A Two Dimensional Mathematical Model of Airborne Infection in an Outpatient Room with an Outlet Ventilation System

Wasu Timpitak, and Nopparat Pochai

***Abstract*—Bacteria and viruses can travel along air currents, remain in the air, or cling to surfaces before being breathed in by another person. They are transferred by tiny respiratory droplets and are referred to as airborne infections. Lack of ventilation or low ventilation rates is associated with increased infection rates or clusters of airborne infections. While normal people remain in the same outpatient room as infectors, this research will utilize a two-dimensional mathematical model for approximating the exhaled air concentration in the specified region with an outlet ventilation system and the risk of infection. As a result, the exhaled air concentration and infection risk are affected by the initial concentration level, the distance of the seating position, and the rate of ventilation. The forward time center space technique is used to estimate the model solution. A good agreement solution is obtained using the forward time center space. The model can also be used as part of an internet of things (IoT) system to explore inventive ways to infection-free zone management. The proposed strategy represents the balance in the air quality management process between the distance of the seating position and the performance of the air ventilation system for optimal outcomes. We illustrate that the suggested technique works in real-world circumstances.**

***Index Terms*—airborne, infectious, diseases, ventilation system.**

I. Introduction

BACTERIA and viruses can travel along air currents, remain in the air, or cling to surfaces before being breathed in by another person. They are transferred by tiny respiratory droplets and are referred to as airborne infections. Lack of ventilation or low ventilation rates is associated with increased infection rates or clusters of airborne infections. In [1], they proposed the governing equation of the model of air quality in a considered area is a time-dependent three-dimensional advection-diffusion equation. The governing equation is approximated using a finite difference approach. They explore wind influx in two scenarios: wind input in only one direction (x) and wind inflow in both directions (x and y). In [2], they proposed a numerical model for air pollution emission control problems with uniform wind velocities and constant diffusion coefficients. The finite difference method is used to solve the atmospheric diffusion equation. This research focused on the sulfur dioxide ambient air quality standard. In [3], air pollution levels are measured employing numerical simulation. The three-dimensional model for detecting air pollution is presented in a high-traffic location beneath the Bangkok sky train station. The estimated air pollutant level indications suggested by user-specified indices of pollutant gases such as PM2.5, SO2, NOx, and CO have been replaced. In [4], air pollution is measured by simulating the release of pollutants from multiple point sources over an industrial area to another place. They proposed a finite difference technique used to estimate the controlling partial differential equation of pollution concentration in the air. The levels of air contaminants influenced by various particular sources are also studied. In [5], several types of high traffic street canyons, a three-dimensional time-dependent air pollutant concentration measurement is proposed. To illustrate the considered model solution, explicit finite difference techniques are applied. The numerical simulations proposed provide positive and pleasant outcomes. In [6], the traffic density of the area of Bangkok is interesting. They proposed the computational simulation of air pollution concentrations in sky train platforms with airflow impediments on busy highways. Two methods for determining air pollutant concentrations were introduced using the finite difference technique. In [7], the study focuses on detecting air pollution in a roadway canyon. To characterize the concentration of air pollution along a roadway canyon, a two-dimensional consistently averages air pollution measurement model is used.

In [8], they propose a mathematical model for predicting the concentration of exhaled air in an area with an outlet ventilation system, as well as the risk of infection in the presence of infectors while normal persons remain in the same room. To estimate the model solution, the adaptive Runge-Kutta technique and the standard fourth-order Runge-Kutta technique are utilized. Because the number of individuals who stay in the space changes with time, the Lagrange interpolating polynomial and cubic splines interpolation are used to represent the number of individuals in the space. In [9], they introduce a risk model of airborne transmission and vaccination efficacy in an outpatient room equipped with a ventilation system. A sufficiently ventilated system and the efficacy of each type of vaccination can reduce the risk of airborne infection in an outpatient room in a hospital. Utilized to help control the risk of airborne infection to the desired level if there is a public vaccination database system. In [10], a numerical model for measuring carbon dioxide concentrations in a room with an open ventilation system is proposed. To approximate the model solution, the traditional fourth-order Runge-Kutta approach is used. The proposed

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model in the air quality management process establishes a balance between the number of people permitted to stay in the room and the capacity of the air ventilation system. In [11], they introduce a mathematical model for the risk analysis of airborne infectious disease in an outpatient room. In the real world, there are four categories of people who stay in outpatient rooms: patients, relatives, workers, and outsiders. Use the fourth-order Runge-Kutta (RK4) function to approximate the model solution. The proposed numerical model can describe the dynamical dispersion of airborne infectious diseases in an outpatient room. Increasing the outlet ventilation rate and decreasing the inlet ventilation rate is an effective method for lowering carbon dioxide concentrations in an outpatient room. Controlling indoor pollutants involves considering a balance of the number of people accessing hospital services and the ventilation rate. In [12], they propose the numerical model of carbon dioxide concentration measurement in a space with an opened ventilation system. The model is used to calculate the concentration of carbon dioxide at any time when the number of persons and the rate of ventilation varies. In this research will utilize a two-dimensional mathematical model for estimating the concentration of exhaled air in an area with an outlet ventilation system and the risk of infection. Using the forward time center space, show that the proposed method applies to real-world situations. In the air quality management process, the proposed model achieves a balance between the distance of the seating position in the room and the potential of the air ventilation system.

