I. INTRODUCTION

A substance that concurrently combines the physical and chemical characteristics of various materials is called a hybrid material. Numerous studies have been conducted by many researchers to study the optical, mechanical, electrical and thermal properties of these hybrid materials. A hybrid nanofluid is prepared by dispersing two different types of nanoparticles in the base fluid, this is done to obtain a wide absorption range, high thermal conductivity, lower extinction, low pressure-drop, pumping power and low frictional losses. It has crucial applications in the automotive industry, in heat pipes, as a coolant in machining processes, in solar systems, in heat exchangers, for nuclear system cooling, in electronic cooling, in the biomedical industry and in the defense field. Magnetohydrodynamics (MHD) is the study of the impacts of a magnetic field on liquids. Such studies are important as the magnetic field exists everywhere in the universe. MHD focuses on an electrically conducting fluid that moves in a magnetic field and can be used to control the heat transfer system. The Lorentz force is created by the MHD effect, which is very useful for controlling cooling structures. Many researchers have employed MHD concepts in different industrial applications such as flow metering, nuclear trash disposal, and extracting geothermal energy. Radiative fluids are a type of fluid that emits energy in the form of radiation. These fluids are critical to the successful functioning of several industrial operations. They are particularly useful in systems with large temperature differences between the environmental fluid and the boundary externals.

Grubka and Bobba [1] studied the heat transfer characteristics of a continuous, stretching surface with variable temperature. Merkin [2] determined the dual solutions of mixed convection flow in a porous medium. Weidman et al. [3] examined the effect of transpiration on self-similar boundary layer flow over moving surfaces. The fluid flows heat and mass transfer and the boundary layers across a sheet constantly contracting as per power law velocity were examined by Fang [4]. Tie-Gang et al. [5], Arifin et al. [6], and Rohini et al. [7] explained the viscous flow over a shrinking sheet with concentration transmission. Zaimi et al. [8] scrutinized boundary layer flow and heat transfer past a permeable shrinking sheet in a nanofluid with radiation effect. Bachok et al. [9] explained the copper-water nanofluid of stagnation-point flow over a permeable stretching/shrinking sheet. Rohini et al. [10] studied an unsteady nanofluid flow of a shrinking sheet with suction using Buongiorno model. Das [11], [12], Awati et al. [13], Hafidzuddin and Nazar [14], Ene et al. [15], and Yao et al. [16] discussed the results of an analytical and numerical study of thermal conduction and heat and mass transmission for the boundary layer flow over a permeable shrinking sheet with power law velocity. Dogonchi et al. [17] adopted an analytical method of Adomian Decomposition for continuous, viscous, incompressible water-based MHD radiative nanofluid flow between two stretchable or shrinkable walls. Das et al. [18] assessed the boundary layer flow of nanofluid over a convectively heated shrinking sheet in the presence of a thermal sink or source. The three-dimensional rotational nanofluid flow and heat transmission were explained by Hayat et al. [19], Rashid et al. [20] determined the flow of water-based alumina and copper nanoparticles along a moving surface with variable temperature. Abbaje et al. [21] scrutinized an incompressible unsteady Powell-Eyring nanofluid flow past a shrinking space with the squeal of thermal radiation and heat production. A multi-domain bivariate spectral quasi-linearization method was used to obtain numerical solutions for nonlinear differential equations that described the transport processes. Thumma et al. [22] used the Maxwell and Brinkman models for the numerical study of radiative hydrodynamic blended convection boundary layer flow of nanofluid across a non-linear inclination shrinking/stretching surface with viscous dissipation and a heat source. Vajravel et al. [23] studied the MHD flow and heat transfer over a slender elastic permeable sheet in a rotating fluid with Hall current. Usman et al. [24] assessed the notable impacts of thermal conductivity varying

The Flow Past a Non-Isothermal Shrinking Sheet with the Effects of Thermal Radiation and Heat Source/Sink

N. Nithya and B. Vennila *

Abstract—This research article explores the effect of thermal radiation and heat source/sink on a hybrid nanofluid comprising of Aluminium oxide (Al₂O₃) and Titanium dioxide (TiO₂) nanoparticles suspended in the base fluid of water (H₂O). The bvp4c algorithm is utilized to solve the governing equations, and the influence of various parameters on velocity and temperature is examined via graphs. It is noted that the magnetic parameter (M) escalates the temperature and increases penetrates the velocity. In addition, the skin friction and heat transfer effects are also discussed

Index Terms—radiative heat, heat source/sink, skin friction, nusselt number, magnetic.

