Equivalent-Input-Disturbance-based Dissipative Control for Linear Time-Delay Systems

Chenhui Wu, Fang Gao*, XuYang Yuan, and Jiajun Wang

Abstract—**Dissipativity and disturbance rejection issues are taken into consideration for linear time-delay systems with disturbance in this study. For linear systems, the Equivalent-Input-Disturbance (EID) method has shown to perform well in terms of disturbance rejection. Therefore, the goal of this research is to achieve satisfactory disturbance rejection performance and dissipativity performance levels based on the EID technique. Initially, the state of the time-delay system is reconstructed using a modified proportional-integral observer. Then, a disturbance-rejection control law that includes disturbance information is designed. To guarantee the time-delay system's stability and achieve the dissipativity performance level, a sufficient condition is attained. The stability condition is used to design a state feedback controller. Finally, the effectiveness of the strategy is illustrated using a numerical example.**

Index Terms—**dissipative control, equivalent input disturbance, time-delay, proportional-integral observer.**

I. Introduction

THE primary focus of control research has been on the disturbance and time-delay of linear systems. The system's performance will suffer if there is a disturbance. The presence of time-delay may have negative effects on the system in many practical systems, including industrial processes, chemical processes, remote controls, economic processes, network controls, population dynamics, etc. In order to address these problems, researchers have made great efforts [1], [2].

Dissipation is an important part of control theory and also a hot topic in the field of control in recent years[3], [4], [5]. The concept of dissipation was first proposed in [6]. The dissipative theory proposes an approach to designing and analysing control systems where input and output are described in energy terms.

Numerous disturbance-rejection approaches have been put forth recently. Active Disturbance Rejection Control (ADRC) techniques are most common among them. The idea of ADRC is to actively estimate the value of the disturbance and then compensate the estimated value into the system to offset the negative effect of the disturbance on the system. In order to enhance the effectiveness of disturbance rejection, the Equivalent-Input-Disturbance (EID) technique was originally presented in cites [7], [8]. The EID technique excludes both matched and mismatched disturbances and does not require prior knowledge of disturbances. In a variety of control systems, the EID approach has shown a satisfactory performance in rejecting disturbances. [9], [10], [11], [12]. A Proportional-Integral Observer (PIO) incorporating an EID estimator to study disturbance-rejection performance in linear systems in [13]. This observer has the advantage of increasing both the flexibility of system design and the precision of system state estimation. Despite the fact that this research shows that the proposed approach performs well in terms of disturbance rejection, the time delay is not considered.

Following the above-mentioned discussion, the dissipative problem of a class of linear time-delay systems is investigated. The state of the time-delay plant is rebuilt using a modified PIO. The performance level of the closed-loop system's stability and dissipativity is guaranteed by a given sufficient condition. The condition is then used to design a state feedback controller. A numerical example is then utilized to show the efficiency of the presented approach.

Notations : $G(s)$ is indicated as the Laplace transforms of $g(t)$. $\begin{bmatrix} C & B \\ B^{\mathrm{T}} & D \end{bmatrix}$ is indicated by $\begin{bmatrix} C & B \\ \star & D \end{bmatrix}$. $A > 0$ is indicated as positive definite matrix.

II. Configuration of EID-based control system

Consider

$$\begin{cases} \dot{x}(t) = H_1 x(t) + H_2 x(t-h) + Ju(t) + J_d w(t), \\ y(t) = Nx(t), \\ z(t) = Nx(t) + Dw(t), \end{cases} \tag{1}$$

where $u(t) \in \mathbb{R}^m$, $x(t) \in \mathbb{R}^n$, $w(t) \in \mathbb{R}^{n_d}$, $z \in \mathbb{R}^p$, $y \in \mathbb{R}^q$, represent the control input, state, exogenous disturbance, control output, and output vectors, respectively. $h > 0$. H, J, J_d, N and D are constant matrices with appropriate dimensions.

Based on the concept of EID [8], we know an effect on the system that is equal to an exogenous disturbance from a control input channel which is characterized as a signal $(w_e(t))$. Fig. 1 shows the structure of the control system based on the EID method and using the modified PIO. Consequently, system (1) switches to the following system (2)

$$\begin{cases} \dot{x}(t) = H_1 x(t) + H_2 x(t-h) + Ju(t) + Jw_e(t), \\ y(t) = Nx(t), \\ z(t) = Nx(t) + Dw(t). \end{cases} \tag{2}$$

We choose the following modified PIO observer to replicate the state of the time-delay system

Manuscript received Dec 12, 2022; revised May 7, 2023. This work was supported by the National Natural Science Foundation of China under Grants 61803002, 62103006.

