho_1, \rho_2, ..., \rho_n\right]^T$ is defined as the scaling error vector.

According to (12), the following result is obtained:

$$\dot{\rho} = lA\rho + \varepsilon' + \sum_{i=1}^{n} \frac{1}{l^{i-1}} H_i \tilde{\theta}_i^T \varphi_i\left(\hat{x}_i\right), \tag{16}$$

where $A=\begin{bmatrix}-d_1 & & \\ -d_2 & & I_{n-1} \\ \vdots & & \\ -d_n & 0 & \cdots & 0\end{bmatrix}$ is a strict Hurwitz matrix, $\varepsilon'=\left[\varepsilon_1,\frac{\varepsilon_2}{l},\cdots\frac{\varepsilon_n}{l^{n-1}}\right]^T$ satisfies $|\varepsilon'|\le\bar{\varepsilon}$ and $\bar{\varepsilon}>0$.

From [12], it is can be known that if A is a strict Hurwitz matrix by designing suitable parameters there exist positive definite matrices $O=O^T$ and $P=P^T$ concluding:

$$AP^T+PA=-2O. \tag{17}$$

The Lyapunov function is chosen for scaling error vector as:

$$V_0=\frac{1}{2}\rho^T P\rho. \tag{18}$$

Thus, $\dot{V}_0$ is obtained as:

$$\begin{aligned}\dot{V}_0&=\rho^T P\left(lA\rho+\varepsilon'+\sum_{i=1}^{n}\frac{1}{l^{i-1}}H_i\tilde{\theta}_i^T\varphi_i\left(\hat{x}_i\right)\right)\\&=-l\rho^T O\rho+\rho^T P\varepsilon'+\rho^T P\sum_{i=1}^{n}\frac{1}{l^{i-1}}H_i\tilde{\theta}_i^T\varphi_i\left(\hat{x}_i\right).\end{aligned} \tag{19}$$

In the above equation, $\varphi_i\left(\hat{x}_i\right)$ is selected as a Gaussian function so that $\varphi_m\left(\hat{x}_m\right)^T\varphi_m\left(\hat{x}_m\right)\le 1$. From Young's inequality, one has:

$$\rho^T P\varepsilon'\le\frac{1}{2}\|P\|^2\|\bar{\varepsilon}\|^2+\frac{1}{2}\|\rho\|^2, \tag{20}$$

$$\rho^T P\sum_{i=1}^{n}B_i\tilde{\theta}_i^T\varphi_i\left(\hat{x}_i\right)\leqslant\frac{n}{2}\|\rho\|^2+\frac{1}{2}\|P\|^2\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i. \tag{21}$$

Thus, from (20) and (21), (19) can be transformed as:

$$\dot{V}_0\leqslant-p_0\|\rho\|^2+\sum_{i=1}^{n}\frac{1}{2}\|P\|^2\tilde{\theta}_i^T\tilde{\theta}_i+\eta, \tag{22}$$

Since $\lambda_{\min}(O)$ is the minimum eigenvalue of O, $p_0=\min\left\{l\lambda_{\min}(O)-\frac{n}{2}-\frac{1}{2}\right\}>0$ and $\eta=\frac{1}{2}\|P\|^2\|\bar{\varepsilon}\|^2$.

IV. FUZZY ADAPTIVE CONTROLLER DESIGN

In this part, to ensure that the system is ultimately stable, an observer-based controller is constructed by using adaptive backstepping approach. The control laws can be designed in each step of the derivation. In order to avoid the burden caused by repeated derivations of the control laws and to conserve system communication resources, we apply command filter and event triggering strategy to the design of the controller. Finally, we perform stability analysis on the designed controller to illustrate the rationality and effectiveness of the controller design method

Based on the method of the command filtered backstepping, we have the following coordinate transformation.

$$\begin{cases}z_1=x_1-\bar{\alpha}_1,\ (\bar{\alpha}_1=y_r)\\ z_i=\hat{x}_i-\bar{\alpha}_i,\\ z_n=\hat{x}_n-\bar{\alpha}_n,\end{cases} \tag{23}$$

where $\bar{\alpha}_i$ represents the output of the command filter associated with α_i. Thus, we define the first-order command filter as:

$$\bar{\alpha}_i+\Im_i\dot{\bar{\alpha}}_i=\alpha_i,\ \left(\bar{\alpha}_i(0)=\alpha_i(0)\right) \tag{24}$$

where $\Im_i>0$ are the parameters that can be designed.

