# Study on the Train Plan of the Cross-line Operation in the Urban Rail Transit Considering the Multi-composition Train

Xianen Yang, Changfeng Zhu, Xuegui Wang, Xuejiao Ma, and Linna Cheng

Abstract—The cross-line operation can promote urban rail transit network operation and realize resource sharing. In this regard, the operation mode of running cross-line trains with different compositions on two rail transit lines is proposed. And based on the analysis of passenger travel behavior, the passenger flow allocation model of cross-line operation is proposed. Considering constraints such as the passing capacity of transfer stations and the full load rate of trains, a multi-objective planning model is established with the operating frequencies of different types of trains and the location of the turnaround station as decision variables. The optimization objective is to minimize passenger travel cost and operating costs. Based on the proposed model, an improved quantum genetic algorithm(IQGA) is designed to solve the model, and the Pareto solution of the train plan is solved. Finally, the rationality of the model and algorithm is verified through a case study. The analysis shows that the operation of cross-line trains can make 44.5% of the cross-line passenger flows reach the destination station directly without transferring. And the cross-line routing shares the passenger flows in a balanced way so that the full load rates of each routing are balanced and the passengers travel more comfortably. The average passenger travel cost gradually increases with the increase of the transfer time perception factor. In addition, the setting of the vehicle capacity has a significant influence on the train plan. When making the train plan, the vehicle capacity should be set scientifically and reasonably.

Index Terms—Train Plan, Cross-line Operation, Passenger Flow Allocation, Quantum Genetic Algorithm(QGA)

#### I. INTRODUCTION

As the scale of the urban rail transit line network is becoming more well-established, the focus of rail transit development in many large and medium-sized cities has

Manuscript received April 21, 2023; revised August 13, 2023.

This work was supported in part by the National Natural Science Foundation of China (No.72161024), "Double-First Class" Major Research Programs, Educational Department of Gansu Province (No.GSSYLXM-04) and Outstanding Graduate Student "Innovation Star" Program of Educational Department of Gansu Province (No.2023CXZX-578).

Xianen Yang is a postgraduate student at School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: xianenyang0623@163.com).

Changfeng Zhu is a professor at School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (Corresponding author, phone: +86 18919891566, e-mail: cfzhu003@163. com).

Xuegui Wang is a Senior Engineer at China Railway First Survey and Design Institute Group, Xian 710000, China. (e-mail: tyywxg@ qq.com).

Xuejiao Ma is a postgraduate student at School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: mxj\_020406@163. com).

Linna Cheng is a doctoral candidate at School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: chenglinnajiao@126. com).

shifted from construction to operation, and the operation of urban rail transit network has become an inevitable trend. Cross-line operation[1] refers to the operation of metro trains from one metro line to another via a liaison line or across the line at a transfer station. It can effectively reduce the transfer pressure at the transfer station and reduce the travel time of cross-line passengers. Moreover, it is of great significance to the sharing of metro network resources and the operation of the metro network[2]. At the end of 2021, Chongqing, China, has already achieved three-line cross-line operation by operating direct cross-line express trains without transfer on the rail ring line, line 4 and line 5[3].

However, cross-line operation still faces a series of challenges and difficulties. Some studies [4][5][6][7][8][9] have studied the conditions and feasibility of cross-line operation in terms of vehicle, limit, signaling, power supply, and track design to achieve cross-line operation. In terms of operation, the scientific and reasonable cross-line train plan is crucial to improving the operational efficiency of the urban rail transit line network.

As a result, a series of detailed studies have been conducted by some scholars in recent years on the train plan for cross-line operation. Among them, the optimization model of the train plan of the urban rail transit network under interconnection mode was proposed by [10][11] to systematically optimize the cross-line operation routes and the number of trains operating in the metro network by considering constraints such as maximum full load rate, the number of vehicles in operation and section transport capacity. Based on the analysis of the characteristics of cross-line operation between metro and suburban railways, a dual-objective planning model for the train plan was established by [12] under cross-line operation to maximize the travel time saved for passengers and minimize the cost increase for metro operation. And a solution algorithm based on fuzzy mathematical programming was designed to solve the model. [13] considered the interests of different operating entities of each line after the cross-line operation. With the objective function of maximizing the increased revenue of metro enterprises and the time saving of passengers after the cross-line operation, the operating frequencies of various trains, cross-line operation routes, and track usage fees were optimized. Unlike the references above.[14] considered the fairness of passenger travel. Based on the analysis of the travel characteristics of passengers on the "Y/T" and "X" intersecting urban rail lines, the operating frequencies of



Fig.1. Diagram of the COM

trains and the turn-around plan were optimized. Reference [15] considered the operation of cross-line express trains on the cross-line routing to reduce the travel time of cross-line passenger flows. A mixed integer non-linear programming model was established with the operating frequencies, stopping plan, and turn-around plan of the cross-line express and slow trains as decision variables and the passenger travel time and operating costs as the objective functions. And a genetic algorithm was designed to solve the optimal train plan. By considering the unevenness of the full load rate of the train and the operation of a multi-group train, models of cross-line operation train plan were established by [1][16] respectively to optimize the operating frequencies, the location of the return station and the number of the train compositions. [17] took into account the matching of capacity and transportation volume based on the aforementioned study. With the objective of maximizing capacity matching and minimizing transport costs, a two-tier planning model for the preparation of the train plan was established, taking into account constraints on passenger services, capacity resources and the train operation organization. Based on passenger flow allocation.[18] considered the optimization model of cross-line train plan at line network to minimize passenger travel and operating costs, and applies simulated annealing algorithm to solve the model.

The above references have done fruitful studies on the train plan of the cross-line operation, which have greatly enriched the research in the field of train plan. However, few references have considered the operation of the multi-group train on cross-line routing. Operating trains of different compositions on the cross-line routing can let the number of train compositions change with the change in cross-line passenger flows. It will increase the operating frequencies of the cross-line routing without increasing operating costs, allowing more cross-line passengers to travel on cross-line trains and reducing the number of transfer passengers.

Based on the above analysis, a cross-line operation mode (COM) of operating multi-group trains on the cross-line routing was proposed. Considering passengers' travel choice behavior for different trains, OD passengers are classified according to the different origins and destinations. With the operating frequencies of different routings, the number of train compositions on the cross-line routing and the location of the return station as the decision variables, the minimization of the operating costs and the travel costs of

passengers on the three routings as the optimization objectives, a bi-objective planning model is established by considering constraints such as train passing capacity, maximum full load rate and station turnaround conditions, and an improved quantum genetic algorithm(IQGA) is designed to solve for the Pareto solutions of the train plan.

## **II. PROBLEM DESCRIPTION**

It is assumed that two urban rail lines connect at the transfer station, and there is a large amount of transfer passenger flows between the two lines. On the one hand, it can result in a high level of passenger flows at the transfer station, which would burden passenger operations at the transfer station. On the other hand, it will increase the travel time of the transfer passenger flow and reduce the comfort level of the travel, so the operation of cross-line trains between the two lines is considered. In addition, to reduce operating costs while better satisfying passenger travel demand, multi-composition trains are considered to be operated on cross-line routing [19]. The cross-line operation model is shown in Fig.1.

As shown in Fig.1, urban rail Line 1 and Line 2 are interconnected at the transfer station  $N(1,n_2)$ . The set of stations is denoted by N, where Line 1 includes  $n_3$  stations and its set of stations is denoted by  $N_1 = \{N(1,1), \dots, N(1,n_3)\}$ , Line 2 includes  $n_4$  stations and its set of stations is denoted as  $N_2 = \{N(2,1), \dots, N(2,n_4)\}$ . The transfer station is denoted both by  $N(1,n_2)$  and by N(2,1). The OD passenger flows between station *i* and station *j* are expressed as  $q_{ij}$ . And  $q_{ij}^{\phi}$  denotes the OD passenger flows from station *i* to station *j* in direction  $\phi$ ,  $\phi$  values are 1 and 2, where 1 represents the down direction and 2 represents the up direction. The set of routings is denoted as  $R = \{R_2 | z=1,2,3\}$ , where  $R_1$  and  $R_2$  denote the local train routings corresponding to Line 1 and Line 2, and  $R_3$ denotes the cross-line routing with the origin station  $N(2,n_4)$ and the turnaround station  $N(1, n_{1+1})$ .

