

# How the Wind Changes the Leslie-Gower Predator-prey System?

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**Abstract**—Under the assumption that the predators better detect the prey species exposed to the wind blowing, a modified Leslie-Gower predator prey system incorporating wind effect is proposed and studied in this paper. Local and global stability properties of the boundary equilibrium and the positive equilibrium are investigated, respectively. For the wind large than the threshold, the prey species will be driven to extinction, and for the limited wind effect, though the density of the prey species will be reduced, both prey and predator species could coexist in a stable state. Our study shows that the wind effect plays a crucial role in the persistence and stability property of the system. Our study also indicates that with the increasing of the other food resource, the final density of the prey species will decrease while the density of predator species will increase. Numeric simulations are run to demonstrate the viability of the major conclusions.

**Index Terms**—Leslie-Gower, predator-prey model, stability, wind effect

## I. INTRODUCTION

THE objective of this study is to examine the dynamic characteristics of the modified Leslie-Gower predator-prey system with the inclusion of wind effect.

$$\begin{aligned} \frac{du}{dt} &= ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)uv - equ, \\ \frac{dv}{dt} &= sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right), \end{aligned} \quad (1)$$

The variables  $u$  and  $v$  represent the population density of prey and predator species, respectively, at a given time  $t$ . The variables in this model include the intrinsic growth rate of the prey species ( $r$ ) and its carrying capacity ( $K$ ), as well as the attack rate of the prey due to the predator ( $\alpha$ ), the fishing effort used to harvest the prey species ( $e$ ), the harvesting coefficient ( $q$ ), the intrinsic growth rate of the predator species ( $s$ ), and the carrying capacity of the predator species ( $\beta\phi(\omega)u + c$ ), which includes an additional food source represented by the variable  $c$ . The selection of the Lotka-Volterra type functional response is denoted by  $\alpha u$ . The quantity of prey apprehended by the predator can be expressed as  $\alpha\phi(\omega)uv$ . The wind efficiency function is defined as  $\phi(\omega) = 1 + \omega$ , where  $\omega$  denotes the wind flow that conforms to the given assumptions: (1) The predator's search efficiency remains constant in the absence of wind,

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denoted by  $\phi(0) = 1$ . (2) The predator's search efficiency increases as wind speed increases, denoted by  $\phi'(\omega) > 0$ .

The ubiquitous presence of the predator-prey dynamic has prompted extensive scholarly investigation([1]-[24]). One famous predator-prey model is proposed by Leslie[25], The model takes the form:

$$\begin{aligned} \frac{dH}{dt} &= (r_1 - a_1P - b_1H)H, \\ \frac{dP}{dt} &= \left(r_2 - a_2\frac{P}{H}\right)P, \end{aligned} \quad (2)$$

where  $H$  and  $P$  are the density of prey species and predator species at the time  $t$ , respectively. Korobeinikov[26] provided a rigorous demonstration that the aforementioned system possesses a solitary coexisting fixed point  $(H^*, P^*)$  that is stable on a global scale. In this model, the carrying capacity of the predator species is proportional to prey abundance. Aziz-Alaoui and Daher Okiye[27] argued that in the case of severe scarcity,  $P$  can switch over to other populations, but its growth will be limited by the fact that its favorite food  $H$  is not available in abundance. Aziz-Alaoui and Daher Okiye[27] proposed the following modified Leslie-Gower predator-prey model:

$$\begin{aligned} \frac{dH}{dt} &= \left(r_1 - b_1H - \frac{a_1P}{H + K_1}\right)H, \\ \frac{dP}{dt} &= \left(r_2 - a_2\frac{P}{H + C}\right)P, \end{aligned} \quad (3)$$

here,  $c$  reflects the other food resource that could be afforded to predator species. By constructing a Lyapunov function, the authors were able to derive a set of conditions that assure the global stability of the positive equilibrium. It is well known that the effect of wind plays important roles on the ecosystem[28]-[32]. For example, research[33] has demonstrated that avian predators exhibit superior detection abilities when it comes to locating reed warbler nests that are exposed to the effects of wind, and this may increase the rate of predation of the reed warbler. Stimulated by this fact, Jawad, Sultan, and Winter[32] recently proposed the following predator-prey model

$$\begin{aligned} \frac{du}{dt} &= ru\left(1 - \frac{u}{K}\right) - \frac{\alpha uv}{\phi(\omega)} - equ, \\ \frac{dv}{dt} &= sv\left(1 - \frac{v}{\phi(\omega) + \beta u + c}\right) - \gamma v. \end{aligned} \quad (4)$$

When

$$0 < \alpha(s - \gamma)(1 + \omega + l) < s(r - eq)(1 + \omega) \quad (5)$$

holds, the system (5) admits a unique positive equilibrium

$E_3(u^*, v^*)$ , where

$$\begin{aligned} u^* &= \frac{k[s(r - eq)(1 + \omega) - \alpha(s - \gamma)(1 + \omega + l)]}{rs(1 + \omega) + \alpha\beta(s - \gamma)}, \\ v^* &= \frac{(s - \gamma)(1 + \omega + \beta u^* + l)}{s}. \end{aligned} \tag{6}$$

The authors have derived the subsequent outcomes with regards to the system’s persistent and global stabilities characteristics.