Lemma 3[13]**:** There is a known constant κ_i, and its relationship to the filter output error is $\|\bar{\alpha}_{i+1}-\alpha_{i+1}\|\le\kappa_i$.

Step 1:

According to (11) and (23), one yields

$$\begin{aligned}\dot{z}_1&=\dot{x}_1-\dot{\bar{\alpha}}_1\\&=\hat{x}_2+l\rho_2+\hat{f}_1(\hat{\bar{x}}_1|\theta^*)+\varepsilon_1-\dot{y}_r\\&=z_2+\bar{\alpha}_2-\alpha_2+\alpha_2+l\rho_2+\hat{f}_1(\hat{\bar{x}}_1|\theta^*)+\varepsilon_1-\dot{y}_r\end{aligned} \tag{25}$$

To address the effect the of the the filter's output error ($\bar{\alpha}_2-\alpha_2$), the compensating signal β_1 is introduced.

$$\dot{\beta}_1=-\gamma_1\beta_1+\bar{\alpha}_2-\alpha_2+\beta_2, \tag{26}$$

where γ_1 is a known constant and its initial value is zero.

Therefore, we define the compensating error as:

$$\omega_1=z_1-\beta_1. \tag{27}$$

By plugging (25) and (26) into (27) yields

$$\begin{aligned}\dot{\omega}_1&=z_2+\bar{\alpha}_2+l\rho_2+\hat{f}_1(\hat{\bar{x}}_1)+\varepsilon_1\\&\quad-\dot{y}_r+\gamma_1\beta_1-\bar{\alpha}_2+\alpha_2-\beta_2\\&=\omega_2+\alpha_2+l\rho_2+\hat{f}_1(\hat{\bar{x}}_1)+\varepsilon_1-\dot{y}_r+\gamma_1\beta_1.\end{aligned} \tag{28}$$

The Lyapunov function constructed for step 1 is the following equation:

$$V=\frac{1}{2}w_1^2+\frac{1}{2a_1}\tilde{\theta}_1^T\tilde{\theta}_1, \tag{29}$$

where $\tilde{\theta}_1=\theta_1^*-\hat{\theta}_1$ and a_1 is a positive parameter that needs to be selected.

Based on the above, we can obtain $\dot{V}_1$ as:

$$\begin{aligned}\dot{V}_1&=\omega_1\dot{\omega}_1-\frac{1}{a_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1\\&=\omega_1(\omega_2+\alpha_2+l\rho_2+\hat{\theta}_1^T\varphi_1(\hat{x}_1)+\varepsilon_1-\dot{y}_r+\gamma_1\beta_1)\\&\quad-\frac{1}{a_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1+\omega_1\tilde{\theta}_1^T\varphi_1(\hat{x}_1).\end{aligned} \tag{30}$$

According to Young's inequality, it gives

$$\begin{aligned}\omega_1 l\rho_2&\le\frac{1}{2}\omega_1^2+\frac{1}{2}l^2\|\rho\|^2.\\ \omega_1\varepsilon_1&\le\frac{1}{2}\omega_1^2+\frac{1}{2}\bar{\varepsilon}^2.\end{aligned} \tag{31}$$

We design the virtual control law α_2 as:

$$\alpha_2=-\hat{\theta}_1^T\varphi_1(\hat{x}_1)+\dot{y}_r-\gamma_1 z_1-\omega_1. \tag{32}$$

Design the adaptive law $\hat{\theta}_1$ as follows.

$$\dot{\hat{\theta}}_1=a_1\omega_1\varphi_1(\hat{x}_1)-b_1\hat{\theta}_1, \tag{33}$$

where b_1 is a design parameter which is positive.

Based on above, one produces

$$\begin{aligned}V_1&\le\omega_1\omega_2-\frac{1}{2}\gamma_1\omega_1{}^2+\frac{b_1}{a_1}\tilde{\theta}_1^T\hat{\theta}_1\\&\quad+\frac{1}{2}l^2\|\rho\|^2+\frac{1}{2}\bar{\varepsilon}^2\end{aligned} \tag{34}$$

Step i:

Consistent with Step 1, from (10) and (23), we have the derivative of z_i as

$$\begin{aligned}\dot{z}_i &= \dot{\hat{x}}_{i+1} + d_i l^i \rho_1 + \hat{f}_i(\hat{\bar{x}}_i) - \dot{\bar{\alpha}}_i \\ &= z_{i+1} + \bar{\alpha}_{i+1} - \alpha_{i+1} + \alpha_{i+1} + d_i l^i \rho_1 + \hat{f}_i(\hat{\bar{x}}_i) - \dot{\bar{\alpha}}_i\end{aligned} \tag{35}$$

To address the effect the of the the filter's output error ($\bar{\alpha}_{i+1} - \alpha_{i+1}$), introduce the β_i as

$$\dot{\beta}_i = \beta_{i+1} + \bar{\alpha}_{i+1} - \alpha_{i+1} - \beta_{i-1} - \gamma_i \beta_i, \tag{36}$$

where γ_i is a known constant and its initial value is zero.