According to the common number of compositions of the metro train (type B), trains of 4, 6, and 8 compositions are operated on the cross-line Routing  $R_3$  [20], and the train of different compositions set is denoted as  $B = \{B_i | i=1, 2, 3\}$ , where  $B_1, B_2$ , and  $B_3$  denote trains of 4, 6 and 8 compositions respectively. The frequencies of trains of 6 compositions on Routing  $R_1$  and  $R_2$  are expressed as  $f_1$  and  $f_2$ . The frequencies of running trains of 4, 6 and 8 compositions on

Routing  $R_3$  are expressed as  $f_3^1$ ,  $f_3^2$ , and  $f_3^3$ .

In summary, the train plan of the cross-line operation mode is mainly to determine the operating frequencies on the three routings, the location of the turnaround station of Routing  $R_3$ on Line 1, so the train plan can be expressed as  $\Omega = \{ f_1, f_2, f_3^1, f_3^2, f_3^3, N(1, n_{l+l}) \}.$ 

## III. MODEL CONSTRUCTION

Firstly, the passenger OD flows on the two lines are assigned to Routings  $R_1$ ,  $R_2$ , and  $R_3$  based on operating frequencies and passenger choice behavior, and then a bi-objective planning model is established with the objective of minimizing travel costs of passengers and train operating costs. The model assumptions are as follows:

a. During the study period, passenger arrivals obey a uniform distribution and the amount of OD passengers is known. Trains of different compositions depart evenly and trains on Routings  $R_1$ ,  $R_2$ , and  $R_3$  depart evenly.

b. The line meets the basic conditions for cross-line operation and the operation of the multi-group trains. Routing  $R_3$  meets the conditions for turning around at the turnaround station on Line 1.

## A. Passenger Flow Distribution

## A.1. Passenger Flow Classification

For the cross-line operation model shown in Fig.1, some OD passenger flows do not have a unique travel path from the origin station to the destination station due to the operation of trains on the cross-line. For example, some cross-line passenger flows travel on cross-line direct trains, while others prefer to reach their destinations by transfer.

Passengers are divided into 16 types according to the departure and arrival station sections of their trips, and the routes available for different types of passenger flows are shown in Table I.

As shown in Table I, after the operation of cross-line trains on different lines, the choice of passenger travel paths is more complex. Among the OD passengers classified according to the different origins and destinations of travel, there are nine types of OD passengers whose travel paths are not unique and only seven types of OD passengers whose travel does not have a choice behavior. For example, for OD passengers with an origin station in Section II and a destination station in Section IV, there are two options in the travel process: arriving at the destination by transfer and taking the cross-line train directly to the destination.

## A.2. Calculation of the Passenger Flow Sharing Ratio

As passenger arrivals are assumed to follow a uniform distribution, and trains depart evenly at each routing.

Therefore, during the study period, the passenger share of each routing is related to the operating frequencies, and most passengers will choose to board the train as soon as it comes. However, some passengers prefer cross-line routing for cross-line passenger flows, and the route choice behavior of this group of passengers is shown in Fig.2.



Fig.2. Diagram of the COM

As shown in Fig.2, for example, for cross-line passenger flows with the origin Station *i* in Section II and the destination Station *j* in Section IV, some passengers waiting for the train before  $t_1$  choose to take the first arriving train and reach their destination station by transferring at the transfer station. Still, some passengers do not want to transfer at the transfer station and prefer to continue waiting and take the cross-line train. Considering the preference of cross-line passenger flows for cross-line Routing  $R_3$ , a preference factor  $\rho$  is introduced to increase the cross-line passenger share, so that the passenger flow allocation result is closer to the actual situation.

For an OD passenger flow  $q_{i,j}^{\phi}$ , the flows  $q_{i,j}^{\phi,R_z,S_k}$  are allocated to the Section  $S_k$  on Routing Rz as calculated by (1).

$$q_{i,j}^{\phi,R_z,S_k} = \tau_{i,j}^{\phi,R_z,S_k} q_{i,j}^{\phi}$$
(1)

Where,  $\tau_{i,j}^{\phi,R_z,S_k}$  represents the proportion of passenger flows from Station *i* to Station *j* in the direction  $\phi$  that are allocated to the Section  $S_k$  on Routing  $R_z$ . The share of the different types of passenger flows on each section of the routing is calculated as shown in Table II.

The passenger flows  $q_{i,j}^{\phi,R_z,S_k}$  allocated to each section of the Routing  $R_z$  are integrated into the OD passenger flows on the Routing  $R_z$ , which is denoted as  $q_{i,j}^{\phi,R_z}$ . Therefore, the cross-sectional passenger flow of each section can be calculated by (2), and the train plan can be determined according to the cross-sectional passenger flow of each section.

		TABLE I							
ROUTES AVAILABLE FOR PASSENGERS									
Section	Ι	П	Ш	IV					
Ι	$R_2$	R <sub>2</sub>	$R_2$	a. $R_2$ - $N(2, 1)$ - $R_1$ b. $R_2$ - $N(2, 1)$ - $R_3$					
П	$R_{I}$	a. <i>R</i> <sub>1</sub> b. <i>R</i> <sub>2</sub>	a. <i>R</i> <sub>2</sub> b. <i>R</i> <sub>1</sub> - <i>N</i> (2, 1)- <i>R</i> <sub>2</sub>	a. $R_1$ b. $R_2 + N(2, 1) - R_3$					
III	$R_2$	$R_2$	$R_2$	a. $R_2$ - $N(2, 1)$ - $R_1$ b. $R_2$ - $N(2, 1)$ - $R_3$					
IV	a. $R_1$ - $N(2, 1)$ - $R_2$ b. $R_3$ - $N(2, 1)$ - $R_2$	a. $R_1$ b. $R_3$ - $N(2, 1)$ - $R_2$	a. <i>R</i> <sub>1</sub> - <i>N</i> (2, 1)- <i>R</i> <sub>2</sub> b. <i>R</i> <sub>3</sub> - <i>N</i> (2, 1)- <i>R</i> <sub>2</sub>	a. <i>R</i> 1 b. <i>R</i> 3					

## Volume 53, Issue 4: December 2023

TABLE II

THE SHARE OF THE DIFFERENT TYPES OF PASSENGER FLOWS ON EACH SECTION OF THE ROUTING																	
с. <i>і</i> :			$R_{I}$				$R_2$			R3			$R_4$				
Sec	tion	Ι	Π	III	IV	Ι	II	III	IV	Ι	Π	III	IV	Ι	Π	III	IV
	$R_1$	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1	0
Ι	$R_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$R_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	R <sub>1</sub>	1	$\frac{f_1}{f_1 + f_3}$	0	0	0	$\frac{f_1}{f_1 + f_3}$	0	0	$\frac{f_1}{f_1 + f_3}$	0	$\frac{f_1}{f_1 + f_3}$	0	$\frac{f_1}{f_1 + f_3}$	0	$\frac{f_1}{f_1 + f_3}$	0
Π	$R_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	R3	0	$\frac{f_3}{f_1 + f_3}$	0	0	0	$\frac{f_3}{f_1 + f_3}$	0	0	$\frac{f_3}{f_1 + f_3}$	0	$\frac{f_3}{f_1 + f_3}$	0	$\frac{f_3}{f_1 + f_3}$	0	$\frac{f_3}{f_1 + f_3}$	0
	$R_{I}$	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0
III	$R_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$R_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$R_{I}$	1	$\frac{f_2}{f_2 + f_3}$	0	0	0	$1 - \frac{\rho f_3}{f_2 + f_3}$	0	0	0	0	0	0	0	0	0	0
IV	$R_2$	0	0	0	$\frac{f_2}{f_2 + f_3}$	0	0	0	$1 - \frac{\rho f_3}{f_2 + f_3}$	0	0	0	0	0	0	0	$1 - \frac{\rho f_3}{f_2 + f_3}$
	R3	0	$\frac{f_3}{f_2 + f_3}$	0	$\frac{f_3}{f_2 + f_3}$	0	$\frac{\rho f_3}{f_2 + f_3}$	0	$\frac{\rho f_3}{f_2 + f_3}$	0	0	0	0	0	0	0	$\frac{\rho f_3}{f_2 + f_3}$

$$\mu_{i,i+1}^{R_z,\phi} = \sum_{i\in N, j\in N} \delta q_{i,j}^{R_z,\phi} \tag{2}$$

Where,  $\delta$  is a 0-1 variable,  $\delta = 1$  when the OD passenger flows on the Routing  $R_z$  contains the interval (i, i+1), otherwise  $\delta = 0$ .