Chenhui Wu is a graduate student of Physics and Electronic Information, Anhui Normal University, Wuhu 241000, China (e-mail: 1151713803@qq.com).

Fang Gao is a professor of Physics and Electronic Information, Anhui Normal University, Wuhu 241000, China, and Anhui Conch Group Company Limited, Wuhu 241000, China (corresponding author to provide phone: +8618155334125; e-mail: gaof1988@126.com).

XuYang Yuan is a graduate student of Physics and Electronic Information, Anhui Normal University, Wuhu 241000, China (e-mail: 2945320321@qq.com).

Jiajun Wang is a graduate student of Physics and Electronic Information, Anhui Normal University, Wuhu 241000, China (e-mail: 3416692349@qq.com).

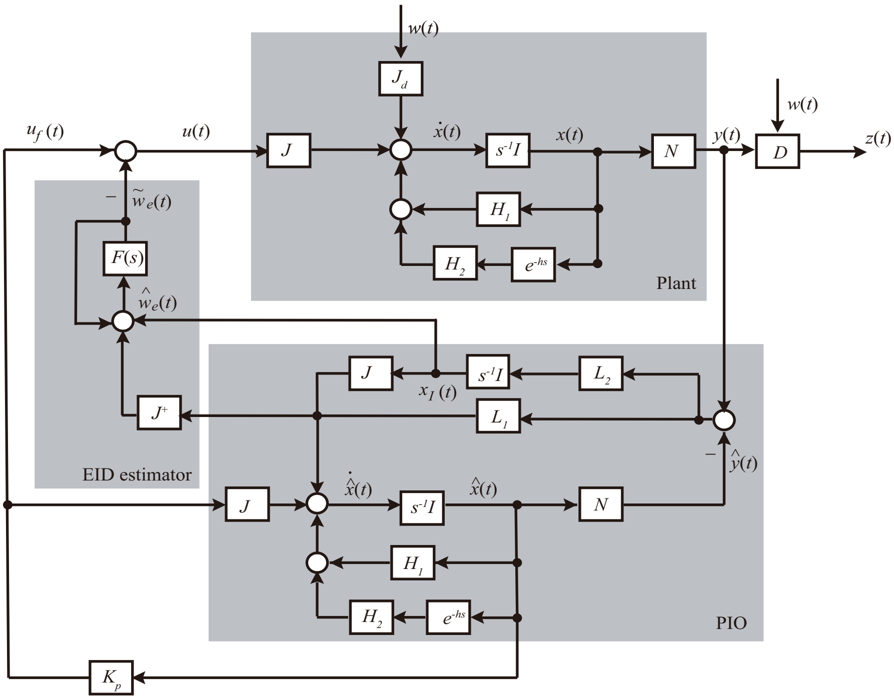

Fig. 1: EID-based control system configuration.

$$\begin{cases} \dot{\hat{x}}(t) = H_1\hat{x}(t) + H_2\hat{x}(t-h) + Ju_f(t) + Jx_I(t) \\ \qquad + L_1(y(t) - \hat{y}(t)), \\ \hat{y}(t) = N\hat{x}(t), \\ \dot{x}_I(t) = L_2(y(t) - \hat{y}(t)), \end{cases} \tag{3}$$

where $\hat{x}(t)$ is an estimate of the state $x(t)$. Letting

$$J^+ = (J^{\mathrm{T}}J)^{-1}J^{\mathrm{T}}, \tag{4}$$

$$\nabla x(t) = x(t) - \hat{x}(t), \tag{5}$$

Substituting this into (2) gives

$$\begin{aligned} \dot{\hat{x}}(t) &= H_1\hat{x}(t) + Ju(t) + Jw_e(t) + H_2\hat{x}(t-h) \\ &\quad + H_1\nabla x(t) - \nabla\dot{x}(t) + H_2\nabla x(t-h). \end{aligned} \tag{6}$$

Introducing a variable $\Delta w(t)$ satisfying

$$J\Delta w(t) = H_1\nabla x(t) - \nabla\dot{x}(t) + H_2\nabla x(t-h), \tag{7}$$

and let

$$\Delta w(t) + w_e(t) = \hat{w}_e(t), \tag{8}$$

by combining equations (7), (8) and (3), the disturbance estimation $\hat{w}_e(t)$ is easily obtained

$$\hat{w}_e(t) = J^+L_1N\nabla x(t) + u_f(t) - u(t) + x_I(t), \tag{9}$$

as detailed in [3].