The compensating error gives

$$\omega_i = z_i - \beta_i. \tag{37}$$

By substituting (34) and (35) into (37) yield:

$$\begin{aligned}\dot{\omega}_i &= \dot{z}_i - \dot{\beta}_i \\ &= \omega_{i+1} + d_i l^i \rho_1 + \hat{f}_i(\hat{\bar{x}}_i) - \dot{\bar{\alpha}}_i \\ &\quad + \gamma_i \beta_i + \alpha_{i+1} + \beta_{i-1}.\end{aligned} \tag{38}$$

Select the following Lyapunov function for step i.

$$V_i = V_{i-1} + \frac{1}{2}\omega_i^2 + \frac{1}{2a_i}\tilde{\theta}_i^T \tilde{\theta}_i. \tag{39}$$

From (39), $\dot{V}_i$ is obtained as:

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + \omega_i \dot{\omega}_i - \frac{1}{a_i}\tilde{\theta}_i^T \dot{\hat{\theta}}_i \\ &= \dot{V}_{i-1} + \omega_i(\omega_{i+1} + d_i l^i \rho_1 + \hat{\theta}_i^T \varphi_i(\hat{x}_i) - \dot{\bar{\alpha}}_i + \gamma_i \beta_i + \alpha_{i+1} + \beta_{i-1}) \\ &\quad - \frac{1}{a_i}\tilde{\theta}_i^T \dot{\hat{\theta}}_i + \omega_i \tilde{\theta}_i^T \varphi_i(\hat{x}_i) - \omega_i \tilde{\theta}_i^T \varphi_i(\hat{x}_i).\end{aligned} \tag{40}$$

Similar to (21), it gives

$$-\omega_i \tilde{\theta}_i^T \varphi_i(\hat{\bar{x}}_i) \le \frac{1}{2}\omega_i^2 + \frac{1}{2}\tilde{\theta}_i^T \tilde{\theta}_i. \tag{41}$$

For system stability, the α_{i+1} is designed as:

$$\alpha_{i+1} = -\hat{\theta}_i^T \varphi_i(\hat{x}_i) + \dot{\bar{\alpha}}_i - \gamma_i z_i - z_{i-1} - d_i l^i \rho_1 - \frac{1}{2}\omega_i, \tag{42}$$

The $\hat{\theta}_i$ for step i is designed as:

$$\dot{\hat{\theta}}_i = a_i \omega_i \varphi_i(\hat{x}_i) - b_i \hat{\theta}_i, \tag{43}$$

where b_i is a positive parameter that needs to be selected..

Based on the above design, it produces

$$\begin{aligned}\dot{V}_i &\le \omega_i \omega_{i+1} + \sum_{m=1}^{i} -\frac{1}{2}\gamma_m \omega_m{}^2 + \sum_{m=1}^{i}\frac{b_m}{a_m}\tilde{\theta}_m \hat{\theta}_m \\ &\quad + \frac{1}{2}l^2 \|\rho\|^2 + \frac{1}{2}\bar{\varepsilon}^2 + \frac{1}{2}\sum_{m=2}^{i}\tilde{\theta}_m{}^T \tilde{\theta}_m.\end{aligned} \tag{44}$$

Step n:

In the last step, the controller with update event triggering strategy is derived.

Similar to Step i, we yield the derivative of z_n as:

$$\begin{aligned}z_n &= \dot{\hat{x}}_n - \dot{\bar{\alpha}}_n \\ &= \text{u} + \text{d}_n l^n \rho_1 + \hat{f}_n(\hat{\bar{x}}_n) - \dot{\bar{\alpha}}_n.\end{aligned} \tag{45}$$

Then, we introduce the compensating signal as:

$$\dot{\beta}_n = -\gamma_n \beta_n - \beta_{n-1}, \tag{46}$$

where γ_n is a known constant.

