Cascade Failure Evolution of Bilayer Rail Transit Network Considering Station Resistance Characteristics

Xuejiao Ma, Changfeng Zhu, Zongfang Wang, and Xianen Yang

Abstract—The multi-layer integrated rail transit network provides the foundation for promoting the metropolitan area’s long-term development. It is possible to efficiently prevent accidents and assure operational safety by simulating the cascade failure phenomenon of network stations after the actual integration operation, analyzing the weak stations of the network and executing targeted traffic relief measures. Based on the perspective of a multi-layer network, a construction method for a bilayer rail transit network was proposed, and the coupled map lattice considering the station resistance characteristics (WH-CML) was proposed by analyzing the differences of stations in each layer. Additionally, the evaluation indicators were proposed for analyzing the results of network cascade failure. The research results reveal that the WH-CML model can significantly slow down the rate of cascade failure in the rail transit network compared to the W-CML model, so traffic control can significantly reduce the risk of failure spreading. The topological association resistance factor is more effective at suppressing the cascade failure spreading of stations, and cascade failure is more likely at stations with greater functional significance than topological significance, and urban rail transit stations are more susceptible to failure than suburban railway stations. Strengthening the resiliency of urban rail transit stations can assist in ensuring network stability more efficiently. The research results can serve as a guide for rail transit integration and secure operation.

Index Terms—Bilayer network, Rail transit, Coupled map lattices (CML), Resistance characteristic, Cascade failure

I. INTRODUCTION

The progressive enhancement of rail transit integration and construction will make the rail transit network connect more smoothly, operate more efficiently, and travel more conveniently. In comparison to road traffic, inclement weather, unanticipated events, and an unreasonable operation organization plan are more likely to result in challenging coordination of capacity resource allocation in a short period, resulting in operational accidents or passenger congestion in the rail transit system [1]. At the same time, due to the rising density of the traffic line and network, rail transit exhibits the characteristics of network operation, the failure of individual stations is very likely to cause cascading failure in the network. The study of the cascade failure evolution process of the rail transit network can serve as guidance for safe operation under network formation conditions.

Since the discovery of the small-world [2] and scale-free properties [3] of complex networks, numerous scholars have devoted themselves to examining the application of complex networks in transportation [4][25]. The main research included transportation network optimization [6][7], transportation network characterization [7][8], traffic dynamics analysis [9][10][11], etc. Realistic complex networks can be categorized as social, informational, technological, or biological [12]. The transportation network belongs to the category of technological networks and is commonly defined in space L, space P, and space R. SEN et al. [13] established an Indian railroad traffic transport network in space P and analyzed its transport characteristics. KURANT et al. [14] compared the network characteristics of a Central European railroad network and a Swiss railroad network in space P and analyzed it's transport characteristics. Diab [16] built a rail transit network for the city of Toronto based on space L, and the impact of outdoor track sections and weather conditions on subway operational delays was analyzed. On the basis of these studies, a number of researchers gradually realized that each transportation system is not an isolated entity and that there are frequent interactions between systems, as a result, they proposed the method for building a multi-layer transportation topological network model. Current approaches to multi-layer network modeling tend to abstract distinct modes of transportation as separate layers [17], or entities performing distinct divisions of labor in the same mode of transportation as separate layers [18]. Based on the multi-layer network theory, Luo et al. [19] and Li [20] developed a bus-metro network model in Beijing and Chongqing, respectively, and revealed the

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small-world and scale-free characteristics of the bus-metro network by analyzing its network characteristics. Xu [21] established an undirected and unweighted Chinese HSR-CA composite network, analyzed the network characteristics of each sub-network and the composite network separately, and concluded that both the sub-network and the composite network are scale-free networks. Currently, the application of multi-layer networks in transportation focuses mainly on heterogeneous bilayer networks consisting of various transportation modes. In the process of building a multi-layer network, the coupling relationship between layers is commonly neglected.

There is a widespread cascading failure in the transportation system [22], and the resulting congestion spreading poses a major safety hazard. Complex networks provide novel approaches for analyzing cascading failures in transportation networks [5] [23], the widely used complex network cascade failure models include the over-percolation model, the load-capacity model, the Couple Map Lattices model, and the binary failure model [24]. Sun [25] proposed a method for analyzing the vulnerability of rail transit networks by combining the effects of topology and passenger flow and simulating the cascading failure phenomenon in rail transit systems through the weighted coupled map lattice (W-CML). Lu [26] improved the traditional CML model by considering the ability of stations to evacuate passengers, then used the improved model to simulate the cascade failure of the rail transit network and analyzed the impact of the cascade failure process on the functionality of network. Zhang [27] analyzed the impact of cascade failure on the connectivity and functionality of rail transit networks using various attack strategies. Ye [33] simulated the rail transit network cascade failure process through a load-capacity model and analyzed the impact of private car restriction policies on the cascade failure process and network vulnerability. Ma [28] improved the traditional CML model based on the Graph Attention Network (GAT) by incorporating the dynamic behavior of passengers, and applied it to the study of cascade failure in urban rail transit. Zhang [30] simulated the cascade failure process of a directionally weighted urban rail transit network using a linear threshold model and analyzed the impact of cascade failure on network robustness under the influence of passenger flow by constructing a comprehensive robustness evaluation index.

