Impact of Channel Slope, Variable magnetic Field and Effective Prandtl Number on MHD Maxwell Fluid in the Presence of Heat Generation and Thermophoresis

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Abstract—The study examines the impact of inclined stretching sheet, aligned magnetic field and effective Prandtl number on MHD Maxwell fluid in the presence of thermophoresis and heat generation. By employing similarity transformations the given partial differential equations are converted into nonlinear ordinary differential equations which are then solved numerically using MATLAB bvp4c technique. The investigations reveal that enlargement of the effective Prandtl number enhances the temperature and concentration of the fluid and suppresses the magnitude of the fluid velocity. It is observed that the fluid concentration is highest when both the channel slope and magnetic field inclination angles assume least values provided the effective Prandtl number is greater or equal to unity or when both inclination angles attain maximum values when the effective Prandtl number is less than unity. Additionally, the fluid temperature profile is greatest when channel slope and magnetic field inclination angles assume inclined stretching sheet, aligned magnetic field and thermophoresis.

Index Terms — Maxwell fluid, effective Prandtl number, inclined channel, thermophoresis.

I. INTRODUCTION

Many industrial processes deal with Newtonian and non-Newtonian fluids. In this study we consider a non-Newtonian model and in particular Maxwell fluid. Maxwell fluids are non-viscous rate-type fluids which predict the fluid elasticity, viscosity and stress relaxation impacts.

Wenchaang et al. [30] examined unsteady viscoelastic Maxwell fluid flow between two parallel plates and Qi and Xu [18] applied Fourier and Laplace transform to find exact solution of an unsteady viscoelastic fluid with fractional derivative Maxwell model. Later, Noor [17] conducted a study to explore MHD Maxwell fluid flow over a stretching surface in the presence of chemical reaction and Soret effect. The influence of convection heat transfer and viscous dissipation on fractional MHD Maxwell fluid flow was considered by Boi et al. [2]. Faqroq et al. [8] explored MHD Maxwell fluid containing nanoparticles caused by an exponentially stretching sheet. During the same period, Wang et al. [29] analyzed oscillatory Maxwell fluid flow passing through a rectangular tube. An investigation on oscillatory fluid passing through a tube of triangular cross section was carried out by Sun et al. [26]. Nadeem et al. [16] disclosed the numerical analysis of MHD Maxwell fluid flow containing nanoparticles flowing past a stretching sheet. Most recently, Fetecau et al. [9] addressed the effect of a constantly oscillating wall on Maxwell fluid flow past a porous channel. During the same period, Loganathan et al. [11] investigated the impact of thermal radiation and Cattaneo-Christov dual diffusion on MHD Maxwell fluid past a heated stretching sheet. Detailed analysis on Maxwell fluid flow has been carried out by numerous researchers, who among them are Haritha et al. [10], Loganathan et al. [12], Loganathan et al. [13], Saleem et al. [21], Sandeep and Solochana [23].

The current investigation also examines the influence of thermal radiation on the Maxwell fluid. The impact of thermal radiation on fluid flow has been reported by several authors. Mgyari et al. [29] and Buzuzi and Makanda [4] revealed that in optically thick medium the impact of the radiation parameter and the Prandtl number on the fluid cannot be studied independently but instead the combined effect of the two parameters called the effective Prandtl number is considered. The surfaces upon which fluid flows vary in their geometry and the magnetic field applied may be normal or oblique depending on the flow geometry configuration. Raju et al. [19], Buzuzi [5], Sandeep and Sugunamma [22], Buzuzi and Makanda [4] and Dadheech et al. [7] carried out studies on aligned magnetic fields. Their studies showed that by raising the inclination angle of the magnetic field, the heat transfer rate is enhanced and the fluid velocity is diminished. Exploration on fluid flow past oblique surfaces have re-
received the attention of numerous researchers. Among them are Alam et al. [1], Uddin [27], Buzuzi et al. [3], Buzuzi [5], Buzuzi and Makanda [4] and Ramzan et al. [20].

