Design Optimization of FSAE Car Steering System

Funing Cai, Hailan Zhao, Binbin Sun*, Lin Sun

Abstract—**The steering system is one of the most important components of the FSAE racing car, and its design directly affects its operational performance. In this thesis, the goal was to provide the maximum lateral force when the racing car was cornering at high speed. To achieve this, it was necessary to study the slip deflection characteristics of the tire, determine the slip angle of the racing car when cornering at high speed, and guide the design of the Ackermann correction coefficient and the steering trapezoid. Use the "three-center theorem" to design the steering disconnection point to reduce bump steer. Optimized by Adams Car software, the actual relationship between inner and outer wheel rotation angles was close to the design value, and the absolute error of the Ackermann percentage was reduced from 8.92% to 4.54%. The steering design was also checked for motion interference to verify its rationality. The dynamic analysis results showed that a reasonable design could reduce the variation of toe angle and improve the high-speed cornering ability and stability of the car.**

Index Terms—**FSAE racing car, cornering characteristics, Ackermann percentage, steering trapezoid, three-center theorem**

I. Introduction

The China Student Formula Competition requires the participating team to design a car with excellent performance that can complete all or part of the competition within one year under the standards of the competition rules[1].

In the development and design process of the FSAE racing car, the steering system directly determines the car's cornering performance and affects its handling stability, which plays an important role in the whole car design process. The parameters of the FSAE racing steering system are designed to differ significantly from those of a family car. For example, the influence of tire slip angle cannot be ignored when turning; 100% Ackermann steering theory does not apply to FSAE racing design; and there is no power steering system when steering. In recent years, scholars at home and abroad have also made a series of progress in the research on the steering system of FSAE racing cars. Zeng Jinhua et al. optimized the design of the steering outer point of the racing car and designed the steering outer point through the steering disconnection point[2]; Song Xueqian et al. designed the steering trapezoidal parameters by analyzing the magnitude of the steering force[3]; Sun Juzeng et al. optimized the design of the steering geometry of the racing car by building an Adams virtual prototype model[4]; Pei Xinzhe et al. designed the steering geometry of the car by studying the slip deflection characteristics of the tires to reduce the overall time of the car driving on the Xiangyang track[5]; At the same time, the foreign TTC Tire Data Testing Organization sorts and tests relevant data from racing tires, guiding designers to design the steering and suspension system of the FSAE racing car. However, the results of the aforementioned studies were considered one-sided in the design of the steering system of racing cars and could not achieve the best steering performance. For example, Sun Jiuzeng, Song Xueqian, and others ignored the influence of slip angle when designing the steering trapezoid. However, when FSAE racing cars are cornering at high speed, the slip angle is relatively large. Neglecting the wheel slip angle will reduce the steering performance of the car and aggravate tire wear.

The objective of this paper is to refine the design of the FSAE racing steering system to ensure maximum lateral force during high-speed cornering. We utilize the TTC tire test data and apply the Magic Formula to ascertain the tire slip angle value. With this tire slip angle as a reference, we shape the Ackermann correction coefficient and steering trapezoidal parameters. This serves to satisfy the racing car's high-speed cornering needs and minimize tire wear. Bump steer is mitigated using the "three-center theorem," guiding the steering and suspension design. Additionally, the Adams Car software is employed to fine-tune hardpoints, ensuring the real-life inner and outer wheel angle relationships align with the design objectives. The final step involves confirming the appropriateness of the steering system design via an interference check analysis.

II. Determination of Ackermann Correction Coefficient K

A. Calculation of front wheel load redistribution during steering

Since FSAE racing cars often corner at high speeds, tires are required to provide sufficient lateral force for the vehicle.

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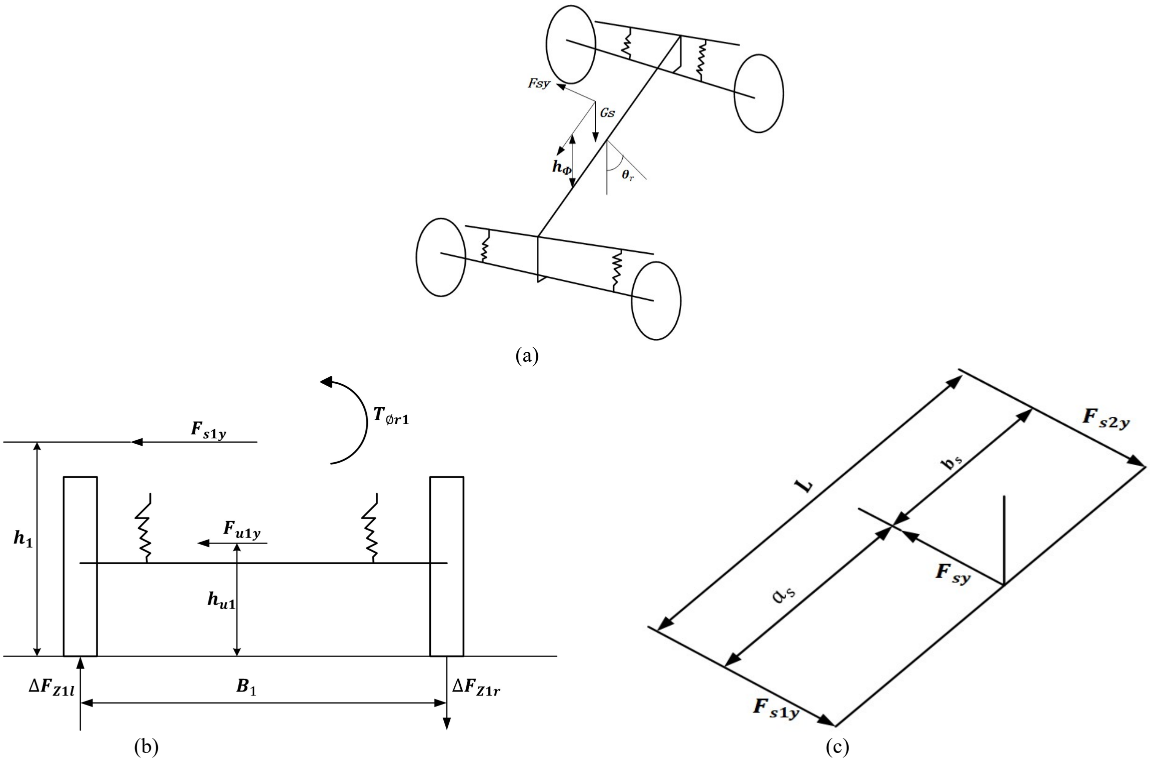