The majority of the above studies were conducted from the perspective of a single-layer network. However, with the gradual improvement of the multi-layer network theory, the focus of cascade failure research has shifted from a single-layer network to a multi-layer network [31]. Buldyrev et al. [32] first simulated the cascade failure process on a bilayer interdependent network, subsequently, a growing number of researchers carried out studies on the cascade failure of multi-layer networks. In transportation research domains, similar theories have also been widely applied. Zhang [29] developed a bilayer interdependent network model for urban public transportation, simulated the cascading failure process through the modified load-capacity model, and verified the feasibility of the urban public transportation interdependent network construction method. Ma [34] improved the traditional CML model by quantifying external perturbations through storm intensity and applied it to the study of cascading failures in bus-subway bilayer networks to analyze the impact of storm intensity on cascading failure results. Wang [35] proposed a method for constructing road-rail intermodal networks, simulated the intermodal network cascade failure process, and analyzed the impact of transshipment and cross-country long-distance transport on the vulnerability of intermodal networks through the CML model. Yin [36] described the urban rail transit network as a multi-layer network by considering the heterogeneity among the lines of the urban rail transit network, and performed cascading failure analysis for the cases of station failure, side failure, and line failure, respectively. Among the above cascade failure models, the CML model is widely used in the study of cascade failure in transportation networks because it accurately represents the chaotic nature of traffic flow. Numerous researchers are currently attempting to improve the traditional CML model, which takes topological factors into account. There have been beneficial investigations regarding weights [34], directions [33][37], external perturbations [26] and station resistance [38]. Currently, research on the consideration of station resistance factors in CML models is in its early stages, with few researches taking into account the combined effects of multiple factors as well as the characteristics of actual transportation networks.

In conclusion, the existing literature has laid a solid foundation for the study of cascade dynamics in bilayer rail transit networks, but more research is still necessary. Specifically: (1) The modeling methods for multi-layer traffic networks proposed in the established literature tend to ignore the coupling relationship between layers; (2) Most scholars treat stations as passive subjects in the application of CML to study cascade failures in transportation networks, ignoring the resistance characteristics of stations; (3) The combined effects of multiple factors on the cascade failure process of transportation networks are yet to be further studied.

This paper proposed the construction method of a bilayer rail transit topology network from the perspective of a multi-layer network. On this basis, the traditional W-CML model was optimized and the coupled map lattice considering the station resistance characteristics (WH-CML) was proposed, then the improved model was applied to simulate the cascade failure phenomenon of the bilayer rail transit network. Finally, indicators of connectivity, effectiveness, and resilience were introduced in order to analyze the network cascade failure process and its key influencing factors.

II. BI LAYER TOPOLOGY NETWORK CONSTRUCTION

A complex network mathematical model based on the Space L is used to describe the topology structure of a bilayer rail transit network (BRTN) with the following abstraction rules:

**Network:** The BRTN is defined as a bilayer, non-directional, weighted network made up of an urban rail transit network and a suburban railway network.

**Node:** Each station of the BRTN acted for a node.
Edge: The edge of the BRTN is consist of an intra-layer edge set and an inter-layer edge set. If station \( i \) and station \( j \) are adjacent stations in at least one train operation and are located in the same layers of the network, then these two stations have connected edges and all connected edges constitute the intra-layer edge set; if station \( i \) and station \( j \) are located in different layers of the network and can be reached by intra-station transfer or the distance of extra-station transfer does not exceed 1km, then these two stations have connected edges and all connected edges constitute the inter-layer edge set.

Weight: The weight of the BRTN is the passenger flow between station \( i \) and station \( j \) zones.