II. Governing Equation

Because an infected person's exhaled absorption occurs particles that can infect the air, carbon dioxide levels could be used instead [13],[14],[15],[16], and [17]. CO_2 concentration in the air of approximately 400 ppm in an area, but when people enter it, exhaled air concentration begins to rise, depending on the rate of ventilation each person, the length of the room, and the proportion of persons in the area [14],[17], and [18].

We suppose that an indoor area, such as a room with a volume of V, begins the day with an environment CO_2 concentration of C_E roughly 400 ppm and is inhabited by the number of individuals(n). Given the presence of infectors, the exhale air concentration that may include airborne contagious particles may tend to rise in the room, determined by the rate of ventilation (Q) and n. We simply assume that persons in the room make a significant contribution to the production of CO_2, which serves as an exhaled air marker. The general equation of the accumulation rate exhaled air concentration in a room with C_E, is equivalent to the exhaled air rate generated by inhabitants plus the rate of C_E, minus Q removes exhaled air [19],

$$V\frac{dC}{dt} = npC_a + QC_E - QC, \tag{1}$$

where C is the exhale air concentration indoor(ppm), C_a is a fraction of the CO_2 contained in inbreathed air and p is the rate of respiration in the room for each person(L/s). t is the duration time and T is the stationery simulation time. Initial condition $C(0) = C_0$ where C_0 is the latent CO_2 concentration.

Currently, there is an epidemic of airborne infectious diseases. Outpatient departments in hospitals, clinics, classrooms, and restaurants are all places where individuals are at risk of infection while there are patients inside. In this research, we want to assess the air quality and the risk of infection while regular people remain in an outpatient room with infectors in the aspect of the two-dimensional space.

We consider airborne infections generated by inhabitants, and the value of Q is assumed by Q_{out}, then this value is named the outlet ventilation rate. In a simple scenario, the number of people in each position are varied and assumed by $n(x, y, t)$. The two-dimensional equation of the accumulation rate exhaled air concentration in a room with outlet ventilation rate becomes :

$$V\frac{\partial C}{\partial t} = n(x, y, t)pC_a + k(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}) - Q_{out}C, \tag{2}$$

for all $(x, y, t) \in \Omega \times \tau$ such that $\Omega = \{(x, y); 0 \leq x \leq L, 0 \leq y \leq W\}$, and $\tau = \{t; 0 \leq t \leq T\}$, where C is the concentration of air exhaled indoors (ppm), k is the diffusion coefficient of CO_2, p is the rate of respiration for each person (L/s) and C_a is a fraction of the airborne infection concentration contained inbreathed air.

A. The percentage of exhaled air in an unstable state

In Eq.(2), calculation of the percentage of exhaled air in a room with an outlet ventilation system under unsteady state conditions, we get

$$f(x, y, t) = \frac{C(x, y, t)}{C_a}. \tag{3}$$

B. The averaged infection-causing concentration of infectious particles

Given $\beta - \mu$ is the rate of survival of airborne infection particles generated by the infector that reaches its target the infected area of the person who is vulnerable to infection at a threshold value (particles per second) as illustrated in Figure 1,

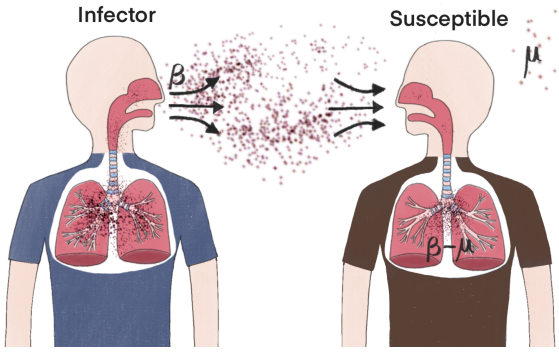

Fig. 1. Movement of airborne infectious particles.

where β is the infector's production rate of total released airborne infection particles and μ is the rate of infected particles death in the air caused by the infector that cannot be embedded in the alveoli layer.