## A.3. Algorithmic Steps for Passenger Flow Allocation

Combined with the above analysis, we propose the following steps for the distribution of passenger flows at each routing after the operation of cross-line trains.

**Step1:** Classification of OD passenger flows according to the origin and destination of the passenger.

**Step 2:** Determining the alternative routes available for each type of OD passenger flow.

**Step 3:** Calculating the passenger share of each type of passenger flow in each section, and the table for calculating the share of different passenger flows in each section of each routing is shown in Table II.

**Step 4:** Inputting OD passenger flow in turn, determining the type of the OD passenger flow and the route available for the travel.

**Step 5:** According to the travel route of the OD passenger flow, the passenger share of each section of the routing is selected in Table II, according to (1), the number of passengers allocated to each routing during the study time period is calculated.

**Step 6:** Determine whether the OD passenger flow has been entered, if so, go to Step 7, otherwise, go to Step 3.

**Step 7:** Integrating the OD passenger flows for each section of the routing and calculating the cross-sectional passenger flows for each routing by (2).

## **B.** Objective Functions

In order to reduce operating costs while meeting passenger

demand, a dual-objective planning model was established with the objective of minimizing travel costs of passengers and operating costs.

## B.1. Travel Costs of Passengers

Considering that the main travel costs for passengers in the process of traveling from their origin station to the destination station are the cost of purchasing a ticket and the cost of travel time [21]. However, the distance traveled remains the same and the fare remains the same, so the passenger travel time costs are minimized as the optimization objective.

The process of a passenger travelling is divided into the processes of waiting for the train, taking the train and transferring to another train. Considering that the passenger's time on the train remains the same, only the passenger's waiting time and transfer time need to be calculated.

Since the waiting times of passengers on the Routings  $R_1$ ,  $R_2$ , and  $R_3$  are involved, the waiting times of passengers on Routings  $R_1$ ,  $R_2$ , and  $R_3$  are calculated separately in the up and down direction. When passenger arrivals obey a uniform distribution and uniform departures, the passenger waiting time for each routing is half of the departure interval for that routing [22], and the passenger waiting times on the three interchanges are denoted by  $T_{R_1}$ ,  $T_{R_2}$  and  $T_{R_3}$  respectively.

The waiting times for up and down passengers on Routing  $R_1$  are denoted as  $T_{R_1}^{up}$  and  $T_{R_1}^{down}$ . The passenger flow types waiting for trains on Routing  $R_1$  are shown in Fig.3.

As shown in Fig.3, in terms of downstream passenger flows, the passenger flows waiting for trains on Routing  $R_1$  are divided into the following three main types, and the waiting times for each type of passenger flow are calculated respectively.



Fig.3. Passenger flow classification on Routing  $R_1$ 

The first type is the downstream passenger flow waiting in Section I. That is, passengers choosing to take the train on the Routing  $R_I$  from Section I to Section I, II, III, and IV, with the waiting time of  $T_{R_I,S_I}^{down}$  which is calculated by (3).

$$T_{R_{1},S_{1}}^{down} = \frac{30}{f_{1}} \left( \sum_{i=N(1,1)}^{N(1,n_{1})} \sum_{j=N(1,i)}^{N(1,n_{1})} q_{ij} + \sum_{i=N(1,1)}^{N(1,n_{1})} \sum_{j=N(1,n_{1}+1)}^{N(1,n_{3})} q_{ij} + \sum_{i=N(1,1)}^{N(1,n_{1})} \sum_{j=N(2,1)}^{N(1,n_{4})} q_{ij} \right)$$
(3)

The second type is the downstream passenger flow waiting in section II. That is, passengers choosing to take the train on the Routing  $R_2$  from Section II to Sections II, III, and IV, with the waiting time of  $T_{R_1,S_2}^{down}$  which is calculated by (4).

$$T_{R_{1},S_{2}}^{down} = \frac{30}{f_{1}} \left[ \frac{f_{1}}{f_{1} + f_{3}} \sum_{i=N(1,n_{1}+1)}^{N(1,n_{2})} \sum_{j=N(1,i)}^{N(1,n_{2})} q_{ij} + \left(1 - \frac{\rho f_{3}}{f_{1} + f_{3}}\right) \sum_{i=N(1,n_{1}+1)}^{N(1,n_{2})} \sum_{j=N(2,i)}^{N(2,n_{4})} q_{ij} + \sum_{i=N(1,n_{1}+1)}^{N(1,n_{2})} \sum_{j=N(1,n_{2}+1)}^{N(1,n_{3})} q_{ij} \right]$$
(4)

The third type is the downstream passengers in Section III who choose to travel on trains on Routing  $R_1$  with a waiting time of  $T_{R_1,S_1}^{down}$  which is calculated by (5).

$$T_{R_{1},S_{3}}^{down} = \frac{30}{f_{1}} \sum_{i=N(1,n_{2}+1)}^{N(1,n_{3})} \sum_{j=N(1,i)}^{N(1,n_{3})} q_{ij}$$
(5)

Therefore, the waiting time for the downstream passenger flow on Routing  $R_i$  is calculated by (6).

$$T_{R_{1}}^{down} = \sum_{i=1}^{3} T_{R_{1},S_{i}}^{down}$$
(6)

Similar to the downward direction, as shown in Fig.3, the upstream passenger flow on Routing  $R_1$  can also be divided into three types. The first type is the upstream passenger flow waiting in Section I,with a waiting time of  $T_{R_1,S_1}^{up}$  which is

calculated by (7).

$$T_{R_1,S_2}^{up} = \frac{f_1}{30} \left( \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(1,1)}^{N(1,n_1)} q_{ij} + \frac{f_1}{f_1 + f_3} \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(1,n_1+1)}^{N(1,n_2)} q_{ij} \right) (7)$$

The second and third types are upstream passenger flows from Sections II and III, calculated by (8) and (9) respectively.

$$T_{R_{1},S_{2}}^{up} = \frac{f_{1}}{30} \left( \sum_{i=N(1,n_{1}+1)}^{N(1,n_{2})} \sum_{j=N(1,1)}^{N(1,n_{1})} q_{ij} + \frac{f_{1}}{f_{1} + f_{3}} \sum_{i=N(1,n_{1}+1)}^{N(1,n_{2})} \sum_{j=N(1,n_{1}+1)}^{N(1,i)} q_{ij} \right) (8)$$

$$T_{R_{1},S_{3}}^{up} = \frac{30}{f_{1}} \left( \sum_{i=N(1,n_{2}+1)}^{N(1,n_{3})} \sum_{j=N(1,1)}^{N(1,n_{2})} q_{ij} + \sum_{i=N(1,n_{2}+1)}^{N(1,n_{3})} \sum_{j=N(2,1)}^{N(1,n_{3})} q_{ij} \right) (9)$$

Therefore, the waiting time for the upstream passenger flows on the Routing  $R_1$  is calculated by (11).

$$T_{R_{\rm l}}^{up} = \sum_{i=1}^{3} T_{R_{\rm l},S_{i}}^{up} \tag{10}$$

Compared to the passenger flows on the Routing  $R_1$ , the classification of passenger flows on the Routing  $R_2$  is relatively simple, and the passengers waiting for trains on  $R_2$  are shown in Fig.4.