Fig. 1. Simplified model diagram of vertical load redistribution

And the Ackermann correction coefficient for traditional family car designs is no longer applicable to the design of steering system parameters for FSAE racing cars. And considering that when the car is cornering at high speed, the four-wheel vertical load is redistributed and the wheels have a large slip angle. Therefore, when designing the Ackermann correction coefficient *K* of the FSAE racing car, the influence of the load transfer amount and the slip angle on the relationship between the inner and outer wheels of the wheel are introduced.

When analyzing the calculation of front wheel load redistribution when the FSAE racing car turns, the racing car can be simplified to the model shown in Fig. 1 [6]. The I-shaped frame represents the body and is hinged on the roll axis.

From Fig. 1, the front wheel load redistribution of the FSAE car is [6]:

$$\theta_r = \frac{a_y m h}{K_f + K_r} \tag{1}$$

$$T_{\emptyset r1} = K_f \theta_r \tag{2}$$

$$\Delta F_{z1l} B_1 = F_{sy} \frac{b_s}{L} h_1 + T_{\emptyset r1} + F_{u1y} h_{u1} \tag{3}$$

$$\Delta F_{z1r} = -\Delta F_{z1l} \tag{4}$$

$$F'_{z1l} = F_{z1l} + \Delta F_{z1l} \tag{5}$$

$$F'_{z1r} = F_{z1r} + \Delta F_{z1r} \tag{6}$$

Where ΔF_{z1l} and ΔF_{z1r} are the variations of the vertical loads on the left and right wheels of the front axle of the racing car, N; F'_{z1l} and F'_{z1r} are the vertical forces on the ground on the left and right wheels of the front axle when the car is in a roll state, N; θ_r is the roll angle of the car, ° ; a_y is the lateral acceleration of the car, m/s^2; *m* is the sprung mass, kg; *h* is the height of the center of mass of the car, m; $\mathrm{K_f}$ and $\mathrm{K_r}$ are the roll angle stiffness of the front and rear suspension, N·m/°; $\mathrm{T}_{\emptyset r1}$ is the restoring moment acting on the front suspension, N·m; $\mathrm{B_1}$ is the front tread, m; F_{sy} is the centrifugal force acting on the racing car, N; $\mathrm{b_s}$ is the horizontal distance from the center of mass of the car to the rear axle, m; *L* is the wheelbase of the racing car, m; h_1 is the height of the front roll center, m; F_{u1y} is the centrifugal force generated by the un-sprung mass of the front axle, N; h_{u1} is the height of the center of mass of the un-sprung mass of the front axle from the ground, m.

Table I FSAE car related parameters

parameters	value
Vehicle quality (passenger) /kg	280
Front axle mass/kg	137.2
rear axle mass/kg	142.8
wheelbase/mm	1560
front tread/mm	1180
rear tread/mm	1160
Front suspension roll angle stiffness/N·m/°	441.7
Rear suspension roll angle stiffness/N·m/°	429.9
Front suspension roll center height/mm	31
Center of mass height/mm	350
Wheel radius/mm	228.6
Tire footprint/mm	222.1
Kingpin distance/mm	1050.1

According to the analysis of the endurance running data of the FSAE car, it was appropriate to take 1.4g as the steering system parameter design for the lateral acceleration of the car when turning. According to the parameters of the racing car in Table I, combined with formulas (1)–(6), it can be obtained that when the car turned to the left with the lateral acceleration of 1.4g, the roll angle was 1.35°, and the vertical load transfer was 657.1N. That is, the vertical load F'_{z1l} of the inner wheel was 28.9N, and the load F'_{z1r} of the outer wheel was 1343.1N.

B. *The value of slip angle*

FSAE racing car needs to provide enough lateral force to resist the centrifugal force it receives when cornering at high speed so that the tires of the car can achieve pure rolling when cornering at high speed. And for the tires of a racing car to provide sufficient lateral force, an appropriate slip angle needs to be determined.

The value of the slip angle can be studied by Pacejka89 tire model (Magic Formula). The Magic Formula is an empirical tire model based on test data. In the common linear range of lateral acceleration less than 0.4g, the fitting accuracy of tire performance is high, and it is also very reliable in the nonlinear range of greater than 0.4g. The Magic Formula is a good model for studying the handling stability of FSAE racing cars.

