A Review of Homogenization and Fractal Methods Applied to the Roughness Analysis of Bonded Materials

Hélène Canot, Philippe Durand, Emmanuel Frenod, Bouchra Hassoune-Rhabbour, Valerie Nassiet

Abstract—**This paper present a review of mathematical and physical approaches of homogenization, fractal and multifractal methods for the study of roughness effect in the bonded materials. In particular, attention is paid to the effects of roughness on materials joined through an adhesive. Firstly, considering the periodic roughness, the different homogenization methods are presented, including mathematical and physical approaches. Secondly, when the rough surface is no longer considered periodic and taking into account multi-scale of asperities, the article focuses on fractal methods. Lot of research has focused on the effect of roughness irregularities on surface contact, friction or wear, crack propagation using fractal theory. Multifractal analysis was introduced in the 80s and has overcome the limitations of fractal analysis. The multifractal analysis of fractal functions is performed using wavelets instead of boxes like in the classical multifractal formalism.**

Index Terms—**Materials, roughness, homogenization, fractals, wavelet, multifractals, adhesion, bonded assembly**

I. Introduction

For a large number of industrial applications, bonding assembly is proving to be a very interesting alternative to other more conventional assembly techniques because it allows in some cases to simplify the design of the assembly, a considerable weight saving, a reduction cost and a more homogeneous distribution of stresses in the joint. However, we must remain cautious because the gluing, to be effective, requires a sometimes restrictive implementation. To obtain a high-performance bonding, a surface treatment to be bonded must be considered. We are particularly interested in polymer / metal bonding assemblies [16]. The quality of a bonded assembly depends on the contact at the interface of the two materials to be assembled. it is necessary to understand the interphase mechanism to control the adhesion and durability of joints. Adhesion is defined as the energy expended to separate the two parts of an assembly if the failure is interfacial. Many factors have a significant influence on the adhesion of glued assemblies. Among the factors that affect grip is roughness. Poorly adapted surface roughness can be detrimental to bonding. It can in particular create interfacial stresses at the level of the bumps and hollows. It can also cause areas of initiation of rupture.

Among those that affect the mechanical, physical or chemical properties of the interfacial bonds responsible for adhesive / substrate adhesion, we are interested in roughness. The roughness of a surface refers to the roughness and variations in altitude that distinguish this surface from a completely smooth surface. The surface roughness can be described qualitatively or quantitatively, which explains the large number of parameters proposed for its description. Among the quantitative measurements are the statistical parameters often used to model roughness: Ra, Rq, Rz. They are based on the calculation of the average peak heights and / or average depths of the valleys measured on the profile. A number of standard roughness criteria are internationally recognized by an ISO standard. Among the measurement tools are those based on optics and in particular those using the principle of light interference. The interferometer and the scanning electron microscope (SEM) are the main tools based on the interference of light.

Many engineering surfaces are rough and induced effects that need particular attention especially in thin films. Various methods of modelling that consider the influence of surface roughness have been developed. In this paper, it is therefore a question of reviewing the mathematical and numerical techniques and concepts which make it possible to model and analyze the phenomena of roughness in the assemblies. More and more researchers in applied mathematics are interested in roughness models and similarly, more and more physicists are using mathematical and numerical methods to study the difficulties associated with these problems. One of the aims is to find an improvement in the adhesion between the substrate and the adhesive.

In order to justify the condition of adhesion generally imposed by the adhesive on the rough walls of the domains, the microscopic details of the walls must be taken into account in the models. However, due to the exorbitant computation costs, the phenomenon described is simplified by replacing the interactions taking place within the system considered, by the description of an average tendency. Such roughness is usually modeled by defining the edges of the domain using periodic functions that oscillate rapidly. One of the approaches, which we will see in this paper, corresponds to

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the homogenization method which amounts to rewriting the problem posed in two others: a local problem and a homogenized problem. The coefficients of the global problem depend on the solution of the local problems. Among the techniques which seem to be the most efficient for taking roughness into account in gluing assembly problems, we will see fractals and the theory of wavelets. These mathematical methods have given rise to a rich literature in the study of fracture, initiation and crack propagation. Modeling of bonded joint began with the problem of two adherents joined by a thin adhesive film of thickness ε. An asymptotic expansion gives a simplified model in which the adhesive is treated as a material surface. Many mathematicians have studied the phenomena of roughness in thin films. They are interested in the case of thin and rough domains of size ε^2 and show that we can see these roughness as a disturbance or as an effect of the main order effect of to fluid mechanics problems. The homogenization method for the problems of roughness effect in thin films and films interfaces have been adapted by physicists in optical, mechanical, wetting ... In particular in the field od adhesion, lubrication, contact, friction.

Fractal analysis is another very popular mathematical model to characterize roughness. The concept of fractal was described by Mandelbrot, and many lines of research have emerged applying fractal analysis for the characterization, description, measurement of rough surfaces. In fact, the fractal approach to roughness does not only result in the search for intrinsic surface parameters but in reconsidering numerous surface phenomena, taking into account the multi-scale nature of roughness. Some researchers have studied the effect of roughness irregularities on contact, friction or wear of a surface using fractal theory [...]. These issues intricate the multifractal nature of the composition of surfaces. Multifractal formalism is the concepts related to the estimation of the spectrum of singularities of a singular mathematical measure whose variations are subject to wide variations. This formalism based on the theory of wavelets was introduced in the 1990s by Mallat [41] [42], Arnéodo [6] [7] [8], Bacry [54] and Muzy [54]. The wavelet transformation is a mathematical tool that appeared in the 1980s in signal analysis and was introduced by two French researchers, Morlet and Grossmann [29] within the framework of the analysis of seismic signals. It consists in decomposing a signal on a set of functions characterized by a position parameter and a scale parameter. The wavelet transform is particularly suitable for analyzing the scale invariance properties of fractal objects. It allows you to zoom in on well-localized structures by adjusting the scale parameter. Singularities and irregular structures often correspond to essential information in the analyzed signal. The local regularity of the signal can then be described by the decrease in the modulus of the wavelet transform across the scales.

In recent years, the researchers combined the two methods for introducing a multifractal analysis of irregular signals based on the wavelet transform modulus maxima (WTMM). The WTMM methodology has been generalized in 2D for multifractal analysis of rough surfaces [22]. The evolution of the Transform Modules Maximas into Wavelet through the analysis scales makes it possible to estimate the Holder exponent $h(x_0)$. This approach becomes more stable than the old standard box-counting methods ... Then it can be applied in the field of fracture mechanics to study the process of crack propagation. Obviously, Wavelet Transform prove to be an effective tool for multiscale characterization of surfaces and their main advantages are their ability to perform local analysis and to reveal singularities.