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with time and nonlinear thermal radiation caused by spinning 
$Cu + Al_2O_3/water$ hybrid nanofluid flow across a three-
dimensional stretched sheet. The solution of the proposed 
model was evaluated using the least squares method (LSM).
Ghadikolaei et al. [25], [26] adopted the numerical tech-
nique (Runge-Kutta-Fehlberg) for hybrid nanofluids $Al_2O_3$
$Cu/water$ and the mixture of ethylene glycol-water/$TiO_2 +
CuO$ over a porous medium and rotating cone by taking 
into account the form factor of the nanoparticles. Khashi et
al. [28] discovered the effect of suction on the magnetic flow
in double stratified micro-polar fluid over a porous surface. 
Dinarvand et al. [31] studied the magnetic flow caused by a non-
Newtonian flow over a stretching sheet by studying the effects visco-
supposed properties, Brownian motion, and thermal radiation.
Khan et al. [36] found dual solutions for radiative stagnation point
of a hybrid nanofluid across a shrinking sheet, based on a
numerical study. Visalakshi et al. [37] explained the skin friction of radiative
hybrid nanofluid flow over a stretching/shrinking sheet. Nithya and
Vennila [38] reported an analytical solution of mass and heat transfer of nanoliquid
flow over a stretching sheet by studying the effects viscous
dissipation and ohmic dissipation.

In this research article, the magnetic flow caused by a non-
isothermal shrinking surface of a hybrid nanofluid ($Al_2O_3 −
TiO_2/H_2O$) is presented. The obtained numerical results are
discussed and showcased in tables and figures. The momentum and thermal fields are significantly influenced by
the radiation, heat sink parameters, and volume fraction.

II. MATHEMATICAL MODEL

The two-dimensional, incompressible laminar flow of a
hybrid nanofluid past a non-isothermal permeable shrinking
sheet is studied by considering $Al_2O_3 − TiO_2/H_2O$ with
the Tiwari and Das model. The sheet velocity is obtained
by $u_\infty(x) = bx$, where $b > 0$ is constant. The governing
equation is written in the usual notation under boundary-
layer approximation. The flow is affected by the combined
action of a transverse magnetic field with a strength of $B_0$
and radiative heat with a heat flux of $q_r$, which is applied
normally to the surface in the y-direction [33]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hf}}{\rho_{hf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hf}}{\rho_{hf} c_p} B_0^2 u \quad (2)$$

subject to:

$$v = v_0, u = \lambda u_\infty(x), \quad T = T_\infty(x) \quad (4)$$

$$u \longrightarrow 0, y \longrightarrow \infty \quad (5)$$

The velocity components in the x and y directions are
denoted by $u$ and $v$ respectively. Considering Rosseland’s
approximation for radiation as

$$q_r = -\frac{\sigma \rho^*}{3K^*} \frac{\partial T^4}{\partial y} \quad (6)$$

Where, $K^*$ and $\sigma^*$ denote the mean absorption coefficient
and Stefan-Boltzmann constant respectively. $T^4$ may be
expressed as a temperature linear function as $T^4 = 4T_\infty^3$, then
the following is given by equation (3).

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho c_p)_{hf}} \left\{ \left[ K_{hf} + \frac{16\sigma \tau_{hf}^4}{3K^*} \right] \frac{\partial^2 T}{\partial y^2} \right\} + \frac{Q_0}{(\rho c_p)_{hf} (T - T_\infty)} \quad (7)$$

The surface temperature is as follows

$$T_\infty(x) = T_\infty + T_0 (x/L)^m \quad (8)$$

Here, $T_0$ is the thermal variable and $L$ is a sheet’s character-
istic length. The surrounding temperature $T_\infty$ is constant
and $m$ is denoted as power-law index, with $m > 0$ specifying
as isothermal sheet and $m > 0$ specifying as non-isothermal
sheet