The Pacejka89 tire model can be described as:

$$Y(x) = y(x) + S_v \tag{7}$$

$$y(x) = D\sin\left(\mathrm{Carctan}\left(\mathrm{B_x} - \mathrm{E}(\mathrm{B_x} - \arctan(\mathrm{B_x}))\right)\right) \tag{8}$$

$$x = X + S_h \tag{9}$$

Where $Y(x)$ is the tire longitudinal force, lateral force or aligning torque; x is the independent variable when considering the horizontal offset factor; X is the longitudinal slip or slip angle; S_v is the vertical offset factor; S_h is the horizontal offset factor; B, C, D, E are all the Magic Formula factors.

The type of tire used in the FSAE racing car in this paper is 18.0×7.5-10 Hoosier. To accurately build the tire dynamics model of the Magic Formula and improve the credibility of the data, the tire test data of the TTC cornering working condition is processed accordingly and then imported into the Adams Car TDFT (tire data and fitting tools) module for Hoosier tire testing. Fit the data of the pure cornering condition, and reasonably modify the parameters to be fitted to make the fitting effect more in line with the actual situation. The operation interface is shown in Fig. 2, and Fig. 3 is the tire fitting curve. It can be seen that the tire test data and the fitting curve have a high degree of coincidence, the fitting effect is good, and it has very good credibility.

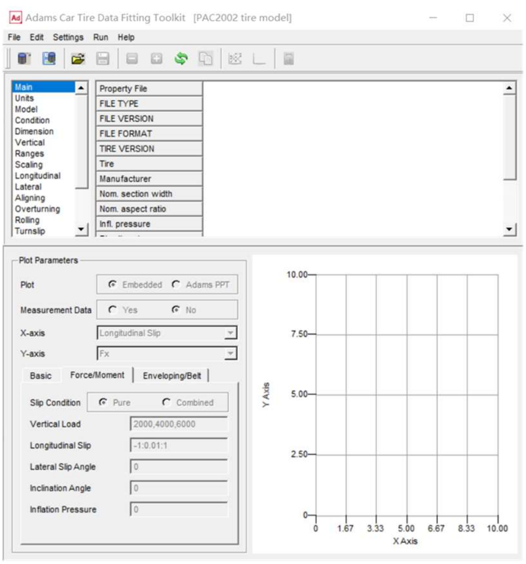


Fig. 2. Adams Car TDFT fitting tool

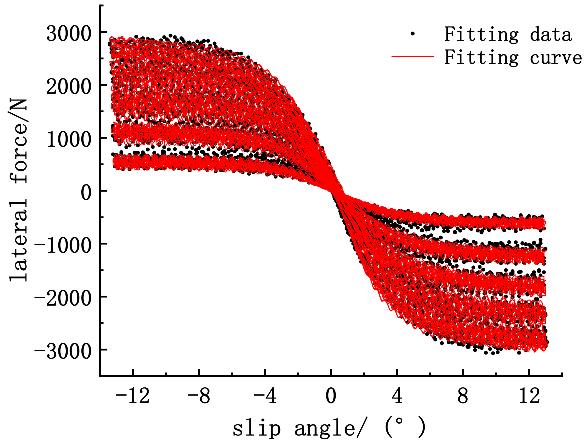


Fig. 3. Tire fitting

Fig. 4 is the tire data processing diagram fitted by the Magic Formula. It can be seen from Fig. 4 that the vertical load of the tire has a great influence on its lateral force. With the increase of the vertical load of the wheel, the peak value of the lateral force of the wheel is also increasing. Therefore, to explore the relationship between lateral force and slip angle, it is necessary to fix the tire's vertical load.

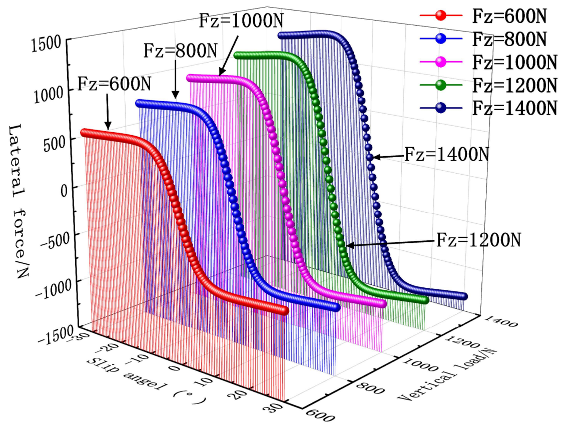


Fig. 4. Tire cornering characteristics

From the previous section, under the lateral acceleration of 1.4g in the steering condition of the racing car, the vertical load on the outer wheel is redistributed to 1343.1N. By fixing the vertical load of the outer wheel at 1343.1N, explore the relationship curve between the slip angle and the tire lateral force under different wheel camber angles, as shown in Fig. 5. It can be seen from the Fig. 5 that under the same wheel load, as the camber angle increases, the peak value of the lateral force decreases continuously. Considering that the car rolls when cornering, the outer wheel will increase the positive camber, which will affect the performance of the tire when cornering. Therefore, according to the actual endurance running data of the racing car and the calculation in the previous section, it is known that setting a negative camber of -1.35° to resist the increased positive camber can make the tire perpendicular to the ground when cornering and exert better performance of the tire. And it can be seen from Fig. 5 that when the camber angle of the wheel is -1.35° and the slip angle of the outer wheel is 15°, the tire can provide the maximum lateral force for the car when cornering. Similarly, under a lateral acceleration of 1.4g, the vertical load on the inner wheel is redistributed to 28.9N. The vertical load of the inner wheel is fixed at 28.9N, and the relationship curve between the slip angle and the tire lateral force under different wheel camber angles is explored. It can be seen from Fig. 6 that when the camber angle of the wheel is -1.35° and the slip angle of the inner wheel is 16°, the tire can provide the maximum lateral force for the car when cornering.