The main idea of this paper is to provide a sort of toolbox to tackle the roughness effect problems of complex surfaces joined by an adhesive. A brief review is given of the various mathematical methods that have been adapted to the problems of surface roughness in the physical domains of adhesion, contact, rupture in bonded assemblies. They are approached according to two approaches: one when the roughnesses are considered periodic, the other when the rough surfaces are characterized by a fractal geometry. In the first part, the homogenization methods applied in different models are reviewed with mathematical and physical approaches. Some of these mathematical models are not directly related to the problem of adhesion of assemblies, but these methods of homogenization can be applied to them. The second part is reserved for models applying fractal geometry. Finally the multifractal analysis is discussed with in particular the application of the wavelet transform modulus maxima method.

II. Homogenization methods

As glued assemblies are made up of adhesives and a thin layer of adhesive, we will first provide a theoretical review of mathematical methods, specifically homogenization methods, for thin adhesive layers. Then we will see in the following paragraphs the study of the effect of roughness in thin layers. Before therefore presenting the mathematical method modelling the mechanical and physical behavior of bonding and roughness interfaces. The first paragraph is dedicated to the mathematical homogenization of an adhesive thin layer with a soft interface. it constitutes an introduction before the study and the modeling of bonded joints on rough surfaces which are studied in the following paragraphs. As the thickness of the joint is small with respect to the other adherents, the adhesive joint is considered as a small parameter ε. One injects the displacement and the stresses indexed by ε into the mechanical problem. Homogenizing the problem consists in studying the limit problem, i.e. studying the asymptotic behavior of the solution when ε tends to 0.

A. Homogenization asymptotic of thin layers

From the problem of thin films in fields such as aeronautics, nuclear ... arose from modeling methods called multiscale methods to respond to scaling problems. Many works have been developed for the mathematical and mechanical modeling of adhesive assemblies based on asymptotic methods [25], [28], [37], [62]. In various works, the modeling of the thin layers of assemblies was based on the homogenization asymptotic method [51], [60], [69].

1) Energy Asymptotical Method: We illustrate this method with the model introduced by Dumont et al. [23] which study the asymptotic first order analysis of two structures bonded together : they consider two cases the gluing of an elastic

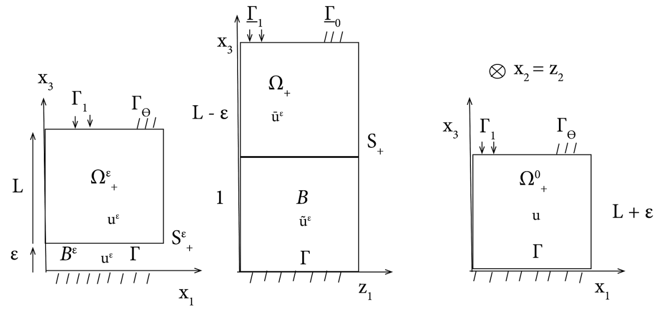

Figure 1. (a) Initial, (b) rescaled, and (c) limit configuration of a solid glued to a rigid base.

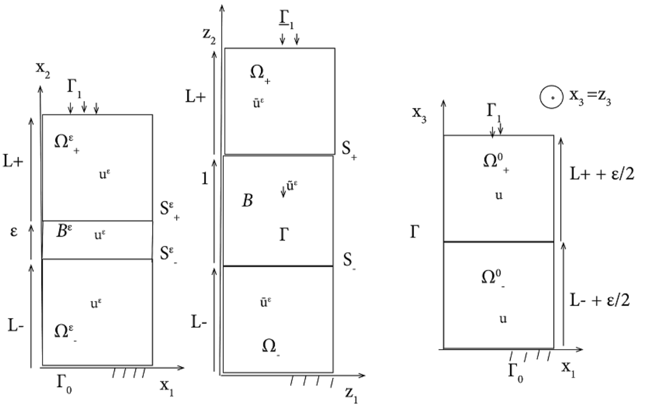

Figure 2. Interphase/interface problem. Left: the initial problem with an interphase of thickness ε. Middle: the rescaled problem with interphase height equal to 1. Right: the limit interface problem.

structure with a rigid body and the gluing of two elastic structures.

They extend the imperfect interface law given in [39] to the case of a very thin interphase whose stiffness is of the same order of magnitude as that of the adherents. And the Lamé's coefficients of the interphase do not depend on the thickness ε of the interphase. For the asymptotic analysis for an elastic body glued to a rigid base, they introduce a small parameter ε which defines the thickness of the glue. Body force f is applied in Ω_+^ε, and the interface Γ is considered as a plane normal to the third direction e_3. The system of equations is given by:

$$
\begin{aligned}
div\sigma^\varepsilon + f &= 0 \text{ in } \Omega_+^\varepsilon \cup B^\varepsilon && (1)\\
\sigma^\varepsilon n &= g \text{ on } \Gamma_1 && (2)\\
u^\varepsilon &= u_d \text{ on } \Gamma_0 && (3)\\
u^\varepsilon &= 0 \text{ on } \Gamma && (4)\\
\sigma^\varepsilon &= A_+ e(u^\varepsilon) \text{ in } \Omega_+^\varepsilon && (5)\\
\sigma^\varepsilon &= \hat{A} e(u^\varepsilon) \text{ in } B^\varepsilon. && (6)
\end{aligned}
$$

where $e(u^\varepsilon)$ is the strain tensor $e_{ij}(u^\varepsilon) = \frac{1}{2}(u_{ij} + u_{ji}), i,j = 1,2,3$ and where A_+,$\hat{A}$ are the elasticity tensors of the deformable adherent and the adhesive, respectively.