The infection-causing concentration of infectious particles in the air, $N(x, y, t)$, is expressed as [19]:

$$N(x, y, t) = \frac{If(x, y, t)(\beta - \mu)}{n(x, y, t)p}, \tag{4}$$

where I is the number of possible infected people inside the room.

A two-dimensional equation for the accumulation rate exhaled air concentration in a room with an outlet ventilation rate is introduced. $f(x, y, t)$ can be calculated for all $(x, y, t) \in \Omega \times \tau$. So, the airborne infectious particle concentration that causes airborne infection under unsteady-state conditions is derived by replacing Eq.(3) into Eq.(4):

$$N(x, y, t) = \frac{IC(x, y, t)(\beta - \mu)}{pC_a}. \tag{5}$$

C. The number of airborne infection particle

Not that all infected particles can reach the alveolar cavity and deposit there; let θ be the proportion of airborne infected particles that penetrate and deposit at the host's location of the infected area. As a result, the number of airborne infection particle is expressed as [19]:

$$\lambda(x, y, t) = pt\theta N(x, y, t), \tag{6}$$

where t is the time consumed in the room got to the moment of infection in the room and $(0 < \theta < 1)$.

D. The probability of airborne infectors

In [19],[20], and [21], they proposed tuberculosis transmission follows a Poisson distribution, the probability of airborne infectors is expressed as:

$$P(x, y, t) = 1 - e^{-\lambda(x,y,t)}, \tag{7}$$

where the probability (P) of susceptible individuals with airborne infectors risk.

III. Numerical technique

The domain is now discretized by dividing the interval $[0, L]$ into M sub-intervals so that $M\Delta x = L$, the interval $[0, W]$ into N sub-intervals so that $N\Delta y = W$, and the time interval $[0, T]$ into R sub-intervals so that $R\Delta t = T$. The grid points (x_i, y_j, t_n) are defined by $x_i = i\Delta x$ for all $i = 0, 1, 2, \ \dots \ , M$, $y_j = j\Delta y$ for all $j = 0, 1, 2, \ \dots \ , N$ and $t_n = n\Delta t$ for all $n = 0, 1, 2, \ \dots \ , R$, in which M, N and R are integers that are positive. Then we can estimate $C\left(x_i, y_j, t_n\right)$ by $C_{i,j}^n$, which is the value of the difference approximation of $C\left(x, y, t\right)$ at point $x = i\Delta x$ $y = j\Delta y$ and $t = n\Delta t$, where $0 \leq i \leq M$ $0 \leq j \leq N$.and $0 \leq n \leq R$. In Eq.(2), we will use the forward time central space finite difference scheme (FTCS).

A. Forward Time Central Space Finite Difference Scheme

$$C\left(x_i, y_j, t_n\right) \cong C_{i,j}^n, \tag{8}$$

$$\left.\frac{\partial C}{\partial t}\right|_{(x_i, y_j, t_n)} \cong \frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t}, \tag{9}$$

$$\left.\frac{\partial^2 C}{\partial x^2}\right|_{(x_i, y_j, t_n)} \cong \frac{C_{i+1,j}^n + C_{i-1,j}^n - 2C_{i,j}^n}{\left(\Delta x\right)^2}, \tag{10}$$

$$\left.\frac{\partial^2 C}{\partial y^2}\right|_{(x_i, y_j, t_n)} \cong \frac{C_{i,j+1}^n + C_{i,j-1}^n - 2C_{i,j}^n}{\left(\Delta y\right)^2}. \tag{11}$$

Substituting Eqs.(8)-(11) into Eq.(2), we get the finite difference equation,

$$V\left(\frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t}\right) = n(x_i, y_j, t_n)pC_a - Q_{out}C_{i,j}^n$$

$$+k\left(\frac{C_{i+1,j}^n + C_{i-1,j}^n - 2C_{i,j}^n}{\left(\Delta x\right)^2} + \frac{C_{i,j+1}^n + C_{i,j-1}^n - 2C_{i,j}^n}{\left(\Delta y\right)^2}\right). \tag{12}$$

for all $i = 0, 1, 2, 3, \ \dots \ , M$, $j = 0, 1, 2, \ \dots \ , N$ and $n = 0, 1, 2, \ \dots \ , R - 1$. Then the explicit finite difference equation becomes

$$C_{i,j}^{n+1} = A_{i,j}^n + \lambda\left(C_{i+1,j}^n + C_{i-1,j}^n\right)$$

$$+\beta\left(C_{i,j+1}^n + C_{i,j-1}^n\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right)C_{i,j}^n, \tag{13}$$

where $A_{i,j}^n = \frac{n(x_i, y_j, t_n)p\Delta t C_a}{V}$, $\lambda = \frac{k\Delta t}{V(\Delta x)^2}$, and $\beta = \frac{k\Delta t}{V(\Delta y)^2}$.