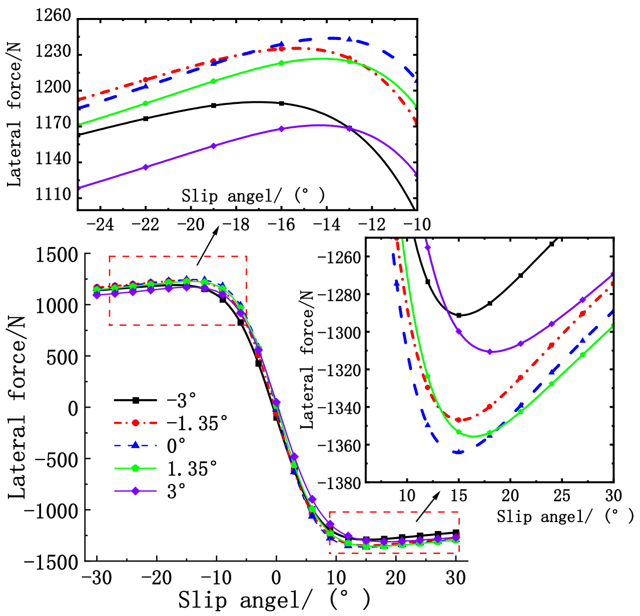


Fig. 5. Effect of wheel camber angle on tire lateral force under the vertical load of 1343.1N

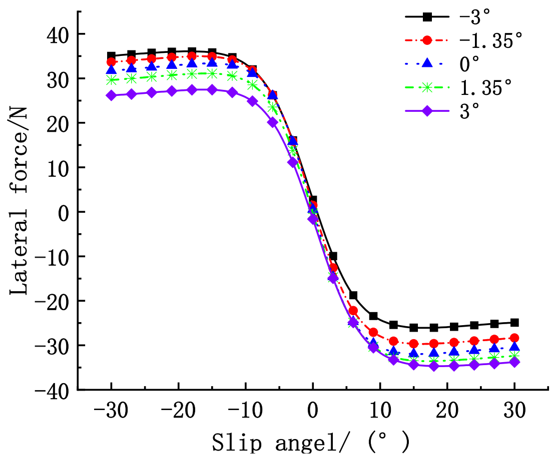


Fig. 6. Effect of wheel camber angle on tire lateral force under the vertical load of 28.9N

C. Relationship between inner and outer wheel rotation angle when slip angle is introduced

Since the FSAE racing car will have a large slip angle when cornering at high speed, this will make the steering center of the car offset by a distance t from the extension line of the rear axle of the car, as shown in Fig. 7 [7]. The relationship between the inner and outer wheel rotation angles is derived by considering the slip angle when the vehicle is turning left. Where O is the steering center; B_1 is the front tread; B_2 is the rear tread; L is the wheelbase; θ_{fsl} is the slip angle of the front left wheel, that is, the slip angle of the inner wheel of the front wheel; θ_{fsR} is the slip angle of the front right wheel, that is, the slip angle of the outer wheel of the front wheel; θ_{Rsl} is the slip angle of the inner wheel of the rear wheel; θ_{RsR} is the slip angle of the outer wheel of the rear wheel.

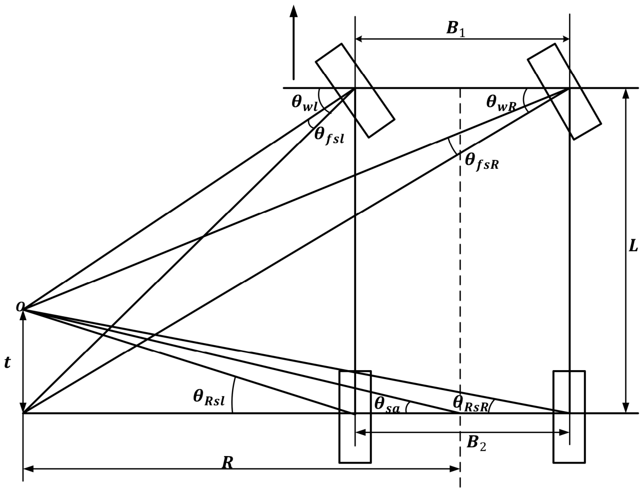


Fig. 7. Relationship between inner and outer wheel rotation angle with the introduction of slip angle