Using asymptotic expansions with respect to the parameter ε and the very small interface, the equilibrium problem is written as a minimization problem of the total energy:

$$
J^\varepsilon(u^\varepsilon) = \frac{1}{2}\int_{\Omega_+^\varepsilon} A_+ e(u^\varepsilon)\cdot e(u^\varepsilon)dx - \int_{\Omega_+^\varepsilon} f\cdot u^\varepsilon - \int_{\Gamma_1} g\cdot u^\varepsilon - \frac{1}{2}\int_{B_+^\varepsilon} \hat{A}e(u^\varepsilon)\cdot e(u^\varepsilon)dx \tag{7}
$$

Moreover, they introduced a change of variables to reformulate the mechanical energy in an interphase domain independent of ε in the glue and the adherent:

$$
(z_1, z_2, z_3) = (x_1, x_2, \frac{x_3}{\varepsilon}) \quad (x_1, x_2, x_3) \in B^\varepsilon \tag{8}
$$

$$
(z_1, z_2, z_3) = (x_1, x_2, x_3 + 1 - \varepsilon) \quad (x_1, x_2, x_3) \in \Omega_+^\varepsilon \tag{9}
$$

After scaling et minimizing the energy, they obtained the following problem at the order zero:

$$
div(A_+(e(u^0))) + \bar{f} = 0 \text{ in } \Omega_+^0 \quad (P_0) \tag{10}
$$

$$
A_+(e(u^0))n = g \text{ on } \Gamma_1 \tag{11}
$$

$$
A_+(e(u^0))n = 0 \text{ on } \partial\Omega_+^0 \ (\Gamma_1 \cup \Gamma) \tag{12}
$$

$$
u^0 = 0 \text{ on } \Gamma \tag{13}
$$

and the problem at the first order:

$$
div(A_+(e(u^1))) = 0 \text{ in } \Omega_+^0, \quad (P_1) \tag{14}
$$

$$
A_+(e(u^1))n = 0 \text{ on } \partial\Omega_+^0 \setminus \Gamma \tag{15}
$$

$$
u^1 = ((\hat{K}^{33})^1(A_+(e(u^0)))n) - u_3^0, \text{ on } \Gamma \tag{16}
$$

And in the case of plane strain in the plane (x_1, x_2), the interface between the glue and the adhesive is a line orthogonal to the direction e_2. They reobtain the problem P_0 and the problem P_1 modified as:

$$
div(A_+(e(u^1))) = 0 \text{ in } \Omega_+^0, \quad (P_1) \tag{17}
$$

$$
A_+(e(u^1))n = 0 \text{ on } \partial\Omega_+^0 \ \Gamma \tag{18}
$$

$$
u^1 = ((\hat{K}^{22})^{-1}(A_+(e(u^0)))n) - u_2^0, \text{ on } \Gamma \tag{19}
$$

where the matrices $\hat{K}^{jl}, j,l = 1,2,3$ are defined by

$$
\hat{K}_{ki}^{jl} := \hat{A}_{ijkl}. \tag{20}
$$

For the case of the gluing of two elastic bodies, satisfying the plane strain hypothesis, the displacement along the interface is replaced by a jump of the displacement across the interface between the two bodies. Numerical experiments have shown the validity of the interface law method when the interphase thickness becomes smaller. The interface law is able to reproduce the mechanical behavior. The numerical scheme, implemented in a finite element, for the adhesion of two deformable bodies, was based on the Nitsche's method.

2) Matched Asymptotic Expansion Method: Another approach successfully applied many times to adhesive assemblies is that of matched asymptotic expansions see [1], [63], which consist in defining two approximations: one, far from the disturbance and a second in the vicinity of the disturbance. The connection of the two approximations defines the approximate solution of the sought field. Adhesive assemblies contain a layer of negligible thickness e compared to a length L characteristic of the assembly. Then a small parameter $\varepsilon = \frac{e}{L}$ is introduced. Two problems are analyzed: one external (outer problem) where the thin layer plays the role of disturbance, and the other "undisturbed" which rests on the homogeneous structure. One then introduces an asymptotic development which restores the behavior far from the joint and which brings into play the parameter ε:

$$u(x,y) = u^0(x,y) + \epsilon u^1(x,y)... \tag{21}$$

The displacement is then characterized by the undisturbed continuous field u^0 with its corrector ϵu^1 in connection with the thin layer. The joint is replaced by an interface of discontinuity, when $\varepsilon \to 0$. For the second development, in the near field, the rapid variation of $u(x,\mathbf{y})^\varepsilon$ is accounted introducing the variable $\mathbf{y} = \frac{y}{\varepsilon}$

$$u(x,\mathbf{y})^\varepsilon = U^0(x,\mathbf{y}) + \epsilon U^1(x,\mathbf{y})... \tag{22}$$

It describes the behavior of u near the thin film.

Due to the separation into two subspace, the question of boundary conditions arises. There are missing boundary conditions in the near field when $y \to 0$, there are missing conditions in the far field when $x \to 0^\pm$. These missing conditions for near and far fields are provided by so-called matching conditions which ensure the continuity of the solutions in an intermediate region. We want the two solutions to correspond in this intermediate region, for this purpose a Taylor expansion is then applied to the outer expansion 21 around y = 0. The specific connection rules linking the two developments give

$$\begin{cases} \lim\limits_{\mathbf{y} \to \pm\infty} (U^{0\pm}(x,\mathbf{y}) - u^{0\pm}(x,0)) = 0, \\ \lim\limits_{\mathbf{y} \to \pm\infty} (U^{1\pm}(x,\mathbf{y}) - \mathbf{y}\frac{\partial u^{0\pm}(x,0)}{\partial y} - u^{1\pm}(x,0)) = 0 \end{cases}$$

This approach therefore allows a multi-scale modeling of structures with thin layers, however to go further we will be interested in works which study the influence of roughness on the quality of the assembly.

B. Study of the influence of roughness in thin layers by homogenization methods: a mathematical approach

The next step is the study of the phenomena of roughness in thin films in particular when the substrate has a rough surface at the interface. In bonding assembly, surface roughness influence is very important. The low thickness of the joint makes all process particularly sensitive to these irregularities, which can have a significant impact on their performance. From the point of view of calculation, the mesh at the scale of these irregularities entails prohibitive calculation costs, so that other approaches are necessary to take into account the average effects of these roughness. Mathematicians have also been interested in the problems of roughness in thin films, in particular if one considers that the roughness is periodic the homogenization methods for periodic structures have been used. Among the classic periodic homogenization methods we have the multiple scale method of A. Bensoussan, JL Lions and G. Papanicolaou [10], the oscillating test functions method of L. Tartar [66], the double scale convergence method of G. Nguetseng [53] and G. Allaire [4] and the method of periodic unfolding of D. Cioranescu et al. [17].

Here, the two models for studying the effects of roughness on thin films by homogenization methods, which will be exposed in the paragraphs below do not directly concern bonded assemblies but rather the mathematical analysis of problems resulting from the fluid mechanics. However the two following models can be applied in adapting it to glued joints.

1) Influence of roughness on the Elrod-Adams model: Many mathematicians have studied influence of roughness in thin flows. G. Bayada, C. Vazquez and S. Martin in [9], [14] studied the influence of surface roughness on the Elrod-Adams model. The surface roughness, assumed to be periodic, was modeled by considering a strongly oscillating height

$$h := h_0(\mathbf{x}, \mathbf{x}/\varepsilon), \quad \varepsilon \leq 1, \tag{23}$$

and the influence of these roughness was studied by techniques multi-scale homogenization. The oscillating function h involves two distinct scales: a slow scale, described by the usual variable $\mathbf{x}$, and a fast scale described by the variable $y = \mathbf{x}/\varepsilon$ with this variable y living on the unit cell $Y :=]0,1[^2$, the ε parameter measuring the frequency of roughness.