Whereas most of the investigations done so far dealt with either aligned magnetic field or flow surface slope and not both, a few researchers have developed detailed analysis on the simultaneous impact of both aligned magnetic field and slope of fluid channel. Sivaraj and Sheremet [25], Buzuzi [5] and Buzuzi, Makanda [4] and Buzuzi [6] are among the few researchers who have investigated the effect of both aligned magnetic field and flow surface slope. Most of these researchers considered the impact of the inclination angles independently. However, Buzuzi [5], Buzuzi and Makanda [4] and Buzuzi [6] considered the simultaneous effect of both aligned magnetic field and flow surface inclination on MHD Newtonian flow. The current investigations seek to address the simultaneous role of both inclination angles on Maxwell fluid flow.

To the best of the author’s knowledge, there is no work covered on aligned magnetic field, effective Prandtl number, channel slope, thermophoresis and heat generation on Maxwell fluid, hence impetus for the current investigation.

II. MATHEMATICAL DESCRIPTION

We consider a steady MHD flow of Maxwell fluid past an inclined stretching sheet in a porous medium with aligned magnetic field, thermophoresis and heat generation. We let the $x-$axis be directed along the inclined sheet and $y-$axis directed perpendicular to it. The stretching sheet is inclined at an angle $\gamma$ from the vertical. A uniform aligned magnetic field of uniform strength $B_0$ is applied at an angle $\alpha_1$ to the flow direction. We assume that the magnetic Reynolds number is sufficiently small as to render the induced magnetic field negligible.

Under these assumptions and Boussinesq approximation, the governing equations for the boundary layer equations of continuity, momentum, energy and concentration can be stated as ([10], [17]):

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = \frac{\partial u_1^2}{\partial x^2} + \frac{\partial u_1 v_1}{\partial x y} + \frac{\partial v_1^2}{\partial y^2}$$  \tag{1}

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + \lambda_t \left[ \frac{u_1}{\partial x} \frac{u_1}{\partial x^2} + \frac{v_1}{\partial x y} \frac{v_1}{\partial y} \right] + 2\lambda_t u_1 v_1 \frac{\partial u_1}{\partial x y} = \frac{\partial^2 u_1}{\partial y^2} - \frac{\nu}{\kappa} u_1$$

$$- \frac{\sigma B_0^2 \sin^2 \alpha_1}{\rho} \left[ u_1 + \lambda_t v_1 \frac{\partial u_1}{\partial y} \right]$$

$$+ g \left[ \beta_p (T_1 - T_\infty) \cos \gamma + \beta_M (C_1 - C_\infty) \cos \gamma \right],$$

$$u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial y} = \frac{\lambda_c}{\rho c_p} \frac{\partial^2 T_1}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_0}{\partial y},$$  \tag{2}

$$+ \frac{Q_1 (T_1 - T_\infty)}{\rho c_p} + \frac{\mu}{\rho c_p} \left( \frac{\partial u_1}{\partial y} \right)^2 + \frac{\sigma B_0 u_1^2 \sin^2 \alpha_1}{\rho c_p}.$$

$$u_1 \frac{\partial C_1}{\partial x} + v_1 \frac{\partial C_1}{\partial y} = D_b \frac{\partial^2 C_1}{\partial y^2} - \frac{\partial}{\partial y} (V'_1 (C_1 - C_\infty))$$  \tag{4}

The corresponding boundary conditions are

$$u_1 = u_1 w = a_1 x, \quad v_1 = 0, \quad T_1 = T_1 w = T_\infty + b_1 x, \quad C_1 = C_1 w = c_1 x \quad \text{at} \quad y = 0,$$

$$u_1 \to 0, \quad T_1 \to T_\infty, \quad C_1 \to C_\infty \quad \text{as} \quad y \to \infty,$$  \tag{5}

where $u_1$ and $v_1$ are the components of dimensional velocities along the $x$ and $y$ directions respectively. $T_1$ is the fluid temperature, $C_1$ is the fluid concentration, $g$ is the gravitational acceleration, $\nu$ is the kinematic viscosity, $\beta_p$ is the coefficient of thermal expansion, $\beta_M$ is the volumetric coefficient of expansion with concentration, $B_0$ is the constant magnetic field intensity, $D_b$ is the coefficient of mass diffusivity, $\kappa$ is the thermal conductivity of the fluid, $\rho$ is the density, $c_p$ is the specific heat at constant pressure, $V'_1$ denotes the thermophoretic velocity and $Q_1$ is the heat generation.

