On Partial Derivative of the Multi-Variable Incomplete H-function

Akanksha Shukla, Shalini Shekhawat and Kanak Modi

Abstract—**In this paper, some formulas for the incomplete H- function of several variables have been established with the help of partial derivatives. On reducing the parameters, these formulas present a large variety of incomplete functions like incomplete Gamma functions, incomplete Wright functions and many more. We have also presented some of the cases here in this paper.**

Index Terms—**Special function, Fractional Calculus, Bessel-Maitland function, Incomplete H- function.**

I. Introduction

SRIVASTAVA and other mathematicians ([3]-[4]) have thoroughly studied the incomplete Gamma-function and incomplete hypergeometric function. After that Srivastava et al. ([5]-[7]) have proposed and defined the incomplete H-function and the incomplete $\overline{H}$ -function. Recently, Bansal et al.([1]-[2]) have studied the incomplete Aleph-function ([8]-[11]), the incomplete I-function and find the integrals of the incomplete H-function ([18]-[19]) respectively. Motive of this paper is to find some formulas for the incomplete H- function of several variables with the help of partial derivatives ([12],[20]-[22]). We have also discussed some particular cases.

II. Definition and Preliminaries

A. Incomplete Gamma H-function

In current segment, we audit essential hypothesis of local fractional math, that has been used in the paper.
The incomplete Gamma H-function is given as follows:

$$ {}^{(\gamma)}H(u_1,...,u_k) = {}^{(\gamma)}H^{0,b;a_1,b_1;.;a_k,b_k}_{p,q;p_1,q_1;.;p_k,q_k}\left(\begin{matrix} u_1 \\ \vdots \\ u_k \end{matrix}\middle| \begin{matrix} (d_1,\alpha_1,...,\alpha_1^{(k)},x),(d_j,\alpha'_j,...,\alpha_j^{(k)})_{2,p}(r'_j,\mu')_{1,p_1},...,(r_j^{(k)},\mu_j^{(k)})_{1,p_k} \\ \vdots \\ (v_j,\beta'_j,...,\beta_j^{(r)})_{1,q}(e'_j,\sigma')_{1,q_1},...,(e_j^{(k)},\sigma_j^{(k)})_{1,q_k} \end{matrix}\right) \tag{1}$$

$$ = \frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_c}\psi(s_1,...,s_k)\prod_{i=1}^{k}\phi_{i'}(s_i)u_i^{s_i}ds_1...ds_k \tag{2}$$

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where $\psi(s_1,...,s_k), \phi_i(s_i),\ i=1,...,k$ are given by:

$$\psi(s_1,...,s_k) = \frac{\Gamma(1-d_1+\sum_{i=1}^{k}\alpha_j^{(1)}t_1,x)}{\prod_{j=b+1}^{p}\Gamma(d_j-\sum_{i=1}^{k}\alpha_j^{(i)}t_i,x)} \times \frac{\prod_{j=2}^{b}\Gamma(1-d_j+\sum_{i=1}^{k}\alpha_j^{(i)}t_j)}{\prod_{j=1}^{q}\Gamma(1-v_j+\sum_{i=1}^{k}\beta_j^{(i)}t_j)} \tag{3}$$

and

$$\phi(s_i) = \frac{\prod_{j=i}^{a_i}\Gamma(e_j^{(i)}-\sigma_j^{(i)}t_j)\prod_{j=1}^{b_i}\Gamma(1-r_j^{(i)}+\mu_j^{(i)}t_i)}{\prod_{j=a+1}^{q_i}\Gamma(1-e_j^{(i)}+\sigma_j^{(i)}t_i)\prod_{j=b_i+1}^{p_i}\Gamma(r_j^{(i)}-\mu_j^{(i)}t_i)} \tag{4}$$