Based on these assumptions, the topology of the BRTN is described as follows:

\[
\text{BRTN} = (L, N, E, W) \tag{1}
\]

Where, \( L = \left\{ l_1(l_1), l_2(l_2) \right\} \) is the set of layers of BRTN, \( l_1(l_1), l_2(l_2) \) are the urban rail transit layer and suburban railway layer respectively; \( N = \left\{ N(l_1), N(l_2) \right\} \) is the set of nodes of BRTN, \( N(l_1), N(l_2) \) are the stations within each layer respectively; \( E = \left\{ E(l_1(l_1)), E(l_1(l_2)) \right\} \), \((n \neq m=1,2)\) is the set of edges of BRTN, \( E(l_1(l_1)), E(l_1(l_2)) \) are the intra-layer and inter-layer edges of the network respectively; \( W = \left\{ W(l_1), W(l_1(l_2)) \right\} \), \((n \neq m=1,2)\) is the set of weights of BRTN, \( W(l_1), W(l_1(l_2)) \) are the intra-layer connected edge weights and inter-layer connected edge weights of the network respectively.

The adjacency matrix \( A^{\text{double}} \) of BRTN is described as

\[
A^{\text{double}} = \begin{bmatrix} A^{(1)} & A^{(2)} \\ A^{(2)} & A^{(2)} \end{bmatrix} \tag{2}
\]

Where, \( A^{(1)} = \begin{bmatrix} a^{(1)}_{11} & \cdots & a^{(1)}_{1N} \\ \vdots & \ddots & \vdots \\ a^{(1)}_{N1} & \cdots & a^{(1)}_{NN} \end{bmatrix} \) and \( A^{(2)} = \begin{bmatrix} a^{(2)}_{11} & \cdots & a^{(2)}_{1M} \\ \vdots & \ddots & \vdots \\ a^{(2)}_{M1} & \cdots & a^{(2)}_{MM} \end{bmatrix} \)

is the intra-layer adjacency matrix. \( N, M \) is the total number of stations in each layer respectively, the values of \( a^{(1)}_{ij}, a^{(2)}_{ij} \) are:

\[
a^{(1)}_{NN} = \begin{cases} 1, & i^{(1)}, j^{(1)} \in E(l_1(l_1)) \\ 0, & i^{(1)}, j^{(1)} \notin E(l_1(l_1)) \end{cases} \tag{3}
\]

\[
a^{(2)}_{MM} = \begin{cases} 1, & i^{(2)}, j^{(2)} \in E(l_1(l_2)) \\ 0, & i^{(2)}, j^{(2)} \notin E(l_1(l_2)) \end{cases} \tag{4}
\]

\[
A^{(2)} = A^{(2)} = \begin{bmatrix} a^{(2)}_{11} & \cdots & a^{(2)}_{1M} \\ \vdots & \ddots & \vdots \\ a^{(2)}_{M1} & \cdots & a^{(2)}_{MM} \end{bmatrix} \tag{5}
\]

is an inter-layer adjacency matrix. The values of \( a^{(2)}_{ij} \) is:

\[
a^{(2)}_{MM} = \begin{cases} 1, & i^{(2)}, j^{(2)} \in E(l_1(l_2)) \\ 0, & i^{(2)}, j^{(2)} \notin E(l_1(l_2)) \end{cases} \tag{6}
\]

The connected edge weight matrix \( W^{\text{double}} \) of BRTN is described as

\[
W^{\text{double}} = \begin{bmatrix} W^{(1)} & W^{(2)} \\ W^{(2)} & W^{(2)} \end{bmatrix}
\]

Where, \( W^{(1)} = \begin{bmatrix} q^{(1)}_{11} & \cdots & q^{(1)}_{1N} \\ \vdots & \ddots & \vdots \\ q^{(1)}_{N1} & \cdots & q^{(1)}_{NN} \end{bmatrix} \), \( W^{(2)} = \begin{bmatrix} q^{(2)}_{11} & \cdots & q^{(2)}_{1M} \\ \vdots & \ddots & \vdots \\ q^{(2)}_{M1} & \cdots & q^{(2)}_{MM} \end{bmatrix} \)

is intra-layer connected edge weight matrix, the value of weight is defined as the cross-sectional passenger flow between station \( i \) and station \( j \), then:

\[
q^{(1)}_{NN} = \begin{cases} q_{ij}, & i^{(1)}, j^{(1)} \in E(l_1(l_1)) \\ 0, & i^{(1)}, j^{(1)} \notin E(l_1(l_1)) \end{cases} \tag{7}
\]

\[
q^{(2)}_{MM} = \begin{cases} q_{ij}, & i^{(2)}, j^{(2)} \in E(l_1(l_2)) \\ 0, & i^{(2)}, j^{(2)} \notin E(l_1(l_2)) \end{cases} \tag{8}
\]

Where, \( q_{ij} \) is the cross-sectional passenger flow between station \( i \) and station \( j \).