B. The initial condition and boundary conditions

1) The initial condition:

$$C(x, y, 0) = g(x, y, 0), \tag{14}$$

for all $0 \leq x \leq L$ and $0 \leq y \leq W$, where $g(x, y, 0)$ is the latent airborne infection concentration. The quantity of exhaled air of one person is 1004.22 ppm, [19]. The quantity of latent exhaled air is 0.008 ppm, [19]. The available people who are seated in a room are assumed by a function,

$$g(x, y, t) = \begin{cases} 0.01, & \text{if} \quad (x, y, t) \in A; \\ 0.00008, & \text{if} \quad (x, y, t) \notin A; \end{cases}$$

where A is a set of seated positions. A is defined by $A = \{(x_i, y_j, t) : x_i$ and y_j are coordinates of seat positions in a room over time $t > 0$, $i = 1, 2, \ \dots \ , K$, $j = 1, 2, \ \dots \ , S$, for all $t > 0$,$\}$ where K is a row number and S is column number i.e. $K \times S$ is a number of seat in the room.

Over time, the exhaled air around the available people seated in a room increases. So, the average exhaled air within 0.5 meters of people who move around the point (x_i, y_j) is defined by a particular example of a two-dimensional Gaussian function,

$$C(x, y, 0) = g(x, y, 0) \times exp\Big(-\Big(\frac{(x - x_i)}{2\sigma_x^2} + \frac{(y - y_j)}{2\sigma_y^2}\Big)\Big), \tag{15}$$

where σ_x and σ_y are the flatten coefficients, and x and y is within 0.5 meters of people who move around the point x_i and y_j.

2) The boundary conditions:

$$\frac{\partial C(0, y, t)}{\partial x} = k_1, \tag{16}$$

$$\frac{\partial C(L, y, t)}{\partial x} = k_2, \tag{17}$$

$$\frac{\partial C(x, 0, t)}{\partial y} = k_3, \tag{18}$$

$$\frac{\partial C(x, W, t)}{\partial y} = k_4. \tag{19}$$

for all $0 \leq x \leq L$, $0 \leq y \leq W$ and $0 \leq t \leq T$, where k_1, k_2, k_3 and k_4 are given rate of change of airborne infection concentration.

3) Boundary conditions approximation: According to the boundary conditions Eqs.(16)-(19). we can divide Eq.(13) into eight cases. The boundary condition is illustrated in Figure 2.

C. case 1: the bottom left corner boundary condition

If $i = 0$ and $j = 0$, substituting the approximate unknown value of the boundary, we can let $C^n_{i-1,j} = C^n_{i+1,j} - 2\Delta x k_1$ and $C^n_{i,j-1} = C^n_{i,j+1} - 2\Delta y k_3$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i+1,j} - \Delta x k_1\right) + 2\beta\left(C^n_{i,j+1} - \Delta y k_3\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (20)$$

D. case 2: the bottom boundary condition

If $1 \leq i \leq M-1$ and $j = 0$, we can let $C^n_{i,j-1} = C^n_{i,j+1} - 2\Delta y k_3$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + \lambda\left(C^n_{i+1,j} + C^n_{i-1,j}\right) + 2\beta\left(C^n_{i,j+1} - \Delta y k_3\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (21)$$

E. case 3: the bottom right corner boundary condition

If $i = M$ and $j = 0$, we can let $C^n_{i+1,j} = C^n_{i-1,j} + 2\Delta x k_2$ and $C^n_{i,j-1} = C^n_{i,j+1} - 2\Delta y k_3$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i-1,j} + \Delta x k_2\right) + 2\beta\left(C^n_{i,j+1} - \Delta y k_3\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (22)$$

F. case 4: the left boundary condition

If $1 \leq j \leq N-1$ and $i = 0$, we can let $C^n_{i-1,j} = C^n_{i+1,j} - 2\Delta y k_1$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i+1,j} - \Delta y k_1\right) + \beta\left(C^n_{i,j+1} + C^n_{i,j-1}\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (23)$$

G. case 5: the upper left corner boundary condition

If $j = N$ and $i = 0$, we can let $C^n_{i-1,j} = C^n_{i+1,j} - 2\Delta x k_1$ and $C^n_{i,j+1} = C^n_{i,j-1} + 2\Delta y k_4$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i+1,j} - \Delta x k_1\right) + 2\beta\left(C^n_{i,j-1} - \Delta y k_4\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (24)$$

H. case 6: the upper boundary condition

If $1 \leq i \leq M-1$ and $j = N$, we can let $C^n_{i,j+1} = C^n_{i,j-1} + 2\Delta y k_4$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + \lambda\left(C^n_{i+1,j} + C^n_{i+1,j}\right) + \beta\left(C^n_{i,j-1} + \Delta y k_4\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (25)$$