The state-space of $F(s)$ is

$$\begin{cases} \dot{x}_F(t) = H_F x_F(t) + J_F\hat{w}_e(t), \\ \tilde{w}_e(t) = N_F x_F(t). \end{cases} \tag{10}$$

And

$$\tilde{W}_e(s) = F(s)\hat{W}_e(s), \tag{11}$$

where $\hat{W}_e(s)$ and $\tilde{W}_e(s)$are the Laplace transform of $\hat{w}_e(t)$ and $\tilde{w}_e(t)$,

The control system's new control law is

$$u(t) = u_f(t) - \tilde{w}_e(t) = K_P\hat{x}(t) - \tilde{w}_e(t). \tag{12}$$

III. Stability Analysis and Design

Definition 1. *If the energy supply function*

$$G(w,z,\tau) \geq \gamma < w,w >_\tau, \forall\tau \geq 0, \tag{13}$$

holds under a zero initial state, where $G(w,z,\tau) =< z,Az >_\tau$ $+2 < z,Bw >_\tau + < w,Cw >_\tau$ *and* $< a,b >_\tau = \int_0^\tau a^{\mathrm{T}}b\mathrm{d}t$ *for any* $\tau \geq 0$*, then system (1) is said to be strictly* $(A,B,C) - \gamma$ $-$ *dissipative. (*$\gamma > 0$*,* $A < 0$*, symmetric matrices* C*, and any real matrix* B*).*

Lemma 1 ([14])**.** *Regarding a specific symmetric matrix*

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12}^{\mathrm{T}} & \rho_{22} \end{bmatrix}, \tag{14}$$

the following are considered equivalent:

(a) $\rho < 0$*;*
(b) $\rho_{11} < 0$ *and* $\rho_{22} - \rho_{12}^{\mathrm{T}}\rho_{11}^{-1}\rho_{12} < 0$*; and*
(c) $\rho_{22} < 0$ *and* $\rho_{11} - \rho_{12}\rho_{22}^{-1}\rho_{12}^{\mathrm{T}} < 0$*.*

Let us suppose that the singular-value decomposition (SVD) of a matrix $\mathfrak{k}$ is as follows

$$\mathfrak{k} = \mathfrak{U}_1 \begin{bmatrix} \mathfrak{V}_1 & 0 \end{bmatrix} \mathfrak{T}_1{}^{\mathrm{T}}, \tag{15}$$

where $\mathfrak{V}_1 > 0$, $\mathfrak{U}_1$ and $\mathfrak{T}_1$ are unitary matrix.

An equivalent condition to the matrix equation $\mathfrak{k}X = \bar{\mathbb{X}}\mathfrak{k}$ is given by the following lemma.

Lemma 2 ([15])**.** *There exists a matrix* $\bar{\mathbb{X}} \in \mathbb{R}^{q\times q}$ *such that* $\mathfrak{k}X = \bar{\mathbb{X}}\mathfrak{k}$ *holds for every* $X \in \mathbb{R}^{n\times n}$*, but only if* X *can be* *broken down into its component parts*

$$X = \mathfrak{T}_1 \begin{bmatrix} \bar{\mathbb{X}}_{11} & 0 \\ 0 & \bar{\mathbb{X}}_{22} \end{bmatrix} \mathfrak{T}_1^{\mathrm{T}}, \tag{16}$$

where $\mathfrak{T}_1 \in \mathbb{R}^{n\times n}$ is a unitary matrix, $\bar{\mathbb{X}}_{11} \in \mathbb{R}^{q\times q}$ and $\bar{\mathbb{X}}_{22} \in \mathbb{R}^{n-q\times n-q}$. This holds true for any matrix with rank$(\mathfrak{k}) = q$ for which $\mathfrak{k} \in \mathbb{R}^{q\times n}$.