where $i=1,...,k\ \&\ u_i\neq 0$
and an empty multiplication is considered as unity. Also $b,p,q,a_i,b_i,p_i,q_i(i=1,...,k)$ are all positive integers such that $0\le b\le p, 0\le q\ 0\le a_i\le q_i; 0\le b_i\le p_i(i=1,...,k)$. $A_j, B_j, C_j^{(i)}, D_j^{(i)}$ and $\alpha_j^{(i)}, \beta_j^{(i)}, \mu_j^{(i)}, \sigma_j^{(i)}(i=1,...,k)$ and are all positive numbers and $d_j, v_j, r_j^{(i)}, e_j^{(i)}$ are complex numbers.
The contour L_i lies in the p- plane and goes from $-\omega\infty$to $+\omega\infty$ with loops such that the poles of $\Gamma^{A_j}(1-d_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_j)\ (j=1,...,b), \Gamma^{C_j^{(i)}}(\Gamma(1-r_j^{(i)}+\mu_j^{(i)}s_j)\ (j=1,...,b_i)$lie to the left of L_i. The incomplete Gamma H-function of k-variable will be analytic if:

$$V_i = \sum_{j=1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} + \sum_{j=1}^{p_i}\mu_j^{(i)} - \sum_{j=1}^{q_i}\sigma_j^{(i)} \le 0,\ i=1,...,k \tag{5}$$

The integral (2) converges absolutely if

$$|\arg(u_i)| < \frac{1}{2}\Delta_i\pi,\ i=1,...,k$$

where

$$\Delta_i = \sum_{j=1}^{b}\alpha_j^{(i)} - \sum_{j=b+1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} + \sum_{j=1}^{a_i}\sigma_j^{(i)} + \sum_{j=1}^{b_i}\mu_j^{(i)} - \sum_{j=b_i+1}^{p_i}\mu_j^{(i)} \tag{6}$$

According to Braaksma ([13]) asymptotic behavior can be expressed in the form given below:

$${}^{(\Gamma)}H(u_1,...,u_k) = 0\left(|u_1|^{\alpha_1},...,|u_k|^{\alpha_k}\right),$$
$$\max(|u_1|,...,|u_k|)\to 0$$
$${}^{(\Gamma)}H(u_1,...,u_r) = 0\left(|u_1|^{\beta_1},...,|u_k|^{\beta_k}\right),$$
$$\min(|u_1|,...,|u_k|)\to\infty$$

where $i = 1, ..., k$;
$\alpha_i = \min_{1\le j\le a^{(i)}} Re\left[\left(\frac{e_j^{(i)}}{\sigma_j^{(i)}}\right)\right]$
and
$\beta_i = \max_{1\le j\le b^{(i)}} Re\left[\left(\frac{r_j^{(i)}-1}{\mu_j^{(i)}}\right)\right]$
Notations used in paper are defined below:

$$X = a_1, b_1; ...; b_k\, ;\ Y = p_1, q_1; ...; p_k, q_k \tag{7}$$

$$A = (d_j; \alpha_j^{(1)}, ..., \alpha_j^{(k)})_{2,p} :\ B = (v_j; \beta_j^{(1)}, ..., \beta_j^{(k)})_{1,q} \tag{8}$$

$$C = (r_j^{(1)}, \mu_j^{(1)})_{1,p_1}; ...; (r_j^{(k)}, \mu_j^{(k)})_{1,p_k}; \tag{9}$$

$$D = (e_j^{(1)}, \sigma_j^{(1)})_{1,q_1}; ...; (e_j^{(k)}, \sigma_j^{(k)})_{1,q_k} \tag{10}$$

Similarly, we described the incomplete gamma multivariable H-function:

$$^{(\gamma)}H(u_1, ..., u_k) = {}^{(\gamma)}H^{0,b;a_1,b_1;.;a_k,b_k}_{p,q;p_1,q_1;.;p_k,q_k}\left(\begin{array}{c|c} u_1 & (d_1, \alpha_1, ..., \alpha_1^{(k)}, x), (d_j, \alpha_j', ..., \alpha_j^{(k)})_{2,p} (r_j', \mu')_{1,p_1}, ..., (r_j^{(k)}, \mu_j^{(k)})_{1,p_k} \\ \vdots & \vdots \\ u_k & (v_j, \beta_j', ..., \beta_j^{(r)})_{1,q} (e_j', \sigma')_{1,q_1}, ..., (e_j^{(k)}, \sigma_j^{(k)})_{1,q_k} \end{array}\right) \tag{11}$$

$$= \frac{1}{(2\pi\omega)^k} \int_{L_1} ... \int_{L_c} \psi'(s_1, ..., s_k) \prod_{i=1}^{k} \phi_{i'}(s_i) u_i^{s_i} ds_1 ... ds_k. \tag{12}$$