I. case 7: the upper right corner boundary condition

If $i = M$ and $j = N$, we can let $C^n_{i+1,j} = C^n_{i+1,j} + 2\Delta x k_2$ and $C^n_{i,j+1} = C^n_{i,j-1} + 2\Delta y k_4$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i-1,j} + \Delta x k_2\right) + 2\beta\left(C^n_{i,j-1} + \Delta y k_4\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (26)$$

J. case 8: the right boundary condition

If $1 \leq j \leq N-1$ and $i = M$, we can let $C^n_{i+1,j} = C^n_{i-1,j} + 2\Delta x k_2$ and by rearranging, we obtain

$$C^{n+1}_{i,j} = A^n_{i,j} + 2\lambda\left(C^n_{i+1,j} + \Delta x k_2\right) + \beta\left(C^n_{i,j+1} + C^n_{i,j-1}\right) + \left(1 - 2\lambda - 2\beta - \frac{\Delta t Q_{out}}{V}\right) C^n_{i,j}, \quad (27)$$

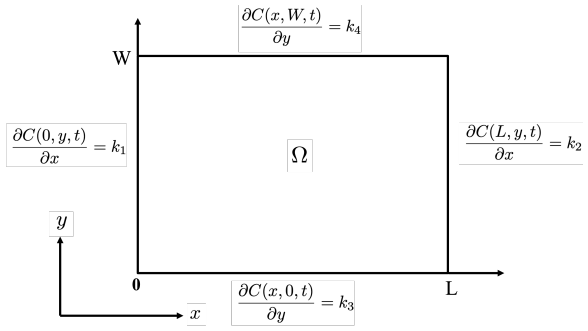

Fig. 2. The boundary condition.

IV. Numerical experiments and results

Assuming that the respiration rate is $p = 0.12\ (L/s)$, and a fraction of the CO_2 concentration contained in inbreathed air $C_a = 0.04$. By employing the forward time central space method Eqs.8-13.

A. Simulation 1: The concentration measurement of exhaled air and the probability of airborne infection when normal people sit in every seat.

The parameters and point of the seat are assumed in Tables I-Table II. The boundary conditions and the initial condition are assumed by Eqs.14-19. Figures 3-5 present the number of people in the room and the initial condition of the latent CO_2 concentration respectively. The approximated solutions are illustrated in Figures 6-7. Figures 8-9 show the line plot of the approximated air exhaled indoor concentration in a room and the line plot of the probability of airborne infection in a room.

TABLE I
PARAMETER

V	Q_{out}	I	θ	σ_x	σ_y	$\beta - \mu$
72	2	1	0.05	0.5	0.5	2
k_1	k_2	k_3	k_4	K	S	k
0	0	0	0	6	5	0.106×10^{-4}

TABLE II
POINT OF THE SEAT

x_i/y_i	1	2	3	4	5
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)

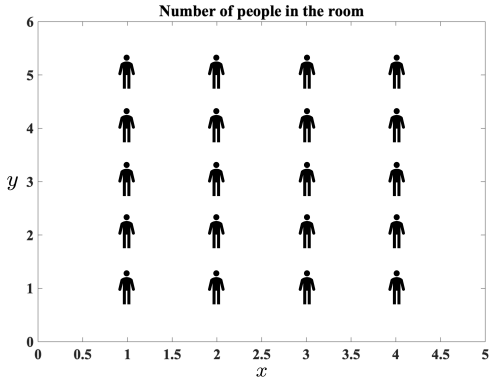

Fig. 3. The number of people in the room.

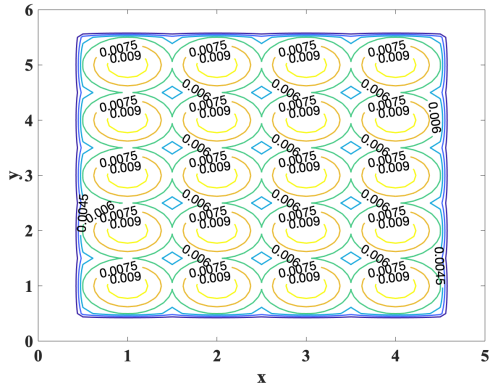

Fig. 4. The contour plot of the latent CO_2 concentration.