The radiant heat flux $q_0$ is written as

$$q_0 = -\frac{4\sigma_1^* \partial T_1^4}{3K_1^* \partial y}$$  \tag{6}

where $\sigma_1^*$ is the Stefan-Boltzmann constant, $K_1^*$ is the mean absorption coefficient. As stated by Buzuzi et al. [4], $T_1^4$ can be expressed as a linear combination of the temperatures to give

$$T_1^4 \approx -3T_\infty^4 + 4T_\infty^3 T_1$$  \tag{7}

and consequently

$$-\frac{1}{\rho c_p} \frac{\partial q_0}{\partial y} = \frac{16\sigma_1^* T_\infty^4}{3\rho c_p K_1^*} \frac{\partial^2 T_1}{\partial y^2} \tag{8}$$

Hence, equation (3) takes the following form

$$u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial y} = \lambda_c \frac{\partial^2 T_1}{\partial y^2} + \frac{16\sigma_1^* T_\infty^4}{3\rho c_p K_1^*} \frac{\partial^2 T_1}{\partial y^2}$$

Equations (2) - (4) and boundary conditions (5) are transformed using the following similarity transformations:

$$\eta = \left( \frac{a_1}{\nu} \right)^{0.5} f_1, \quad u_1 = a_1 x f'_1, \quad v_1 = -(a_1 \nu)^{0.5} f_1,$$

$$T_1 - T_\infty = (T_1 w - T_\infty) T(\eta), \quad C_1 - C_\infty = (C_1 w - C_\infty) C(\eta),$$

where $f_1(\eta)$, $T(\eta)$, $C(\eta)$ are the velocity, temperature and concentration distribution fields. Equations (2)- (4)
reduces to the following non-linear, coupled ordinary differential equations:

\[ f_1''' + (M_p A_1 \sin^2 \alpha_1 + 1) f_1 f_1'' - f_1' f_1''' = 0 \]  
\[ + 2 A_1 f_1 f_1' f_1''' - A_1 f_1' f_1''' - (K_{pp} + M_p \sin^2 \alpha_1) f_1' \]
\[ + \beta_1 (T \cos \gamma + G_{MT} C \cos \gamma) = 0 \]  
\[ T'' + E_{pr} \left[ f_1 T' + E_c (M_p f_1'' + f_1''') + Q_h T \right] = 0 \]  
\[ C_1'' + S_c [f_1 C_1' + \tau_1 (T'' + C_1'')] - K_m C = 0 \]

such that the following boundary conditions are satisfied:

\[ f_1'(\eta) = 1, \quad f_1(\eta) = 0, \quad T(\eta) = 1, \quad C(\eta) = 1 \quad \text{at} \quad \eta = 0, \]
\[ f_1'(\eta) = 0, \quad T(\eta) = 0, \quad C(\eta) = 0, \quad \text{as} \quad \eta \to \infty, \]

where the prime (') represents differentiation with respect to \( \eta \) and \( E_{pr} = P_r/(1 + 4 R_p^2) \) denotes the effective Prandtl number, \( P_r = \nu \beta_1 c_p / \lambda_c \) is the Prandtl number, \( R_p = 4 \nu^2 T_\infty^3 \lambda_c / \kappa_1^3 \) represents the radiation parameter, \( K_{pp} = \nu / K^* a_1 \) is the permeability parameter, \( \beta_1 \) represents the local buoyancy number, \( M_p = \sigma \beta_3^5 / \rho a_1 \) is the Hartman number, \( S_c = \nu / D_b \) is the Schmidt number, \( A_1 = a_1 \lambda \) denotes the Deborah number, \( Q_h = Q_{a1} a_1 \rho c_p \) represents the heat generation parameter, \( G_T = \frac{g \beta(T_1 - T_\infty)}{\nu^2} \) denotes the thermal Grashof number, \( G_M = \frac{\beta M (C_1 - C_1)}{\beta T (T_1 - T_\infty)} \) is the solutal Grashof number, \( G_{MT} = G_M / G_T = \frac{\beta M (C_1 - C_1)}{\beta T (T_1 - T_\infty)} \) represents the buoyancy ratio, \( \alpha_1 \) denotes the magnetic field inclination angle, \( \gamma \) represents the channel inclination angle, \( K_m = K_1 S_c / a_1 \) is the chemical reaction parameter and \( \tau_1 = (T_1 - T_\infty) / K_1 \) represents the thermophoretic parameter where \( K_1 \) is the thermophoretic coefficient and \( T_{rc} \) is the reference temperature.