Thus, instead of calculating the solution of the direct problem at fixed ε, we will be interested in determining an approximate solution by studying the asymptotic when $\varepsilon \to 0$.

In their work, the homogenization of the Reynolds equation was extended to take into account the non-linearity introduced by the Elrod-Adams model. They described the homogenized models taking into account surface defects whose scale ratios vary significantly with the following form $h := h_0(\mathbf{x}, \mathbf{x}/\varepsilon, \mathbf{x}/\varepsilon^2...)$. They generalized the study in order to take into account nonlinear phenomena such as the elastic deformation of the surfaces which confine the flow and the piezoviscosity of the fluid which makes the problem non-local

$$h := h_0(\mathbf{x}, \mathbf{x}/\varepsilon) + \int_\Omega k(\mathbf{x}, \mathbf{z}) p(\mathbf{z}) dz, \quad \mu = \mu_0 e^{\alpha p} \tag{24}$$

with h_0, the rigid contribution of the spacing between surfaces, k the Hertz kernel function which depends on the type of contact considered and which aims to weight the effects of high pressure on the elastic deformation of surfaces and $\alpha \geq 0$ le piezoviscosity parameter.

2) Influence of roughness at main order: In their work [15] [18] Chupin et al. show that even small roughness can have an effect on the main order. Thus, in a thin domain of order ε and roughness of order ε^2 they approach the Stokes equations by a modified Reynolds equation. The modification is explicitly given according to the roughness, they show the flow can be accelerated by the roughness. The limit problem is justified thanks to the double scale convergence. Indeed, the problem has two small scales: that related to the thin domain and that related to roughness. The height is of the form

$$h^{\varepsilon}(x) = \varepsilon h_1(x) + \varepsilon^2 h_2(\frac{x}{\varepsilon^2}) \tag{25}$$

on the domain defined by:

$$\Omega^{\varepsilon} = \{(x,z) \in R \times R;\;\; 0 < x < 1 \;\; et \;\; 0 < z < h^{\varepsilon}\}. \tag{26}$$

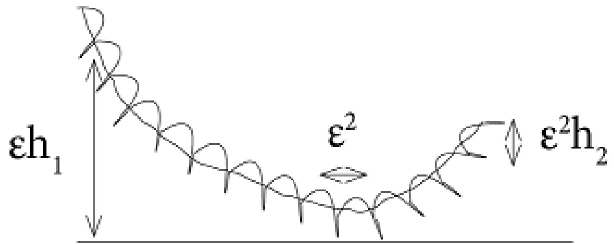

Figure 3. The domain

The Stockes equations considered are:

$$\begin{cases} -\Delta u^{\varepsilon} + \nabla p^{\varepsilon} = 0 \;\; \text{on } \Omega^{\varepsilon} \\ div(u^{\varepsilon}) = 0, \\ u^{\varepsilon}|_{z=0} = (s,0) \;\; \text{and} \;\; u^{\varepsilon}|_{z=h^{\varepsilon}} = (0,0). \end{cases}$$

First, they start to scale only the macroscopic domain and not the roughness, the slow variable x is separated from the fast oscillation variable $X = \frac{x}{\varepsilon^2}$ by setting $h(x,X) = \varepsilon h_1(x) + \varepsilon^2 h_2(X)$. Next, the change of variable $Z = \frac{z}{h(x,X)}$ is performing. Then the following development is injected into the problem II-B2:

$$\begin{aligned} u^{\varepsilon} &= u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + ..., && (27)\\ w^{\varepsilon} &= w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + .. && (28)\\ p^{\varepsilon} &= \frac{1}{\varepsilon^2} p_0 + \frac{1}{\varepsilon} p_1 + p_2 ... \varepsilon^2 u_2 + ... && (29) \end{aligned}$$

The following theorem D. Bresch, C. Choquet, L. Chupin, T. Colin, and M. Gisclon is obtained:

Theorem: Let, $(u^{\varepsilon}, w^{\varepsilon}, p^{\varepsilon})_{p>0}$ be a series of Stokes solutions II-B2 in the domain Ω^{ε}. The suite brought up to scale $\{(u^{\varepsilon}, \frac{w^{\varepsilon}}{\varepsilon}, \varepsilon^2 p^{\varepsilon} \circ (x, h^{\varepsilon} Z)\}_{\varepsilon>0}$ converges double scale to the weak solution (u_0, w_1, p_0) of the following generalized Reynolds system:

$$\begin{cases} -\partial^2_{ZZ} u_0 + h_1^2 \partial_x p_0 + MZ\partial_Z u_0 = 0, \\ \partial_Z p_0 = 0, \\ \partial_x(h_1 u_0) + \partial_Z(w_1 - Z\partial_x h_1 \cdot u_0) = 0 \end{cases}$$

with $M = \int_{\mathbb{T}} |h_2'(X)|^2 dX$.

For the proof, after having given the estimations on the speed field then on the pressure:

$$\|u^{\varepsilon}\|_{L^2(\Omega)} \leq 1, \;\; \|\nabla u^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon}, \quad \|\partial_Z u^{\varepsilon}\|_{L^2(\Omega)} \leq 1 \tag{30}$$

$$\|p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon^2}, \;\; \|\nabla u^{\varepsilon}\|_{H^{-1}(\Omega)} \leq \frac{1}{\varepsilon}, \quad \|\nabla p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon^2}, \|\partial_Z p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon}, \tag{31}$$

One can then extract the subsequences which two-scale converge towards the following limits, for $p_0 \in L^2(\Omega, L^2(\mathbb{T}))$, $u_0 \in L^2(\Omega, H^1(\mathbb{T}))$ and $w_0 \in L^2(\Omega, L^2(\mathbb{T}))$

$$\|u^{\varepsilon}\|_{L^2(\Omega)} \leq 1, \;\; \|\nabla u^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon}, \quad \|\partial_Z u^{\varepsilon}\|_{L^2(\Omega)} \leq 1 \tag{32}$$

$$\|p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon^2}, \;\; \|\nabla u^{\varepsilon}\|_{H^{-1}(\Omega)} \leq \frac{1}{\varepsilon}, \quad \|\nabla p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon^2}, \|\partial_Z p^{\varepsilon}\|_{L^2(\Omega)} \leq \frac{1}{\varepsilon}, \tag{33}$$