The weight value of the passenger flow between station \( i \) and station \( j \) is defined as follows:

\[
q^{(1)}_{NN} = \begin{cases} q_{ij}^{(1)}, & i^{(1)}, j^{(1)} \in E(l_1(l_1)) \\ 0, & i^{(1)}, j^{(1)} \notin E(l_1(l_1)) \end{cases} \tag{9}
\]

Where, \( q_{ij}^{(1)} \) and \( q_{ij}^{(2)} \) are passenger exchange flow at station \( i \) and station \( j \), \( s_{ij}^{(1)}, s_{ij}^{(2)} \) are the strength of station \( i \) and station \( j \) respectively.

The abstract rules of the rail transit topology network are shown in Fig.1.

III. ANALYSIS OF CASCADING FAILURES IN A BILAYER RAIL TRANSIT NETWORK BASED ON WH-CML

A. Improved WH-CML cascade failure model

In the application of traffic cascade dynamics, the W-CML model generally regards traffic nodes as passive, non-resisting subjects. However, in the actual traffic network, traffic control measures can confer a certain resistance to failure on a station, with the resistance of the station reflecting the resistance characteristics. It is further
improved by introducing a resistance characteristic factor[38] \((H, [H] \in (0,1))\) to characterize the ability of the station to resist failure. A comparison of the network failure process between the W-CML model and the WH-CML model is shown in Fig.2.

![Fig. 2. Comparison of network failure process between the W-CML model and WH-CML model](image)

A straightforward indication of the magnitude of the resistance of the station in terms of degree. As shown in Figure 2, the WH-CML model is able to effectively avoid failure due to station resistance while also avoiding failure to continue spreading to neighboring stations. The cascade failure process of the bilayer rail transit network considering the station resistance characteristics is shown in Fig.3.

![Fig. 3. The cascade failure process of the bilayer rail transit network](image)

As shown in Fig.3, when the station fails, the passenger flow from the failed station will arrive at the adjacent station alongside the train operation, and the overloaded passenger flow may lead to the secondary failure of the adjacent station. In actual operation, measures such as flow restriction can be taken to resist the occurrence of failure phenomena, if the resistance is successful, the failure effect will not propagate further; if the resistance fails, the failure effect will cascade throughout the bilayer rail transit network.

The time-varying state evolution law of the station \(x_i^{(n)}\) is revealed by introducing a chaotic logistic mapping function \(f\left[x_i^{(n)}(t)\right]\), i.e.

\[
x_i^{(n)}(t+1) = f\left[x_i^{(n)}(t)\right] = \frac{1}{H}\left[1 - \varepsilon_1 - \varepsilon_2\right] f\left[x_i^{(n)}(t)\right] + \frac{\varepsilon_1}{1} \sum_{j=1}^{N} a_i j f\left[x_j^{(n)}(t)\right] / k(f^{(n)}) + \frac{\varepsilon_2}{1} \sum_{j=1}^{N} w_j(t) f\left[x_j^{(n)}(t)\right] / s(f^{(n)})
\]

(11)

Where, \(x_i^{(n)}(t+1), x_i^{(n)}(t)\) are the state of station \(i\) in layer \(n\) of the bilayer rail transit network at moment \(t+1\) and moment \(t\), to determine whether the station \(i\) is in the failed state; \(k(f^{(n)}), s(f^{(n)})\) are the degree and strength of station \(i, a_{ij}\) reflects the connection relationship between station \(i\) and station \(j\), if both station \(i\) and station \(j\) are located in the layer \(n\) of the network, then \(a_{ij} = a_{ji} = a_{ij}^{(n)}\); if station \(i\) and station \(j\) are located in the layer \(n\) and layer \(m\) of the network, then \(a_{ij} = a_{ji} = a_{ij}^{(m)}\). \(H\) is the resistance characteristic factor, the larger \(H\) is, the stronger the ability to resist the station failure. Considering that in actual operation, the more important transportation hubs often take longer to return to normal after a failure, the topological association resistance factor and functional association resistance factor are defined to portray their ability to resist failure propagation:

\[
H_d^{(n)} = \sum_{j=1}^{N} k_j^{(n)} - k_i^{(n)}
\]

(12)

\[
H_s^{(n)} = \sum_{j=1}^{N} s_j^{(n)} - s_i^{(n)}
\]

(13)

Where, \(H_d^{(n)}\) is the topological association resistance factor, \(k_i^{(n)}, k_j^{(n)}\) are the degree of station \(i\) station \(j\) in layer \(n\); \(H_s^{(n)}\) is the functional association resistance factor, \(s_i^{(n)}, s_j^{(n)}\) are the strength of station \(i\) station \(j\) in layer \(n\); \(h_d^{(n)}\), \(h_s^{(n)}\) are the topological basis resistance factor and functional basis resistance factor of layer \(n\), respectively, quantifying the total resistance magnitude imposed on each layer network.