**Theorem A.** Assume that the boundary equilibriums conditions hold, then the system (4) is uniformly persistent.

**Theorem B.**  $E_3$  is globally asymptotically stable in  $R_{\pm}^2$ , whenever it exists.

In order to demonstrate the practicability of the principal findings, the authors employed the subsequent collection of parameters.

$$\begin{aligned} r &= 5, k = 20, \alpha = 0.8, e = 0.03, q = 0.02, \\ s &= 4, \beta = 0.3, l = 0.2, \gamma = 0.001. \end{aligned}$$

Then for  $\omega = 0$ , i.e., for the system without wind flow, the system admits a positive steady state  $E_3(8.24, 3.67)$ , and for  $\omega = 5$ , the system admits a positive steady state  $E_3(14.02, 11.2)$ . From the numeric simulations (Fig.1 and 2 in [32]), global asymptotic stability of the positive steady state can be observed. The authors have stated that the population densities of both prey and predator exhibit an increase up to a certain threshold in the presence of wind flow, whereas a decrease is observed in their absence. In an ecological context, the presence of wind can hinder the prey’s ability to detect the proximity of predators. Consequently, the predator’s ability to detect the prey is enhanced, thereby increasing its effectiveness. This conclusion is intriguing. The phenomenon whereby the presence of wind impedes the prey’s ability to detect the proximity of predators has been observed. Consequently, the likelihood of the prey being captured is heightened. In general, this will result in the reduce of the prey species’ density, which stands in opposition to the numerical example provided above. By carefully checking the model (4), we found the model may have the following drawbacks:

The first one is the term  $\frac{\alpha uv}{\phi(\omega)}$  in the first equation of system (4), from an ecological point of view, which means that each predator species could capture  $\frac{\alpha u}{\phi(\omega)}$  of prey species, the term  $\frac{\alpha u}{\phi(\omega)}$  is the decreasing function of  $\omega$ , which means that with the increasing wind effect, the chance for the prey species to be detected by predator species is decreasing, which is a contract to the observation in Klimczuk et al.[33].

The second one is the term  $\frac{v}{\phi(\omega) + \beta u + c}$  in the second equation of system (4). The denominator of this item is the capacity of the predator species. Without the wind effect, it ought to be degenerate to the term  $\beta u + c$ , but, with this expression, it degenerates to the term  $1 + \beta u + c$ . That is, system (4) could not degenerate to the modified Leslie-Gower model for the case  $\omega = 0$ . Maybe a more appropriate assumption should take the form  $\phi(\omega)\beta u + c$ , then if  $\omega = 0$ , i.e., for the system without wind effect, the system could be degenerate to the modified Leslie-Gower model. Also, with

such an expression, one could easily see that the capacity of the predator species is increasing as a function of the wind effect. Such an assumption is a coincidence with the observation: in the presence of blowing wind, The ability of the attacker to find the food improves.

The third one is the term  $\gamma v$  in system (4), here, the authors said  $\gamma$  is the death rate of predator species. However, generally speaking,  $s$  in the second equation of system (4) is defined as the intrinsic growth rate of the predator species, which is equal to the birth rate minus the death rate of the predator species, hence, one need not consider the death rate of the predator species separately.

The preceding discourse has provided impetus for us to put forth the system denoted by equation (1). In system (1), compared to system (4), we replace  $\frac{\alpha uv}{\phi(\omega)}$  in (4) with the term  $\alpha\phi(\omega)uv$ , obviously, this term is the strictly increasing function of  $\omega$ . We also replace the term  $\phi(\omega) + \beta u + c$  in (4) with the term  $\beta\phi(\omega)u + c$ . One could easily see that if  $\omega = 0, q = 0$  then system (1) is reduced to the system

$$\begin{aligned} \frac{du}{dt} &= ru\left(1 - \frac{u}{K}\right) - \alpha uv, \\ \frac{dv}{dt} &= sv\left(1 - \frac{v}{\beta u + c}\right), \end{aligned} \tag{7}$$

which is the modified Leslie-Gower predator-prey model.

It is of interest to determine whether the dynamic behaviors exhibited by the system described in equation (1) align with the observation made in reference [33] regarding the heightened ability of aerial predators to detect reed warbler nests that are exposed to wind currents. Our aim is to conduct a comprehensive analysis of the dynamic characteristics of the system (1), with the objective of demonstrating that our findings are more aligned with empirical observations.

The subsequent sections of the document are organized in the following manner: The next section will examine the positivity and boundedness of the system (1). In Section 3, the existence and local stability of the equilibria of the system (1) will be explored. Section 4 will investigate the extinction property of the system, while Section 5 will focus on the global stability property of the positive equilibrium. Finally, Section 6 will examine the persistent property of the system. Subsequently, the impact of wind effect is deliberated in Section 7. Section 8 includes numeric simulations that demonstrate the practicality of the primary findings. The present manuscript concludes with a succinct discourse.

## II. POSITIVITY AND BOUNDEDNESS OF SYSTEM (1)

With regard to the positivity of the system represented by equation (1), the following outcome has been obtained. **Theorem 2.1.** *The domain  $R_{\pm}^2 = \{(u, v) | u > 0, v > 0\}$  is invariant with respect to (1).*

**Proof.** Since from (1) it follows that

$$\begin{aligned} u(t) &= u(0) \exp \left\{ \int_0^t \left( r \left( 1 - \frac{u}{K} \right) - \alpha \phi(\omega) v - eq \right) dt \right\}, \\ v(t) &= v(0) \exp \left\{ \int_0^t \left( s \left( 1 - \frac{v}{\beta \phi(\omega) u + c} \right) \right) dt \right\}. \end{aligned} \tag{8}$$

The lemma’s assertion can be readily derived for all values of  $t$  within the interval  $[0, +\infty)$ .