Fig.4. Passenger flow classification on Routing  $R_2$ 

As shown in Fig.4, Routing  $R_2$  mainly serves passengers on Section IV and the downstream passenger waiting time  $T_{R_2}^{down}$  is the waiting time for downstream passengers in Section 4 who choose to travel on trains on  $R_2$ , calculated by (11).

$$T_{R_2}^{down} = \frac{30}{f_2 + f_3} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(2,i)}^{N(2,n_4)} q_{ij}$$
(11)

The upstream passenger waiting time is the upstream passenger flow waiting in Section IV. That is, passengers choosing to take the train on the Routing  $R_2$  from Section IV to Sections I, II, and III, with a waiting time of  $T_{R_2}^{up}$  which is calculated by (12)

calculated by (12)

$$T_{R_2}^{up} = \frac{30}{f_2 + f_3} \left( \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_3)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(2,1)}^{N(2,i)} q_{ij} \right) \quad (12)$$

The passengers waiting for trains on  $R_3$  are shown in Fig.5.



Fig.5. Passenger flow classification on Routing R3

As shown in Fig.5, in terms of downstream passenger flows, the passenger flows waiting for trains on Routing  $R_3$  are divided into the following two main types, and the waiting times for each type of passenger flow are calculated respectively.

The first type is the downstream passenger flow waiting in Sections II. That is, passengers choosing to take the train on the Routing  $R_3$  from Section II to Section IV, with the waiting time of  $T_{R_1,S_2}^{down}$  which is calculated by (13).

$$T_{R_3,S_2}^{down} = \frac{30}{f_3} \left[ \frac{f_3}{f_1 + f_3} \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(1,i+1)}^{N(1,n_2)} q_{ij} \right]$$

$$\frac{\rho f_3}{f_1 + f_3} + \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(2,1)}^{N(2,n_4)} q_{ij} \right]$$
(13)

The second type is the downstream passenger flow waiting in Section II with a waiting time of  $T_{R_3,S_4}^{down}$  which is calculated by (14).

$$T_{R_3,S_4}^{down} = \frac{30}{f_2 + f_3} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(2,i)}^{N(2,n_4)} q_{ij}$$
(14)

Therefore, the waiting time for the downstream passenger flow on Routing  $R_3$  is calculated by (15).

$$T_{R_3}^{down} = T_{R_3,S_2}^{down} + T_{R_3,S_4}^{down}$$
(15)

The upstream passenger flow on Routing  $R_3$  is divided into flow departing from Section II and within Section IV, with waiting times calculated by (16) and (17) respectively.

$$T_{R_3,S_4}^{up} = \frac{30}{f_2 + f_3} \left( \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_3)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(2,1)}^{N(2,i)} q_{ij} \right) \quad (16)$$

$$T_{R_3,S_2}^{up} = \frac{30}{f_2 + f_3} \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(1,n_1+1)}^{N(1,i)} q_{ij}$$
(17)

Therefore, the waiting time for the upstream passenger flow on the Routing  $R_3$  is calculated by (18)

$$T_{R_3}^{up} = T_{R_3,S_2}^{up} + T_{R_3,S_4}^{up}$$
(18)

In summary, the passenger waiting time  $T_w$  can be calculated by (19).

$$T_{w} = \sum_{i=1}^{3} \left( T_{R_{i}}^{up} + T_{R_{i}}^{up} \right)$$
(19)

Depending on the passenger's travel path, most passengers transfer at transfer station  $N(1, n_2)$ , while a small number of passengers transfer at a station in Section II, which is a same-station transfer. Passengers transferring at a transfer station must go through two processes: interchange walking and waiting for the train. However, passengers transferring at the same station only have to consider the time spent waiting for the transfer train. The types of transfer passengers on the two lines are shown in Fig.6.



Fig.6. The types of transfer passengers

As shown in Fig.6, the transfer passenger flows are divided into transfer passenger flows from Line 1 to Line 2 and transfer passenger flows from Line 2 to Line 1. The transfer passenger flows from Routing  $R_2$  to Routing  $R_1$  also includes the same station transfer from Section IV to Section I at the turnaround station.

For passengers transferring at the transfer station  $N(1, n_2)$ ,

the average transfer travel time is assumed to be  $T_{tra}$ . While passengers are perceived to be taking longer than they actually are during the transfer, a further transfer time perception factor  $\lambda$  is introduced.

The transfer time of the transfer passenger flows from Line 1 to Line 2 is denoted as  $T_{1-2}$ , and consists of the transfer walk time  $T_{1-2}^{tra}$  and the waiting time  $T_{1-2}^{w}$  of the transfer passenger flow, calculated by the (20) and (21) respectively.

$$T_{1-2}^{tra} = T_{tra} \lambda \left[ \sum_{i=N(1,1)}^{N(1,n_1)} \sum_{j=N(2,1)}^{N(1,n_4)} q_{ij} + \left( 1 - \frac{\rho f_3}{f_1 + f_3} \right) \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(2,1)}^{N(2,n_4)} q_{ij} + \sum_{i=N(1,n_2+1)}^{N(1,n_3)} \sum_{j=N(2,1)}^{N(1,n_4)} q_{ij} \right]$$

$$T_{1-2}^{w} = \frac{30}{f_2 + f_3} \left( \sum_{i=N(1,1)}^{N(1,n_1)} \sum_{j=N(2,1)}^{N(1,n_4)} q_{ij} + \sum_{i=N(1,n_2+1)}^{N(1,n_3)} \sum_{j=N(2,1)}^{N(1,n_4)} q_{ij} \right)$$

$$+ \frac{30}{f_2} \left( 1 - \frac{\rho f_3}{f_1 + f_3} \right) \sum_{i=N(1,n_1+1)}^{N(1,n_2)} \sum_{j=N(2,1)}^{N(2,n_4)} q_{ij}$$
(20)
(21)

The transfer time of the transfer passenger flows from Line 2 to Line 1 is expressed as  $T_{2-1}$  and consists of the transfer travel time  $T_{2-1}^{tra}$  for the transfer passenger flow, the waiting time  $T_{2-1}^{w}$  and the transfer time  $T_{t}^{w}$  for passengers transferring at the same station at station  $N(1, n_{1+1})$ , calculated by (22), (23) and (24) respectively.

$$T_{2-1}^{rra} = T_{tra} \lambda \Big[ \frac{f_2}{f_2 + f_3} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_1)} q_{ij} + \frac{(22)}{f_2 + f_3} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,n_2)}^{N(1,n_2)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,n_2+1)}^{N(1,n_3)} q_{ij} \Big]$$

$$T_{2-1}^w = \frac{30}{f_1} \Big[ \frac{f_2}{f_2 + f_3} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_1)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,n_2+1)}^{N(1,n_3)} q_{ij} \Big]$$

$$+ \Big(1 - \frac{\rho f_3}{f_2 + f_3} \Big) \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,n_1+1)}^{N(1,n_2)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,n_2+1)}^{N(1,n_3)} q_{ij} \Big]$$

$$T_i^w = \frac{30 f_3}{f_1(f_2 + f_3)} \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_1)} q_{ij}$$

$$(23)$$

In summary, the passenger transfer time  $T_{tra}$  can be calculated by (25).

$$T_{tra} = T_{1-2} + T_{2-1} + T_t^w$$
(25)

he passenger unit time value  $\cot \beta$  is introduced to convert passenger waiting time and passenger transfer time into passenger travel time value cost, denoted as  $C_l$ , which is calculated by (26).

$$C_1 = \beta (T_w + T_{tra}) \tag{26}$$

## **B.2.** Operating Costs

The operating frequencies, the number of train compositions and the turnaround plan directly influence the vehicle kilometers travelled during the study period.