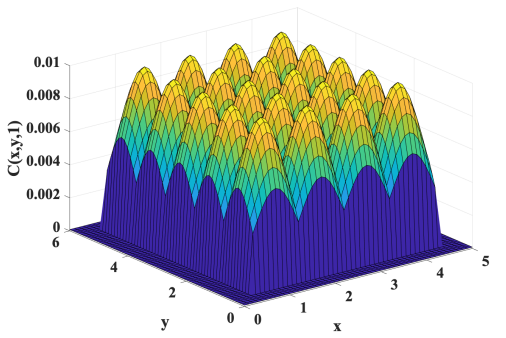

Fig. 5. The surface plot of the latent CO_2 concentration.

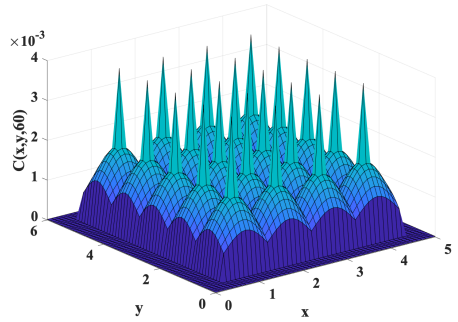

Fig. 6. The surface plot of the approximated air exhaled indoor concentration around the people who stays in the room at t=60.

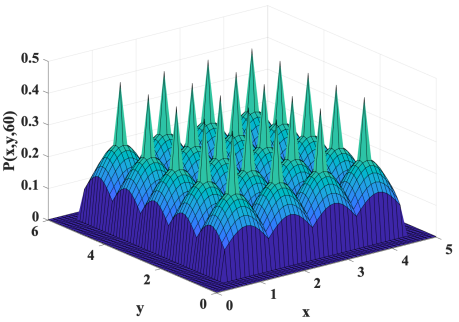

Fig. 7. The surface plot of the probability of airborne infection around the people who stays in the room at t=60.

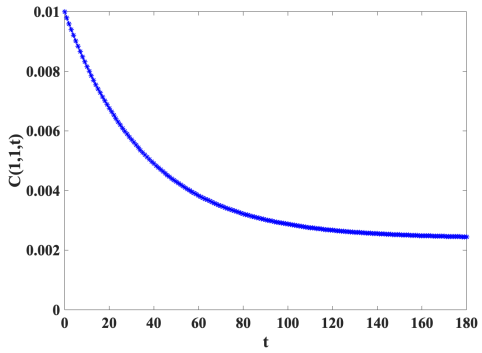

Fig. 8. The line plot of the approximated air exhaled indoor concentration in a room at x=1, y=1.

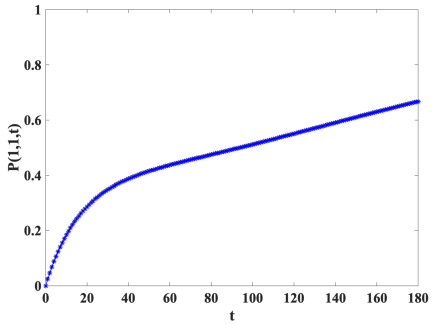

Fig. 9. The line plot of the probability of airborne infection in a room at x=1, y=1.

B. Simulation 2 : The concentration measurement of exhaled air and the probability of airborne infection when normal people are not seated in the room for all seats.

The parameters and point of the seat are assumed in Tables III-IV. The boundary conditions and the initial condition are

assumed by Eqs.14-19. Figures 10-11 present the number of people in the room and the initial condition of the latent CO_2 concentration respectively. The approximated solutions are illustrated in Figures 12-13. Figure 14 shows the line plot of the probability of airborne infection in a room.

TABLE III
PARAMETER

V	Q_{out}	I	θ	σ_x	σ_y	$\beta - \mu$
72	2	1	0.05	0.5	0.5	2
k_1	k_2	k_3	k_4	K	S	k
0	0	0	0	6	5	0.106×10^{-4}

TABLE IV
POINT OF THE SEAT

x_i/y_i	1	2	3	4	5
1	(1,1)	–	–	–	(1,5)
2	–	–	(2,3)	–	–
3	–	(3,2)	–	(3,4)	–
4	–	–	(4,3)	–	(4,5)

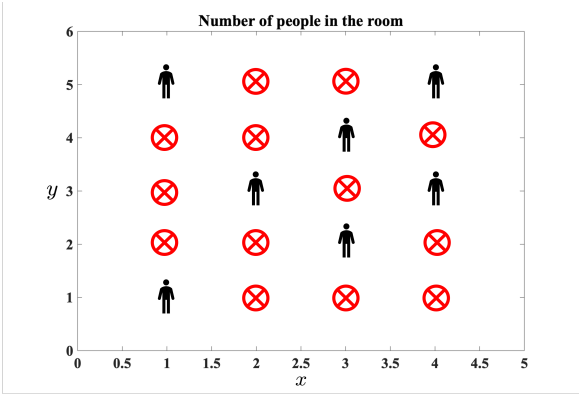

Fig. 10. The number of people in the room.