$$\psi'(s_1, ..., s_k) = \frac{\gamma(1-d_1+\sum_{i=1}^{k}\alpha_j^{(1)}t_1, x)}{\prod_{j=b+1}^{p}\Gamma(d_j-\sum_{i=1}^{k}\alpha_j^{(i)}t_i, x)} \times \frac{\prod_{j=2}^{b}\Gamma(1-d_j+\sum_{i=1}^{k}\alpha_j^{(i)}t_j)}{\prod_{j=1}^{q}\Gamma(1-v_j+\sum_{i=1}^{k}\beta_j^{(i)}t_j)} \tag{13}$$

B. Bessel-Maitland function

The Bessel-Maitland function $J_\vartheta^\xi(x)$ is defined by the following series representation:

$$J_\vartheta^\xi(x) = \sum_{l=0}^{\infty} \frac{(-x)^n}{l!\Gamma(\xi l + \vartheta + 1)} \tag{14}$$

Generalization of Bessel-Maitland function defined by Singh et al. ([10]) is as follows:

$$J_{\vartheta,y}^{\xi,\zeta}(x) = \sum_{l=0}^{\infty} \frac{(\zeta)_{yl}(-x)^l}{l!\Gamma(\xi l + \vartheta + 1)} \tag{15}$$

where $\vartheta, \zeta, \xi \in C$, $Re(\vartheta) \ge -1$, $Re(\zeta) \ge 0$, $Re(\xi) \ge 0$ and $\vartheta \in (0, 1) \cup N$ and $(\zeta)_0 = 1$.
Generalization of Pochhammer symbol is defined as $(\zeta)_{\vartheta l} = \frac{\Gamma(\zeta + l\vartheta)}{\Gamma(\zeta)}$.
Later on, Ghayasuddin and Khan ([8]), introduced new generalization of Bessel-Maitland, which is defined as follows:

$$J_{\vartheta,\zeta,\chi}^{\xi,y,z}(x) = \sum_{l=0}^{\infty} \frac{(\zeta)_{yl}(-x)^n}{\Gamma(\xi l + \vartheta + 1)\,(\chi)_{zl}} \tag{16}$$

where $\xi, \zeta, \vartheta, \chi \in C$, $Re(\xi) \ge 0$, $Re(\vartheta) \ge -1$, $Re(\chi) \ge 0$, $Re(\zeta) \ge 0$; $y, z > 0$and$y < R(\phi) + z$.

Khan et al. ([9]-[11]) introduced a new extension of Bessel-Maitland function which is defined by:

$$J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}(x) = \sum_{l=0}^{\infty} \frac{(\xi)_{\tau l}(\zeta)_{yl}(-x)^n}{\Gamma(l\theta + \phi + 1)\,(\chi)_{zl}(\vartheta)_{\rho l}}. \tag{17}$$

Where $\phi, \theta, \vartheta, \tau, \rho, \xi, \chi, \zeta \in C$; $R(\theta) > 0, R(\tau) > 0, R(\vartheta) > 0, R(\rho) > 0, R(\phi) \ge -1, R(\xi) > 0, R(\zeta) > 0, R(\chi) > 0$; $y, z > 0$,and$y < R(\phi) + z$.

The whole article is divided into four parts. Section 1 deals with the introduction and pre-requisite part of the paper. Part 2 is having the main results of the paper including all the theorems while next segment is having some particular cases of the main results and the last section is having the conclusion of the article. At last we have also acknowledged the authors and researchers whose papers were found extremely helpful in completion of the presented article. So the last part is dedicated to the references.

III. Main Results

In this section we define four partial derivative formula involving multivariable incomplete H-function and Bessel Maitland function.