One can then extract the subsequences which two-scale converge towards the following limits, for $p_0 \in L^2(\Omega, L^2(\mathbb{T}))$, $u_0 \in L^2(\Omega, H^1(\mathbb{T}))$ and $w_0 \in L^2(\Omega, L^2(\mathbb{T}))$

$$\varepsilon^2 p^{\varepsilon} \rightharpoonup 2p_0, \quad u^{\varepsilon} \rightharpoonup 2u_0 \;\; \text{and} \quad w^{\varepsilon} \rightharpoonup 2w_0. \tag{34}$$

Then by passing to the limit in Stokes equations, they show that the limit (u_0, w_1, p_0) satisfies the system II-B2, with the vertical speed

$$w_1 = \int_{\mathbb{T}} \eta_1 dX \tag{35}$$

C. Study of the influence of roughness in thin layers by homogenization methods: a physical approach

The paragraph reviews, in the recent literature, the study of the influence of roughness in adhesion problems thanks to homogenization. In general, mechanics and physicists use homogenization methods differently: the most popular approach is a representative volume element, i.e. a sample of size larger than the heterogeneity but still small compared to the size of the overall producing domain, in mean, the effective properties of the material. However, physicists have applied in their work the different methods of homogenization, asymptotic analysis and interface models, specifically, in the area of the adhesion, in the mechanical contact issues, the adhesive and cohesive failure for the fracture model in material and for rough interface. What we will see in the following paragraphs.

1) Influence of roughness in adhesion problems: The implementation of numerical modeling and the use of homogenization methods in the work of G. Bresson in [12] allowed the evaluation of the local and effective properties of the structural adhesive, with a view to its application on a prototype space launcher. Significant surface analysis work has been carried out to set up a stable and efficient assembly process: indeed, the adhesion of glues on the aluminum substrate strongly depends on the composition and the roughness of the surface to be glued. An increase in the surface roughness to be bonded should lead to an increase in the contact surface see [72]. However, too much roughness decreases the penetrating capacity of the adhesive, which increases the formation of voids and introduces additional stress concentrations see [24]. The roughness parameters [27] presented: R_a, arithmetic mean roughness, R_p, peak, R_v, valley, R_t, max profile height, R_q, root mean square roughness of the profile, R_z, arithmetic mean of the 5 highest peaks and the 5 deepest troughs, R_{ku}, amplitude distribution curve flattening factor (kurtosis), R_{sk}, skewness factor, make it possible to estimate the shape of the profiles generated by the treatment (here sandblasting). This led to the conclusion that it was wise to increase the roughness of the substrate in order to increase the surface on which the glue applied.

The author used the homogenization method, the aim being to predict the mechanical behavior of an epoxy matrix material reinforced by particles but weakened by air bubbles in the matrix. The average local mechanical properties of the microstructure are evaluated in order to determine the elastic properties of the matrix reinforced by the particles. Mori-Tanaka's homogenization model [52] is preferred to that of the self-consistent model [38]. The Mori-Tanaka model [11] is defined for a heterogeneous medium, for example a particulate medium, consisting of inclusions distributed isotropically behaving, on average, like isolated inclusions in an infinite matrix. This method makes it possible to analyze the heterogeneity of the mechanical properties of the adhesive as well as the volume fraction of porosities which makes it possible to estimate the effective local stress leading to the overall stress in the adhesive. The porosities redistribute the stresses to nearby elements in the adhesive joint.

In [70] Verhoosel et. al proposed a computational homogenisation framework to model cohesive and adhesive failure in materials with complex microstructures. They considered the Hill-Mandel energy condition to define the homogenization relations. For the cohesive crack, the averaging relations are applied to a finite element model representing the bulk material in the vicinity of a crack. For the adhesive crack, the material in the adhesive layer is represented by the micro-scale model. For both cases the macroscopic traction-opening relations are only defined on macroscopic cracks, the proposed averaging scheme can be regarded as a homogenisation procedure applied along the macroscopic cracks.

However, physicists and mechanics have adapted mathematical methods of periodic homogenization to their structural roughness problems.

[13] developed a tool, thanks multi-scale homogenization method, of combined effects of surface roughness and lubricant rheology on the hydrodynamic contact of inclined slider bearing. The pad surface is rough and stationary but the lower surface is assumed to be smooth and moving. The rheological behavior of the lubricant flowing between the two surfaces is performed by the V.K. Stokes couple stress fluid model. The behavior of a non-Newtonian polar fluid in stationary, isothermal and laminar flow regimes is described by the equation:

$$\nabla \cdot G(h,l)\nabla p = \Lambda \frac{\partial h}{\partial x_1} \quad \text{on} \quad D = (0,L)\times(0,B) \tag{36}$$

where

$$G(h,) = h^3 - 12l^2h + 24l^3 \tanh\big(\frac{h}{2l}\big) \tag{37}$$

and

$$\Lambda = 6\mu U \tag{38}$$

where $l = \sqrt{\frac{\eta}{\mu}}$ is the parameter of the stress couple, with μ the dynamic viscosity of the lubricant and η the physical constant of the fluid. The pressure field unsatisfying 36 the equation and the following Dirichlet limiting condition:

$$p = 0 \quad \text{on} \partial D \tag{39}$$

The authors used multiple scale homogenization method for study the effects of surface roughness on hydrodynamic contact performance. To homogenize the modified Reynolds equation, local coordinates are introduced $(y_1, y_2) = (\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon})$ with $(y_1, y_2) \in Y = (0,1)\times(0,1)$ and the thickness of the film is expressed by:

$$h(x_1,x_2,y_1,y_2) = h_0(x_1,x_2) + h_1(y_1,y_2) \tag{40}$$

The asymptotic expansion of the pressure

$$p(x_1,x_2,y_1,y_2) = p_0(x_1,x_2) + \varepsilon p_1(x_1,x_2,y_1,y_2) + \varepsilon^2 p_2(x_1,x_2,y_1,y_2) + ... \tag{41}$$

where $p_1, p_2)$ are periodicals in relation to variables (y_1, y_2), and the following differentiation rule

$$\nabla = \nabla_x + \frac{1}{\varepsilon}\nabla_y \tag{42}$$

lead to the homogenized problem:

$$\nabla_x \cdot (\mathbf{A}(x)\nabla p_0) = \nabla_x \cdot \tag{43}$$

and the cellular problem

$$\nabla_y \cdot (G(h,l))\nabla_y v_1 = -\frac{\partial G}{\partial y_1} \quad \text{on} \quad Y \tag{44}$$

$$\nabla_y \cdot (G(h,l))\nabla_y v_2 = -\frac{\partial G}{\partial y_2} \quad \text{on} \quad Y \tag{45}$$

$$\nabla_y \cdot (G(h,l))\nabla_y v_3 = \Lambda\frac{\partial h}{\partial y_1} \quad \text{on} \quad Y \tag{46}$$

Homogenized pressure satisfies $p_0 = 0$ on the border ∂D. Numerical simulations were performed for transverse, longitudinal and anisotropic roughness patterns, and various values of the couple stress parameter.