The smaller the operating frequencies, the smaller the number of the train composition, and the shorter the Routing  $R_3$  length, the smaller the vehicle kilometers traveled. The smaller the number of vehicle kilometers traveled, the lower the operating costs for the company while keeping fixed costs constant. Therefore, the number of vehicle kilometers traveled on the three routings is the primary indicator to measure operating costs.

The vehicle kilometers traveled on Routing  $R_1$  is expressed as  $S_{R_1}$ . Since Routing  $R_1$  operates trains of a fixed composition,  $S_{R_1}$  is only related to the frequency  $f_1$  of Routing  $R_1$  and is calculated by (27).

$$S_{R_1} = f_1 \sum_{i=N(1,1)}^{N(1,n_2-1)} s(i,i+1)$$
(27)

Where, s(i, i+1) denotes the interval distance from station *i* to station *i*+1, km.

Similarly, the vehicle kilometers traveled on Routing  $R_2$  are denoted as  $S_{R_2}$  which is calculated by (28).

$$S_{R_2} = f_2 \sum_{i=N(2,1)}^{N(2,n_4-1)} s(i,i+1)$$
(28)

However, as Routing  $R_3$  operates trains of different compositions, the vehicle kilometers travelled are related to the operating frequencies, the number of train compositions and the location of the turnaround station. The vehicle kilometers travelled on Routing  $R_3$  are expressed as  $S_{R_3}$ 

which is calculated by (29).

$$S_{R_3} = (f_3^1 + f_3^2 + f_3^3) \sum_{i=N(1,n_1+1)}^{N(1,n_2)} s(i,i+1)$$

$$+ (f_3^1 + f_3^2 + f_3^3) \sum_{i=N(2,1)}^{N(2,n_4-1)} s(i,i+1)$$
(29)

In summary, the vehicle kilometers travelled  $S_R$  on the three routings during the study period is calculated by (30).

$$S_R = 2(S_{R_1} + S_{R_2} + S_{R_3})$$
(30)

According to the study [23], introducing the unit vehicle operating cost  $c_{run}$ , the business operating costs  $C_2$  can be calculated by (31).

$$C_2 = c_{run} S_R \tag{31}$$

## C. Constraints

The constraints of the model are as follows.

(1) The full load rate for each section of each routing is less than the maximum full load rate of the train.

In order to ensure the quality of service of the urban railway, usually, the maximum full load rate of the train is limited, assuming that the trains do have a high full load rate of  $\mu_{max}$ . In order to ensure that the train's full load rate does not exceed the maximum full load rate, only the transport demand of the section with the highest passenger flow needs to be met. The cross-sectional full load rate for Routings  $R_1$ ,  $R_2$ , and  $R_3$  are calculated by (32) (33) (34) respectively.

$$\mu_{i,i+1}^{R_1,\phi} = \frac{\mu_{i,i+1}^{R_1,\phi}}{6f_1Q}$$
(32)

$$\mu_{i,i+1}^{R_2,\phi} = \frac{\mu_{i,i+1}^{R_2,\phi}}{6f_2Q} \tag{33}$$

$$\mu_{i,i+1}^{R_3,\phi} = \frac{\mu_{i,i+1}^{R_3,\phi}}{(4f_3^1 + 6f_3^2 + 8f_3^3)Q}$$
(34)

where, *Q* indicates the vehicle capacity of the metro.

Therefore, the constraint on the train's full load rate can be expressed by (35).

$$\max \mu_{i,i+1}^{R_z,\phi} \le \mu_{\max} \qquad z \in \{1,2,3\}$$
(35)

(2) Passing capacity constraints at turnaround and transfer stations.

$$f_1 + f_2 + f_3^1 + f_3^2 + f_3^3 \le N_{\max}^{tra}$$
(36)

$$f_1 + f_3^1 + f_3^2 + f_3^3 \le N_{\text{max}}^{turn}$$
(37)

where,  $N_{\max}^{tra}$  and  $N_{\max}^{turn}$  denote maximum passing capacity at transfer and turnaround stations respectively.

(3) Constraints on operating frequencies.

$$0 \le f_1 \le f_1^{\max} \tag{38}$$

$$0 \le f_2 \le f_2^{\max} \tag{39}$$

$$0 \le f_3^1 + f_3^2 + f_3^3 \le f_3^{\max}$$
(40)

where,  $f_1^{\text{max}}$ ,  $f_2^{\text{max}}$  and  $f_3^{\text{max}}$  denote maximum operating frequencies at Routing  $R_1$ ,  $R_2$ , and  $R_3$  respectively.

(4) Restraint of turnaround station.

The turnaround station must be set in a reasonable interval, so the constraint on the position of the turnaround station is expressed as (41).

$$\sum_{i=N(1,1)}^{N(1,n_2)} \sum_{j=N(2,1)}^{N(2,n_4)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_2)} q_{ij} \ge q_{\min}$$
(41)

(5) Cross-line passenger flows meet the conditions for operating cross-line trains.

In order to make the operation of cross-line trains realistic and economical, the cross-line passenger flow needs to meet the conditions for the operation of cross-line trains. The minimum passenger flow requirement is expressed as and the condition that the cross-line passenger flow satisfies to operate cross-line trains can be expressed by (42).

$$\sum_{N(1,1)}^{N(1,n_2)} \sum_{j=N(2,1)}^{N(2,n_4)} q_{ij} + \sum_{i=N(2,1)}^{N(2,n_4)} \sum_{j=N(1,1)}^{N(1,n_2)} q_{ij} \ge q_{\min}$$
(42)

(6) Integer constraints.

$$f_1, f_2, f_3^1, f_3^2, f_3^3, n_1 \in \mathbb{Z}$$
(43)

#### IV. ALGORITHM DESIGN

The NSGA-II algorithm can efficiently solve the multi-objective planning problem[24] through a fast non-dominated sorting approach and a crowding degree selection strategy[25]. The quantum genetic algorithm (QGA) makes the algorithm more stable by encoding quantum bits and facilitating the jumping out of locally optimal solutions [26]. Therefore, an improved quantum genetic algorithm (IQGA) is proposed by introducing the fast non-dominated sorting operator and the crowding degree selection strategy in NSGA-II into the classical quantum genetic algorithm.

#### A. Encoding and decoding

We use quantum bit encoding, and as the decision variables include operating frequencies  $f_1$ ,  $f_2$ ,  $f_3^1$ ,  $f_3^2$ ,  $f_3^3$ , and turnaround station location  $N(l, n_{l+l})$ , they are encoded in



Fig.7. Diagram of encoding and decoding

the form of segmented chromosomes, which are encoded and decoded as shown in Fig7.

As shown in Fig.7, the chromosome length is a total of 30 bits, with each 5 bits from left to right representing the encoding of a decision variable.

Based on this encoding, a three-stage decoding approach is designed. In the decoding process, the quantum bit encoding is first transformed into a binary encoding by means of quantum collapse. Secondly, the binary encoding is decoded into an actual number encoding every five bits from left to right. Finally, the real numbers are mapped to the range of decision variables. The formula for the mapping is shown in (44), and the final decoding yields the decision variable corresponding to the chromosome[27].

$$x_{ac} = \left[\frac{x_{ac}^{\max} - x_{ac}^{\min}}{x_{de}^{\max}}\right] \times x_{de} + x_{ac}^{\min}$$
(44)

Where,  $x_{ac}$  represents the actual value of the decision variable after decoding,  $x_{ac}^{\min}$  and  $x_{ac}^{\max}$  represent the upper and lower limits of the independent variable respectively,  $x_{de}^{\max}$  represents the maximum value after the binary string is decoded into a decimal value,  $x_{de}$  represents the actual value after the binary string is decoded into a decimal value.