We define the following compensated error:

$$\omega_n = z_n - \beta_n \tag{47}$$

By substituting (45) and (46) in (45), it gives

$$\begin{aligned}\dot{\omega}_n &= \dot{z}_n - \dot{\beta}_n \\ &= \text{u} + \text{d}_n l^n \rho_1 + \hat{f}_n(\hat{\bar{x}}_n) - \dot{\bar{\alpha}}_n + \gamma_n \beta_n + \beta_{n-1}.\end{aligned} \tag{48}$$

The α_{n+1} is designed as follows:

$$\alpha_{n+1} = -\hat{\theta}_n{}^T \varphi_n(\hat{x}_n) + \dot{\bar{\alpha}}_n - \gamma_n z_n - z_{n-1} - d_n l^n \rho_1 - \frac{1}{2}\omega_n. \tag{49}$$

In this case, an event triggering strategy which has the fixed threshold is introduced. This strategy is considered when the controller error reaches a set value. Thus, we design the following adaptive controller.

$$\delta(t) = \alpha_{n+1} - \bar{\hbar}\tanh(\frac{\omega_n \bar{\hbar}}{\Psi}). \tag{50}$$

Meanwhile, we design the adaptive law $\hat{\theta}_n$ as:

$$\dot{\hat{\theta}}_n = a_n \omega_n \varphi_n(\hat{x}_n) - b_n \hat{\theta}_n. \tag{51}$$

The event triggering form is constructed as follows

$$u(t) = \delta(t_k), \quad \{\forall t \in [t_k, t_{k+1})\} \tag{52}$$

$$t_{k+1} = \inf\{t \in R \,\|\, \ell(t) \ge \hbar\|\}, \tag{53}$$

where Ψ, $\hbar$, and $\bar{\hbar}$ are all positive parameters and it has $\bar{\hbar} > \hbar$. The controller error is specified as $\ell(t) = \delta(t) - u(t)$. In the above formula, $t_k\ (k \in Z^+)$ represent the updating time. When the error $\ell(t)$ reaches the critical threshold b, the event will be updated to the latest time t_{k+1}. The controller will be adjusted to $u(t_{k+1})$. In the condition of $t \in [t_k, t_{k+1})$, the controller will be kept as $\delta(t_k)$.

Design the Lyapunov function for the last step as:

$$V = V_0 + V_{n-1} + \frac{1}{2}\omega_n^2 + \frac{1}{2a_n}\tilde{\theta}_n^T \theta_n. \tag{54}$$

From above, the derivative of V gives

$$\begin{aligned}\dot{V} &= \dot{V}_0 + \dot{V}_{n-1} + \omega_n \dot{\omega}_n - \frac{1}{\gamma_n}\tilde{\theta}_n^T \dot{\hat{\theta}}_n \\ &= \dot{V}_0 + \dot{V}_{n-1} + \omega_n(\text{u} + \text{d}_n l^n \rho_1 + \hat{\theta}_n{}^T \varphi_n(\hat{x}_n) - \dot{\bar{\alpha}}_n + \gamma_n \beta_n + \beta_{n-1}) \\ &\quad - \frac{1}{a_n}\tilde{\theta}_n{}^T \dot{\hat{\theta}}_n + \omega_n \tilde{\theta}_n{}^T \varphi_n(\hat{x}_n) - \omega_n \tilde{\theta}_n{}^T \varphi_n(\hat{x}_n).\end{aligned} \tag{55}$$

From (52) and (53), we know that when the time is in the range between t_k and t_{k+1}, then there is $|\delta(t) - u(t)| \le \hbar$. Therefore there is a function $\tau(t)$ that satisfies the following rules between trigger times t_k and t_{k+1}.

$$\begin{cases}\tau(t_k) = 0, \\ \tau(t_k + 1) = 1, \\ |\tau(t)| \le 1.\end{cases} \tag{56}$$

Then, $\delta(t)$ is transformed as:

$$\delta(t) = u(t) + \tau(t)\hbar. \tag{57}$$

From Young's inequality, one gives

$$-\omega_n \tilde{\theta}_n^T \varphi_n(\hat{\bar{x}}_n) \le \frac{1}{2}\omega_n^2 + \frac{1}{2}\tilde{\theta}_n^T \tilde{\theta}_n. \tag{58}$$

$$\sum_{i=1}^{n}\tilde{\theta}_i^T \hat{\theta}_i \le -\frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2}\sum_{i=1}^{n}\|\theta_i^*\|^2. \tag{59}$$