Thanks to homogenization methods, they concluded: that multi-scale homogenization is efficient for all three roughness patterns, and surface roughness and stress torques due to the presence of polymeric additives in the lubricant have significant effects on the hydrodynamic performance of the contact.

In [56] J. Orlik used homogenization methods for a physical problem: a contact problem of two elastic bodies with periodically rough surfaces at the interface is considered. She takes the size of the micro-peaks and valleys very small compared with the macro-size of the bodies. The period of the micro-roughness on the contacting interface is of order ε.

She considers the equilibrium equations and constitutive elastic relations with contact and boundary conditions. She develops a method that allows deriving a macro-contact condition on the interface using the two-scale asymptotic homogenization procedure that takes into account the micro-geometry of the interface layer and the stiffnesses of materials of both domains. A two-scale algorithm for the solution of the contact problem was performed to solve homogenized problem by the Finite element method and the cell-problem. The averaged contact stiffness obtained allows the replacement of the interface layer in the macro-model by the macro-contact condition. The advantages of two-scale homogenization in the mechanical sense are:

- The fact of starting with the frictionless contact micro-problem with a rough interface, but ending up with the macro-problem containing friction.
- Two-scale homogenization allows to reduce contact problem with two different size scales on the micro and macro-levels to a single-scale problem.

III. Fractals, multifractals and wavelets methods

A. Fractal measurment of rugosity

The modeling of roughness in physics calls upon various mathematical theories, fractals, spectral analysis and its derivations, Fourier transform, wavelet transform. More concretely, the analysis of deterministic signals is linked to classical analysis, a field in which the functions are piecewise "regular". The notion of regularity is related to the notion of differentiability. We can immerse the deterministic functions (the classical functions) in a larger space which also model the impulses: this is the space of distributions of L. Schwartz [61]. In this space, certain discontinuous functions are differentiable..... For its functions, long considered classic, the notion of roughness has no place.In the study of random signals, there are many examples of "pathological" signals, white noise, Brownian motion,... these signals are part of a new class of functions which have the property of being non-differentiable everywhere. They are special cases of fractals, so named by B. Mandelbrot [46] at the very beginning of computer science.

Fractal geometry, is a the right tools to model the roughness of a shape. In this theory, the notion of dimension of topological spaces can be generalized. Its value can be non-integer. In fractal theory, the dimension is calculated by another method: we talk about similarity or scale dimension.

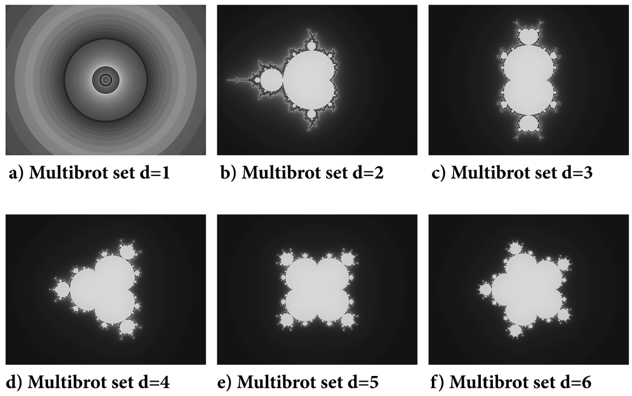

Figure 4. Fractal images from Multibrot sets for various exponents of d

This method restores the classical (topological) dimension for regular objects (rectifiable curve), regular "pavable" surface...), but also adapts to irregular objects. It makes it possible to make measurable objects that would not be measurable in the classical sense. The classical dimension of a piece of wool of negligible thickness is one. but from the point of view of the scale dimension, this is not always exact: if we compress a piece of wool into a ball that we place in a cube, or any other regular volume, its dimension n is no longer 1 but 3. The similarity dimension therefore leads to consider intermediate dimensions, and to measure intermediate physical objects between curve, surface, volumes... and can be non-integer. An object can be of infinite measure if the dimensional space in which it is measured is not adapted. On the other hand, it will be of finite measure if its scale dimension corresponds to it. In this review, we will revisit the notion of fractal, the calculation of the fractal dimension and the resulting roughness measurement, we will then talk about spectral analysis and in particular the Fourier transform, and the Wavelet transform as well as the link with fractals. We will also evoke a generalization of fractals: multifractals which make it possible to stick to natural phenomena as closely as possible.

B. Simalarity dimension

The similarity dimension generalizes the classical dimension of regular objects. A good piece of curve will also have a scale dimension equal to 1, a good piece of surface, a scale dimension equal to 2....etc. However, a rough object contained in a plane or more generally a piece of regular surface may have a dimension between 1 and 2, the excess of dimension compared to 1 characterizes the roughness. Similarly, an object contained in a straight line or a piece of regular curve, may have a dimension between 0 and 1, we then speak of dust. One method for calculating the scale dimension is the box dimension, more precisely a ball of wool compressed in a cubic box, will have a box size of 3.

C. Fractals applications

B. N. J. Persson and E. Tosatti have studied in [58] the influence of surface roughness on the adhesion of elastic solids. They consider, as in reality, different length scales for surfaces roughness. They take the case that the roughness surfaces are described by a self-affine fractal, and they show that the adhesion force may be strongly reduced or may vanish when the fractal dimension D_f is greater than $2,5$. They consider the block-substrate pull-off force as a function of roughness, they find a partial detachment transition preceding the full detachment with single scale roughness. They found in good agreement with experimental data, that the partial detachment results in a very substantial reduction in the pull-off force prior to full detachment.

In [68] X. Yin, K. Komvopoulos derived an adhesive wear model of rough surfaces in normal contact based on plasticity-induced wear behavior that accounts for adhesion between interacting asperities. In this paper, the equivalent rough fractal surface is assumed to be isotropic and self-affine, then the 3D surface topography is represented by a (2D) surface profile.