The proof of Theorem 2.1 is finished.

**Theorem 2.2** All solutions  $u(t)$  and  $v(t)$  that satisfy the system (1) and the initial conditions  $u(0) > 0, v(0) > 0$  exhibit uniform boundedness.

**Proof.** Utilizing the first equation of the system (1), it can be inferred that.

$$\frac{du}{dt} \leq ru\left(1 - \frac{u}{K}\right). \tag{9}$$

Applying Lemma 2.3 in [34] to above inequality leads to

$$\limsup_{t \rightarrow +\infty} u(t) \leq K. \tag{10}$$

Hence, for any positive constant  $\varepsilon > 0$  sufficiently small, there exists a  $T_1 > 0$  such that for all  $t \geq T_1$ ,

$$u(t) < K + \varepsilon. \tag{11}$$

For  $t \geq T_1$ , it follows from the second equation of the system (1) and (11) that

$$\begin{aligned} \frac{dv}{dt} &= sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right) \\ &\leq sv\left(1 - \frac{v}{\beta\phi(\omega)(K + \varepsilon) + c}\right). \end{aligned} \tag{12}$$

Applying Lemma 2.3 in [34] to the above inequality leads to

$$\limsup_{t \rightarrow +\infty} v(t) \leq \beta\phi(\omega)(K + \varepsilon) + c. \tag{13}$$

Since  $\varepsilon$  is a small positive constant, setting  $\varepsilon \rightarrow 0$  in (13) leads to

$$\limsup_{t \rightarrow +\infty} v(t) \leq \beta\phi(\omega)K + c. \tag{14}$$

(10) and (14) show that  $u(t)$  and  $v(t)$  of the system (1) with the initial conditions  $u(0) > 0, v(0) > 0$  are uniformly bounded. The demonstration of Theorem 2.2 has been concluded.

### III. THE EXISTENCE AND LOCAL STABILITY OF THE EQUILIBRIA OF THE SYSTEM (1)

With regards to the presence of equilibria in the system (1), the following outcome has been observed.

**Theorem 3.1.** System (1) always admits the vanishing equilibrium point  $E_0(0, 0)$  and the prey free equilibrium point  $E_2(0, c)$ . If

$$r > eq \tag{15}$$

holds, the predator-free equilibrium point  $E_1(\bar{u}, 0)$  exists, where

$$\bar{u} = \frac{k(r - eq)}{r}.$$

Also, there exists a unique positive equilibrium  $E_2(u^*, v^*)$ , if

$$r > eq + \alpha c \phi(\omega) \tag{16}$$

holds, where

$$u^* = -\frac{k(\alpha c \omega + \alpha c + eq - r)}{\alpha \beta k \omega^2 + 2\alpha \beta k \omega + \alpha \beta k + r}$$

and  $v^*$  satisfies

$$v^* = \beta\phi(\omega)u^* + c.$$

**Proof.** The system's equilibria (1) fulfill the equation

$$\begin{aligned} ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)uv - equ &= 0, \\ sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right) &= 0. \end{aligned} \tag{17}$$

From the second equation of (17),  $v = 0$  or  $v = \beta\phi(\omega)u + c$  can be derived. When  $v = 0$  is substituted into the first equation of (17), the result is

$$ru\left(1 - \frac{u}{K}\right) - equ = 0. \tag{18}$$

Equation (18) has solutions  $u_1 = 0$  and  $u_2 = \frac{k(r - eq)}{r}$ . Therefore, the system represented by equation (1) possesses a vanishing equilibrium point denoted as  $E_0(0, 0)$ . Additionally, the existence of a predator-free equilibrium point denoted as  $E_1(\bar{u}, 0)$  is guaranteed if the condition  $r > eq$  is satisfied.

Substituting  $v = \beta\phi(\omega)u + c$  into the first equation of (17) produces

$$ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)u(\beta\phi(\omega)u + c) - equ = 0. \tag{19}$$

Given the premise stated in equation (16), it is evident that a unique positive solution exists for equation (19). It follows that the system represented by equation (1) possesses a unique positive equilibrium denoted as  $E_2(u^*, v^*)$ , which is unique.

This ends the proof of Theorem 3.1.

**Theorem 3.2.** The vanishing equilibrium point  $E_0(0, 0)$  is unstable; Assume that

$$r < \alpha\phi(\omega)c + eq \tag{20}$$

holds, the prey free equilibrium point  $E_2(0, c)$  is locally asymptotically stable. The predator-free equilibrium point  $E_1(\bar{u}, 0)$  is unstable; If

$$r > \alpha\phi(\omega)c + eq \tag{21}$$

holds, the positive equilibrium  $E_3(u^*, v^*)$  is locally asymptotically stable.