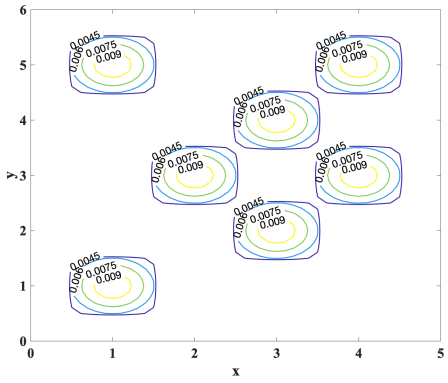

Fig. 11. The contour plot of the latent CO_2 concentration.

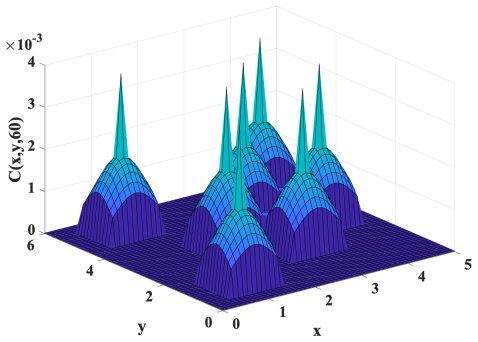

Fig. 12. The surface plot of the approximated air exhaled indoor concentration in a room around the people who stays in the room at t=60.

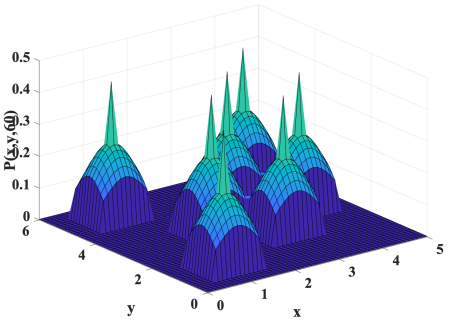

Fig. 13. The surface plot of the probability of airborne infection around the people who stays in the room at t=60.

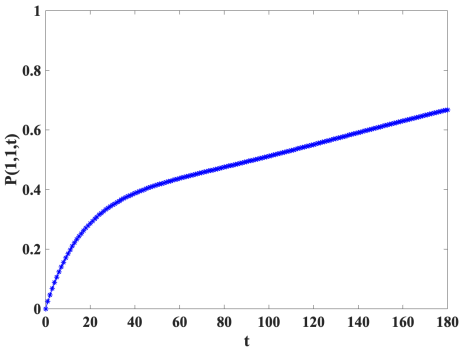

Fig. 14. The line plot of the probability of airborne infection in a room at x=1, y=1.

C. Simulation 3 : The concentration measurement of exhaled air and the probability of airborne infection when different outlet ventilation levels.

The parameters and point of the seat are assumed in Tables V-VI. The approximated solutions are illustrated in Figures 15-17. Figure 18 shows the line plot of the comparison probability air exhaled indoor concentration.

TABLE V
PARAMETER

V	Q_{out}	I	θ	σ_x	σ_y	$\beta - \mu$
72	2, 4, 6	1	0.05	0.5	0.5	2
k_1	k_2	k_3	k_4	K	S	k
0	0	0	0	6	5	0.106×10^{-4}

TABLE VI
POINT OF THE SEAT

x_i/y_i	1	2	3	4	5
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)

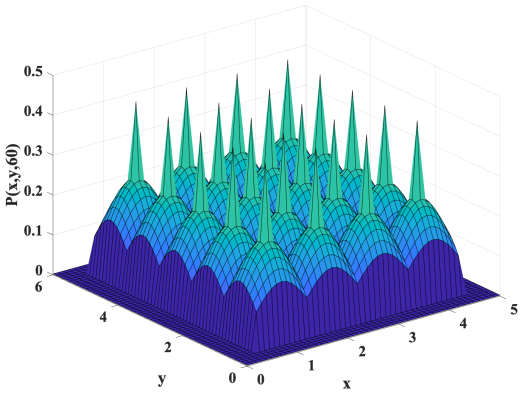

Fig. 15. The surface plot of the probability of airborne infection around the people who stays in the room at t=60 with the outlet ventilation rate $Q_{out} = 2$.

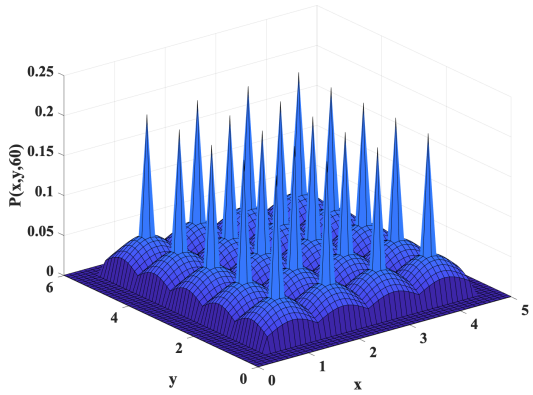

Fig. 16. The surface plot of the probability of airborne infection around the people who stays in the room at t=60 with the outlet ventilation rate $Q_{out} = 4$.