Theorem-1

$$\prod_{i=1}^{m}[D_x(fx + gy + h) - \mu_i] \times \left\{\begin{array}{l}(fx + gy + h)^\lambda \times \\ ^{(\Gamma)}H[u_1(fx + gy + h)^{n_1}, ..., u_k(fx + gy + h)^{n_k}]\end{array}\right\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx + gy + h)^n]$$
$$= f^m(fx + gy + h)^\lambda \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx + gy + h)^n]$$
$$^{(\Gamma)}H^{0,b+k;X}_{p+k,q+k;Y}\left(\begin{array}{c|c} u_1(fx + gy + h)^{n_1} & (d_1, \alpha_1, ..., \alpha_1^{(r)}, x), A : (\eta_i - \lambda - nl - 1, n_1, ..., n_j) : C \\ . & . \\ . & . \\ u_k(fx + gy + h)^{n_k} & B : (\eta_i - \lambda - nl, n_1, ..., n_j) \quad : D \end{array}\right),$$

provided that $D_x = \frac{\partial}{\partial x}, \lambda > 0, n_j > 0$ for $j = 1, ..., k. f, g, h$ are complex numbers,$f \ne 0, g \ne 0, m$ is a positive integer. Also $|\arg u_i(fx + gy + h)^{n_i}| < \frac{1}{2}\Delta_i\pi$ where Δ_i is defined by (6); $\mu_i = f\eta_i$ for $i = 1, ..., m$.

Proof: Let P_1 is the L.H. S. of Theorem 1, then

$$\prod_{i=1}^{m}[D_x(fx + gy + h) - \mu_i] \times \left\{\begin{array}{l}(fx + gy + h)^\lambda \times \\ ^{(\Gamma)}H[u_1(fx + gy + h)^{n_1}, ..., u_k(fx + gy + h)^{n_k}]\end{array}\right\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx + gy + h)^n]$$

$$P_1 = \prod_{i=1}^{m}\{D_x(fx + gy + h)^{\lambda+1} \times {}^{(\Gamma)}H[u_1(fx + gy + h)^{n_1}, ..., u_k(fx + gy + h)^{n_k}] - \mu_i(fx + gy + h)^\lambda \times {}^{(\Gamma)}H[u_1(fx + gy + h)^{n_1}, ..., u_k(fx + gy + h)^{n_k}]\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx + gy + h)^n]$$

$$P_1=\prod_{i=1}^{m}\{f(\lambda+1)(fx+gy+h)^{\lambda}$$
$$\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\psi(s_1,...,s_k)$$
$$\left[\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}\,ds_1...ds_k\right]$$
$$\times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}+$$
$$\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}(fx+gy+h)^{\lambda+1}$$
$$D_x\frac{1}{(2\pi\omega)^k}\int_{L_1}..\int_{L_k}\psi(s_1,..,s_k)\times$$
$$\left[\prod_{i=1}^{k}\psi_i(s_i)[u_i^{s_i+l}(fx+gy+h)^{n_is_i+nl}]\,ds_1...ds_k\right]$$
$$-\mu_i(fx+gy+h)^{\lambda}\times$$
$${}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}\times$$
$$\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}.$$

or

$$P_1=f^m(fx+gy+h)^{\lambda}\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\psi(s_1,...,s_k)$$
$$\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}$$
$$\times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$
$$\times\prod_{i=1}^{k}(\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i)\,ds_1...ds_k.$$

Now, using the relation

$$\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i=\frac{\Gamma\left(\lambda+2+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}{\Gamma\left(\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}$$

We can easily get the R.H.S. of the theorem by interpreting the terms in form of Mellin Barnes type contour integral defined by (2).

Theorem-2

$$\prod_{i=1}^{m}[D_y(fx+gy+h)-\omega_i]\left\{(fx+gy+h)^{\lambda}\right.$$
$$\times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}[-u_j(fx+gy+h)^n]=g^m(fx+gy+h)^{\lambda}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}[-u_i(fx+gy+h)^n]$$
$${}^{(\Gamma)}H^{0,b+m;X}_{p+m,q+m;Y}\left(\begin{array}{c}u_1(fx+gy+h)^{n_1}\\ \vdots\\ u_k(fx+gy+h)^{n_k}\end{array}\middle|\begin{array}{c}(a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m},A:C\\ \vdots\\ (a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m},A:C\end{array}\right)\tag{18}$$

provided that $D_y=\frac{\partial}{\partial y}$, $\lambda>0, n_j>0$ for $j=1,...,k. f,g,h$ are complex numbers, $f\neq0, g\neq0, m$ is a positive integer . Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_i\pi$, where Δ_i is defined by (6); $\omega_i=g\eta_i$ for $i=1,...,m$.