According to the relationship of trigonometric functions:

$$\cot(\theta_{wl}-\theta_{fsl}) = \frac{R-\frac{B_1}{2}}{L-t} \tag{10}$$

$$\cot(\theta_{wR}-\theta_{fsR}) = \frac{R+\frac{B_1}{2}}{L-t} \tag{11}$$

$$\cot\theta_{Rsl} = \frac{R-\frac{B_2}{2}}{t} \tag{12}$$

$$\cot\theta_{RsR} = \frac{R+\frac{B_2}{2}}{t} \tag{13}$$

In order to facilitate the derivation and analysis, the slip angles of the two rear wheels are set equal to the midpoint slip angle θ_{sa} of the rear axle, that is:

$$\cot\theta_{sa} = \frac{R}{t} \tag{14}$$

From (14) it follows that:

$$t = \frac{R}{\cot\theta_{sa}} \tag{15}$$

$$R = t\cot\theta_{sa} \tag{16}$$

Substituting (15) into (10) and (16) into (11) gives:

$$R\left[\frac{\cot(\theta_{wR}-\theta_{fsR})}{\cot\theta_{sa}}\right] = \frac{-B_1}{2} + L\cot(\theta_{wR}-\theta_{fsR}) \tag{17}$$

$$t\left[\cot(\theta_{wl}-\theta_{fsl}) + \cot\theta_{sa}\right] = \frac{B_1}{2} + L\cot(\theta_{wl}-\theta_{fsl}) \tag{18}$$

Let $C=\frac{B_1}{2L}$, from (17)–(18) we get:

$$\cot(\theta_{wl}-\theta_{fsl}) = \cot(\theta_{wR}-\theta_{fsR})\frac{\cot\theta_{sa}-C}{\cot\theta_{sa}+C} - 2C\frac{\cot\theta_{sa}}{\cot\theta_{sa}+C} \tag{19}$$

From the calculation of the tire fitting data in the section II. B, it can be seen that when the FSAE racing car is cornering at high speed, the outer wheel slip angle θ_{fsR}=15°, and the inner wheel slip angle θ_{fsl}=16°. Bring them into (19) to get:

$$\cot(\theta_{wl} - 16°) = \cot(\theta_{wR} - 15°)\frac{\cot\theta_{sa} - C}{\cot\theta_{sa} + C} -2C\frac{\cot\theta_{sa}}{\cot\theta_{sa} + C} \quad (20)$$

Equation (20) is the relationship between the inner and outer wheel rotation angle when considering the tire slip angle during high-speed cornering of the FSAE car in this thesis.

D. Target inner and outer wheel rotation angle relationship

When the FSAE racing car is turning, to ensure the pure rolling motion of its wheels, all the wheels need to move around the same center. Assuming the wheels are rigid, the relationship between the inner wheel rotation angle θ_n and the outer wheel rotation angle θ_w is as follows:

$$\theta_n = arctan\frac{Ltan\theta_w}{L-Btan\theta_w} \quad (21)$$

Where B is the distance between the extension line of the kingpin axis on both sides and the intersection point of the ground; L is the wheelbase of the racing car. Satisfying the above-mentioned rotation angle relationship is called standard Ackermann steering.

Due to the slip angle of the car when cornering at high speed, the relationship between the inner and outer wheel angles no longer conforms to the standard Ackermann steering relationship. At this point, the Ackermann correction coefficient K is introduced, and its definition is as follows:

$$K = \frac{\theta_1-\theta_w}{\theta_n-\theta_w} \times 100\% \quad (22)$$

Where θ_1 is the actual inner wheel rotation angle; θ_n is the inner wheel rotation angle conforming to the standard Ackermann steering; θ_w is the outer wheel rotation angle.

By combining equations (21) and (22), the essay obtains the rotation angle relationship that introduces the Ackermann correction coefficient:

$$\theta_1 = Karctan\frac{Ltan\theta_w}{L - Btan\theta_w} + (1 - K)\theta_w \quad (23)$$

In order to design the target inner and outer wheel angle relationship when the FSAE racing car corners at high speed, it is necessary to select an appropriate Ackermann correction coefficient K so that when the outer wheel angle of the car is constant, the inner wheel rotation angle θ_1 in formula (23) approaches infinitely close to the inner wheel rotation angle θ_{wl} in formula (20) considering the slip angle.

Through the actual running data of the racing car, the analysis of the wheel angle and lateral acceleration can be obtained: When the wheel angle is below 10°, the steering relationship is just established, and the lateral acceleration is the smallest; the wheel angle of 10°–20° belongs to the high-speed cornering stage of the racing car and its lateral acceleration is the largest; the wheel angle above 20° belongs to low-speed and large-angle cornering and its lateral acceleration is the next. Therefore, to make θ_1 and θ_{wl} as close as possible, a weighting factor $\omega(\theta)$ is introduced, and its optimization function is as follows:

$$f(x) = \sum_{\theta=0}^{27}\omega(\theta)\left|\frac{\theta_{wl}-\theta_1}{\theta_1}\right| \quad (24)$$

Take the weighting factor as:

$$\omega(\theta) = \begin{cases} 0.5, & 0° < \theta \leq 10° \\ 1.5, & 10° < \theta \leq 20° \\ 1, & 20° < \theta \leq 27° \end{cases} \quad (25)$$

$f(x)$ is the accumulated error between the actual internal wheel rotation angle and the theoretical internal wheel rotation angle when taking different Ackermann correction coefficients. In the optimization calculation, K is taken from 0 to 1, and the relationship between the Ackermann correction coefficient and the accumulated error is shown in Fig. 8. It can be seen from the Fig. 8 that when the Ackermann correction coefficient is 46%, the error is the smallest, which is 18.381; that is, when K=46%, θ_1 and θ_{wl} are closest. Substituting K=46% into Equation (23), the equation can obtain the target steering relationship with the Ackermann correction coefficient:

$$\theta_1 = 0.46arctan\frac{Ltan\theta_w}{L-Bta\ _w} + 0.54\theta_w \quad (26)$$