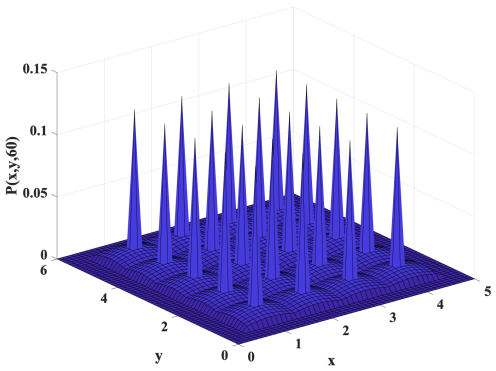

Fig. 17. The surface plot of the probability of airborne infection around the people who stays in the room at t=60 with the outlet ventilation rate $Q_{out} = 6$.

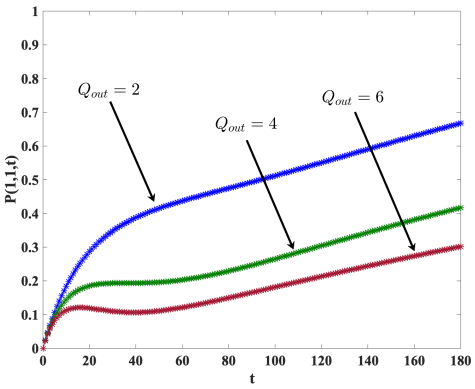

Fig. 18. The line plot of the comparison probability air exhaled indoor concentration in a room at x=1, y=1 when the difference outlet ventilation rate.

V. DISCUSSION

In simulation 1, the concentration measurement of exhaled air and the risk of airborne infection when normal people sit in every seat. Figure 6 shows the approximation of the concentration of exhaled air around the people who stays in the room. We can see that the point where the person sits has a carbon dioxide concentration distributed around it. Figure 7 shows the probability of airborne infection. We can see that the risk of airborne infection is spread across the person's sitting position. Figure 8 shows the line plot of the approximated air exhaled indoor concentration in a room at x = 1 and y = 1. We can see that when time increases, the concentration of exhaled air is reduced. Figure 9 shows the line plot of the probability of airborne infection in a room at x = 1 and y = 1. We can see that when time increases, the probability of airborne infection increases.

In simulation 2, the concentration measurement of exhaled air and the risk of airborne infection are proposed when normal people are not seated in the room for all seats. Figure 12 shows the approximation of the concentration of exhaled air when normal people are not seated in the room for all seats. We can see that the point where the person sits has a carbon dioxide concentration distributed around it. Figure 13 shows the probability of airborne infection. We can see that the risk of airborne infection is spread across the person's sitting position. Figure 14 shows the line plot of the probability of airborne infection in a room at $x = 1$ and $y = 1$. We can see that when time increases, the probability of airborne infection increases.

In simulation 3, the concentration measurement of exhaled air and the risk of airborne infection are proposed when different outlet ventilation levels. Figures 15-17 show the approximation of the concentration of exhaled air when outlet ventilation levels are $2, 4$ and 6 respectively. We can see that when the outlet ventilation level increases, the probability of airborne infection around the people who stays in the room is reduced. Figure 18 shows the line plot of the comparison probability air exhaled indoor concentration in a room at $x = 1$ and $y = 1$. We can see that when the outlet ventilation level increase, the probability of airborne infection is reduced.

VI. Conclusion

A two-dimensional mathematical model for predicting the concentration of exhaled air in a room with an outlet ventilation system is introduced. The risk of infection when healthy people remain in the same room as infected people is considered. To predict the exhaled air concentration, we set a 1-meter distance between persons in the room to assess the reduction of the risk of infection associated with social distancing measures. We introduce the initial condition-setting approach, and boundary condition techniques to adjust the case of the study. A two-dimensional Gaussian function is used to determine the initial conditions. So that the initial carbon dioxide value corresponds to the individual seated in each location within the enclosed area. As a result, the initial concentration level, the distance between persons in the room, the number of users, and the ventilation rate all impact the exhaled air concentration and infection risk. The forward time center space method is used to estimate the model solution. The proposed strategy represents a balance in the air quality management process between the distance of individuals allowed to stay in the room and the performance of the air ventilation system. We demonstrate that the suggested strategy is effective in real-world scenarios.

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