Proof: Let P_1 is the left hand side of Theorem 2, then

$$\prod_{i=1}^{m}[D_y(fx+gy+h)-\omega_i]\left\{(fx+gy+h)^{\lambda}\times\right.$$
$${}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}[-u_i(fx+gy+h)^n]$$

or

$$P_1=\prod_{i=1}^{m}\{D_y(fx+gy+h)^{\lambda+1}\times$$
$${}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]$$
$$-\omega_i(fx+gy+h)^{\lambda}\times$$
$${}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}[-u_i(fx+gy+h)^n]$$

or

$$P_1=\prod_{i=1}^{m}\{D_y(fx+gy+h)^{\lambda+1}$$
$$\times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]$$
$$-\omega_i(fx+gy+h)^{\lambda}$$
$$\times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$

or

$$P_1=\prod_{i=1}^{m}\{g(\lambda+1)(fx+gy+h)^{\lambda}$$
$$\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\psi(s_1,...,s_k)$$
$$\left[\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}\,ds_1...ds_k\right]$$
$$\times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}+$$
$$\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}(fx+gy+h)^{\lambda+1}$$
$$D_y\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\psi(s_1,...,s_k)$$
$$\times\left[\prod_{i=1}^{k}\psi_i(s_i)[u_i^{s_i+l}(fx+gy+h)^{n_is_i+nl}]\,ds_1...ds_k\right]$$
$$-\omega_i(fx+gy+h)^{\lambda}$$
$$\times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},..,u_k(fx+gy+h)^{n_k}]\}$$
$$\times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$

$$P_1=g^m(fx+gy+h)^{\lambda}\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\psi(s_1,..,s_k)$$
$$\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}$$
$$\times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$
$$\times\prod_{i=1}^{k}(\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i)\,ds_1..ds_k$$

Now, using the relation

$$\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i=\frac{\Gamma\left(\lambda+2+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}{\Gamma\left(\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}$$

We can easily get the R.H.S. of (17) by interpreting the terms in form of Mellin Barnes type contour integral defined by (2).

Theorem-3

$$\prod_{i=1}^{m}[(fx+gy+h)D_x-\mu_i]\Big\{(fx+gy+h)^{\lambda}\times {}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\Big\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n]$$
$$= f^m(fx+gy+h)^{\lambda}\times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n] \times {}^{(\Gamma)}H^{0,b+k;X}_{p+k,q+k;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1} \\ . \\ . \\ u_k(fx+gy+h)^{n_k}\end{array}\middle| \begin{array}{c}(d_1,\alpha_1,...,\alpha_1^{(r)},x),A:(\eta_i-\lambda-nl,n_1,....,n_j):C \\ . \\ . \\ B:(1-\lambda+\eta_i-nl,n_1,....,n_j)\quad :D\end{array}\right) \tag{19}$$

provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0, n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers,$f\neq 0, g\neq 0, m$ is a positive integer .Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_i\pi$, where Δ_i is defined by (6); $\mu_i=f\eta_i$ for $i=1,...,m$.

Proof: Let P_1 is equal to the left hand side of Theorem 3, then

$$\prod_{i=1}^{m}[(fx+gy+h)D_x-\mu_i] \Big\{(fx+gy+h)^{\lambda}\times {}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\Big\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n]$$

$$P_1=\prod_{i=1}^{m} f\lambda(fx+gy+h)^{\lambda} \frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k) \left[\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}\,ds_1...ds_k\right] \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$
$$+\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}(fx+gy+h)^{\lambda+1} D_x\frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k) \times\left[\prod_{i=1}^{k}\psi_i(s_i)[u_i^{s_i+l}(fx+gy+h)^{n_is_i+nl}]\,ds_1...ds_k\right]$$
$$-\mu_i(fx+gy+h)^{\lambda} \times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\} \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$

or

$$P_1=f^m(fx+gy+h)^{\lambda} \frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k) \prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i} \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}} \times\prod_{i=1}^{k}(\lambda+\sum_{j=1}^{k}n_js_j+nl-\eta_i)\,ds_1...ds_k$$

Now, using the relation

$$\lambda+\sum_{j=1}^{k}n_js_j+nl-\eta_i=\frac{\Gamma\left(\lambda+1+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}{\Gamma\left(\lambda+\sum_{j=1}^{k}n_js_j+nl-\eta_i\right)}$$

We can easily get the R.H.S. of (18) by interpreting the terms in form of Mellin Barnes type contour integral defined by (2).