**Proof.** The system's Jacobian matrix (1) is computed as

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}, \tag{22}$$

where

$$\begin{aligned} J_{11} &= r\left(1 - \frac{u}{K}\right) - \frac{ru}{k} - \alpha(1 + \omega)v - eq, \\ J_{12} &= -\alpha(1 + \omega)u, \\ J_{21} &= \frac{sv^2\beta(1 + \omega)}{(\beta(1 + \omega)u + c)^2}, \\ J_{22} &= s\left(1 - \frac{v}{\beta(1 + \omega)u + c}\right) - \frac{sv}{\beta(1 + \omega)u + c}. \end{aligned} \tag{23}$$

The Jacobian matrix concerning the equilibrium point  $E_0(0, 0)$  for the system (1) is given

$$J(E_0(0, 0)) = \begin{pmatrix} r - eq & 0 \\ 0 & s \end{pmatrix}. \tag{24}$$

The eigenvalues of  $J(E_0)$  are  $\lambda_1 = r - eq$ ,  $\lambda_2 = s > 0$ . Thus,  $E_0(0, 0)$  is unstable.

The Jacobian matrix related to the system (1) evaluated at the predator-free equilibrium point  $E_1(\bar{u}, 0)$  is being referred to

$$J(E_1(\bar{u}, 0)) = \begin{pmatrix} r\left(1 + \frac{eq-r}{r}\right) - r & \frac{\alpha(1+\omega)k(eq-r)}{r} \\ 0 & s \end{pmatrix}.$$

The eigenvalues of  $J(E_1)$  are  $\lambda_1 = r\left(1 + \frac{eq-r}{r}\right) - r$ ,  $\lambda_2 = s > 0$ . Thus,  $E_1(\bar{u}, 0)$  is unstable.

The Jacobian matrix of the system represented by equation (1) evaluated at the prey-free equilibrium point  $E_2(0, c)$  is presented as follows:

$$J(E_2(0, c)) = \begin{pmatrix} r - \alpha(1+\omega)c - eq & 0 \\ s\beta(1+\omega) & -s \end{pmatrix}.$$

The aforementioned expression denotes the eigenvalues of  $J(E_2)$ , which are given by  $\lambda_1 = r - \alpha(1+\omega)c - eq$  and  $\lambda_2 = -s$ , where  $\lambda_2$  is negative. Therefore, it can be concluded that  $E_2(0, c)$  exhibits local asymptotic stability if and only if the inequality (20) is satisfied.

The Jacobian matrix pertaining to the system (1) in relation to the equilibrium point  $E_3(u^*, v^*)$  is being referred to

$$J(E_3(u^*, v^*)) = \begin{pmatrix} A_{11} & A_{12} \\ s\beta(1+\omega) & -s \end{pmatrix}, \tag{25}$$

where

$$A_{11} = \frac{(\alpha(1+\omega)c + eq - r)r}{k\beta(1+\omega)^2\alpha + r},$$

$$A_{12} = \frac{\alpha(1+\omega)k(\alpha c\omega + \alpha c + eq - r)}{\alpha\beta k\omega^2 + 2\alpha\beta k\omega + \alpha\beta k + r}.$$

Then, under the assumption that (21) and (25) hold, we have

$$\begin{aligned} DetJ(E_3(u^*, v^*)) &= -s(\alpha(1+\omega)c + eq - r) > 0 \end{aligned}$$

and

$$TrJ(E_3(u^*, v^*)) = A_{11} - s < 0.$$

Therefore, both eigenvalues of  $J(E_3(u^*, v^*))$  have negative real parts, and  $E_3(u^*, v^*)$  is locally asymptotically stable.

This ends the proof of Theorem 3.2.

#### IV. EXTINCTION

In the previous section, we showed that under the assumption (20) holds, the prey free equilibrium point  $E_2(0, c)$  is locally asymptotically stable. An intriguing issue is to find out the sufficient conditions that guarantee the globally asymptotically stable of the prey free equilibrium point  $E_2(0, c)$ . Indeed, we have the following result. An intriguing issue is to find out the sufficient conditions that guarantee

the globally asymptotically stable of the prey free equilibrium point  $E_2(0, c)$ . The aforementioned outcome has been obtained.

**Theorem 4.1.** *Given the condition expressed in Equation 26*

$$r < \alpha\phi(\omega)c + eq, \tag{26}$$

*it can be concluded that the prey free equilibrium point denoted by  $E_2(0, c)$  is global asymptotic stability.*

**Proof.** For  $\varepsilon > 0$  enough small, without loss of generality, we assume that

$$0 < \varepsilon < c - \frac{r - eq}{\alpha\phi(\omega)},$$

then inequality (26) implies that

$$r < \alpha\phi(\omega)(c - \varepsilon) + eq \tag{27}$$

holds.

From the second equation of the system (1), we have

$$\begin{aligned} \frac{dv}{dt} &= sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right) \\ &\geq sv\left(1 - \frac{v}{c}\right) \end{aligned} \tag{28}$$

Applying Lemma 2.3 in [34] to the above inequality leads to

$$\liminf_{t \rightarrow +\infty} v(t) \geq c. \tag{29}$$

For any sufficiently small positive constant  $\varepsilon > 0$ , there exists a  $T_1 > 0$ , such that for all  $t > T_1$ ,

$$v(t) > c - \varepsilon. \tag{30}$$

For  $t > T_1$ , we know from (30) and the first system equation (26) that

$$\begin{aligned} \frac{du}{dt} &= ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)uv - equ \\ &\leq ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)u(c - \varepsilon) - equ \\ &< (r - \alpha\phi(\omega)(c - \varepsilon) - eq)u. \end{aligned} \tag{31}$$

Hence,

$$u(t) < u(T_1) \exp\left\{\left(r - \alpha\phi(\omega)(c - \varepsilon) - eq\right)(t - T_1)\right\} \rightarrow 0 \tag{32}$$

as  $t \rightarrow +\infty$ .