We have

$$\begin{aligned} \dot{\hat{x}}(t) &= H_1\hat{x}(t) + H_2\hat{x}(t-h) + JK_p\hat{x}(t) + L_1N\nabla x(t) \\ &\quad + Jx_I(t), \\ \nabla\dot{x}(t) &= (H_1 - L_1N)\nabla x(t) + H_2\nabla x(t-h) - Jx_I(t) \\ &\quad - JN_Fx_F(t) + J_dw(t), \\ \dot{x}_I(t) &= L_2N\nabla x(t), \\ \dot{x}_F(t) &= H_Fx_F(t) + J_FJ^+L_1N\nabla x(t) + J_FN_Fx_F(t) \\ &\quad + J_Fx_I(t). \end{aligned}$$

Letting

$$\psi(t) = \begin{bmatrix} \hat{x}^{\mathrm{T}}(t) & \nabla x^{\mathrm{T}}(t) & x_I^{\mathrm{T}}(t) & x_F^{\mathrm{T}}(t). \end{bmatrix}^{\mathrm{T}} \tag{17}$$

So,

$$\dot{\psi}(t) = \bar{H}_1\psi(t) + \bar{H}_2\psi(t-h) + \bar{J}_dw(t), \tag{18}$$

where

$$\bar{H}_1 = \begin{bmatrix} H_1 + JK_p & L_1N & J & 0 \\ 0 & H_1 - L_1N & -J & -JN_F \\ 0 & L_2N & 0 & 0 \\ 0 & J_FJ^+L_1N & J_F & H_F + J_FN_F \end{bmatrix},$$

$$\bar{H}_2 = \begin{bmatrix} H_2 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{J}_d = \begin{bmatrix} 0 \\ J_d \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{N} = \begin{bmatrix} N & N & 0 & 0 \end{bmatrix}.$$

Suppose that the SVD of the matrix N is as follows

$$N = U_2 \begin{bmatrix} V_2 & 0 \end{bmatrix} T_2^{\mathrm{T}}, \tag{19}$$

where V_2 is a positive definite matrix and U_2 and T_2 are unitary matrices.

Letting T_2 be

$$T_2 = \begin{bmatrix} \bar{T} & \hat{T} \end{bmatrix}, \tag{20}$$

the theorem obtained in this paper is as follows.

Theorem 1. *M, A, B, C are given matrixs, assume that there exist positive-definite matrices Y_1, Y_2, Y_3, Y_4, X_1, X_{11}, X_{22}, X_3, X_4, and appropriate matrices W_1, W_2, and W_3, such that the following inequality holds*

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & X & \Psi_{15} \\ \star & -Y & 0 & 0 & 0 \\ \star & \star & \Psi_{33} & 0 & 0 \\ \star & \star & \star & -Y & 0 \\ \star & \star & \star & \star & A^{-1} \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} X_1 & 0 & 0 & 0 \\ \star & X_2 & X_2M & 0 \\ \star & \star & X_3 & 0 \\ \star & \star & \star & X_4 \end{bmatrix} > 0, \tag{22}$$

where

$$\Psi_{11} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \star & \star & \Phi_{33} & \Phi_{34} \\ \star & \star & \star & \Phi_{44} \end{bmatrix},$$

$$\begin{aligned} \Phi_{11} &= H_1X_1 + JW_1 + X_1H_1^{\mathrm{T}} + W_1^{\mathrm{T}}J^{\mathrm{T}}, \\ \Phi_{12} &= W_2N + JM^{\mathrm{T}}X_2, \\ \Phi_{13} &= W_2NM + JX_3, \\ \Phi_{22} &= H_1X_2 + X_2H_1^{\mathrm{T}} - W_2N - N^{\mathrm{T}}W_2^{\mathrm{T}} \\ &\quad - JM^{\mathrm{T}}X_2 - X_2MJ^{\mathrm{T}}, \\ \Phi_{23} &= H_1X_2M - W_2NM - JX_3 + N^{\mathrm{T}}W_3^{\mathrm{T}}, \\ \Phi_{24} &= -JN_FX_4 + N^{\mathrm{T}}W_2^{\mathrm{T}}J^{+\mathrm{T}}J_F^{\mathrm{T}} + X_2MJ_F^{\mathrm{T}}, \\ \Phi_{33} &= W_3NM + M^{\mathrm{T}}N^{\mathrm{T}}W_3^{\mathrm{T}}, \\ \Phi_{34} &= M^{\mathrm{T}}N^{\mathrm{T}}W_2^{\mathrm{T}}J^{+\mathrm{T}}J_F^{\mathrm{T}} + X_3J_F^{\mathrm{T}}, \\ \Phi_{44} &= H_FX_4 + X_4H_F^{\mathrm{T}} + J_FN_FX_4 + X_4N_F^{\mathrm{T}}J_F^{\mathrm{T}}, \end{aligned}$$