The dimensionless form of the skin friction, heat transfer rate and mass transfer rate are, respectively, 
\[ \text{Re}^{1/2} C_f = (1 + A_1) f_1'(0), \]
\[ \text{Re}^{-1/2} N_u = -(1 + 4 R_p^2) T'(0), \]
\[ \text{Re}^{-1/2} S_h = -C'(0), \]
where \( \text{Re} \) is the Reynolds number, \( C_f \) is the skin friction coefficient, \( N_u \) is the local Nusselt number and \( S_h \) is the local Sherwood number.

III. SOLUTION APPROACH

To solve the coupled system of ODE’s (10) - (12) we first convert them to first order ODE’s as follows. Let
\[ f_1 = y_1, \quad f_1' = y_2, \quad f_1'' = y_3, \quad T = y_4, \quad T' = y_5, \quad C = y_6 \]
and \( C' = y_7 \). The resulting first order ODE’s are

\[ \frac{df_1}{d\eta} = y_2, \]
\[ \frac{d^2 f_1}{d\eta^2} = y_3, \]
\[ y_3 = \frac{1}{1 - A_1 y_1^2} \left[ y_2^2 + (K_{pp} + M_p \sin^2 \alpha_1) y_2 \right] \]
\[ - \frac{\beta_1}{1 - A_1 y_1^2} \left[ y_4 \cos \gamma + G_{MT} C \cos \gamma \right], \]
\[ \frac{dT}{d\eta} = y_5, \]
\[ y_6 = E_{pr} \left[ y_1 y_5 + E_c (M_p y_2^2 + y_2^3) + Q_h y_4 \right], \]
\[ \frac{dC}{d\eta} = y_7, \]
\[ y_7 = K_m y_6 - S_c y_3 y_7 + S_c \tau_1 \left[ y_5 y_7 - E_{pr} y_6 y_1 y_5 \right] \]
\[ + S_c \tau_1 E_c y_3 y_6 y_5 \left[ M_p y_2^2 + y_2^3 \right] + S_c E_{pr} \tau_1 Q_h y_1 y_4 \]

with the following corresponding conditions:
\[ y_2(0) = 1, \quad y_1(0), \quad y_4(0) = 1, \quad y_6(0) = 1, \quad y_7(0) = y_6(10). \]

All numerical asymptotic solutions are accurately obtained as long as \( \eta_{max} = 10 \).

IV. RESULTS AND DISCUSSION

By employing MATLAB software bvp4c the numerical solution to the non-linear ordinary differential equations (10) – (12) are found. Figures 1 – 33 portray the behaviour and impact of the parameters that include the Deborah number \( A_1 \), Hartman number \( M_p \), porosity parameter \( K_{pp} \), local buoyancy parameter \( \beta_1 \), buoyancy ratio \( G_{MT} \), chemical reaction parameter \( K_m \), Eckert number \( E_c \), Schmidt number \( S_c \), heat generation parameter \( Q_h \), effective Prandtl number \( E_{pr} \), magnetic field inclination angle \( \alpha_1 \), channel inclination angle \( \gamma \) and thermophoretic parameter \( \tau_1 \) on the velocity, temperature, concentration, skin friction, heat transfer rate and mass transfer rate.