(32) shows that there exists a  $T_2 > T_1$  such that

$$u(t) < \varepsilon \text{ for all } t \geq T_2. \tag{33}$$

For  $t \geq T_2$ , it follows from the second equation of the system (1) and (33) that

$$\begin{aligned} \frac{dv}{dt} &= sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right) \\ &\leq sv\left(1 - \frac{v}{\beta\phi(\omega)\varepsilon + c}\right) \end{aligned} \tag{34}$$

Applying Lemma 2.3 in [34] to above inequality leads to

$$\limsup_{t \rightarrow +\infty} v(t) \leq \beta\phi(\omega)\varepsilon + c. \tag{35}$$

Since  $\varepsilon$  is sufficiently small positive constant, setting  $\varepsilon \rightarrow 0$  in (35) leads to

$$\limsup_{t \rightarrow +\infty} v(t) \leq c. \tag{36}$$

(29) together with (36) leads to

$$\limsup_{t \rightarrow +\infty} v(t) = c. \tag{37}$$

The global asymptotic stability of the prey free equilibrium point  $E_2(0, c)$  can be demonstrated by combining equations (32) and (37), provided that assumption (26) is satisfied. The demonstration of Theorem 4.1 has been concluded.

V. GLOBAL ASYMPTOTICAL STABILITY OF POSITIVE EQUILIBRIUM

This section aims to investigate the global stability property of the positive equilibrium of the system (1). Indeed, we have the following result:

**Theorem 5.1.** *The positive equilibrium  $E_3(u^*, v^*)$  is globally asymptotically stable if*

$$r > \alpha\phi(\omega)c + eq \tag{38}$$

*holds. This means that the positive equilibrium is globally asymptotically stable as long as it exists.*

**Proof.** Previously, we showed that  $E_0$  and  $E_1$  are all unstable. Also, under the assumption that (38) holds, from the proof of Theorem 3.2, we know that  $E_2(0, c)$  is also unstable. Demonstrating the global asymptotic stability of  $E_3(u^*, v^*)$  can be achieved by establishing the absence of a limit cycle within the first quadrant of the system. The Dulac function  $B(u, v) = u^{-1}v^{-1}$  shall be taken into consideration.

$$\begin{aligned} & \frac{\partial(PB)}{\partial u} + \frac{\partial(QB)}{\partial v} \\ = & \frac{r(1 - \frac{u}{K}) - \frac{ru}{K} - \alpha(1 + \omega)v - eq}{uv} \\ & - \frac{ru(1 - \frac{u}{K}) - \alpha(1 + \omega)uv - equ}{u^2v} \\ & + \frac{s}{(\beta(1 + \omega)u + c)u} \\ = & - \frac{\beta\omega r u^2 + \beta r u^2 + cru + skv}{ukv(\beta u \omega + \beta u + c)} < 0, \end{aligned} \tag{39}$$

where

$$\begin{aligned} P(u, v) &= ru\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)uv - equ, \\ Q(u, v) &= sv\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right). \end{aligned} \tag{40}$$

According to Dulac's Theorem[35], it can be concluded that there exists no orbit that is closed within the first quadrant. Given that the solutions of system (1) are confined and  $E_3$  represents a distinct positive equilibrium point, it can be inferred that the global asymptotic stability of  $E_3$  can be established through the utilization of the Poincare-Bendixson theorem.

The demonstration of Theorem 5.1 has been completed.

**Remark 5.1.** It follows from Theorem 5.1 that once the positive equilibrium exists, it is globally asymptotically stable, which means that the system (1) has no bifurcation at  $E_3$ .

VI. UNIFORM PERSISTENCE

The uniform persistence of the system (1) denotes the long-term survival of all system species. Mathematically, this indicates that the rigorously positive trajectories of the system (1) that begin in  $R_2+$  will remain within a tightly closed set  $D$  in the first quadrant. From a mathematical point of view, in the previous section, we showed that once the positive equilibrium exists, it is globally asymptotically stable, hence, all the solutions of the system (1) with a positive initial value will approach  $E_3$  as  $t \rightarrow +\infty$ , which means the uniform persistence of the system. However, here we would like to take the method used in Jawad et al[32] to give another way to prove the uniform persistence of the system (1). Specifically, we have the following outcome.

**Theorem 6.1.***If*

$$r > eq + \alpha c\phi(\omega) \tag{41}$$

*holds true, then system (1) is uniformly persistent.*

**Proof.** It is observed that the equilibrium points are the sole potential omega limit sets of the system (1) situated on the periphery of the  $uv$ -plane. It is enough to show that the boundary equilibrium  $E_0, E_1$  and  $E_2$  could not be the  $\omega$ -limit set of the system (1). Consider the function  $w(u, v) = u^a v^b$ , where  $a$  and  $b$  are positive constants. Obviously,  $w(u, v) > 0$  for all  $(u, v) \in R_2+$ , and  $w(u, v) \rightarrow 0$  as  $u \rightarrow 0$  and  $v \rightarrow 0$ . Then

$$\begin{aligned} \dot{\phi}(u, v) &= \frac{\dot{w}}{w} = a\left(r\left(1 - \frac{u}{K}\right) - \alpha\phi(\omega)v - eq\right) \\ &+ b\left(s\left(1 - \frac{v}{\beta\phi(\omega)u + c}\right)\right). \end{aligned} \tag{42}$$