$$\Psi_{12} = \begin{bmatrix} H_2Y_1 & 0 & 0 & 0 \\ 0 & H_2Y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{13} = \begin{bmatrix} -X_1N^{\mathrm{T}}B - X_1N^{\mathrm{T}}AD \\ J_d - X_2N^{\mathrm{T}}B - X_2N^{\mathrm{T}}AD \\ -M^{\mathrm{T}}X_2N^{\mathrm{T}}B - M^{\mathrm{T}}X_2N^{\mathrm{T}}AD \\ 0 \end{bmatrix},$$

$$\Psi_{14} = \begin{bmatrix} X_1 & 0 & 0 & 0 \\ 0 & X_2 & X_2M & 0 \\ 0 & M^{\mathrm{T}}X_2 & X_3 & 0 \\ 0 & 0 & 0 & X_4 \end{bmatrix},$$

$$\Psi_{15} = \begin{bmatrix} X_1N^{\mathrm{T}} \\ X_2N^{\mathrm{T}} \\ M^{\mathrm{T}}X_2N^{\mathrm{T}} \\ 0 \end{bmatrix},$$

$$\Psi_{33} = -D^{\mathrm{T}}AD - D^{\mathrm{T}}B - B^{\mathrm{T}}D - C + \gamma I,$$

the system (18) is strictly $(A,B,C) - \gamma -$dissipative.

$$X_2 = \begin{bmatrix} \bar{T} & \hat{T} \end{bmatrix} \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} \begin{bmatrix} \bar{T}^{\mathrm{T}} \\ \hat{T}^{\mathrm{T}} \end{bmatrix}. \tag{23}$$

Then, the parameters of the controller are

$$K_P = W_1X_1^{-1}, \tag{24}$$

$$L_1 = W_2U_2V_2X_{11}V_2^{-1}U_2^{\mathrm{T}}, \quad L_2 = W_3U_2V_2X_{11}^{-1}V_2^{-1}U_2^{\mathrm{T}}. \tag{25}$$

Proof: Choose a Lyapunov functional candidate to be

$$V(\psi_t) = \psi^{\mathrm{T}}(t)P\psi(t) + \int_{t-h}^{t} \psi^{\mathrm{T}}(s)S\psi(s)\mathrm{d}s, \tag{26}$$

where $P > 0$ and $S > 0$, here P and S are defined by $P = X^{-1}$, $S = Y^{-1}$, where

$$X = \begin{bmatrix} X_1 & 0 & 0 & 0 \\ \star & X_2 & X_2M & 0 \\ \star & \star & X_3 & 0 \\ \star & \star & \star & X_4 \end{bmatrix},$$

$$Y=\begin{bmatrix} Y_1 & 0 & 0 & 0 \\ \star & Y_2 & 0 & 0 \\ \star & \star & Y_3 & 0 \\ \star & \star & \star & Y_4 \end{bmatrix}.$$

Calculating the derivative of $V(\varphi_t)$, yields

$$\dot{V}(\psi_t)=2\psi^{\mathrm{T}}(t)P\dot{\psi}(t)+\psi^{\mathrm{T}}(t)S\psi(t)-\psi^{\mathrm{T}}(t-h)S\psi(t-h). \tag{27}$$

So, let

$$\Omega=\dot{V}(\psi_t)-Z^{\mathrm{T}}AZ-2Z^{\mathrm{T}}Bw-w^{\mathrm{T}}(C-\gamma I)w \tag{28}$$

we obtain

$$\Omega=\begin{bmatrix} \psi(t) \\ \psi(t-h) \\ w(t) \end{bmatrix}^{\mathrm{T}} \Xi \begin{bmatrix} \psi(t) \\ \psi(t-h) \\ w(t) \end{bmatrix}, \tag{29}$$

where

$$\Xi=\begin{bmatrix} \Gamma_{11} & P\bar{H}_2 & \Gamma_{13} \\ \star & -S & 0 \\ \star & \star & \Gamma_{33} \end{bmatrix},$$

$\Gamma_{11}=P\bar{H}_1+\bar{H}_1{}^{\mathrm{T}}P+S-\bar{N}^{\mathrm{T}}A\bar{N}$,
$\Gamma_{13}=P\bar{J}_d-\bar{N}^{\mathrm{T}}B-\bar{N}^{\mathrm{T}}AD$,
$\Gamma_{33}=-D^{\mathrm{T}}AD-D^{\mathrm{T}}B-B^{\mathrm{T}}D-C+\gamma I$.