Theorem-4

$$\prod_{i=1}^{m}[(fx+gy+h)D_y-\omega_i] \Big\{(fx+gy+h)^{\lambda}\times {}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\Big\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n]$$
$$=g^m(fx+gy+h)^{\lambda} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n] \times{}^{(\Gamma)}H^{0,b+k;X}_{p+k,q+k;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1} \\ . \\ . \\ u_k(fx+gy+h)^{n_k}\end{array}\middle| \begin{array}{c}(d_1,\alpha_1,...,\alpha_1^{(r)},x),A:(\eta_i-\lambda-nl,n_1,....,n_j):C \\ . \\ . \\ B:(1-\lambda+\eta_i-nl,n_1,....,n_j)\quad :D\end{array}\right) \tag{20}$$

provided that $D_y=\frac{\partial}{\partial y}$, $\lambda>0, n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers,$f\neq 0, g\neq 0, m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_i\pi$ where Δ_i is defined by (6); $\omega_i=g\eta_i$ for $i=1,...,m$.

Proof: Let P_1 is the L.H. S. of Theorem 4, then

$$\prod_{i=1}^{m}[(fx+gy+h)D_y-\mu_i]\Big\{(fx+gy+h)^{\lambda}\times {}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(ax+by+c)^{n_k}]\Big\} \times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}[-u_i(fx+gy+h)^n]$$

$$P_1=\prod_{i=1}^{m}\{g\lambda(fx+gy+h)^{\lambda} \frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k) \left[\prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i}\,ds_1...ds_k\right] \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}$$
$$+\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}}(fx+gy+h)^{\lambda+1} D_y\frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k)\times \left[\prod_{i=1}^{k}\psi_i(s_i)[u_i^{s_i+l}(fx+gy+h)^{n_is_i+nl}]\,ds_1...ds_k\right]$$
$$-\omega_i(fx+gy+h)^{\lambda}\times {}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\} \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}},$$

or

$$P_1=g^m(fx+gy+h)^{\lambda} \frac{1}{(2\pi\omega)^k}\int_{L_1}...\int_{L_k}\psi(s_1,...,s_k) \prod_{i=1}^{k}\psi_i(s_i)[u_i(fx+gy+h)^{n_i}]^{s_i} \times\sum_{l=0}^{\infty}\frac{(\xi)_{\tau l}(\zeta)_{yl}[u_i(fx+gy+h)^n]^l}{\Gamma(l\theta+\phi+1)(\chi)_{zl}(\vartheta)_{\rho l}} \times\prod_{i=1}^{k}(\lambda+\sum_{j=1}^{k}n_js_j+nl-\eta_i)\,ds_1...ds_k.$$

Now, using the relation

$$\lambda+\sum_{j=1}^{k} n_j s_j+nl-\eta_i=\frac{\Gamma\left(\lambda+1+\sum_{j=1}^{k} n_j s_j+nl-\eta_i\right)}{\Gamma\left(\lambda+\sum_{j=1}^{k} n_j s_j+nl-\eta_i\right)}$$

. We can easily get the R.H.S. of (19) by interpreting the terms in form of Mellin Barnes type contour integral defined by (2).

IV. Special Cases

Gupta and Mittal ([14]) defined the H-function of two variables, on applying same concept for multivariable H-function, which gives the Gamma incomplete H-function of two variables and Gamma incomplete H- function of two variables. Here some of the special cases are defined by using Theorem 1.

A. Corollary 1

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]\Big\{(fx+gy+h)^{\lambda}$$
$$^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},u_2(fx+gy+h)^{n_2}]\Big\}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[-u_i(fx+gy+h)^{n}\right]$$
$$=f^m(fx+gy+h)^{\lambda}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[-u_i(fx+gy+h)^{n}\right]$$
$$\times{}^{(\Gamma)}H^{0,b+2;X}_{p+2,q+2;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \cdot\\ \cdot\\ u_2(fx+gy+h)^{n_2}\end{array}\right.$$
$$\left|\begin{array}{c}(d_1,\alpha_1^{(1)},\alpha_1^{(2)},x),A:(\eta_i-\lambda-nl-1,n_1,n_2):C\\ \cdot\\ \cdot\\ B:(\eta_i-\lambda-nl,n_1,n_2)\quad :D\end{array}\right)$$

provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0,n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0,g\neq 0,m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (6); $\mu_i=f\eta_i$ for $i=1,...,m$. Under the same condition as Theorem (1). Similarly, we can have defined the incomplete Gamma function of two variables.