Now, adopting the similarity transformation

$$\psi = \sqrt{\nu f(\eta)} \quad \frac{\partial \psi}{\partial \eta} = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \sqrt{\frac{\alpha}{\nu f}} \quad (9)$$

$\psi$ is the stream function, where $v = -\frac{\partial \psi}{\partial x}$ and $u = \frac{\partial \psi}{\partial y}$, then:

$$u = axf'(\eta), \quad v = -\sqrt{\nu f}(\eta) \quad (10)$$
In equation (10), setting \( \eta = 0 \), we can obtain

\[ v_0 = -\sqrt{\frac{\sigma f}{\rho \nu}} \]  

(11)

Here, the sheet’s permeability is determined by the constant mass flux, \( f(0) = S \), where, \( S > 0 \) (suction) and \( S < 0 \) (injection). Using the equations (9) and (10) in equations (2) and (3), we get

\[ \frac{\mu h_{nf}/\mu_f}{\rho_{nf}/\rho_f} f'''' + f'' + \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} M f' = 0 \]  

(12)

\[ \frac{1}{Pr} \left( k_{nf}/k_f + \frac{4}{3} R \right) \theta'' + \left( \frac{\rho C_p}{\rho C_p_f} f(\theta' - m f' \theta) + Q \theta \right) = 0 \]  

(13)

subject to the following

\[ f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad \text{at} \ \eta = 0 \]  

(14)

\[ f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \text{at} \ \eta \rightarrow \infty \]  

(15)

Take into account that the indicators for a stretched surface as \( \lambda > 0 \) and a shrinking surface as \( \lambda < 0 \), where

\[ Pr = \frac{(\rho C_p)_f}{\rho_f}, \quad R = \frac{4\sigma^2 T^3}{k_f^2}, \quad M = \frac{\sigma_f}{\sigma_f} B^2 \]

The coefficient of the skin friction \( C_f \) and local Nusselt number \( Nu_x \) are described by

\[ C_f = \frac{\mu h_{nf}}{\mu_f} \frac{\theta}{u_w^2} \left( \frac{\partial u}{\partial y} \right)_{y=0} \]

\[ Nu_x = \frac{x}{k_f(T_w - T_\infty)} \left( \frac{k_{nf}}{k_f} \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} + (\theta_T)_{y=0} \right) \]

using equation (8) and (13) we can obtain,

\[ Re_x^{1/2} C_f = \frac{\mu h_{nf}}{\mu_f} f'''(0) \]  

(16)

\[ Re_x^{-1/2} Nu_x = -\left( \frac{k_{nf}}{k_f} + \frac{4}{3} R \right) \theta'(0) \]  

(17)

\[ Re_x = u_w(x)/\nu_f \]

represents the local Reynolds number

**Numerical Procedure**

The non linear couple ODEs (12) and (13) with boundary condition (14) and (15) and were solved using bvp4c technique. Now converting higher order non linear ordinary differential equations into system of first order ODE, by introducing new variables

\[ f = y(1); \quad f' = y(2); \quad f'' = y(3); \quad \theta = y(4); \quad \theta' = y(5) \]

\[ dy = \left[ \begin{array}{c} y(2) \\ y(3) \\ \left( \frac{\mu h_{nf}/\mu_f}{\rho_{nf}/\rho_f} \right) \left( (y(2))^2 - y(1) \cdot y(3) + y(3) \right) \\ \left( \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} \right) \cdot M \cdot y(2) \\ y(5) \\ Pr \left( \frac{k_{nf}}{k_f} + \frac{4}{3} R \right) \\ \left( \frac{(\rho C_p)_h/n}{\rho C_p_f} \left( (y(1)y(5) - my(2)y(4)) - Qy(4) \right) \right) \end{array} \right] \]

with boundary condition: \( f(0) = ya(1) = S; \quad f'(0) = ya(2) = \lambda; \quad \theta(0) = ya(4) = 1; \quad f'(\infty) = yb(2); \quad \theta'(\infty) = yb(4) \)

**III. RESULT AND DISCUSSION**

The effect of \( \phi_1, \phi_2 \) for \( Al_2O_3 - TiO_2 \) and \( S, M, m, R, Q \) are analysed and further discussed. \( Pr \) is taken as 6.8(water) and \( \phi \) ranges from 0 to 0.02, where \( 0 \leq \phi_1 \leq 0.02 \), \( 0 \leq \phi_2 \leq 0.022 \), \( 1.9 \leq S \leq 1.7 \), \( 0.1 \leq M \leq 0.3 \), \( 1 \leq m \leq 2 \), \( 5 \leq R \leq 6 \), \( 0.1 \leq Q \leq 0.3 \).