If $\Xi<0$, means

$$\dot{V}(\psi_t)-Z^{\mathrm{T}}AZ-2Z^{\mathrm{T}}Bw-w^{\mathrm{T}}(C-\gamma I)w<0, \tag{30}$$

with zero initial state, integrating both sides of (30) from 0 to t, yields

$$G(w,z,\tau)\geq\gamma<w,w>_{\tau}\ \forall\tau\geq 0, \tag{31}$$

thus, the system (18) is strictly $(A,B,C)-\gamma-$dissipative.

According to Lemma 1, Ξ can be equivalent to

$$\begin{bmatrix} \phi_{11} & P\bar{H}_2 & \phi_{13} & I & \bar{N}^{\mathrm{T}} \\ \star & -S & 0 & 0 & 0 \\ \star & \star & \phi_{33} & 0 & 0 \\ \star & \star & \star & -S^{-1} & 0 \\ \star & \star & \star & \star & A^{-1} \end{bmatrix}<0, \tag{32}$$

$\phi_{11}=P\bar{H}_1+\bar{H}_1{}^{\mathrm{T}}P$,
$\phi_{13}=P\bar{J}_d-\bar{N}^{\mathrm{T}}B-\bar{N}^{\mathrm{T}}AD$,
$\phi_{33}=-D^{\mathrm{T}}AD-D^{\mathrm{T}}B-B^{\mathrm{T}}D-C+\gamma I$.

Pre-and post-multiplying (32) by $\mathrm{diag}\{P^{-1},S^{-1},I,I,I\}=\mathrm{diag}\{X,Y,I,I,I\}$ yield

$$\begin{bmatrix} \xi_{11} & \bar{H}_2Y & \xi_{13} & X & X\bar{C}^{\mathrm{T}} \\ \star & -Y & 0 & 0 & 0 \\ \star & \star & \xi_{33} & 0 & 0 \\ \star & \star & \star & -Y & 0 \\ \star & \star & \star & \star & A^{-1} \end{bmatrix}<0, \tag{33}$$

$\xi_{11}=\bar{H}_1X+X\bar{H}_1{}^{\mathrm{T}}$,
$\xi_{13}=\bar{J}_d-X\bar{N}^{\mathrm{T}}B-X\bar{N}^{\mathrm{T}}AD$,
$\xi_{33}=-D^{\mathrm{T}}AD-D^{\mathrm{T}}B-B^{\mathrm{T}}D-C+\gamma I$.

Applying Lemma 2 to (19) we obtain

$$\bar{X}_2=U_2V_2X_{11}V_2^{-1}U_2^{\mathrm{T}}, \tag{34}$$

with

$$NX_2=\bar{X}_2N. \tag{35}$$

Letting

$$W_1=K_PX_1,\ W_2=L_1\bar{X}_2,\ W_3=L_2\bar{X}_2, \tag{36}$$

and substituting (18) into (33) yield (21).

When disturbance $w(t)=0$, the system (18) is asymptotically stable if $\dot{V}(\psi_t)<0$. Note that, if $\Xi<0$ holds, the $\dot{V}(\psi_t)<0$.

So, the closed-loop system (18) is asymptotically stable if LMI (21) holds.

This completes the proof. □

IV. Simulation

Now we assume each parameter of the plant

$$H_1=\begin{bmatrix} -2 & 0 \\ 0 & -5 \end{bmatrix},\ H_2=\begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.3 \end{bmatrix},$$
$$J=\begin{bmatrix} 2 \\ 1 \end{bmatrix},J_d=\begin{bmatrix} 2 \\ 1 \end{bmatrix},\ M=\begin{bmatrix} 0 \\ -8 \end{bmatrix},$$
$$N=\begin{bmatrix} 2 & 1 \end{bmatrix},\ D=1,\gamma=0.1,A=-1,$$
$$B=1,C=1,h=1.$$

The disturbance

$$d(t)=0.1\tanh(t)+0.8\sin(0.5\pi t). \tag{37}$$

The parameters of the $F(s)$ are set as follows $J_F=100$, $H_F=-101$, $N_F=1$.