The truncated segment is approximated by an asperity with a spherical cap shape with a base radius r' which is equal to one-fourth of the asperity's base wavelength and height equal to the local interference δ define by (Yan and Komvopoulos, [67])

$$\delta = 2G^{(D-2)}(\ln\gamma)^{\frac{1}{2}}(2r')^{(3-D)} \tag{47}$$

where D and G are the fractal dimension and fractal roughness, respectively, and γ ($\gamma > 1$) controls the density of frequencies in the surface profile. These asperity contacts, fundamental in contact mechanics, follow an island distribution see Mandelbrot [44]. This island distribution obeys the power-law relationship:

$$N(a') = (\frac{a'_L}{a'}) \tag{48}$$

where $N(a')$ is the number of asperities with truncated areas larger than a', a'_L is the largest truncated contact area at a given global interference h. And the total truncated area is expressed as

$$S' = (\frac{D-1}{(3-D})a'_L[1-(\frac{a'_S}{a'_L})^{\frac{(3-D)}{2}}] \tag{49}$$

The adhesive wear analysis of rough surfaces in normal contact indicates that both the wear rate and the wear coefficient depend on the elastic–plastic material properties, fractal parameters, surface energies, material compatibility, interfacial adhesion, and total normal load through the total truncated contact area. Numerical simulation was performed revealed the effects of material properties, roughness, surface compatibility, and environmental conditions on the adhesive wear rate and wear coefficient. They concluded that plastic deformation at asperity contacts is controlled by the critical truncated contact area, which depends on the elastic–plastic material properties, roughness, and work of adhesion of the contacting surfaces. Particulary, the wear rate and the wear coefficient decrease with the interfacial adhesion and increase with the roughness of the contacting surfaces. The material properties, surface roughness, and work of adhesion that depends on the surface energies of the contacting surfaces of the contacting surfaces affect the adhesive wear coefficient.

More recently, thanks to fractal dimension in, [59] E.Saborowski *et al* evaluated the interlaminar shear and tensile strength of mechanically interlocked polymer–metal interfaces.

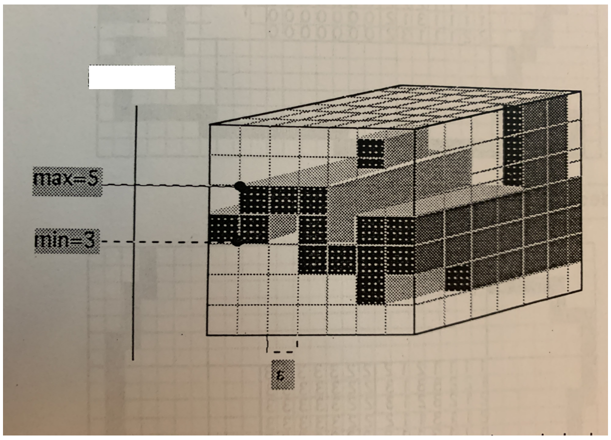

Figure 5. Box-counting algorithm.

The fractal dimension of the interface line D is given by

$$D = \frac{1}{k}\sum_{i=1}^{k} d_i \tag{50}$$

where $k = \log_2 r_{max}$ and the individual box sizes d are difined by

$$d_i = \log_2 n_{i+1} - \log_2 n_i \tag{51}$$

The image is divided in squares of size r, then for each r_i correspond a certain number of squares n_i. Roughness measurements were carried out with a stylus. They concluded that tactile measured surface roughness slope is an appropriate measure for coarse structures, but not for undercut, densely arranged, and small-scaled profile elements. Whereas fractal dimension is an appropriate, scale-independent measure for describing the surface structure.

In his work [64], C.Secrieru characterizes the topography of the fracture surface of materials using the fractal dimension. From the comparative study of the methods of calculating the fractal dimension applicable to fracture surfaces, effective methods have been highlighted: the Box Counting method, the oscillations method and the Hurst exponent method. In the case of Charpy specimens and for hardened XC 65 and 316L stainless steel steels, the determination of the fractal dimension of the fracture surfaces by the classic island method (Slit Island) was applied. It has been shown in this work that the energy at failure by a single shock varies inversely with the fractal dimension.

D. Multifractals analysis

We have seen the efficiency of fractal analysis methods to describe roughness, however the fractal dimension is not always sufficient to describe the irregularity. For a complex object, the fractal dimension alone cannot characterize the complexity completely. We then introduce the local fractal dimension to describe the fluctuations in roughness at each point Stoyan et al [65], when the fractal dimension changes from one point to another, we say that the object is multifractal. Multifractal analysis first appeared with Mandelbrot's multiplicative cascade models for the study of energy dissipation in the context of turbulence. He also observed in 1984 [45] that the fracture surfaces exhibit properties of magnitude scale-invariances. The concept of fractal geometry has been shown to be an effective tool for fractographic study. In [36] Jing et al studied the morphology of the fracture surfaces of certain metallic and ceramic materials and concluded that these surfaces exhibit a multifractal character.

The bases of multifractal formalism were introduced in 1985 by Parisi and Frisch [26] in their article where they introduce the very notion of multifractality and allow the introduction of the Holder exponent. Their goal was to calculate the spectrum of singularities not directly from its definition, but rather from auxiliary quantities that can be easily estimated numerically. With the appearance of the wavelet transform, and the development of numerical computational tools, Mallat and Hwang [40] were able to implement a theory which links the evolution of local Maxima of the Moduli of the Wavelet Transform (WTMM) through the analysis scales and the Hölder exponent.

1) The wavelet transform modulus maxima method: The WTMM method based on the continuous wavelet transform and developed in the early 90s by Arneodo and his collaborators, helps to determine the singularity or multifractal spectrum of the signal. The local maxima modulus of continuous wavelet transform WTMM of the signal gives the Hölder exponent estimation. Decoster in his thesis work [21] presented the theory, the numerical implementation and the application to the statistical analysis of multifractal rough surfaces of the 2D WTMM method.