**Velocity profile:**

![Fig. 2: The effect of \( \phi_1, \phi_2 \) on velocity profile](image)

![Fig. 3: The effect of \( S \) on velocity profile](image)
in the lower solution and a decrement in the upper solution versus the decreased number of $S$. Physically, the variations in the suction parameter are inversely proportional to $f'(\eta)$. It is consistent with the physical reality that when $S$ increases, the momentum boundary layer tends to adhere to the stretching sheet, disrupting the flow momentum. In Figure 4, there is an increase in the first solution of $f'(\eta)$ as $M$ expands the hybrid nanofluid flow. The presence of the magnetic field in an electrically conducting fluid boosts the Lorentz force, which is a resistive force. This force provides resistance in opposition to the motion of the fluid particle, lowering the fluid’s velocity. The synchronization of the magnetic and electrical fields caused by the Lorentz force tends to slow the speed of the conducting fluid near the boundary layer. In short, increasing $M$ raises $f'(\eta)$, which increases the frictional drag produced on the sheet surface. However, about the second solution in Figure 4, $f'(\eta)$ was discovered to be diminishing as $M$ increased from 0.1 to 0.3. The velocity of the hybrid nanofluid reduces as it passes through the shrinking sheet, increasing the thickness of the momentum boundary layer. This unusual tendency could be attributed to the shrinking sheet event, which restricted the infiltration of hybrid nanofluid molecules.

**Temperature profile:**

Figure 5 depicts the trend of temperature distribution for solid volume fraction of hybrid nanofluid. Physically, the nanoparticles dissipate energy in the form of heat. Furthermore, we discovered that the first solution of $\theta(\eta)$ rises when the nanoparticle volume percentage increases. The higher concentration of nanoparticles improves the thermal conductivity of hybrid nanofluid. As expected, increasing thermal conductivity of the hybrid nanofluid has a positive impact on fluid temperature as the nanoparticles volume fraction grows. But, we may get a opposite result for second solution of thermal distribution.

The influence of the magnetic field coefficient on the temperature pattern is depicted graphically in Figure 6. It suggests an improvement in $\theta(\eta)$ with $M$. This is because the created Lorentz force (as a result of $M$ changes) increases the friction between the fluid layers, which improves the temperature profile. It is obvious that the fluid temperature and boundary layer thickness rise with higher values of this parameter.

Figure 7 depicts the temperature profiles behavior, when different thermal radiation parameters ($R$) are used. This variable occurs exclusively in the thermal Equation (13), and it is decoupled from momentum Equation (12); thus, changing the value of $R$ has no effect on the velocity profile. The thermal boundary layer width improves with $R_d$ in both solutions, implying that the temperature gradient at the surface is lower for larger $R$ quantities. Physically, the radiation parameter measures the distribution of thermal radiation due to the flow of heat through conduction. Higher values of this radiation parameter thus show the dominance of thermal radiation over conduction. Therefore, a large amount of heat energy is emitted in the system due to radiation, which leads to an increase in the temperature. This indicates that the temperature of the fluid $\theta(\eta)$ increases because of the presence of high radiation. As demonstrated in Figure 8, the effect of the heat generation/absorption parameter on the fluid temperature distribution results in a substantial enhancement.
in the values of variable. Physically, in the case of heat generation, there is a rise in the transfer and thermal spread of the fluids, which raises the temperature of the fluid as well as the temperature and thickness of the boundary layer as the values of the heat source type grow. Figure 9 shows the thermal field of both solutions are boosted, when the non-isothermal parameter rises.