By calculating the LMI of Theorem 1, the parameter of the controller is

$$K_P=\begin{bmatrix} -51.9590 & -25.2116 \end{bmatrix},$$

and

$$L_1=\begin{bmatrix} 21.3686 & 10.0790 \end{bmatrix}^{\mathrm{T}},\ L_2=48.0322.$$

We compared the proposed method to the Sliding-Mode-Observer-Equivalent-Input-Disturbance (SMO-EID) to demonstrate its efficacy [16]. In comparison to the traditional EID method, the SMO-EID method adds a sliding-mode control law based on the traditional Luenberger observer (Fig.2), which improves disturbance rejection performance.

The switching function and sliding-mode control law are designed as follows:

$$\Xi(\bar{s}(t))=\arctan\bar{s}(t)=\arctan(y(t)-\hat{y}(t)),\ K_s=40.$$

According to Fig.3, our method's peak-peak value in steady state is approximately 0.0165, nevertheless, the peak-peak value of the SMO-EID method is about 0.5187. It is clear that the presented method accomplishes disturbance rejection more successfully than the SMO-EID method since it reduces the peak-peak value of the steady state to 31 times.

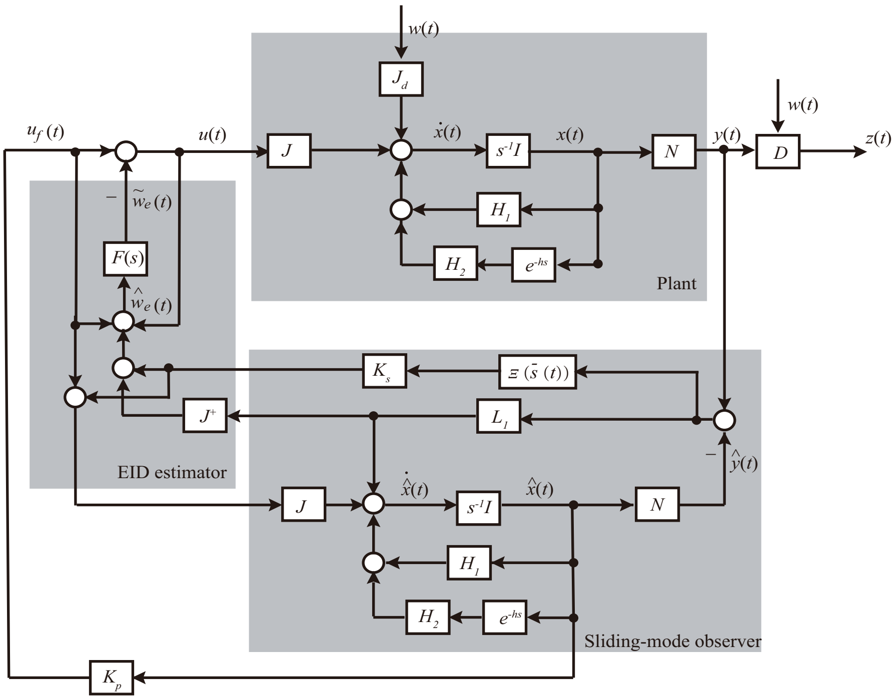

Fig. 2: Configuration of SMO-EID control system.

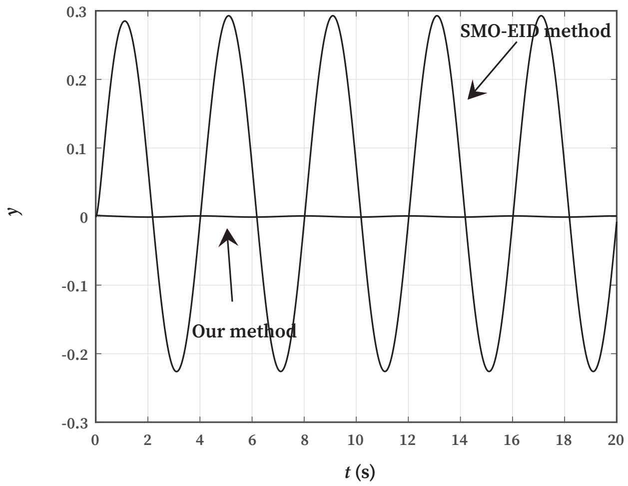

Fig. 3: System output for SMO-EID method and our method.

V. Conclusion

This paper is the first time to consider the dissipation of time-delay systems based on a modified PIO. The traditional Luenberger observer is replaced by a modified PIO and the time delay is also considered. The state of the time-delay system is reconstructed using the observer. A sufficient stability criterion is obtained and the parameters of the controller are designed on the basis of the stability criterion. The efficiency of the presented method is demonstrated by comparisons between it and SMO-EID and EID approaches.