Then one could easily verify that

$$\begin{aligned} \phi(E_0) &= a(r - eq) + bs > 0, \\ \phi(E_1) &= bs > 0, \\ \phi(E_2) &= a\left(r - eq - \alpha c\phi(\omega)\right) > 0, \end{aligned} \tag{43}$$

for all  $a, b > 0$ . (43) shows that  $E_0, E_1$  and  $E_2$  could not be the  $\omega$ -limit set of the system (1). According to the Gard method [36], system (1) is uniformly persistent

VII. THE INFLUENCE OF THE WIND EFFECT AND OTHER FOOD RESOURCE

We will discuss this in three aspects.

(1) Threshold for the extinction or persistence of the prey species

Theorem 4.1 shows that if

$$r < \alpha\phi(\omega)c + eq \tag{44}$$

holds, which is equivalent to

$$\omega > \frac{r - eq}{\alpha c} - 1 \tag{45}$$

holds, then the prey-free equilibrium point  $E_2(0, c)$  is globally asymptotically stable, which means the extinction of the prey species. Thus, the threshold of the wind effect for the extinction or persistence of the prey species is  $R_0 = \frac{r - eq}{\alpha c} - 1$ .

Excessive wind impact may result in the eradication of the prey population. The reason is that with the increase of the wind effect, more and more prey species will be exposed to the wind blowing, and this may increase the rate of predation of the predator species.

(2) The influence of wind effect on the positive equilibrium.

Set  $A = \phi(\omega) = 1 + \omega$ , then

$$\begin{aligned} u^* &= \frac{k(A\alpha c + eq - r)}{A^2\alpha\beta k + r}, \\ v^* &= \beta\phi(\omega)u^* + c \\ &= \frac{A\beta ekq - A\beta kr - cr}{A^2\alpha\beta k + r}. \end{aligned} \tag{46}$$

Then the positive equilibrium  $E_3(u^*, v^*)$  satisfies

$$\begin{aligned} \frac{du^*}{d\omega} &= \frac{k\alpha(A^2\alpha\beta ck + 2A\beta ekq - 2A\beta kr - cr)}{(A^2\alpha\beta k + r)^2} \\ &= \frac{k\alpha[(A^2\alpha\beta k - r)c + 2A\beta k(eq - r)]}{(A^2\alpha\beta k + r)^2}. \end{aligned} \tag{47}$$

Hence, if

$$r > \alpha\beta kA^2, \tag{48}$$

which is equivalent to

$$\omega < \sqrt{\frac{r}{\alpha\beta k}} - 1 \tag{49}$$

holds, then

$$\frac{du^*}{d\omega} < 0,$$

that is, if (48) and (49) hold, then the density of prey species  $u^*$  is the decreasing function of wind effect  $\omega$ .

Also,

$$\begin{aligned} \frac{dv^*}{d\omega} &= \frac{\beta k(A^2\alpha\beta ekq - A^2\alpha\beta kr - 2A\alpha cr - eqr + r^2)}{(A^2\alpha\beta k + r)^2} \\ &= \frac{k\beta[(A^2\alpha\beta k - r)(eq - r) - 2A\alpha cr]}{A^2\alpha\beta k + r)^2}. \end{aligned} \tag{50}$$

Hence, if

$$(A^2\alpha\beta k - r)(eq - r) > 2A\alpha cr \tag{51}$$

holds, then

$$\frac{dv^*}{d\omega} > 0,$$

that is, if (51) holds, then the density of predator species  $v^*$  is the increasing function of wind effect  $\omega$ .

If

$$(A^2\alpha\beta k - r)(eq - r) < 2A\alpha cr \tag{52}$$

holds, then

$$\frac{dv^*}{d\omega} < 0,$$

that is, if (52) holds, then the density of predator species  $v^*$  is the decreasing function of wind effect  $\omega$ .

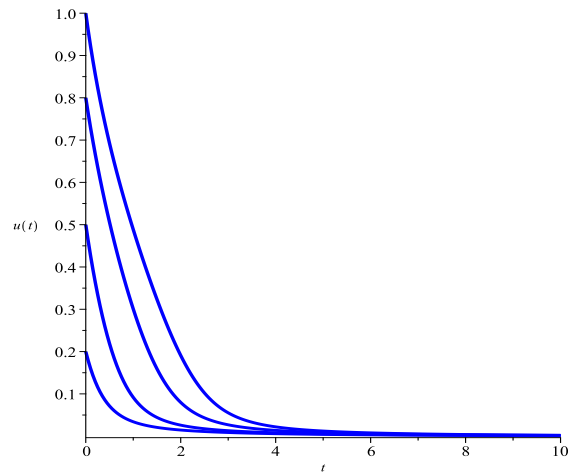


Fig. 1. Dynamic behaviors of the first component  $u(t)$  of the system (54), the initial condition  $(u(0), v(0)) = (1, 0.1), (0.8, 0.2), (0.2, 0.8)$  and  $(0.5, 0.5)$ , respectively. Here we choose  $\omega = 2$ .

(3) The influence of other food resource

From inequality (16) one could easily see that  $c$  plays an important role in the permanence and stability of the system.