B. Corollary 2

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]$$
$$\Big\{(fx+gy+h)^{\lambda}\times{}^{(\Gamma)}H[u(fx+gy+h)^{n}]\Big\}$$
$$\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[-u_i(fx+gy+h)^{n}\right]$$
$$=f^m(fx+gy+h)^{\lambda}\times J^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[-u_i(fx+gy+h)^{n}\right]$$
$$\times{}^{(\Gamma)}H^{0,b+2;X}_{p+2,q+2;Y}\left(u(fx+gy+h)^{n}\right.$$
$$\left|\begin{array}{c}(d_1,\alpha_1^{(1)},x),A:(\eta_i-\lambda-nl-1,n_1):C\\ \cdot\\ \cdot\\ B:(\eta_i-\lambda-nl,n_1)\quad :D\end{array}\right)$$

Provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0,n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0,g\neq 0,m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (6); $\mu_i=f\eta_i$ for $i=1,...,m$. Under the same condition as Theorem (1) Similarly, we can have defined the incomplete Gamma function of one variable.

Particular Cases

1. On replacing ϕ by $\phi-1$ in theorem (1) we get

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]$$
$$\Big\{(fx+gy+h)^{\lambda^{(\Gamma)}}$$
$$\times H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$\times E^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[u_i(fx+gy+h)^{n}\right]$$
$$=f^m(fx+gy+h)^{\lambda}\times E^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[u_j(fx+gy+h)^{n}\right]$$
$$^{(\Gamma)}H^{0,b+m;X}_{p+m,q+m;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \vdots\\ u_k(fx+gy+h)^{n_k}\end{array}\right.$$
$$\left|\begin{array}{c}(a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m}A:C\\ \vdots\\ B,(\eta_i-\lambda-nl,n_1,....,n_j)_{i=1,m}\ :D\end{array}\right)$$

Provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0,n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0,g\neq 0,m$ is a positive integer.
Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (1.6); $\mu_i=f\eta_i$ for $i=1,...,m$, where $E^{\xi,\tau,\zeta,y}_{\phi,\theta,\vartheta,\rho,\chi,z}\left[u_i(fx+gy+h)^{n}\right]$ is Mittag-Leffler function defined by Khan and Ahmad [15].

2. On replacing ϕ by $\phi-1$ and $\xi=\vartheta=\tau=\rho=1$ in theorem (1) we get

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]\Big\{(fx+gy+h)^{\lambda}$$
$$\times{}^{(\Gamma)}H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\Big\}$$
$$\times E^{\zeta,\chi,y}_{\phi,\theta,z}\left[u_j(fx+gy+h)^{n}\right]$$
$$=f^m(fx+gy+h)^{\lambda}\times E^{\zeta,\chi,y}_{\phi,\theta,z}\left[u_j(fx+gy+h)^{n}\right]$$
$$^{(\Gamma)}H^{0,b+m;X}_{p+m,q+m;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \vdots\\ u_k(fx+gy+h)^{n_k}\end{array}\right|$$
$$\left.\begin{array}{c}(a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m}A:C\\ \vdots\\ B,(\eta_i-\lambda-nl,n_1,....,n_j)_{i=1,m}\ :D\end{array}\right)$$

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]$$
$$\{fx+gy+h)^{\lambda}$$
$$H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$\times E^{\zeta,\chi,y}_{\phi,\theta,z}\left[u_j(fx+gy+h)^{n}\right]$$
$$=f^m(fx+gy+h)^{\lambda}\times E^{\zeta,\chi,y}_{\phi,\theta,z}\left[u_j(fx+gy+h)^{n}\right]$$
$$^{(\Gamma)}H^{0,b+m;X}_{p+m,q+m;Y}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \vdots\\ u_k(fx+gy+h)^{n_k}\end{array}\right.$$
$$\left|\begin{array}{c}(a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m}A:C\\ \vdots\\ B,(\eta_i-\lambda-nl,n_1,....,n_j)_{i=1,m}\ :D\end{array}\right)$$

Provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0,n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0,g\neq 0,m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (1.6); $\mu_i=f\eta_i$ for $i=1,...,m$, where $E^{\zeta,\chi,y}_{\phi,\theta,z}\left[u_i(fx+gy+h)^{n}\right]$ is Mittag-Leffler function defined by Salim and Faraz [16].