References

[1] W. B. Chen, F. Gao, S. Y. Xu, Y. M. Li, and Y. M. Chu, "Robust Stabilization for Uncertain Singular Markovian Jump Systems via Dynamic Output-Feedback Control," *Systems & Control Letters*, vol. 171, pp. 105433, 2023.

[2] L. Yao, X. F. Jiang, M. C. Wang, and Y. W. Zhang, "An Improved Stability Criterion for a class of Linear Systems with Interval Time-Varying Delay," in *Lecture Notes in Chinese Control and Decision Conference 2018*, pp. 2801-2805.

[3] A. J. van der Schaft, "Cyclo-Dissipativity Revisited," *IEEE Transactions on Automatic Control*, vol. 66, no. 6, pp. 2920-2924, 2020.

[4] P. N. Kohler, M. A. Muller and F. Allgower, "Approximate Dissipativity of Cost-Interconnected Systems in Distributed Economic MPC," *IEEE Transactions on Automatic Control*, vol. 68, no. 4, pp. 2170-2182, 2022.

[5] R. Saravanakumar, G. Rajchakit, C. K. Ahn, and H. R. Karimi, "Exponential Stability, Passivity, and Dissipativity Analysis of Generalized Neural Networks with Mixed Time-Varying Delays," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 2, pp. 395-405, 2017.

[6] J. C. Willems, "Dissipative Dynamical Systems Part I: General Theory," *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 321-351, 1972.

[7] J. H. She, M. X. Fang, Y. Ohyama, H. Hashimoto, and M. Wu, "Improving Disturbance-Rejection Performance based on an Equivalent-Input-Disturbance Approach," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 1, pp. 380-389, 2008.

[8] J. H. She, X. Xin and T. Yamaura, "Analysis and Design of Control System with Equivalent-Input-Disturbance Estimation," in *Lecture Notes in IEEE International Conference on Control Applications 2006*, pp. 1463-1469.

[9] Q. Zhong, K. Wang, K. Mao, B. T. Dong, and Q. Kuang, "Fault-Tolerant Control of Demagnetization for Ltra-High-Speed PMSM based on Improved Equivalent-Input-Disturbance Approach," in *Lecture Notes in 25th International Conference on Electrical Machines and Systems 2022*, pp. 1-5.

[10] Q. C. Mei, J. H. She, Z. T. Liu, and M. Wu, "Estimation and Compensation of Periodic Disturbance Using Internal-Model-Based Equivalent-Input-Disturbance Approach," *Information Sciences*, vol. 65, no. 182205, 2022.

[11] Y. W. Du, W. H. Cao, J. H. She, M. Wu, M. X. Fang, and S. Kawata, "Disturbance Rejection and Control System Design Using Improved Equivalent Input Disturbance Approach," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 4, pp. 3013-3023, 2019.

[12] P. Yu, K. Z. Liu, X. D. Liu, J. H. She, and X. L. Li, "Error-Driven-Based Performance Analysis of Nonlinear Equivalent-Input-Disturbance Approaches," in *Lecture Notes in IEEE/ASME International Conference on Advanced Intelligent Mechatronics 2022*, pp. 915-920.

[13] M. Wu, F. Gao, P. Yu, J. H. She, and W. H. Cao, "Improve Disturbance-Rejection Performance for an Equivalent-Input-Disturbance-Based Control System by Incorporating a Proportional-Integral Observer," *IEEE*

Transactions on Industrial Electronics, vol. 67, no. 2, pp. 1254-1260, 2019.
[14] P. P. Khargonek, I. R. Petersen and K. M. Zhou, "Robust Stabilization of Uncertain Linear Systems: Quadratic Stabilizability and H_∞ Control Theory," *IEEE Transactions on Automatic Control*, vol. 35, no. 3, pp. 356-361, 1990.
[15] D. W. C. Ho and G. Lu, "Robust Stabilization for a class of Discrete-Time Nonlinear System via Output Feedback: the Unified LMI Approach," *International Journal of Control*, vol. 76, no. 2, pp. 105-115, 2003.
[16] W. J. Cai, "Disturbance-Rejection and Tracking Control for Quadrotor UVAs based on Equivalent-Input-Disturbance Approach," *China University of Geosciences*, 2020.