Considering that the topological association resistance factor and the functional association resistance factor have varying degrees of influence on the station, introducing a weighting coefficient \(\eta\), the integrated resistance characteristic factor of station \(i\): \(H = \eta H_d^{(n)} + (1 - \eta) H_s^{(n)}\)

(14)

If the state of station \(i\) remains \(0 \leq x_i^{(n)}(t) \leq 1\) then station \(i\) is in a normal state, if the state of station \(i\) is \(x_i^{(n)}(t) \geq 1\) at moment \(t+1\), then the station is in a failed state, and at any time thereafter \(x_i^{(n)}(t) = 0\). The state values of other stations develop according to equation (11).

As the station state error value increases exponentially with the number of iterations, the initial state value of the station influences the accuracy of the WH-CML model, the values are defined as the cross-sectional full load ratio of...
arriving trains in adjacent intervals at station $i$ in the unit period $l$ preceding the occurrence of failed:

$$x_i^{(t)}(t-1) = \frac{1}{k} \sum_{l} y_i^{(t)}(l) C_i^{(t)}(l)$$  \hspace{1cm} (15)$$

Where, $y_i^{(t)}(l)$ is the cross-sectional passenger flow of each adjacent section at station $i$ of layer $n$ of the network in unit period $l$, $C_i^{(t)}(l)$ is the sum of the capacity of all rail trains passing through station $i$ of layer $n$ of the network in unit period $l$.

B. Cascade failure analysis method for bilayer rail transit network

In order to analyze the cascade failure process in a bilayer rail transit network, an external disturbance $R$ ($R \geq 1$) is applied to station $i$ at moment $t$ to simulate the failure of a single station. The combined effect of network topology and passenger flow distribution may lead to cascade failures at adjacent stations at the next moment, which in turn may cause cascade failure phenomena in the network. The cascade failure model considering the disturbance parameter $R$ is

$$x_i^{(t)}(t) = \frac{1}{H} \left[ (1 - \varepsilon_i - \varepsilon_j) f \left[ x_i^{(t)}(t-1) \right] + \varepsilon_i \sum_{j \in \text{neigh}(i)} a_{ij} f \left[ x_j^{(t)}(t-1) \right] / k^{(t)} + \varepsilon_j \sum_{i \in \text{neigh}(j)} w_{ij} f \left[ x_j^{(t)}(t-1) \right] / s^{(t)} \right] + R \hspace{1cm} (16)$$

As a result of the perturbation parameter, the state value of station $i$ at moment $t$ becomes $x_i^{(t)}(t) \geq 1$, a failed state, then at moment $t+1$ the failed station $i$ is removed from the network and marked $x_i^{(t)}(t+1) = 0$.

Since the model has a propagation mechanism, all stations connected to station $i$ are affected by the state of station $i$ at moment $t$, which may cause a new round of station failures, and the state values of these stations can be obtained according to equation (11).

The cascading failure process of network stations can lead to changes in the global nature of the rail transit network. Construct connectivity, effectiveness, and resilience indicators to evaluate the outcomes of network cascade failures, reveal the impact of W-CML and WH-CML cascade failure models on network:

Connectivity indicator:

$$G(t) = \frac{n_i^{(1)}(t) + n_i^{(2)}(t)}{N_i^{(1)} + N_i^{(2)}} \hspace{1cm} (17)$$

Where, $G(t)$ is the proportion of failed stations to the total stations number in the network at moment $t$, which reflects the impact of the cascade failure process on the global connectivity of the network; $n_i^{(1)}(t)$, $n_i^{(2)}(t)$ are the number of stations in the urban rail transit layer and suburban railway layer that are in the failed state at moment $t$, respectively; $N_i^{(1)}$, $N_i^{(2)}$ are the total number of stations in urban rail transit layer and suburban railway layer, respectively. When no additional stations in the network fail, the final cascade failure size is given by $N_f$:

$$N_f = \lim_{t \to \infty} \left( n_i^{(1)}(t) + n_i^{(2)}(t) \right) \hspace{1cm} (18)$$

Effectiveness indicator:

$$E(t) = \frac{1}{N(N-1)} \sum_{i,j} d_{ij}(t) \hspace{1cm} (19)$$

Where, $E(t)$ is the average network efficiency at moment $t$, which reflects the impact of the cascade failure process on the shortest path between stations; $N$ is the number of stations in the bilayer rail transit network, $d_{ij}(t)$ is the length of the shortest path between station $i$ and station $j$ at moment $t$. When no path exists between station $i$ and station $j$, $d_{ij}(t) = +\infty$.

Resilience indicator:

$$D(t) = -\sum_{i} p_i(t) \cdot \log_2 p_i(t) \hspace{1cm} (20)$$

Where, $D(t)$ is the network Shannon entropy at moment $t$, which reflects the impact of the cascade failure process on station resilience; $p_i(t)$ is the ratio of the degree value of station $i$ to the total degree value of the network at moment $t$. The greater the amount of average information that can be delivered after the failure of a station in the network, the longer the repair time and the more difficult it is to return to normal.