D. Ait Aouit and A. Ouahabi in [3] introduced a multifractal analysis to discriminate the irregular fracture signals of materials based on the continuous Wavelet Transform Modulus Maxima method (WTMM). The goal is is to define in each fracture profile point the velocity variation law. Fracture surfaces are considered as an anisotropic fractal sets (multifractal). To determine the multifractal spectrum, using WTMM method, they calculate the wavelet transform (WT) of the signal at multiple scales, find the local maxima of wavelet transforms in each scales and chain the wavelet maxima across scales. The partition function in terms of WTMM coefficients is defined by:

$$Z(q,s) = \sum_{l \in L(s)} (\sup_{(u,s') \in l, s' \leq s} |W_X(u,s')|^q), \qquad (52)$$

where $W_X(u,s)$ is is the continuous wavelet transform of the signal X:

$$\forall u \in R, \forall s > 0, \qquad (53)$$

$$W_X(u,s) = \frac{1}{\sqrt{s}} \int_R X(x) \psi(\frac{x-u}{s}) dx. \qquad (54)$$

q the order of moments and l is the line of local maxima. The scaling function $\tau(q)$ measuring the asymptotic decay of $Z(q,s)$ at fine scales s is given by:

$$\tau(q) = \lim inf_{s \to 0} \frac{\log Z(q,s)}{\log s} \qquad (55)$$

which means that

$$Z(q,s) \sim s^{\tau(q)} \qquad (56)$$

Finally, the spectrum of singularities $D(\alpha)$ is defined by :

$$D(\alpha) = \inf_{q \in R} (q\alpha - \tau(q) + c) \qquad (57)$$

where the exponent α measures the local singularity force and $\alpha = \frac{d\tau(q)}{dq}$

Then they applied the WTMM method to the study of crack propagation. They have calculated the fractal dimensions of the 1D profiles according to the distance from the site of the crack initiation.

Three profiles were distinguished in the $3D$ digitized fracture surface: zone 1 correspond to the crack initiation zone, zone 2 is the crack propagation one, and zone 3 corresponds to final rupture zone. Multifractal analysis of these surfaces was investigated: the multifractal spectra was estimated on the three crack zones with the Box-Counting method and compared to the WTMM. The WTMM method allowed a better characterization and differentiation of these zones, it is considered as an efficient numerical tool for fracture irregularity processing and for detect with better accuracy the values of the singularities stamped by the cracking.

In [57] the authors used multifractal functions to characterize vibratory signals whose regularity may change abruptly from one point to the next, because the study investigates the use of vibration measurements to perform the tool-failure detection. Using WTMM method applied to vibratory signals, Ouahabi *et al.* have shown that the vibratory response acquired during machining process has a multifractal behavior (the multifractal spectra $f(\alpha)$ was calculated from vibratory responses acquired during tool life). This made it possible to characterize the wear of the tools by multifractal analysis, and it is shown that it is an efficient tool wear monitoring system.

2) The wavelet leaders method: The wavelet leaders $l(j,k)$ was initiated by Jaffard et al. [31], defined from the discrete wavelet coefficients $d(j,k)$ see [33]. The construction of leaders is carried out from small to large scales. The leader $l(j,k)$ is defined as the maximum, in absolute value, of the coefficients $d(j,k1), d(j,k), d(j,k+1)$, as well as of all the parents of these three coefficients at finer scales.

In [35] Jaffard et al; compare mathematically multifractal formalisms based on the wavelet transform modulus maxima approach and on wavelet leader approach. They illustrate the theoretical comparison between WTMM and the wavelet leaders approach with an application in image processing fractography. With the discrete wavelet transform, $a = 2^j$, the partition function is given by:

$$Z(j,q) = \frac{1}{n} \sum_{k=1}^{n_j} |d_x(j,k)|^q, \qquad (58)$$

where n_j denotes the number of wavelet coefficients d_x available at scale 2^j.

$$Z(j,q) = \frac{1}{n}\sum_{k=1}^{n_j} |l_x(j,k)|^q, \qquad (59)$$

where n_j denotes the number of wavelet coefficients l_x available at scale 2^j. Then the spectrum of singularity of process X in R^d is :

$$D(h) = \min_{q\neq 0}(d + qh - \tau(q)) \qquad (60)$$

Authors showed that wavelet leader brings substantial theoretical, conceptual and practical improvements.

3) The wavelet packet transform method: The wavelet packet transform developed by Coifman in 1992 [20] was born from the desire to adapt to the time-frequency characteristics of signals. The wavelet method is a generalization of the wavelet decomposition, which offers a richer signal analysis. A transform level is indexed by three parameters: position, scale, and frequency. For an orthogonal wavelet function, the method generates a set of bases, called wavelet packet bases. Each of these bases offers a particular possibility of encoding the signal, while preserving its global energy, and an exact reconstruction.

Xiao Wang et al. investigated in [71] wavelet packet transform (WPT) for surface roughness characterization and surface texture extraction. Surface textures are analysed and separated by using wavelet packet transform in 2D simulation and they noticed that the reconstructed roughness and waviness coincide well with the original ones. They calculated the profile (Ra, Rq, Rc, Rku, and Rsk) and areal (Sa, Sq, Sv, Sp, Sku, and Ssk) roughness parameters of different surfaces to quantify the characterization results and they compared with those measured by a profile meter. They shown that extracted textures clearly exhibit the surface structure and the basic tool marks, performing texture classification and surface estimation. They concluded that WPT can characterizing surface finishes, including evaluating surface roughness and extracting surface texture.

IV. Conclusion

Various mathematical and physical models for the characterization of rough surfaces in adhesion problems of glued assemblies have been reviewed in this paper. These models included two different approaches to explain the roughness effect on the adhesion in thin films. Some models presented do not directly concern the adhesion of assemblies but could however be adapted to this problem and to other physical situations. The first approach is based on the homogenization method corresponding to a periodic rough surface. Conversely, in the second approach, the surface with multi-scale properties is characterized by fractal geometry.

Homogenization methods are highly valued techniques for studying the influence of roughness both among physicists and mathematicians. This method has once again recently demonstrated its effectiveness: one of the advantages, for example, of two-scale homogenization procedure, is that it reduces the problem with two different size scales on the micro and macro-levels to a single-scale problem. Now, if we consider that the surface is fragmented and discontinuous, this is the fractal geometry which is the powerful tool for the characterization of roughness. The advantage of fractal geometry compared to the homogenization method is that it incorporates the irregularities of the rough surface. The fractal analysis of rough surfaces in the context of glued assemblies is based on the fractal dimension. But for a complex structure, the fractal dimension alone cannot characterize the complexity completely specifically with a multifractal behaviour. Then to detect the multi in multifractal scaling the multifractal analysis is used to compute a singularity spectrum, a set of fractal dimensions. In order to determine the spectrum of singularities there exists an efficient and accurate numerical algorithm: the wavelet transform modulus maxima (WTMM) which makes it possible to determine the singularity spectrum $D(h)$ via the scaling exponents of some partition functions (q) defined on the skeleton. Using wavelet transform, the WTMM method has already made it possible to better understand many different problems and has been generalized for 2D signal for multifractal analysis of rough surfaces. However, other methods are discussed which seem very effective, it appears that to compute the multifractal spectra, the discrete wavelet techniques based on the wavelet leaders are the most robust.

The purpose of this article was to provide a detailed overview of the mathematical toolbox for mechanics and physicists, in therange of rough assembly surfaces. The different methods discussed will be relevant to this area of research.

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