3. On replacing ϕ by $\phi-1$ and $\xi=\vartheta=\tau=\rho=\chi=z=1$ in theorem (1) we get

$$\prod_{i=1}^{m}\left[D_x(fx+gy+h)-\mu_i\right]\Big\{(fx+gy+h)^{\lambda(\Gamma)}$$
$$\times H[u_1(fx+gy+h)^{n_1},...,u_k(fx+gy+h)^{n_k}]\}$$
$$\times E_{\phi,\theta}^{\zeta,y}\left[u_j(fx+gy+h)^n\right]$$
$$=f^m(fx+gy+h)^{\lambda}E_{\phi,\theta}^{\zeta,y}\left[u_j(fx+gy+h)^n\right]$$
$$^{(\Gamma)}H_{p+m,q+m;Y}^{0,b+m;X}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \vdots \\ u_k(fx+gy+h)^{n_k}\end{array}\right.$$
$$\left|\begin{array}{c}(a_1,\alpha_1,...,\alpha_1^{(k)},x),(\eta_i-\lambda-nl-1,n_1,....,n_j)_{i=1,m}A:C\\ \vdots \\ B,(\eta_i-\lambda-nl,n_1,....,n_j)_{i=1,m}:D\end{array}\right)$$

Provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0, n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0, g\neq 0, m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (6); $\mu_i=f\eta_i$ for $i=1,...,m$, where $\times E_{\phi,\theta}^{\zeta,y}\left[u_i(fx+gy+h)^n\right]$ is Mittag-Leffler function defined by Shukla and Prajapati [17,18].

C. Corollary 3

$$\prod_{i=1}^{m}\left[(fx+gy+h)D_x-\mu_i\right]\Big\{(fx+gy+h)^{\lambda(\Gamma)}$$
$$\times H[u_1(fx+gy+h)^{n_1},u_2(fx+gy+h)^{n_2}]\}$$
$$\times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}\left[-u_j(fx+gy+h)^n\right]$$
$$=f^m(fx+gy+h)^{\lambda}\times J_{\phi,\theta,\vartheta,\rho,\chi,z}^{\xi,\tau,\zeta,y}\left[-u_j(fx+gy+h)^n\right]$$
$$^{(\Gamma)}H_{p+2,q+2;Y}^{0,b+2;X}\left(\begin{array}{c} u_1(fx+gy+h)^{n_1}\\ \vdots \\ u_2(fx+gy+h)^{n_2}\end{array}\right|$$
$$\left.\begin{array}{c}(a_1,\alpha_1^{(1)},\alpha_1^{(2)},x),(\eta_i-\lambda-nl,n_1,n_2)_{i=1,m}A:C\\ \vdots \\ B,(1-\lambda+\eta_i-nl,n_1,n_2)_{i=1,m}:D\end{array}\right)$$

Provided that $D_x=\frac{\partial}{\partial x}$, $\lambda>0, n_j>0$ for $j=1,...,k.f,g,h$ are complex numbers, $f\neq 0, g\neq 0, m$ is a positive integer. Also $|\arg u_i(fx+gy+h)^{n_i}|<\frac{1}{2}\Delta_j\pi$ where Δ_j is defined by (6); $\mu_i=f\eta_i$ for $i=1,...,m$.

V. Conclusion

In our current article, we have tried to find some new formulas for the incomplete H- function of several variables. Those relations have been obtained by using partial derivatives. These formulae give a wide range of incomplete functions when the parameters are reduced, including incomplete Wright functions and incomplete Gamma functions. In this paper, we have also discussed some particular and special cases too.

Author's Contribution: Shalini Shekhawat led the study, interpreted results and arranged the required literature for study. Akanksha Shukla wrote the manuscript and did all the numerical calculations. Kanak Modi created the study site map and formatted closing script. Each author read and approved closing script.

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