C. Simulation process for cascading failure analysis of bilayer rail transit network

Based on the WH-CML model, the simulation algorithm flow for cascade failure analysis of the bilayer rail transit network is obtained as follows:

Step 1: Construct a bilayer topology network based on the actual rail transit network and load the passenger flow, then generate the connection matrix $E$ and the connected edge weight matrix $W$;

Step 2: Analyze the basis resistance factor based on the state of topology network, and generate the topological association resistance factor $H_j^{(t)}$ and functional association resistance factor $H_j^{(t)}$ of each station;

Step 3: The initial state of the station $x_i^{(t)}(t-1)$ is determined by the initial cross-sectional passenger flow between the adjacent zones of each station;

Step 4: At moment $t$, an external perturbation $R$ is applied to the station $x_i^{(t)}$, and its state value is updated to $x_i^{(t)}(t) = x_i^{(t)}(t-1) + R$;

Step 5: At moment $t+1$, update the state values of the stations according to the WH-CML model, mark the stations with $x_i^{(t)}(t) \geq 1$ as failed and remove them from the network, calculate the evaluation indicator $G(t)$, $E(t)$ and $D(t)$ to record the current state of the network;

Step 6: Repeat step 5 until the number of failed stations no longer changes, propagation of cascade failure ends, and the cascade failure results are output.
IV. EXAMPLE ANALYSIS

The Beijing rail transit network is selected for simulation analysis, and its topology, which consists of 324 nodes and 365 edges is shown in Fig.5, and the network failure cascade propagation phenomenon is simulated based on the WH-CML model.

The stations with maximum degree value and average degree value in the urban rail transit system and suburban railway system were selected to analyze the influence of station topological significance on cascade failure results, while the stations with maximum strength and average strength were selected to analyze the influence of station functional significance on cascade failure results. The selected station attributes are as follows:

<table>
<thead>
<tr>
<th>Network</th>
<th>Characteristic</th>
<th>Serial number</th>
<th>Degree value</th>
<th>Strength value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suburban Railway</td>
<td>The maximum degree</td>
<td>317</td>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>The average degree</td>
<td>312</td>
<td>2</td>
<td>420</td>
</tr>
<tr>
<td></td>
<td>The maximum strength</td>
<td>313</td>
<td>2</td>
<td>630</td>
</tr>
<tr>
<td></td>
<td>The average strength</td>
<td>309</td>
<td>1</td>
<td>210</td>
</tr>
<tr>
<td>Urban Rail Transit</td>
<td>The maximum degree</td>
<td>23</td>
<td>6</td>
<td>99170</td>
</tr>
<tr>
<td></td>
<td>The average degree</td>
<td>212</td>
<td>2</td>
<td>12073</td>
</tr>
<tr>
<td></td>
<td>The maximum strength</td>
<td>97</td>
<td>4</td>
<td>131617</td>
</tr>
<tr>
<td></td>
<td>The average strength</td>
<td>148</td>
<td>2</td>
<td>31181</td>
</tr>
</tbody>
</table>

The W-CML and WH-CML models were used to simulate the results of cascade failure, respectively. Take $R=5$, $\varepsilon_1=\varepsilon_2=0.25$, $\eta=0.5$. Considering the difference between the total number of stations in each layer of the network and ensuring the visibility of the simulation effect, the initial topological association resistance factor and functional association resistance factor of the suburban railway layer and urban rail transit layer are set as: $[h^{(d)}, h^{(s)}] = [h^{(d)}, h^{(s)}] = [0.2, 0.01]$. The final simulation results for each station are shown in Fig.6-8.
As shown in Fig.6-8, the WH-CML model has a considerable inhibitory effect on network cascade failure, indicating that traffic control after a failure occurs can significantly mitigate the risk of failure propagation. In the case where resistance characteristics were considered, only station 313 with the maximum strength among the suburban railway stations was attacked, and the failure cascade spread to the entire network, while only station 212 with lower strength and degree values among the urban rail transit stations was attacked, and the failure did not spread. This demonstrates that urban rail transit stations are vital to the rail transit network, and that appropriate management mechanisms should be established immediately after a failure occurs.

In light of the fact that coupling a single-layer network to a bilayer network and adjusting the relevant model parameters can affect failure propagation, a parametric perturbation analysis is required to assess the impact of each factor on cascade failure.