According to (49), (50), (51), (55), (57), (58), (59), and

Lemma 1, one has:

$$\dot{V} \le \dot{V}_0 - \sum_{i=1}^{n} \gamma_i \omega_i^2 - \sum_{i=1}^{n} \frac{b_i}{2a_i} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2} l^2 \|\rho\|^2 + \frac{1}{2}\bar{\varepsilon}^2 + \sum_{i=1}^{n} \frac{b_i}{2a_i} \|\theta_i^*\|^2 + \frac{1}{2}\sum_{m=2}^{n} \tilde{\theta}_m^T \tilde{\theta}_m + 0.2785\Psi. \tag{60}$$

Substituting (19) into (59) gives

$$\dot{V} \le -\sum_{i=1}^{n} \gamma_i \omega_i^2 - \sum_{i=1}^{n} (\frac{b_i}{2a_i} - \frac{1}{2}\|P\|^2) \tilde{\theta}_i^T \tilde{\theta}_i - (p_0 - \frac{1}{2} l^2) \|\rho\|^2 + \frac{1}{2}\bar{\varepsilon}^2 + \sum_{i=1}^{n} \frac{b_i}{2a_i} \|\theta_i^*\|^2 + \frac{1}{2}\sum_{m=2}^{n} \tilde{\theta}_m^T \tilde{\theta}_m + 0.2785\Psi + q \tag{61}$$

$$\le -\sum_{i=1}^{n} \gamma_i \omega_i^2 - \sum_{m=2}^{n} \Upsilon_m \tilde{\theta}_m^T \tilde{\theta}_m - \Upsilon_1 \tilde{\theta}_1^T \tilde{\theta}_1 - \Lambda \|\rho\|^2 + b.$$

where $\Upsilon_1 = \frac{b_1}{2a_1} - \frac{1}{2}\|P\|^2$, $\Upsilon_m = \frac{b_m}{2a_m} - \frac{1}{2}\|P\|^2 - \frac{1}{2} (m = 2,3,...,n)$ $\Lambda = (p_0 - \frac{1}{2} l^2)$, and $b = \frac{1}{2}\bar{\varepsilon}^2 + \sum_{i=1}^{n} \frac{b_i}{2a_i} \|\theta_i^*\|^2 + 0.2785\Psi + q$.

Appropriate parameters are designed to make $\Upsilon_1 > 0$, $\Upsilon_m > 0$, and $\Lambda > 0$. Then, we set $k = \min\{2\Lambda / \lambda_{\min}(\ ), 2\gamma_i, 2\Upsilon_i a_i\},\ =1,..,n$. Thus (61) has the new form as

$$\dot{V} \le -kV + b. \tag{62}$$

Thus, from(57) it produces

$$0 \leqslant V(t) \leqslant V(0) e^{-k(t-t_0)} + \frac{b}{k}, \tag{63}$$

where t_0 is the initial time. From (63), it can prove ω_1^2 will be bounded by a function and it will converges to a compact set $\Omega = \left\{ z_1 \middle| z_1^2 \le \frac{2b}{k} = 2\times((q+0.2785\Psi)/k) \right\}$ at a rate of k by exponential form.

To prove that z_i is bounded, β_i also needs to be proven to be bounded. To prove the conclusion, we again choose the Lyapunov equation as:

$$V_{n+1} = \sum_{i=1}^{n} \frac{1}{2} \beta_i^2. \tag{64}$$

From (26), (36), and (46), the derivative of the above yields

$$\begin{aligned} \dot{V}_{n+1} &= \sum_{i=1}^{n} \beta_i \dot{\beta}_i \\ &= \beta_1(-\gamma_1\beta_1 + \bar{\alpha}_2 - \alpha_2 + \beta_2) + \beta_2(-\gamma_2\beta_2 + \bar{\alpha}_3 - \alpha_3 + \beta_3 - \beta_1) + \\ &\quad ... + \beta_n(-\gamma_n\beta_n - \beta_{n-1}) \\ &= -\sum_{i=1}^{n} \gamma_i \beta_i^2 + \sum_{i=1}^{n-1} \beta_i (\bar{\alpha}_{i+1} - \alpha_{i+1}). \end{aligned} \tag{65}$$

From Young's inequality, it gives

$$\sum_{i=1}^{n-1} \beta_i \kappa_i \le \frac{1}{2}\sum_{i=1}^{n-1} \beta_i^2 + \frac{1}{2}\sum_{i=1}^{n-1} \kappa_i^2. \tag{66}$$

From **Lemma3** and (66), (65) can be obtained as:

$$\dot{V}_{n+1} \le -\sum_{i=1}^{n} (\gamma_i - \frac{1}{2}) \beta_i^2 + \frac{1}{2}\sum_{i=1}^{n-1} \kappa_i^2. \tag{67}$$

The above inequality will be chosen by suitable parameters satisfying $\gamma_i > \frac{1}{2}$. Similar to (62) and (63), we can prove that β_i is convergent to a compact set ultimately. Due to $z_i = \omega_i + \beta_i$, the tracking errors z_i will be bounded finally. According to the above, x_i, α_i, and $\bar{\alpha}_i$ can all be proven to be bounded apparently. Thus, all the signals of the CLS are bounded.