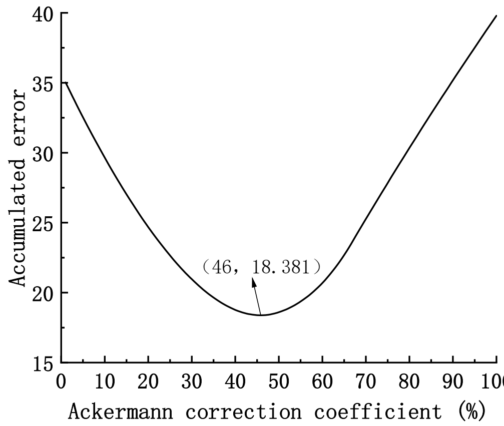


Fig. 8. Accumulation of Ackermann percentage error

III. STEERING GEOMETRY DESIGN

A. Steering trapezoidal parameter design

The design of the steering trapezoid parameters determines the relationship between the angles of the inner and outer wheels of the FSAE car during actual steering. In order to make the racing car play its best when turning, the design goal is to make the relationship between the left and right wheel angles determined by the steering trapezoid as close as possible to the target angle relationship corrected by the Ackermann correction coefficient.

In this paper, considering factors such as the layout space of the racing car and the matching design of the decoupled suspension, a front-disconnected steering trapezoidal design is adopted. When turning left, its schematic diagram is shown in Fig. 9, where A is its calculation operator; N is its kingpin distance; F is the distance from the disconnection point; and p is its distance from the rack axis to the front axle [7,8].

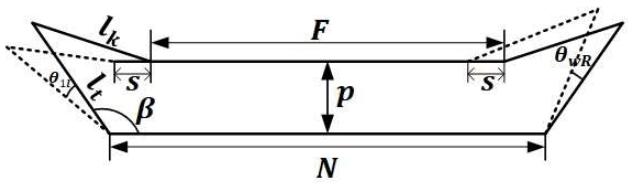


Fig. 9. Steering trapezium

According to the relationship of trigonometric functions:

$$A = \frac{N - F}{2} \tag{27}$$

$$s = l_t \cos(\beta - \theta_{wR}) + \sqrt{{l_k}^2 - [l_t \sin(\beta - \theta_{wR}) - p]^2} - A \tag{28}$$

$$\theta_{1l} = \arctan\frac{p}{A-s} + \arccos\frac{{l_t}^2 + p^2 + (A-s)^2 - {l_k}^2}{2l_t\sqrt{p^2 + (A-s)^2}} - \beta \tag{29}$$

Combine formulas (22) and (23) to get:

$$\begin{cases} D = l_t \cos(\beta - \theta_{wR}) \\ E = \sqrt{{l_k}^2 - [l_t \sin(\beta - \theta_{wR}) - p]^2} \\ \theta_{1l} = \arctan\dfrac{p}{2A - D - E} \\ \quad + \arccos\dfrac{{l_t}^2 + p^2 + [2A - D - E]^2 - {l_k}^2}{2l_t\sqrt{p^2 + [2A - D - E]^2}} \\ \quad -\beta \end{cases} \tag{30}$$

Where s is the rack travel; θ_{1l} is the actual inner wheel rotation angle; θ_{wR} is the outer wheel rotation angle; l_k is the length of the steering tie rod; l_t is the length of the trapezoidal arm; β is the bottom angle of the trapezoid.

Equation (30) is the actual relationship between the inner and outer wheel rotation angles, determined by the steering trapezoid. The design goal is to make the actual inner wheel rotation angle θ_{1l} close to the target inner wheel rotation angle θ_1 in formula (26) when the outer wheel rotation angle is constant. The results of the trapezoidal parameters obtained through analysis and calculation are shown in Table II.

Table II Parameters related to steering trapezoid

parameters	value
Disconnection point distance/mm	444.48
Rack travel/mm	29.88
Distance from rack axis to front axle/mm	29.98
Tie rod/mm	319.04
Trapezoidal arm/mm	64
Bottom angle of the trapezoid/°	103.55
Kingpin distance/mm	1049.29

B. *Matching design of steering linkage and suspension linkage*

Due to the problem of space layout, the lower A-arm of the front suspension cannot always be on the same level as the steering tie rod during the process of the wheel jumping up and down. This means the upper A-arm, lower A-arm, and steering tie rod of the front suspension cannot have a common instantaneous center of motion. It will cause bump steer when the front wheels jump up and down, which will affect the driving stability and handling stability of the car.

In the process of steering hardpoint design, the "three-center theorem" is used to determine the position of the steering disconnection point in the unequal-length double A-arm suspension to reduce bump steer. The basic idea of determining the disconnection point is: according to the geometric design of the front suspension, find the instantaneous center of motion of the movement track of the steering knuckle arm hinge point, and the position of the disconnection point coincides with this. As shown in Fig. 10, it is a schematic diagram using the " three-center theorem " to determine the disconnection point. In Fig. 10, A is the inner end point of the lower A-arm; B is the inner end point of the upper A-arm; C is the upper-end point of the kingpin; D is the lower-end point of the kingpin; and H is the hinge point of the trapezoidal arm and the tie rod. Extending CB and DA, the intersection point is E, and this point is the instantaneous center of suspension oscillation. Extend DC and AB to intersect at K and connect EK. Connect HE, record the included angle between HE and DA as α, construct JH and JE as shown in Fig. 10, and make the included angle between JE and EK also be α. Make a straight line through J and B and intersect HE at point F. Point F is the disconnection point.