A. Influence of external disturbances and station resistance on the cascade failure process

The urban rail transit stations with the maximum degree value (23) and the maximum strength (97) were selected to further analyze the results of cascade failure simulation using W-CML and WH-CML models under different external disturbances, respectively. The simulation results of the station with the maximum degree are shown in Fig.9-11 and the simulation results of the station with the maximum strength are shown in Fig.12-14.

![Fig. 9. Proportion of failed stations when station 23 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model](image)

![Fig. 10. Average network efficiency when station 23 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model](image)
Fig. 11. Shannon entropy of network when station 23 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model.

Fig. 12. Proportion of failed stations when station 97 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model.

Fig. 13. Average network efficiency when station 97 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model.
As shown in Fig. 9-11, as the external disturbance increases, the network failure rate and failure size increase, while the average efficiency and entropy values decrease. Under the same external disturbances, the ratio of cascade failure, as well as the rate of decrease in average network efficiency and entropy value, are substantially slower in the WH-CML model than in the W-CML model. It can be seen that traffic control measures at the station can inhibit the process of failure propagation, allowing the operation department more time to repair the failure and resume normal operation. At $R \leq 2.5$, the failure propagates only in a small region, and the average network efficiency and entropy values are maintained at a high level, demonstrating that traffic diversion measures for stations can be more effective in preventing the propagation of cascade failures when the external disturbances are small.

As shown in Fig. 12-14, as the external disturbance increases, the network failure rate and size, average efficiency, and entropy value for the station with the maximum strength follow the same trend as when the failure occurs at the station with the maximum degree, while the cascade failure threshold is lower but the propagation rate is slower when the failure occurs at the station with the maximum degree compared to the station with the maximum degree. Simultaneously, it can be observed that the final network entropy value is significantly higher when the failure occurs at the station with the maximum strength, indicating that the network is more resilient after the cascade failure caused by the station with the maximum strength, which can restore the normal state more rapidly.

B. Influence of topological network coupling coefficient on the cascade failure process

The topological network coupling coefficient $\varepsilon_1$ depicts the closeness of the topological relationship between stations, and the larger $\varepsilon_1$ is, the greater the interaction between stations. The urban rail transit stations with the maximum degree value (23) and the maximum strength (97) were selected, and the W-CML and WH-CML models were applied to cascade failure simulation under the condition of $R=2$, and the simulation results are shown in Figures 15 and 16.

Fig. 14. Shannon entropy of network when station 97 is attacked by different external disturbances (a) for the W-CML model (b) for the WH-CML model

Fig. 15. Proportion of failed stations for different topological network coupling coefficient (a) when station 23 is attacked (b) when station 97 is attacked
As shown in Fig. 15-16, the cascade failure rate and failure scale increase as the topological network coupling coefficient rises. The main reason is that the increase in topological network coupling coefficient leads to an increase in station-to-station interaction, which makes the cascade failure phenomenon more likely to occur.

Comparing the cascade failure laws exhibited by the WH-CML and W-CML models with different topological network coupling coefficients, it can be seen that the failure is more difficult to propagate and the cascade failure scale is smaller under the condition that the resistance characteristic factor is applied to the station. It is further demonstrated that applying resistance measures to the station can effectively suppress the occurrence of cascade failure.

C. Influence of passenger flow distribution coupling coefficient on the cascade failure process

The passenger flow distribution coupling coefficient $\varepsilon_2$ depicts the closeness of passenger flow interaction between stations, and the larger $\varepsilon_2$ is, the more frequent the passenger flow interaction between stations. The urban rail stations with the maximum degree value (23) and the maximum strength (97) were still selected, and the W-CML model and WH-CML model were applied to cascade failure simulation under the condition of $R=2$, and the simulation results are shown in Figures 17 and 18.

As shown in Fig. 17-18, as the passenger flow distribution coupling coefficient increases, the cascade failure rate and
failure scale also increase, similar to the pattern exhibited by the topological network coupling coefficient. This is mainly due to the fact that the increase in passenger flow distribution coupling coefficient leads to more frequent station-to-station passenger interactions and the easier influx of passenger flow from failed stations to normal stations, thus making the cascade failure phenomenon more likely to propagate.

D. Influence of the resistance factor weighting coefficient on the cascade failure process

Considering the variability of the topological association resistance factors and functional association resistance factors on stations, the cascade failure process when the urban rail transit station with the maximum degree value (23) and the maximum strength (97) is attacked under the condition of $R=3$ is taken as an example, to analyze the influence of the weighting coefficient in the WH-CML model on the cascade failure process, and the simulation results are shown in Fig. 19.

As shown in Fig. 19, the cascade failure size in the first 10 time steps decreases gradually with the increase of $\eta$ for both the station with the maximum degree and the maximum strength, indicating that the topological association resistance factor has a stronger inhibitory influence on the cascade failure propagation of the station. It demonstrates that, under identical initial conditions, the resistance implemented according to the topological association resistance factor has a greater impact on the initial cascade failure propagation of the rail transit network, allowing the operation department to remove the failure as soon as possible at the early stage of the failure propagation to ensure the normal operation.