#### B. Quantum Rotation Gate

The quantum rotation gate is a specific measure for the quantum genetic algorithm to increase the diversity of the population [28]. An important Q-gate is the quantum rotation gate given by the rotation of the qubit by an angle  $\Delta\theta$  in the Bloch sphere.

$$U = \begin{pmatrix} \cos \Delta \theta & -\sin \Delta \theta \\ \sin \Delta \theta & \cos \Delta \theta \end{pmatrix}$$
(45)

The new qubit is given by (46).

$$Q' = \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \begin{pmatrix} \cos \Delta \theta & -\sin \Delta \theta \\ \sin \Delta \theta & \cos \Delta \theta \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$
(46)

Where,  $(\alpha_i, \beta_i)^T$  and  $(\alpha'_i, \beta'_i)^T$  are the probability amplitude before and after the update of the i-th qubit revolving gate of the chromosome;  $\Delta \theta$  is the angle of rotation, which uses the strategy shown in [29].

## C. Improved Elite Retention Strategy

In NSGA-II, the next generation of individuals is selected by merging N parent populations and N child populations and selecting the top N individuals with high dominance rank as the next generation population. However, this elite retention strategy is to retain all elite individuals, which may lead to fast convergence or convergence to a locally optimal solution. Therefore, referring to the existing reference [30], an improved elite retention strategy is shown in Fig.8.



Fig.8. Diagram of the improved elite retention strategy

As shown in Fig.8, the parameter  $\sigma$  was set to add the first  $N \times \sigma$  individuals directly to the new population, and the remaining  $N \times (1-\sigma)$  individuals were selected randomly from the plane of suboptimal solutions.



Fig.9. IQGA algorithm flowchart

## D. Algorithm Flow

The fast non-dominated sorting operator and the operator of congestion selection are introduced into the quantum genetic algorithm to solve the multi-objective planning problem, and the overall algorithmic flow of the algorithm is similar to that of NSGA-II. Therefore, the flow chart of the improved quantum genetic algorithm proposed in this paper is shown in Fig.9.

## V.CASE STUDY

#### A. Case Background

In order to validate the model and algorithm, two urban rail lines in a city were selected as a case study. The layout of the station and line is shown in Fig.10. Line 1 and Line 2 both include 12 stations, with the two lines articulating at station 10. Using the passenger flow data from 8:00-9:00 on a weekday in 2019 as input for the analysis, the OD passenger flows between the two lines are shown in Fig.11. The total cross-line passenger flows from Line 1 to Line 2 are 21,539, and the total cross-line passenger flows from Line 2 to Line 1 are 22,225. The model parameters were taken as shown in Table III.





Fig.11. OD passenger flows

#### B. Optimal Train Plan

The model parameters, passenger flow conditions, and algorithm parameters are input into the model, and the Pareto solution distribution of the train plan is solved by the improved quantum genetic algorithm as shown in Fig. 12, and some of the optimal train plans are shown in Table IV.

	TABLE III Model parameter	s	
Symbols	Meaning of the symbols	Numerical values	Unit
ρ	Cross-line routing preference factor	1.1	-
λ	Transfer time perception factor	1.2	-
β	Passenger Time Value Costs	30	yuan/h
Crun	Unit vehicle running costs	8	yuan/km
Q	Vehicle capacity	210	people/ vehicle
Ttra	Transfer travel time	5	min



Fig.12. Pareto solution set

TABLE IV

SOME OF THE OPTIMAL TRAIN PLANS									
No.	$f_1$	$f_2$	$f_3^1$	$f_{3}^{2}$	$f_{3}^{3}$	N	<i>C</i> <sub>l</sub> (yuan)	C2(yuan)	
1	12	13	6	5	3	5	180344.29	107126.69	
2	12	11	6	5	3	6	188820.78	96483.65	
3	11	8	6	5	3	6	200284.90	87274.75	
4	10	8	6	5	3	6	207264.92	84002.11	
5	11	7	6	4	3	6	209079.45	82569.41	
6	10	8	6	3	3	6	220284.76	78548.93	
7	9	8	6	4	3	6	222008.36	78002.88	
8	9	7	3	3	3	6	259526.84	67844.35	

As can be seen from Fig.12, the solution yields 35 Pareto solutions, which are much smaller than the initially set population of individuals because some Pareto solutions are repeated during the iterative process of the algorithm. Each Pareto solution's operating and passenger travel costs satisfy a non-dominated relationship. The minimum value of passenger travel cost for different solutions is 180,344.29 yuan, which corresponds to the maximum value of the operating costs of 107,126.69 yuan. The maximum value of passenger travel cost is 259,526.84 yuan, corresponding to a minimum operating cost value of 67,844.35 yuan. Therefore, the decision maker can choose the train plan that focuses on passenger or corporate benefits, depending on the conditions of the decision.

As can be seen from Table III, the operating frequencies of Routing  $R_1$  vary from 9pairs/h to 12pairs/h, the operating frequencies of Routing  $R_2$  vary from 7pairs/h to 13pairs/h, the operating frequencies of trains in 4 vehicles of Routing R3 vary from 3pairs/h to 6pairs/h, the operating frequencies of trains in 6 vehicles of Routing  $R_3$  vary from 3pairs/h to 5pairs/h, and the operating frequencies of trains in 8 vehicles of Routing  $R_3$  are 3pairs/h. The higher the operating frequencies, the lower the travel costs of passengers and the higher the operating costs. The majority of the options have a return station at Station 5. The Routing  $R_3$  has a return station at either Station 5 or Station 6, due to the fact that the number of passengers crossing from Station 1 to Station 4 on Line 1 to Line 2 is too low to run cross-line trains.

## C. Algorithm Analysis

IQGA is a practical algorithm for solving multi-objective planning problems. We set the population size to 200 and the number of iterations to 100 and use the average passenger travel cost and the average business operating cost of all individuals in the population as the target values for convergence. The iteration curves of the average passenger travel cost and the average business operating cost are shown in Fig.13 and Fig.14.







Fig.14. Iterative chart of average operating costs

As shown in Fig.13 and Fig.14, IQGA converges well, and the two objective functions converge in the 33rd and 38th generations, respectively. The final average passenger travel cost converges to 207,806 yuan and the business operating cost converges to 86,554 yuan. The overall convergence of the algorithm is relatively smooth, and the improved elite retention strategy makes the algorithm more characteristic of jumping out of the local optimal solution.

## VI. RESULTS ANALYSIS

## A. Analysis of the Benefits of Cross-line Operation

The operation of cross-line trains can effectively shorten the travel time of cross-line passengers and reduce the pressure on transfer stations. The statistics of passenger flows after the operation of cross-line trains are shown in Table V.

TABLE V								
THE STATISTICS OF PASSENGER FLOWS								
Indicators	Line 1 to Line2	Line 2 to Line1						
Total passenger flows(people)	21539	22225						
Direct cross-line passenger flows(people)	7662	11858						
Transfer passenger flows(people)	13877	10367						
Transfer time savings (min)	38310	59290						
Direct traffic sharing rate (%)	0.356	0.534						

As shown in Table V, the total numbers of cross-line passengers from Line 1 to Line 2 are 21,539, while the total numbers of cross-line passengers from Line 2 to Line 1 are 22,225. With the cross-line trains, the cross-line passengers shared by Routing  $R_3$  included 7,662 passengers from Line 1 to Line 2 and 11,858 passengers from Line 2 to Line 1. The total times saved between Line 1 and Line 2 during the study period was 97,600 minutes, with the cross-line routing sharing 35.6% of the cross-line passengers from Line 1 to Line 2 and 53.4% of the cross-line passengers from Line 2 to Line 1, as well as reducing transfer pressure at the transfer station by 44.5%.