Then, we should prove a $t^* > 0$ exists and satisfies $t_{k+1} - t_k \ge t^* (\forall k \in Z^+)$. From $\ell(t) = \delta(t) - u(t)$, the following formula can be obtained.

$$\frac{d}{dt}|\ell| = \frac{d}{dt}(\ell * \ell)^{\frac{1}{2}} = sign(\ell)\dot{\ell} \le |\dot{\delta}|. \tag{68}$$

Thus the derivative of $\delta(t)$ yields

$$\dot{\delta} = \dot{\alpha}_{n+1} - \frac{\bar{\hbar}\dot{\omega}_n}{\cosh^2(\frac{\omega_n \bar{\hbar}}{\Psi})}. \tag{69}$$

From the above, we can know that $\dot{\delta}$ is continuous and bounded. Therefore, there is a constant $\Im > 0$ satisfying $|\dot{\delta}| \le \Im$. It can be noted that $\ell(t_k) = 0$ and $\lim_{t \to t_{k+1}} \ell(t) = \hbar$. So we have $t^* \ge \hbar / \Im$. Based on above, we can avoid the Zeno-behavior [17] effectively.

V. Simulation Proof

The validity of this controller will be tested in this section. For better argument, a physical second-order nonlinear system is used for simulation. The system is provided as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u + \frac{1}{M}(\frac{mmgl\sin(x_1)}{2}), \\ y = x_1 \end{cases} \tag{70}$$

where $mm = 0.05$, $l = 0.5$, $M = 1$, $g = 9.8$ are system parameters.

In the simulation, we set $y_r(t) = \sin(t)$. The controller parameters are $\gamma_1 = 1.5$, $\gamma_2 = 0.5$. The high gain observer parameters are $d_1 = 2$, $d_2 = 1.5$, and $l = 2$. The command filter parameters are $\mu_2 = 0.05$. The adaptive parameters are $a_1 = 0.8$, $a_2 = 0.8$, $b_1 = 80$, $b_2 = 80$. The event-triggering strategy with fixed threshold parameters are $\bar{\hbar} = 10$, $\hbar = 0.5$, $\Psi = 8$. The initial condition $x_1(0) = 1.5$, $x_2(0) = 1.5$, $\hat{x}_1(0) = 1.5$, $\hat{x}_2(0) = 1.5$, $\beta_1(0) = 0$, $\beta_2(0) = 0$, $\hat{\theta}_1(0) = 0$, $\hat{\theta}_2(0) = 0$, $\bar{\alpha}_2(0) = 0$. Then, we choose the following fuzzy membership function.

$$\varphi_{F_i^Z}(\hat{x}_i) = \exp\left(-\frac{(\hat{x}_i + 3 - Z)^2}{4}\right), \tag{71}$$

where $i = 1,2$ and $Z = 1,2,3,4,5$.

By selecting the above parameters, we can get the following simulation results. Fig.1 displays the tracking situation between the output y and the y_r under the designed controller. From fig.1, it is known that y can effectively follow the reference signal.

Fig.2 and Fig.3 perform the observation performance of the high gain observer respectively. From the figures, we can know that the proposed controller can estimate the immeasurable states. As a result, the controller can be used in more applications. Fig.4 plots the curves of the actual control law $\delta(t)$ and event triggering control law $u(t)$. Fig.5 shows the event triggering intervals. The number of triggering events is 172. From them, it can be known that the event triggering strategy can free up the storage of system communication resources and thus save network space.