E. Influence of the basis resistance factor on the cascade failure process

According to the analysis of the discussion in (4), it is known that the topological association resistance factor $H^{(1)}_d$ has a greater inhibitory effect on the propagation of cascade failure, therefore, the variation of the topological basis resistance factor $H^{(1)}_d$ in different layers is analyzed further for the variability of its influence on the overall cascade failure process. Taking the urban rail transit station with the maximum degree (23) as an example, the influence of $H^{(1)}_d$, $H^{(2)}_d$ on the cascade failure results was verified by adjusting the initial topological basis resistance coefficient set $[b^{(1)}_d, b^{(2)}_d] = [0.2, 0.01]$ by $\pm 20\%$, respectively, under the condition of $R=3$, and the result is shown in Fig. 20.

As shown in Fig. 20, the network cascade failure process is more sensitive to the changes of $H^{(2)}_d$ in both speed and scale, indicating that the resistance of the urban rail transit layer is essential for maintaining the safety and stability of the overall rail transit network, further confirming the significance of the urban rail transit in the rail transit system.

Fig. 19. Influence of resistance factor weighting coefficient on cascade failure processes (a) for station 23 (b) for station 97

Fig. 20. Influence of basis resistance factor on cascade failure processes

As shown in Fig. 20, the network cascade failure process is more sensitive to the changes of $H^{(2)}_d$ in both speed and scale, indicating that the resistance of the urban rail transit layer is essential for maintaining the safety and stability of the overall rail transit network, further confirming the significance of the urban rail transit in the rail transit system.
F. Influence of bilayer network coupling on the cascade failure process

The WH-CML model is applied to analyze the change in connectivity and the number of failed stations for the single-layer suburban railway network, the single-layer urban rail transit network, the bilayer rail transit network, the suburban railway layer (layer 1) in the bilayer rail transit network, and the urban rail transit layer (layer 2) in the bilayer rail transit network, respectively. The stations with the maximum degree and strength of each network were selected for simulation. The variation of the network connectivity indicator by time step for different networks are shown in Fig.21-22. The variation of network failure stations by time step for different networks are shown in Fig.23-24.

As shown in Fig.21-24, the cascade failure process and results of urban rail transit stations in single-layer and bilayer networks are the same, whereas the scale of cascade failure of suburban railway stations in bilayer network is significantly higher than that in the single-layer network. This is primarily attributable to the sparse distribution of lines in the single-layer suburban railway network and the low degree of coupling between stations. However, in the bilayer network, the failure of urban rail transit stations will lead to the successive failure of suburban railway stations with inter-layer connected edges, stations are affected by the state of stations on the same layer and neighboring layers at the same time, thereby accelerating the propagation of cascade failure in the suburban railway network.
V. CONCLUSIONS

By constructing a bilayer rail transit network and simulating the cascade failure process using the WH-CML model that considers station resistance characteristics, the following conclusions were drawn:

1. The WH-CML model has a stronger inhibitory effect on network cascade failure propagation than the W-CML model, indicating that traffic control after a failure can significantly reduce the risk of failure propagation.

2. The station with the maximum strength is more susceptible to cascade failure than the station with the maximum degree in both resisted and unrestricted conditions, indicating that the station with high functional significance is more vulnerable than the station with high topological significance.

3. Cascade failures are more likely to spread in urban rail transit stations than in suburban railway stations. It can be seen that urban rail transit stations are crucial to the bilayer rail transit network, and practical and effective control measures should be implemented as soon as a failure occurs to ensure the stability of the rail transit network.

4. The increase of topological network coupling coefficient and passenger flow distribution coupling coefficient both increase the cascade failure rate and failure scale subsequently. The main reason is that the increase in topological network coupling coefficient leads to an increase in station-to-station interactions; the increase in passenger flow distribution coupling coefficient leads to more frequent station-to-station passenger flow interactions, thus easier cascade failure.

5. The topological association resistance factor is more effective in suppressing the station cascade failure propagation. Under the same initial conditions, the resistance implemented based on the topological association resistance factor has a greater impact on the initial cascade failure propagation of the rail transit network, making it easier for the operations department to troubleshoot the failure as soon as possible in the early stage of the failure propagation.

6. Network coupling increases the risk of cascade failure of non-critical stations in the single-layer network before coupling. The reason is that network coupling adds interlayer edges to the single-layer network, and stations are affected by the state of stations on the same layer and neighboring layers at the same time, which makes them more prone to cascade failure.

REFERENCES


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