The current calculated numerical values for the heat transfer rate for \( R_h = M_p = A_1 = K_{pp} = Q_h = K_m = E_c = E_{pr} = S_c = 0.1 \), \( Q_h = 0.5 \), \( \tau_1 = 0.2 \), \( E_{pr} = 0.5 \), \( \alpha_1 = 90 \) and \( \gamma = 0 \). Figures 1, 11 and 33 illustrates the impact of the Deborah number on the velocity, temperature and concentration of the fluid. It is observed that escalation of the Deborah number results in the decline of the velocity. Unless specified, the parameter values used in numerical calculations are \( A_1 = 0.2 \), \( M_p = K_{pp} = K_m = 1 \), \( \beta_1 = G_{MT} = E_c = S_c = 0.1 \), \( Q_h = 0.5 \), \( \tau_1 = 0.2 \), \( E_{pr} = 0.5 \), \( \alpha_1 = 90 \) and \( \gamma = 0 \). Figures 1, 11 and 33 illustrates the impact of the Deborah number on the velocity, temperature and concentration of the fluid. It is observed that escalation of the Deborah number results in the decline of the velocity.
Table I: Numerical values of the heat transfer rate \(-T'(0)\) for \(R_p = M_p = A_1 = K_{pp} = Q_h = K_m = E_c = 0\)

<table>
<thead>
<tr>
<th>(P_r)</th>
<th>Wang [28]</th>
<th>Nadeem et al. [16]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.4539</td>
<td>0.4582</td>
<td>0.4544</td>
</tr>
<tr>
<td>2.0</td>
<td>0.9114</td>
<td>0.9114</td>
<td>0.9114</td>
</tr>
<tr>
<td>7.0</td>
<td>1.8954</td>
<td>1.8954</td>
<td>1.8954</td>
</tr>
<tr>
<td>20</td>
<td>3.3539</td>
<td>3.3539</td>
<td>3.3539</td>
</tr>
</tbody>
</table>

distribution, rise of the temperature distribution and no effect on the concentration distribution.

Figure 1: The impact of \(A_1\) on velocity

Figure 2: The effect of \(\alpha_1\) and \(\gamma\) on velocity

Figure 3: The influence of \(\beta_1\) on velocity

The role of the magnetic field inclination angle and the channel inclination angle on the velocity, temperature and concentration are depicted in Figures 2, 12, 23, 24, 25 and 26. Figure 2 reveals that the velocity diminishes with increasing values of \(\alpha_1\) and \(\gamma\). The velocity rises more if \(\gamma > \alpha_1\) for any given values of the inclination angles. Temperature profile grows with rising \(\gamma\) values and declines with increasing \(\alpha_1\) values as demonstrated in Figure 12. As is the case with velocity, temperature profiles are bigger whenever \(\gamma > \alpha_1\) for any given values of \(\gamma\) and \(\alpha_1\). The temperature distribution is greatest when \(\gamma\) value is maximal and \(\alpha_1\) is minimal, vice versa.

Figures 25 and 26 demonstrates that the concentration is magnified with escalating \(\alpha_1\) and \(\gamma\) values provided \(E_{pr} < 1\). However, opposite behaviour is displayed whenever \(E_{pr} \geq 1\). Furthermore, Figures 23 and 24 demonstrates that for any \(\alpha_1\) and \(\gamma\) values the concentration profile is larger whenever \(\alpha_1 > \gamma\).

The impact of the inclination angles \(\alpha_1\) and \(\gamma\) on the temperature is a function of the \(E_{pr}\) value as depicted in Figures 14 and 15. As clearly portrayed in Figure 14 the temperature profile is suppressed for escalating values of \(\alpha_1\) provided \(E_{pr} \leq 0.5\), whereas the temperature profile appreciates for enlarged \(\alpha_1\) values provided \(E_{pr} > 0.5\). Figure 14 depicts the impact of \(\alpha_1\) on temperature for the case \(E_{pr} = 0.5\) and \(E_{pr} = 2\). The influence of \(\gamma\) on the temperature of the fluid for varying \(E_{pr}\) values is demonstrated in Figure 15. The graphical display reveal that the temperature profile is boosted with increasing \(\gamma\) values for \(E_{pr} > 0.6\) and declines for values of \(E_{pr} \leq 0.6\). In particular, Figure 15 portrays effect for \(\gamma\) on temperature for the case \(E_{pr} = 0.6\) and \(E_{pr} = 2\).

The effect of \(\beta_1\) on the velocity, temperature and concentration is depicted in Figures 3, 13 and 29 respectively. Figure 3 demonstrates that the rise in the value of \(\beta_1\) enhances the velocity close to the wall with opposite behaviour witnessed further away from the wall. On the contrary, enlargement of \(\beta_1\) suppresses the temperature closer to the wall with reversed effect further away from the wall. The concentration on the other hand, declines with escalating values of \(\beta_1\) throughout the boundary layer region as portrayed in Figure 29.