## B. Full Load Rate Analysis

The train's full load rate affects the comfort of passengers during their travel. Too high a full load rate will reduce passenger comfort during travel, and too low a full load rate will result in empty trains and increase operating costs. In order to optimize passenger travel costs and operating costs in a balanced way, the train plan  $\Omega$ ={11, 8, 6, 5, 3, 6} in the Pareto solutions is chosen as the object of study, and the full load rate of trains in each routing in the down direction of this train plan is analyzed as an example. The full load rate of trains in the down direction of Routings  $R_1$ ,  $R_2$ , and  $R_3$  are shown in Fig.15, Fig.16, and Fig.17.







Fig.17. The full load rate of trains in the down direction of Routing  $R_3$ 

As seen in Fig.15, Fig.16, and Fig.17, on the one hand, the full load rates of Routing  $R_1$ ,  $R_2$ , and  $R_3$  in each section do not exceed the maximum full load rate limit, except for the full load rates of section 8 and section 9 on Routing R1, which are 112.7% and 112.5%, respectively, which exceed the full load rate limit. The reason is that Section II to Section III has more passenger flows, Section 8 and Section 9 on routing  $R_1$  have increased passenger flows, so they become restricted intervals.

On the other hand, the average full load rates of each section of Routing  $R_1$ ,  $R_2$ , and  $R_3$  are 0.618, 0.68, and 0.619 respectively, so the average number of passengers in each metro vehicle is between 126 and 127, and the overcrowding of the vehicle is minor. Moreover, only trains of Routing  $R_1$  operate between stations 1 and 6, but the full load rate of each section is small, which indicates that the location of the turnaround station of Routing  $R_3$  is more reasonable and will not cause the problem that trains of Routing  $R_1$  and  $R_3$  operate empty in section I.

In summary, after the cross-line trains are operated, Routing  $R_3$  shares the cross-line passenger flows between  $R_1$ and  $R_2$ , and the full load rate of the three routings reaches a balanced level. It can be seen that the model proposed in this paper can provide technical support for the formulation of the cross-line operation train plan.

## C. vehicle capacity sensitivity analysis

Vehicle capacity refers to the maximum number of passengers per unit vehicle when the vehicle is fully loaded. Usually, a square meter can carry passengers 4 or 6 people. However, in practice, when the full load rate is 70%-80%, passengers are already in a more crowded state. Therefore, the train capacity can be adjusted appropriately to consider the

travel comfort of passengers when the train plan is made. In the Pareto solution, different train plans correspond to a different number of vehicles, and the effect of a different number of vehicles and vehicle capacity on the full load rate is analyzed as shown in Fig.18.



Fig.18. Schematic diagram of vehicle capacity sensitivity analysis

As shown in Fig.18, with the increase of the number of vehicles operated in different train plans, the average full load rate of each section of the three routings gradually increases under the same train capacity, and the higher the train capacity is set under the same train plan, the smaller the full load rate is.

The calculated minimum average full load rate is 19.4% when the number of operating vehicles is 228. The vehicle capacity is 330 people per vehicle. The calculated maximum average full load rate is 53.9% when the number of operating vehicles is 150 and the vehicle capacity is 180 people per vehicle. As can be seen, the setting of train capacity directly affects the full load rate and has a more significant impact on the development of the train plan. Therefore, the train capacity should be determined and adjusted scientifically and reasonably for the vehicle conditions and operation of different lines when making the train plan.

## D.Sensitivity analysis of transfer time perception factor

In real life, passengers often perceive the transfer time longer than the actual time. Still, each person's perception is different, so it is necessary to conduct a sensitivity analysis of the transfer time perception factor. The transfer time perception factor is taken to be in the range of 1 to 2 with a step size of 0.1, and the effect of the change of the transfer time perception factor on the average passenger travel costs and operating costs of all solutions in the Pareto solution set is studied. The sensitivity analysis figure of the transfer time perception factor is shown in Fig. 19.

As shown in Fig.19, the average travel cost of passengers in the Pareto solution set gradually increases with the increase of the transfer time perception factor. During the change of transfer time perception factor from 1 to 2, the average passenger travel cost increases from 203,060.8 to 229,762.9, an increase of 13.1%. However, the average operating cost has been fluctuating within a specific range. Therefore, it can

be concluded that the transfer time perception factor has an impact on the average travel cost of passengers of the train plan, and the average travel cost of passengers gradually increases with the increase of the transfer time perception factor, but the transfer time perception factor has a negligible impact on the average operating cost.



Fig.19. Sensitivity analysis of the transfer time perception factor

#### E. Sensitivity analysis of transfer time

Due to the difference in the size of the station, the design of the pedestrian flow line in the station, the congestion level in the station and other factors, the transfer time of passengers in different stations is not the same. The main purpose of cross-line operation is to shorten the travel time of passengers, so it is necessary to analyze the sensitivity of the transfer time of passengers in the station. Combining the observed transfer times in different stations, we select the passenger transfer times from 1 to 10 minutes, and the changes of the average travel costs of passengers and the average operating costs with the transfer time in the Pareto solution are shown in Fig.20.



Fig.20 Sensitivity analysis of the transfer time

As shown in Fig.20, the average passenger travel costs and the average operating costs increase gradually with the increase in transfer time. In the process of changing the transfer time from 1 minute to 10 minutes, the average passenger travel costs increase from 182,342.37 to 236,012.31 yuan, and the average operating costs increase from 84,789.54 yuan to 86,644.35 yuan. The reason for this is that with the increase in transfer time, the passenger waiting time becomes longer, so the passenger travel cost increases. As the passengers' transfer time becomes longer, more cross-line trains are run to meet the needs of cross-line passengers, so the average operating cost increases, which also reflects the reasonableness of the model in this paper.

### VII. CONCLUSIONS

A passenger flow allocation model is proposed for the complex characteristics of passenger flow after the operation of cross-line trains, and a multi-objective planning model is established based on passenger flow allocation with minimum travel costs of passengers and operating costs. Through the case study, the following conclusions can be drawn.

(1) The operation of cross-line trains can effectively shorten the travel time of cross-line passengers and reduce the pressure on the transfer station. After operating cross-line trains, 35.6% of the cross-line passenger flow from Line 1 to Line 2 and 53.4% from Line 2 to Line 1 can reach the destination station directly while reducing the transfer station transfer pressure by 44.5%.

(2) After operating cross-line trains, the local routings and the cross-line routing share the passenger flows in a balanced way. The average full load rate of each section of Routing R1, R2, and R3 is 61.8%, 68%, and 61.9% respectively, so the average number of passengers in each metro vehicle is between 126 and 127, and the train is less crowded.

(3) In the process of making the train plan, the setting of vehicle capacity is crucial to passenger comfort, and the vehicle capacity should be determined and adjusted scientifically and reasonably for the vehicle conditions and operation of different lines. The average passenger travel costs increase with the increase in the transfer time perception factor, but the perceived transfer time factor does not affect the average business operating costs. The average passenger travel costs and the average operating costs increase gradually with the increase in transfer time.

(4) The Improved Quantum Genetic Algorithm (IQGA) is an efficient algorithm for solving multi-objective planning problems, and it has good convergence and jumps out of local optimal solutions.

As mentioned above, cross-line operation is an essential mode of urban rail transit network operation, and it is of great theoretical and practical significance to the urban rail transit organization. In this paper, we have not yet considered the operation of " cross-stopping" cross-line express trains. Still, we will consider the operation of cross-line express trains in future studies to further reduce the travel time of cross-line passengers.

#### REFERENCES

- Q.W. Zeng, Q.Y. Peng, "Study on the Cross-line Train Plan in Urban Rail Transit Considering the Nulti-group Train," *Journal of Railway Science and Engineering*,2022.DOI: https://doi.org/10.19713 /j.enki. 43-1423/u.T20220780
- [2] B.H. Mao,Z.Y. Gao, "Progress of Operational Resources Sharing Methods for Urban Rail Networks," *Journal of Transportation Systems Engineering and Information Technology*, vol. 18, no. 3, pp. 1-8,2018.
- [3] M. Le,C.X. Wen,N.N. Wang, J. Yang,Y. Jiang, "Distribution Design of Cross Line Operation between Metro Line 4 and the Circle Line in Chongqing City," *Urban Mass Transit*, vol. 23, pp. 14-17,2020.