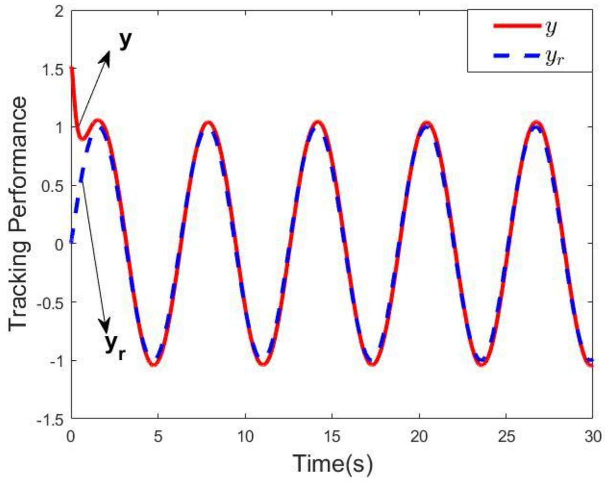

Fig.1. Controller tracking situation

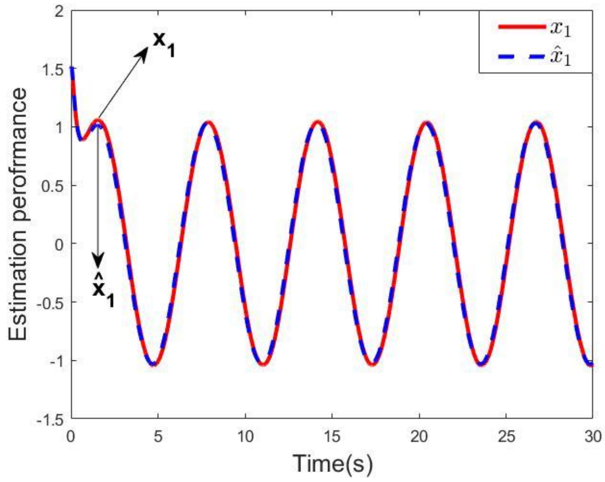

Fig.2. Observation performance of x_1

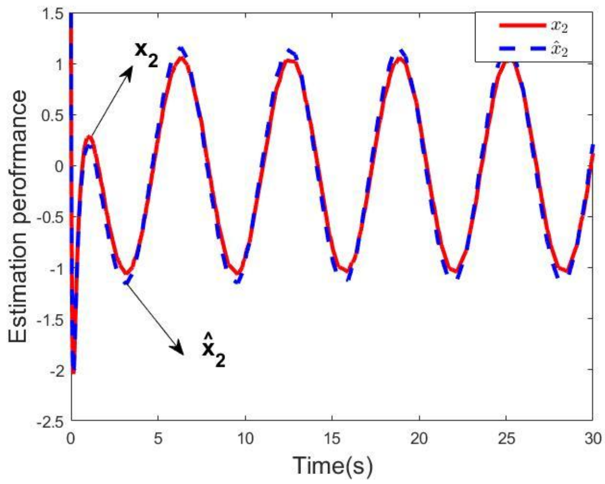

Fig.3. Observation performance of x_2

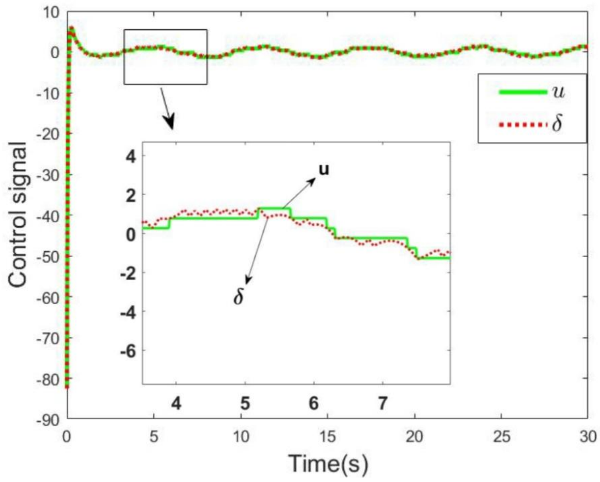

Fig. 4. Curve of the control signal

Fig.6 illustrates the systematic tracking error. According to the figure, the controller can achieve good tracking performance. Fig.7 shows the curve of the compensating signals β_1 and β_2. Obviously, the compensation signals are bounded and eventually oscillate within a fixed region. Fig.8 describes the variation of the adaptive laws. Based on the figures, it can be shown that the controller is appropriate for this nonlinear system.