Figure 4: The impact of \(E_{pr}\) on velocity

Figures 4, 16 and 30 illustrates the impact of \(E_{pr}\) on the velocity, temperature and concentration distribution, respectively. It is observed that increasing values of \(E_{pr}\) declines the velocity profile and magnifies the temperature and concentration profiles.

The influence of \(G_{MT}\) on the velocity, temperature and concentration distribution is depicted in Figures 5, 17 and 27, respectively. It is clearly revealed that the velocity of fluid is enhanced as \(G_{MT}\) value grows. Conversely, the temperature and concentration profiles are suppressed as \(G_{MT}\) is enlarged. The temperature exhibit opposite
The effect of $K_m$ on velocity is evident that the concentration profile grows with increasing $K_m$. Figures 7, 19 and 29 demonstrates the influence of $K_{pp}$ on the velocity, temperature and concentration profiles respectively. It is noticed that escalating values of $K_m$ suppresses the velocity and concentration distribution. On the other hand, the temperature profile grows with increasing values of $K_m$.

Figures 6, 18 and 28 are sketched to portray the impact of $K_m$ on the velocity, temperature and concentration profiles respectively. It is noticed that escalating values of $K_m$ suppresses the velocity and concentration distribution. On the other hand, the temperature profile grows with increasing values of $K_m$.

Figures 7, 19 and 29 demonstrates the influence of $K_{pp}$ on the velocity, temperature and concentration distribution. It is evident that the concentration profile grows with increasing values of $K_{pp}$. The temperature profiles also grows near the wall ($\eta \leq 5$) and exhibit opposite behaviour further away from the wall. Figure 19 reveals that the the magnitude of the velocity is lowered as $K_{pp}$ increases near the boundary wall ($\eta \leq 2$) and opposite trend is observed at free stream.

The effect of $Q_h$ on the magnitude of the velocity, temperature and concentration of the fluid is exhibited in Figures 9,
21 and 27 respectively. The increasing value of $Q_h$ enhances the velocity and temperature of the fluid. It is also noted that magnifying the value of $Q_h$ grows the concentration profile provided $E_{pr} > 0.5$ with opposite behaviour witnessed when $E_{pr} < 0.5$. The role of $M_p$ on the fluid temperature and concentration is displayed in Figures 8, 20 and 30 respectively. Figure 30 reveals that increasing the magnitude of $M_p$ enhances the fluid concentration. Figures 8 and 20 show that escalating values of $M_p$ magnifies the fluid temperature and diminishes the fluid velocity close to the wall and exhibits opposite behaviour at free stream.

Figures 2, 12, 23, 24, 25 and 26 demonstrate the behaviour of the fluid velocity, fluid temperature and fluid concentration when the inclination angles $\alpha_1$ and $\gamma$ are varied at specific $E_{pr}$ values. The fluid velocity is suppressed as $\alpha_1$ and $\gamma$ values are enlarged. Figure 2 reveals that the velocity diminishes with increasing values of $\alpha_1$ and $\gamma$. The velocity rises more if $\gamma > \alpha_1$ for any given values of the inclination angles. Temperature profile grows with rising $\gamma$ values and declines with increasing $\alpha_1$ values as demonstrated in Figure 12. Similar to the observation on velocity, the temperature profile is bigger whenever $\gamma > \alpha_1$ for any given $\gamma$ and $\alpha_1$ values. The temperature distribution is greatest when $\gamma$ value is maximal and $\alpha_1$ value is minimal, vice versa. Figures 25 and 26 demonstrates that the fluid concentration is magnified with escalating $\alpha_1$ values and $\gamma$ values provided $E_{pr} < 1$. However, opposite behaviour is shown when $E_{pr} \geq 1$. Furthermore, Figures 23 and 24 demonstrates that for any set of $\alpha_1$ and $\gamma$ values the fluid concentration profiles are more pronounced whenever $\alpha_1 > \gamma$.