By computation, one has

$$\begin{aligned} \frac{du^*}{dc} &= -\frac{k(1 + \omega)\alpha}{(1 + \omega)^2\alpha\beta k + r} < 0, \\ \frac{dv^*}{dc} &= \frac{r}{(1 + \omega)^2\alpha\beta k + r} > 0. \end{aligned} \tag{53}$$

With the increasing of other food resource, the predator species could increasing its density, and the more predator species, the more food is needed, this finally lead to the decreasing of the density of prey species.

### VIII. NUMERIC SIMULATIONS

**Example 8.1.** Let us now examine the following model:

$$\begin{aligned} \frac{du}{dt} &= u(1 - u) - \phi(\omega)uv - \frac{1}{2}u, \\ \frac{dv}{dt} &= v\left(1 - \frac{v}{\phi(\omega)u + \frac{1}{4}}\right), \end{aligned} \tag{54}$$

In this case,  $r = K = \alpha = \beta = q = s = 1, e = \frac{1}{2}, c = \frac{1}{4}$ . corresponds to system (1). then one could see that if  $\omega > 1$ , then

$$r = 1 < \frac{1}{2} + \frac{1}{4}(1 + \omega) = eq + c\alpha\phi(\omega)$$

holds, Theorem 3.2 states that the prey-free equilibrium  $E_2(0, \frac{1}{4})$  is globally asymptotically stable. This assertion is support by Numeric simulations (Fig.1-2) (here we choose  $\omega = 2$ ).

**Example 8.2.** Let us now examine the subsequent model:

$$\begin{aligned} \frac{du}{dt} &= u(2 - u) - \phi(\omega)uv - \frac{1}{4}u, \\ \frac{dv}{dt} &= v\left(1 - \frac{v}{\phi(\omega)u + \frac{1}{4}}\right), \end{aligned} \tag{55}$$

In accordance with equation (1), we assign the values  $r = K = \alpha = \beta = q = s = 1, e = \frac{1}{4}, c = \frac{1}{4}$ . Upon analysis, it

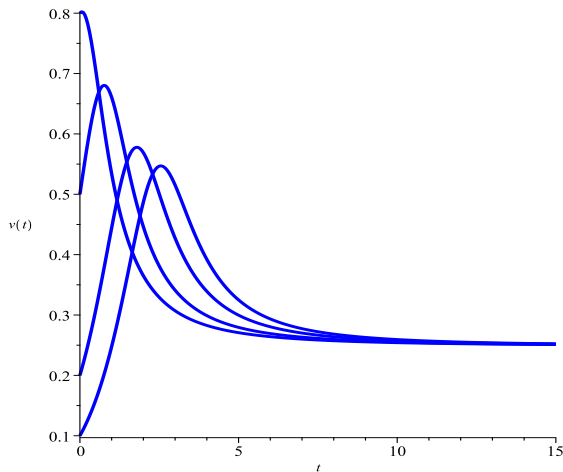


Fig. 2. Dynamic behaviors of the second component  $v(t)$  of system (54), the initial condition  $(u(0), v(0)) = (1, 0.1), (0.8, 0.2), (0.2, 0.8)$  and  $(0.5, 0.5)$ , respectively. Here we choose  $\omega = 2$ .

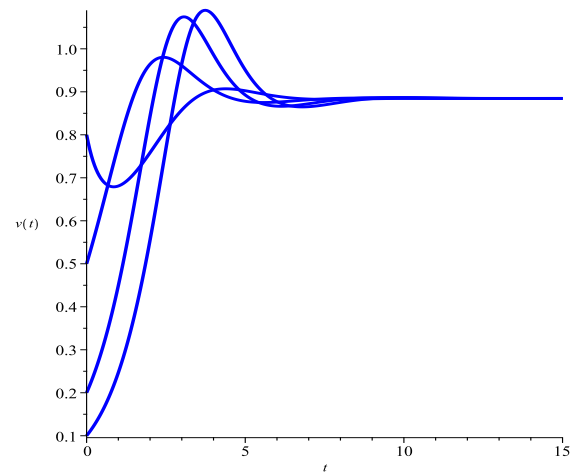


Fig. 4. Dynamic behaviors of the second component  $v(t)$  of the system (55), the initial condition  $(u(0), v(0)) = (1, 0.1), (0.8, 0.2), (0.2, 0.8)$  and  $(0.5, 0.5)$ , respectively. Here we choose  $\omega = 2$ .

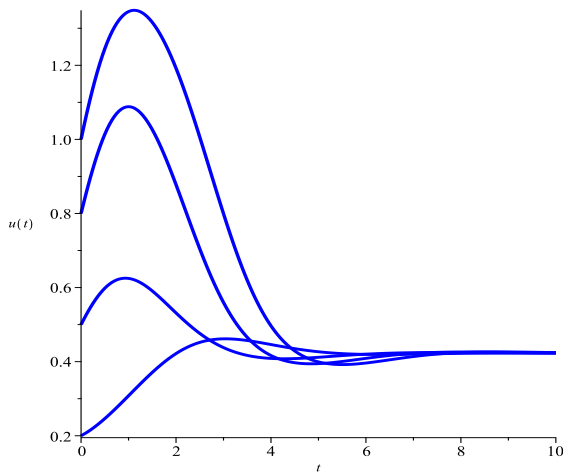


Fig. 3. Dynamic behaviors of the first component  $u(t)$  of the system (55), the initial condition  $(u(0), v(0)) = (1, 0.1), (0.8, 0.2), (0.2, 0.8)$  and  $(0.5, 0.5)$ , respectively. Here we choose  $\omega = 2$ .