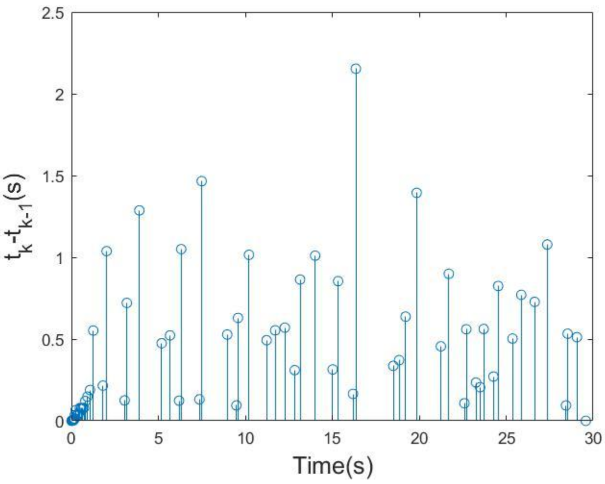

Fig. 5. Time intervals of triggered events

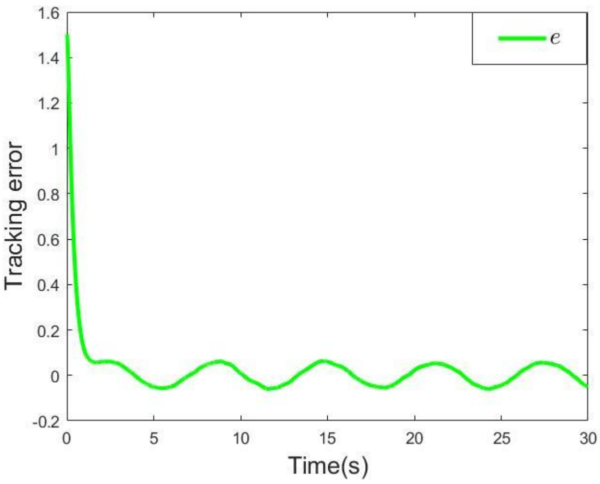

Fig. 6. Tracking error of the system

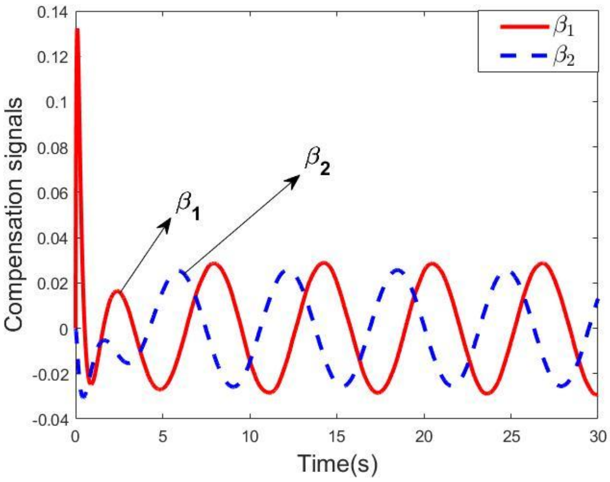

Fig.7. Curve of the compensating signals

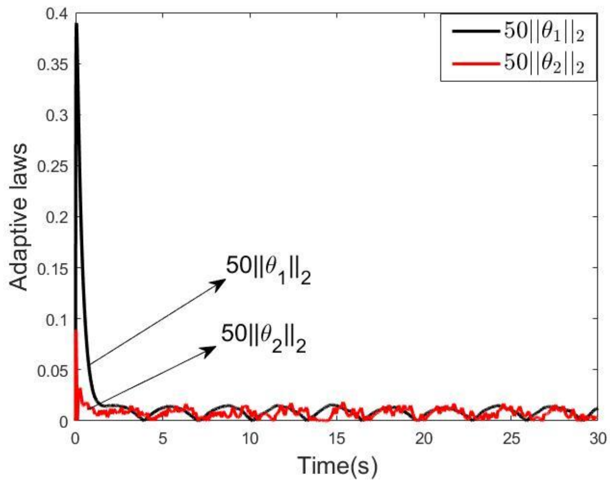

Fig.8. Curve of adaptive laws

VI. CONCLUSION

For the tracking control problem of nonlinear system, a novel adaptive controller based on high gain observer is developed in this paper. Firstly, since the nonlinear system contains unknown parameters to be designed, the adaptive control based on backstepping is adopted. Then, for the existence of the uncertainty, a high gain observer with FLS is introduced in the controller. In addition, the command filter backstepping is used in derivation. It can avoid the problem of excess complexity caused by repeated partial derivatives in the procedure of finding control laws effectively. Furthermore, a event triggering strategy is proposed to conserve network resources. The controller allows all signals in the CLS to be bounded and achieves the tracking goal by converging the tracking error ultimately. The effectiveness and reasonableness of the controller was finally proven by observing the simulation results.

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