The influence of the thermophoretic parameter $\tau_1$ on the velocity, temperature and concentration profile is depicted in Figures 10, 22 and 32 respectively. It is clear from Figure 10 that varying $\tau_1$ has no effect on the velocity of the fluid. However, enlarged $\tau_1$ values have the effect of retarding the temperature of the fluid as depicted in Figure 22. The concentration of the fluid is raised with escalating values of $\tau_1$ provided $E_{pr} < 1$ and opposite behaviour is witnessed when $E_{pr} > 1$.

The impact of the parameters $A_1$, $\alpha_1$, $\gamma$, $G_{MT}$, $E_{pr}$, $Q_h$, $\tau_1$, $M_p$, $S_c$, $E_c$, $\beta_1$, $K_{pp}$ and $K_m$ on the skin friction $-f_1''(0)$, heat transfer $-T'(0)$ and mass transfer rate $-C'(0)$ is portrayed in Tables II, III, IV and V. We infer that, the skin friction is enhanced with the enlargement of the parameters $\gamma$, $A_1$, $\alpha_1$, $K_{pp}$, $K_m$, $M_p$, $S_c$ and $E_{pr} < 1$. On the contrary, the skin friction declines with the escalation of the parameters $Q_h$, $\tau_1$, $E_c$, $\beta_1$ and $E_{pr} > 1$.

It is also deduced from the tables that the heat transfer rate is elevated with increase in the parameters $K_{pp} > 1$, $E_{pr}$, $\beta_1$ and $\tau_1$. On the other hand, the magnitude of the heat transfer rate declines with enlarged values of the parameters $A_1$, $\alpha_1$, $\gamma$, $K_{pp} < 1$, $M_p$, $S_c$, $E_c$ and $E_{pr} > 1$. Furthermore, the mass transfer rate rises with growing $E_{pr} > 1$, $K_{pp} < 1$, $Q_h$, $K_m$, $M_p$, $S_c$, $E_c$ and $A_1$. The mass transfer rate is suppressed by enlarging $\gamma$, $K_{pp} > 1$, $E_{pr}$, $\beta_1$ and $\tau_1$. It is also noted that the impact of $G_{MT}$ on the skin friction, heat transfer rate and mass transfer rate is negligible.

V. CONCLUSIONS

In this study, we have investigated the impact of the channel slope, aligned magnetic field, effective Prandtl number and thermophoresis on MHD Maxwell fluid. The effect of the
Figure 16: The influence of $E_{pr}$ on the temperature

Figure 17: The effect of $G_{MT}$ on temperature

Figure 18: The influence of $K_{in}$ on temperature

Figure 19: The impact of $K_{pp}$ on temperature

Figure 20: The effect of $M_p$ on temperature

Figure 21: The impact of $Q_h$ on temperature

Figure 22: The effect of $\tau_1$ on temperature

Figure 23: The impact of $\alpha_1$ and $\gamma$ on concentration for $E_{pr} = 0.5$

Various parameter on the velocity, temperature, concentra-
Figure 24: The impact of $\alpha_1$ and $\gamma$ on concentration for $E_{pr} = 1$

Figure 25: Effect of $\alpha_1$ on concentration for varying $E_{pr}$

Figure 26: Effect of $\gamma$ on concentration for varying $E_{pr}$

Figure 27: The impact of $Q_{MT}$ and $Q_h$ on concentration

Figure 28: The effect of $K_{m}$ on concentration

Figure 29: The effect of $K_{pp}$ and $\beta_1$ on concentration

Figure 30: The impact of $M_p$ on Concentration

Figure 31: The impact of $Q_h$ with varying $E_{pr}$ on Concentration
1. The magnitude of the fluid velocity is enhanced with rise in the value of $\beta_1$, $A_1$, $Q_b$, $M_p$ for $\eta > 3$, $K_m$ and $G_{MT}$ and $E_{pr}$ and declines with the enlargement of $K_m$, $M_p$ for $\eta < 3$, $K_{pp}$ for $\eta < 3.2$, $\gamma$, $\alpha_1$, and $E_{pr}$.