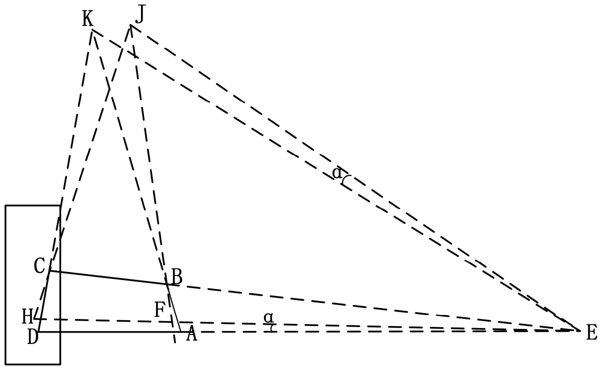


Fig. 10. Three-center theorem determines disconnection point

Ⅳ. Simulation Analysis of Steering and Suspension System

A. *Model building of FSAE racing car suspension and steering system*

According to the relevant hardpoint data of the steering and suspension systems, a virtual prototype model of the FSAE racing car was built in Adams Car software for the kinematics simulation analysis of the car. As shown in Fig. 11, the virtual prototype model used here is a combination of the unequal-length double A-arm decoupling suspension and steering system kinematics model. Before the dynamics simulation, the model parameters need to be modified to make the built virtual prototype model consistent with the actual racing car model so that the dynamics simulation was credible.

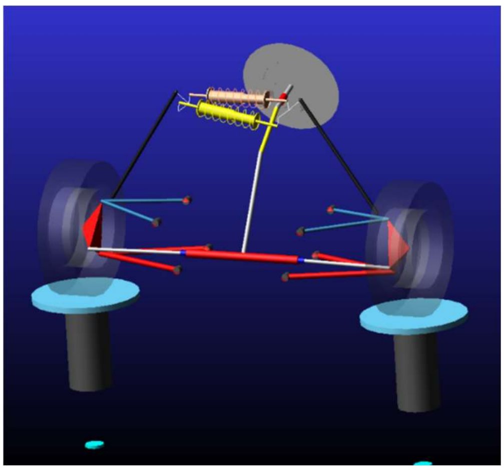
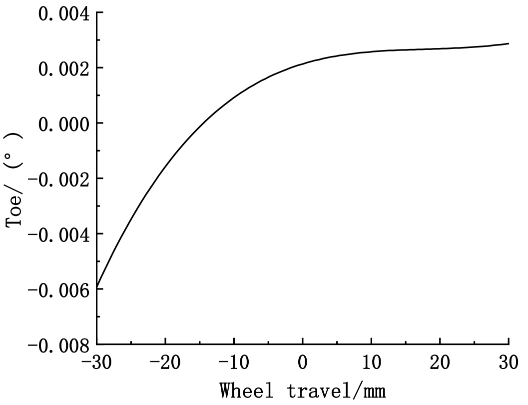
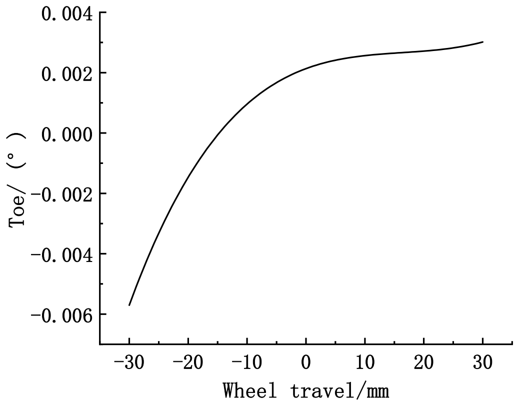


Fig. 11. Suspension and steering assembly virtual prototype



(a) Variation of toe angle in parallel wheel jump



(b) Variation of toe angle in opposite wheel jump

Fig. 13. Wheel jump analysis

B. Dynamics simulation analysis of steering and suspension system

Use the Adams Car software to analyze the steering conditions of the racing car and simulate the change in the Ackermann percentage when the racing car is turning. From the previous analysis, it can be seen that the wheel rotation angle in the region of 10°–20° belongs to the high-speed cornering stage of the racing car, and its lateral acceleration is the largest. That is to say, to enable the car to corner at high speed, the Ackermann percentage should coincide with the Ackermann correction coefficient in this wheel angle range. Fig. 12 shows the variation of the absolute value of the difference between the actual Ackermann percentage and the Ackermann correction coefficient with the wheel angle. Fig. 12 shows that the absolute value of the error increases as the wheel angle increases; the maximum error is 8.92%; the error is relatively large; and further optimization is required to reduce it.

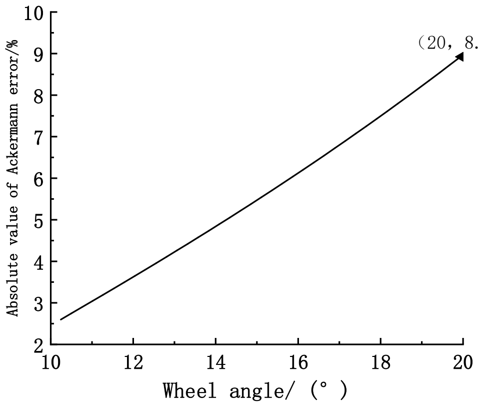


Fig. 12. Ackermann percentage error curve with wheel angle

Analyze the parallel wheel jump and opposite wheel jump with jump travel of ±30mm using Adams Car. As shown in Fig.13, the variation range of the toe angle was -0.00593° to -0.00287° with a variation of 0.0088° under parallel wheel jump, and the variation range of the toe angle was -0.0057° to -0.00302° with a variation of 0.00872° under opposite wheel jump. The change of the toe angle was small in both parallel and opposite wheel jump, meeting the design requirements of the racing car, and no further optimization was necessary. It can be concluded that the matching design problem of the steering system and the suspension system can be handled well by using the "three-center theorem".