- [4] L.F. Liu,S. Lu,L.C. Jiang, "Research on the Inter-connectivity Technology between Suburban Railway and Built Subway," *Journal* of Railway Engineering Society, vol. 39, no. 8, pp. 93-97,2022.
- [5] Z.B. Jiang, M.L. Su, H.Y. Li, S.F. Xu, "Connotation and Technical Features of Rail Transit Interconnection," *Urban Rapid Rail Transit*, vol. 34, no. 3, pp. 46-51,2021.
- [6] M. Wang,J. Di,M.W. Chen,H.Z. Yao,Z.C. Li, "Research on the Key Technology of Through Operation of Suburban Railway and Urban Metro," *China Railway Science*, vol. 43, no. 6, pp. 161-174,2022.
- [7] C. Wang, "Signal System for the Cross-line Operation of Guangzhou Metro Line 14 and Line 21," *Urban Mass Transit*, vol. 20, no. 1, pp. 36-38,2017.
- [8] Q.H. Liang, J.Q. Wang, Y. Zhang, "Technologies for Route Design for Joint Operation of Metro Lines with One-platform Interchange," *Urban Rapid Rail Transit*, vol. 28, no. 5, pp. 16-20,2015.
- [9] H.B. Gong, P.K. Pei, X.M. Chen, Y. Zhang, K.F. Ma, "Study on Matching Relationship Between Depot Scale Allocation and Operation Organization Under Cross-Line Operation of Urban Rail Transit," *Railway Transport and Economy*, vol. 45, no.6, pp. 132-138, 2023.
- [10] M.S. Li,H.C. Li, "Optimal Design of Subway Train Cross-Line Operation Scheme Based on Passenger Smart Card Data," *Sustainability*,2022.DOI: https://doi.org/10.3390/su 14116420
- [11] J.S. Huang,Y. Chen,A.Y. Zhang,Q. Zhou, Y. Bai,B.H. Mao, "Optimization of Urban Rail Network Operation Scheme Considering Interconnection," *Journal of Railway Science and Engineering*, 2022.DOI: 10.19713/j.cnki.43-1423/u.T20220960
- [12] M.G. Li,J.F. Liu,X.H. Liu,J.H. Li, "Train Planning for Through Operation between Subway and Suburban Railway," *Journal of Transportation Systems Engineering and Information Technology*, vol. 17, no. 6, pp. 178-184,2017.
- [13] X.P. Li,Y. Bai,F. Li,Y. Chen,J.F. Liu, "Compilation of Train Cross-line Service Scheme of Rail Transit with Multi-operators," *Journal of Railway Science and Engineering*, 2023. DOI: 10.19713/ j.cnki. 43-1423/u. T20221948
- [14] L.S. Sun,H.B. Lu,X. Yan,D.W. Kong,J. Shao, "Fairness-oriented Train Service Design for Urban Rail Transit Cross-line Operation," *Physica A*, vol. 2022, no.606, Article ID.128124,2022.
- [15] A.A. Yang,B. Wang,J.L. Huang,C. Li, "Service Replanning in Urban Rail Transit Networks: Cross-line Express Trains for Reducing the Number of Passenger Transfers and Travel Time," *Transportation Research Part C*, vol. 2022, no.115, Article ID.102629,2022.
- [16] R.B. Wei,S.P. Jia,B.H. Mao,Z.S. Xiao,Y.Q. Wang, "Train Planning Optimization for Through Operation of Urban Rail Transit Considering the Balance of Load Factors," *Journal of Harbin Institute* of *Technology*, vol. 45, no. 3, pp. 68-77,2023.
- [17] F. Yan,X.M. Yao,P. Zhao,M. Han,K.Qiao, "Modelling Train Operation Plan for Urban Rail Transit under Interconnection Operational Mode," *Journal of Railway Science and Engineering*, vol. 45, no. 2, pp. 13-21,2023.
- [18] L. Chen,X.Y. Duan,Y. Bai, "Optimizing Cross-line Train Service Planning in Urban Rail Transit Network," *Urban Rapid Rail Transit*, vol. 36, no. 3, pp. 59-64,2023.
- [19] Y.P. Rong,X.C. Zhang,Y. Bai,D.J. Xu, "Optimization for Train Plan of Urban Rail Transit Based on Hybrid Train Formation," *Journal of Transportation Systems Engineering and Information Technology*, vol.16, no.5,pp. 117-122,2016.
- [20] Z.Y. Li, J. Zhao, Q.Y. Peng, "Train Service Design in an Urban Rail Transit Line Incorporating Multiple Service Routes and Multiple Train Compositions," *Transportation Research Part C: Emerging Technolo- gies*, vol.123, Article ID. 102959, 2021.
- [21] D.D. Yu,B.M. Han,Q. Zhang,X.M. Yao,F.L. Liu, "Optimization Method for Train Plan of Urban Rail Transit Based on the Flexible Length of Train Formation," *Journay of Beijing Jiaotong University*, vol.39, no.6,pp. 21-31,2015
- [22] C.J. Dai, Y.Z. Li, Z.S. Zhan, H. Chai, "Optimization of Train Operation Scheme for Urban Rail Transit Considering Dynamic Passenger Demand and Full-length & Short-turn Routing Modes," *China Railway Science*, vol.39, no.2, pp. 128-136, 2018.
- [23] Y.Z. Dai,S. Zhang, "Analysis of Full-length and Short-turn Routing of Urban Rail Transit Considering Carbon Emission," *Journal of Railway Science and Engineering*,2022.DOI: https://doi.org/10.19713 /j.cnki. 43-1423/u.T20220149
- [24] L.X. Dang,Y.E. Hou,Q.S. Liu,Y.F. Kong, "A Hybrid Metaheuristic Algorithm for the Bi-objective School Bus Routing Problem," *IAENG International Journal of Computer Science*, vol.46, no.3, pp. 409-416, 2019.

- [25] L.N. Wang, C.K.P. Zhang, "Greenhouse Microclimate Control Strategy to Reduce Costs Based on Improved NSGA-II Optimal Method," *Engineering Letters*, vol.29, no.3, pp. 818-823, 2021.
- [26] K.H. Han, J.H. Kim, "Quantum-inspired Evolutionary Algorithm for a Class of Combinatorial Optimization," *IEEE Transactions on Evolutionary Computation*, vol.6, no. 6, pp. 580-593, 2002.
- [27] C.F. Zhu, X.E. Yang, Z.F. Wang, J.H. Fang, J. Wang, L.N. Cheng, "Optimization for the Train Plan with Flexible Train Composition Considering Carbon Emission," *Engineering Letters*, vol. 31, no.2, pp562-573, 2023.
- [28] Y. Rubio,C. Olvera,O. Montiel, "Quantum-Inspired Evolutionary Algorithms on IBM Quantum Experience," *Engineering Letters*, vol.29, no.4, pp. 1573-1584, 2021.
- [29] B. Wang,W.W. Zhao,S.J. Liu,J. Ke,H. Wu, "Integrated Energy Management of Highway Service Area Based on Improved Multi-objective Quantum Genetic Algorithm," *Power System Technology*, vol.46, no.5, pp. 1742-1751, 2022.
- [30] X.L. Du,N. Zhang,F.Y. Meng,J.H. Wang, "An Improved NSGA2 Algorithm for Solving Flexible Workshop Scheduling," *Modular Machine Tool & Automatic Manufacturing Technique*, vol.579, no.5, pp. 182-186, 2022.



Xianen Yang was born in Gansu, China, in 1999. He obtained his Bachelor's degree in Traffic and Transportation from Southwest Jiaotong University, Chengdu, China, in the year 2021. He is currently pursuing his master's degree at Lanzhou Jiaotong University, majoring in Traffic and Transportation. His research interests are mainly in urban rail transit operation organization optimization.