**Skin friction:**

Figure 10 depicts the behavior of the skin friction coefficient $f'\prime(0)$ with mass transfer specification $S$ for various values of the volume fraction parameter $\phi_{hnf}$ over a shrinking sheet. The skin friction coefficient $f'\prime(0)$ increases as the parameter $\phi_{hnf}$ is increased. For increasing nanoparticle volume fraction of the fluid, the voriticity generation due to shrinking sheet velocity is moderately diminished and thus, with the imposed suction, that vorticity remains confined to the area of a boundary layer. The skin friction values for various parameters, including suction, volume fraction, and magnetic parameters are presented in Table I. The problem of wall friction coefficient of hybrid nanofluid flow model is discussed.

**TABLE I: Values of skin friction $Re^{1/2}C_f$ with $\lambda = -1$**

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$M$</th>
<th>$S$</th>
<th>$Re^{1/2}C_f$ first solution</th>
<th>$Re^{1/2}C_f$ second solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.015</td>
<td>0.025</td>
<td>0.03</td>
<td>1.269915</td>
<td>0.535805</td>
</tr>
<tr>
<td>0.001</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>1.101958</td>
<td>0.719166</td>
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<tr>
<td>0.001</td>
<td>0.1</td>
<td>0.4</td>
<td>0.5</td>
<td>1.521834</td>
<td>0.253231</td>
</tr>
<tr>
<td>0.001</td>
<td>0.15</td>
<td>0.18</td>
<td>1.9</td>
<td>1.527765</td>
<td>0.417984</td>
</tr>
</tbody>
</table>

**Heat transfer:**

Further, we made comparison with earlier published articles by Grubka and Boba [1] and Ishandar et al. [33] in the absence of thermal radiation, magnetic field and heat
TABLE II: Values of heat transfer $Re_{x}^{-1/2}Nu_x$ under different values of $\lambda = -1, S = 1.9, and \phi_{hnf} = 0.02$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$R$</th>
<th>$\mathcal{M}$</th>
<th>$Q$</th>
<th>first solution</th>
<th>second solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>11.1739</td>
<td>11.2267</td>
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<td>0.5</td>
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<td>10.181165</td>
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<td>0.2</td>
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<td>0.3</td>
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</tr>
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<td>10.21489</td>
<td>9.65529</td>
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</table>

Resource surface methodology

The RSM is a statistical method that is frequently used to establish empirical relationships between several inputs and numerous responses. It is a very effective technique for analyzing a multi-variable and multi-response process since it offers information about the most and least dominating elements that influence the answers. Additionally to modeling and analysis, RSM offers process optimization.

The RSM approach was used in this instance as well to examine the impact of the input values ($\phi_1, \phi_2, R, Q$) on the outcomes (first and second solutions of the Nusselt number), leading to the development of an empirical relationship between them and process optimization. In order to do this, a second-order polynomial was used, which also reveals the elements that influence the answers. Additionally to equation (18) provides the RSM-based second-order mathematical relation that was utilised in this work.

$$Y = \beta_0 + \sum_{i=1}^{p} \beta_i X_i + \sum_{i=1}^{q} \beta_j X_j + \sum_{i=1}^{p} \beta_{ij} X_i X_j$$

where $\beta_0$ is a constant term, coefficients $\beta_1, \beta_2, \beta_3, \ldots, \beta_p$ and $\beta_{12}, \beta_{13}, \ldots, \beta_{pq}$ are linear and quadratic terms, while $\beta_{12}, \beta_{13}, \ldots, \beta_{pq}$ are interacting terms.

The complete experimental plan created using the faceted-centered CCD is shown in Table VI. The development of an empirical model for multi-variable and multi-response issues frequently uses RSM. It can also be successfully applied to process optimization.

$$Response = \beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_4 D + \beta_5 A^2 + \beta_6 B^2 + \beta_7 C^2 + \beta_8 D^2 + \beta_9 AB + \beta_{10} AC + \beta_{11} AD + \beta_{12} BC + \beta_{13} BD + \beta_{14} CD$$

The fitting model was predicted using a full quadratic polynomial that took into account every component of the response surface. The model was subjected to analysis of variance (ANOVA), which is used to assess the model's relevance as well as the importance of each term in the model. The ANOVA tables IV and V provide the degree of freedom, mean square, and sum of squared deviations for the model as well as for each term. The 95% confidence interval for RSM means that the model or the term is considered to be significant if the p value (equivalent to the F value) of the term or of the model is determined to be less than 0.05. The figure 11 and 12 shows the residual fitted graph according the first and second solution of Hybrid Nanofluid’s flow of nussel number. Contour plot of this model flow can be represented as figures 13 to 16.
### TABLE VI: Central Composite Design