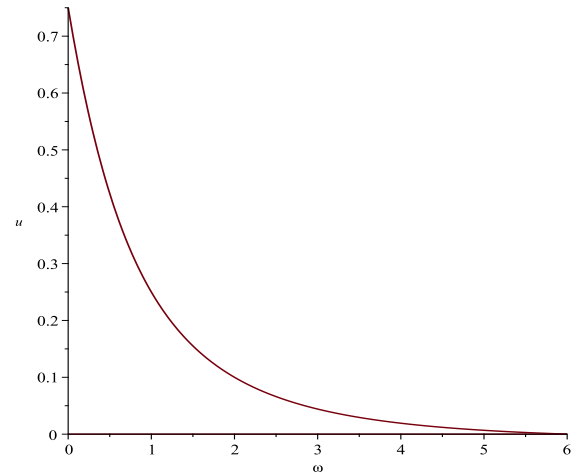


Fig. 5. Relationship of  $u^*$  and  $\omega$  in the system (55).

can be observed that when  $\omega < 6$ , the inequality  $r = 2 > \frac{1}{2} + \frac{1}{4}(1+\omega) = eq + c\alpha\phi(\omega)$  is satisfied. As per Theorem 5.1, this implies that the positive equilibrium  $E_3(u^*, v^*)$  is globally asymptotically stable. The assertion is further corroborated by numerical simulations, as depicted in Figures 3 and 4.

Also, from (48), it follows that for  $\omega < \sqrt{2} - 1$ ,  $u^*$  is the decreasing function of  $\omega$ , however, it follows from (55) that  $u^*$  satisfies the equation

$$(2 - u)u - (1 + \omega)u(u\omega + u + \frac{1}{4}) - \frac{u}{4} = 0.$$

Numeric simulation (Fig.5) shows that for  $\omega \in (0, 6)$ ,  $u^*$  is the decreasing function of  $\omega$ .

**Example 8.3.** Let us now examine the subsequent model:

$$\begin{aligned} \frac{du}{dt} &= u(2 - u) - 2uv - \frac{1}{4}u, \\ \frac{dv}{dt} &= v\left(1 - \frac{v}{2u + c}\right), \end{aligned} \tag{56}$$

Here, all the other coefficients are the same as system (55), only with  $\omega = 1$  and take  $c$  as the variable. then one could see that if  $c < \frac{7}{8}$ , then

$$r = 2 > \frac{1}{2} + 2c = eq + c\alpha\phi(\omega)$$

holds, it follows from Theorem 5.1, the global asymptotic stability of the positive equilibrium  $E_3(u^*, v^*)$  has been established. By computation, one could find that  $u^* = \frac{7}{20} - \frac{2c}{5}$ . Numeric simulation (Fig.6) shows that for  $c \in (0, 1)$ ,  $u^*$  is the decreasing function of  $c$ .

### IX. CONCLUSION

Wind effect plays important roles in the relationships of species, however, only recently did scholars begin to establish mathematical modeling to investigate the influence of wind effect[28]-[32]. Jawad, Sultan, and Winter[32] proposed a system (4), and they gave a thorough analysis of the dynamic behaviors of the system. However, from the numeric simulation in [32], we found that with the increase of the



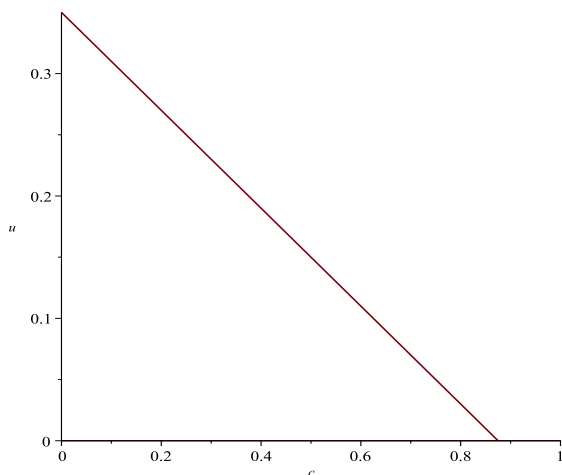


Fig. 6. Relationship of  $u^*$  and  $c$  in the system (56).

wind effect, the densities of predator and prey species are all increasing, and we argued that such a phenomenon is not consistent with the relationship between the aerial predators and the reed warbler, The exposure of reed warbler nests to wind blowing may result in heightened detection by aerial predators, potentially leading to an increased predation rate for this species.

To solve this issue, we proposed the system (1), which, to some extent, overcomes the drawback of the system (4). We showed that the wind effect may accelerate the extinction chance of the prey species (Theorem 4.1), however, for the limited wind effect, though the density of the prey species could be reduced (see Section 7 for more detailed discussion), two species could coexist in a stable state (Theorem 5.1). The main findings are demonstrated through numerical simulations to establish their viability.

So far, there is very little literature on how wind effects affect population growth[28]-[32]. We think it is necessary to propose some more suitable models, for example, incorporating the functional response to the system (1) and then investigating the dynamic behaviors of the system. We leave this for future investigation.

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