2. The magnitude of the fluid temperature is elevated with increment in the value of the parameters, $K_{pp}$ for $\eta < 5$, $M_p$ for $\eta < 4$, $\beta_1$ for $\eta > 3.3$, $K_m$, $A_1$, $Q_b$, $E_{pp}$, $\alpha_1$ for $E_{pp} > 0.5$, $\gamma$ for $E_{pp} < 1$ and $G_{MT}$ for $\eta > 5$ and is lowered with rise in the values of $\alpha_1$ for $E_{pp} < 0.5$, $\gamma$ for $E_{pp} < 1$, $K_{pp}$ for $\eta > 5$, $\beta_1$ for $\eta < 3.3$, $G_{MT}$ for $\eta < 5$ and $M_p$ for $\eta > 4$.

3. The fluid concentration rises with escalation of the parameters $K_{pp}$, $M_p$, $E_{pp}$, $Q_b$ for $E_{pp} < 0.5$, $\alpha_1$ for $E_{pp} < 1$, $\gamma$ for $E_{pp} < 1$ and $\tau_1$ for $E_{pp} < 5$ and diminishes with the growing values of $\beta_1$, $G_{MT}$, $K_m$, $Q_b$ for $E_{pp} < 0.5$, $\alpha_1$ for $E_{pp} < 1$, $\gamma$ for $E_{pp} > 1$ and $\tau_1$ for $E_{pp} > 0.5$.

4. The optimal value of the fluid velocity is attained when both $\alpha_1$ and $\gamma$ are maximal. On the other hand, the fluid temperature is greatest when $\gamma$ is maximum and $\alpha_1$ is minimal. The fluid concentration is highest when either both $\alpha_1$ and $\gamma$ are minimal provided $E_{pp} > 1$ or when both $\alpha_1$ and $\gamma$ are maximal provided $E_{pp} < 0.5$. For any combination of $\alpha_1$ and $\gamma$ values the velocity and temperature profiles are bigger whenever $\gamma > \alpha_1$. On the contrary, the concentration profile is bigger whenever $\alpha_1 > \gamma$.

5. Escalating the value of the parameters $\gamma$, $A_1$, $\alpha_1$, $K_{pp}$, $K_m$, $M_p$, $S_p$ and $E_{pp} < 1$ have the effect of enhancing the skin friction. On the other hand, the skin friction diminishes with increasing value of $Q_b$, $\tau_1$, $E_c$, $\beta_1$ and $E_{pr} > 1$.

6. The heat transfer rate is elevated with enlarged values...
of $K_{pp}>1$, $E_{pr}$, $\beta_1$ and $\tau_1$ and is reduced with rising value of $A_1$, $\alpha_1$, $\gamma$, $K_{pp}<1$, $M_p$, $S_t$, $E_c$ and $E_{pr} > 1$.

7. The mass transfer rate grows with increasing values of $E_{pr} >1$, $K_{pp}<1$, $Q_h$, $K_m$, $M_p$, $S_t$, $E_c$ and $A_1$ and is reduced with increasing value of $\gamma$, $K_{pp}>1$, $E_{pr}$, $\beta_1$ and $\tau_1$. There is considerable agreement between our present investigation and findings in previously published literature. Outstanding findings in this study are that for enlarged values of $Q_h$, the fluid concentration is enhanced provided $E_{pr} > 0.5$. The fluid concentration also rises with increasing values of $\gamma$ and $\alpha_1$ whenever $E_{pr} < 1$. On the other hand, the magnitude of the fluid temperature is greatest when $\gamma$ is maximal and $\alpha_1$ is minimal. The temperature profile is boosted when the values of $\alpha_1$ and $\gamma$ are raised for higher $E_{pr}$ values ($E_{pr} > 0.5$ for $\alpha_1$ and $E_{pr} > 1$ for $\gamma$) and the temperature deprecates for lesser $E_{pr}$ values. Furthermore, it is noticed that for any given set of $\gamma$ and $\alpha_1$ values the magnitude of the fluid velocity and the fluid temperature is greater whenever $\gamma > \alpha_1$. Also for any combination of the inclination angles $\alpha_1$ and $\gamma$ the fluid concentration is greater whenever $\alpha_1 > \gamma$. The effect of $\tau_1$ on the temperature and concentration profile is a function of the $E_{pr}$ value and it depends on whether the $E_{pr}$ value is greater or less than unity.

REFERENCES