<table>
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<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>φ₁</th>
<th>φ₁</th>
<th>R</th>
<th>Q</th>
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**Regression Equation**

First solution:

\[
\text{first solution} = 0.011 + 2.36\phi_1 + 2.17\phi_2 + 0.7192R - 1.524Q - 11.0\phi_1 \ast \phi_1 - 11.0\phi_2 \ast \phi_2 \\
+ 0.06085R \ast R - 0.294Q \ast Q + 9.5\phi_1 \ast \phi_2 + 0.385\phi_1 \ast R - 0.41\phi_1 \ast Q \\
+ 0.367\phi_2 \ast R - 0.41\phi_2 \ast Q - 0.0969R \ast Q
\]

Second solution:

\[
\text{second solution} = 0.012 + 2.34\phi_1 + 2.16\phi_2 + 0.7190R - 1.525Q - 10.7\phi_1 \ast \phi_1 - 10.7\phi_2 \ast \phi_2 \\
+ 0.06088R \ast R - 0.291Q \ast Q + 9.6\phi_1 \ast \phi_2 + 0.386\phi_1 \ast R - 0.42\phi_1 \ast Q \\
+ 0.367\phi_2 \ast R - 0.41\phi_2 \ast Q - 0.0968R \ast Q
\]
Fig. 11: Residual plot of 1st solution

Fig. 12: Residual plot of 2nd solution
Fig. 13: Contour plot of 1st solution Vs $\phi_1, R$

Fig. 14: Contour plot of 1st solution Vs $\phi_1, \phi_2$
Fig. 15: Contour plot of 2nd solution Vs $\phi_1, \phi_2$

Fig. 16: Contour plot of 2nd solution Vs Q, R
IV. CONCLUSION

This research paper discusses the skin friction of the MHD hybrid nanofluid over a shrinking surface in the presence of thermal radiation. Furthermore, it investigates the properties of hybrid nanofluid in heat transmission with the impact of a heat source/sink. The established problem with these physical impacts had significant importance. A formal numerical technique is adopted to demonstrate the influence of all physical parameters on the dual velocity and temperature profiles. The conclusions derived from the findings of the current work are summarized as below:

• For both solutions of horizontal velocity, the components are improved with an increase in the nanoparticle volume fraction.
• In the first and second solutions, the velocity distribution reduces with an increase in the values of suction and magnetic parameters.

It is observed that there is enhancement in the values of Re and 1/√Cf in the first solution with an increasing volume fraction. The influence of different (φ1, φ2) values of the suction parameter (S) is examined. It is noted that higher values of M, R, and Q trend to boost the thermal delineation.

We involved the careful application of the resource surface approach, which enabled us to investigate the importance of key parameters such as volume fraction, thermal radiation, heat source/sink. Through this meticulous analysis, we aimed to gain valuable insights into their roles and effects within our study, understanding of the phenomena under investigation.

REFERENCES


[34] T. Maranna, U. S. Mahalashwari, L. M. Perez, O. Manca,“Flow of viscoelastic ternary nanofluid over a shrinking porous medium with heat


**NOMENCLATURE**

\( u, v \) Velocity components along \( x \) and \( y \) axis  
\( \phi_1 \) Volume fraction of Copper Nanoparticle  
\( \phi_2 \) Volume fraction of Aluminium oxide Nanoparticle  
\( hnf \) Hybrid nanofluid  
\( nf \) Nanofluid  
\( f \) Base fluid  
\( R \) Thermal radiation  
\( M \) Magnetic field  
\( m \) Non isothermal parameter  
\( S \) Suction parameter  
\( \lambda \) Shrinking parameter  
\( \nu \) Dynamic viscosity  
\( \mu \) Kinematic viscosity  
\( \rho \) Density  
\( k \) Thermal conductivity  
\( \sigma \) Electrical conductivity  
\( C_p \) Specific heat capacity  
\( T_w \) Wall temperature  
\( T_\infty \) Ambient temperature