Ⅴ. OPTIMAL DESIGN OF STEERING SYSTEM

From the dynamics simulation in the previous section, it was necessary to optimize the Ackermann percentage change curve during steering. When defining the design variables, the coordinates of the outer point of the steering tie rod were chosen considering that changing the steering disconnection point determined by the "three-center theorem" would make the toe angle larger and affect the driving and handling stability of the car. Considering the layout space of the FSAE racing car and the interference with the suspension system, the coordinate values of the design variables cannot change too much, and the constraint range of the variables was controlled between -2mm and 2mm. The optimization goal was:

$$f(x) = \min\left(\sum_{\theta=10}^{20} |k(\theta) - K|\right) \tag{31}$$

Where $k(\theta)$ was the Ackermann percentage during steering; K was the Ackermann correction coefficient.

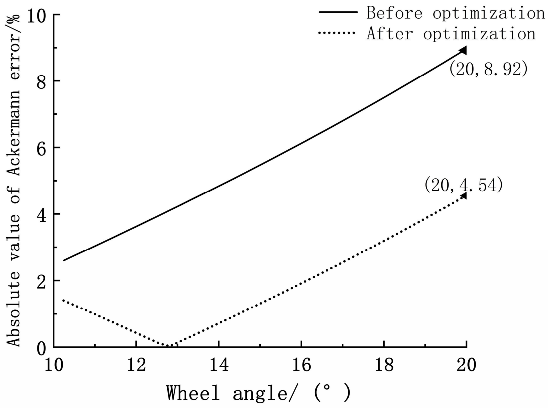
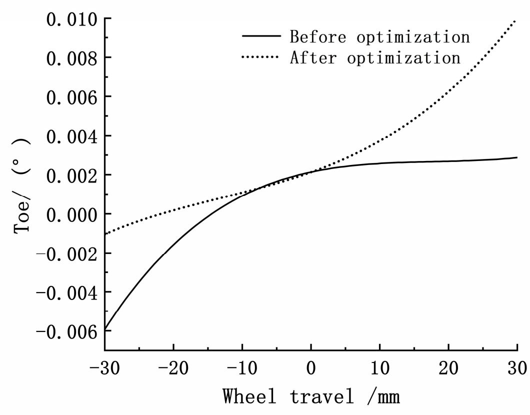
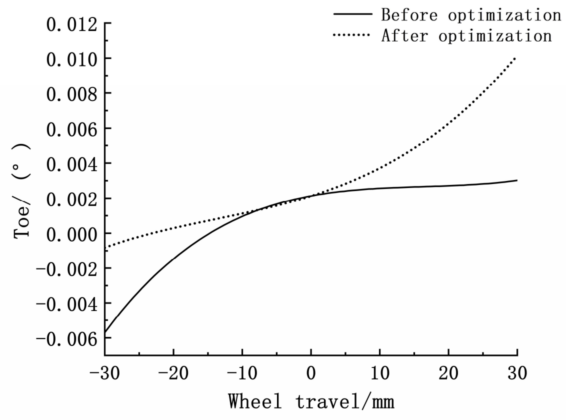


Fig. 14. Variation curves of Ackermann percentage error with wheel rotation angle before and after optimization

Modified the hardpoint coordinates several times in Adams Car and then analyzed the steering conditions to make the value of $f(x)$ within the design requirements. The hardpoint coordinates before and after optimization were shown in Table III. As seen in Table III, the factor that has



(a) Variation of toe in parallel wheel jump before and after optimization



(b) Variation of toe in opposite wheel jump before and after optimization

Fig. 15. Comparison of wheel jump analysis before and after optimization

the greatest effect on the Ackermann percentage during steering is the y-coordinate of the outer point of the tie rod.

Table III Hardpoints before and after steering outer point optimization

Design variables	Before/mm	After/mm
X coordinate of outer point	-62.217	-62.217
Y coordinate of outer point	-540.924	-538.924
Z coordinate of outer point	-52.545	-52.545

The absolute value curve of the Ackermann percentage error before and after optimization was shown in Fig.14. After optimization, the absolute value of the Ackermann percentage error was significantly reduced, and the maximum error was reduced from 8.92% to 4.54%. Therefore, the optimization was effective.

After optimizing the steering hardpoints, the suspension characteristics may change, especially the toe angle. Analyze the parallel wheel jump and opposite wheel jump with jump travel of ±30mm using Adams Car. As shown in Fig.15, the range of toe angle variation changed after hardpoint optimization. The variation of parallel wheel jump increased from 0.0088° to 0.01104° and the variation of opposite wheel jump increased from 0.00872° to 0.011022°, but they remained within the reasonable design requirements.

VI. Interference Analysis

Through the analysis of the above paper, the steering hardpoints were initially determined. By building a simple suspension and steering assembly model in Catia, a suitable kinematic pair was added to make the rim rotate around the kingpin, as shown in Fig.16.

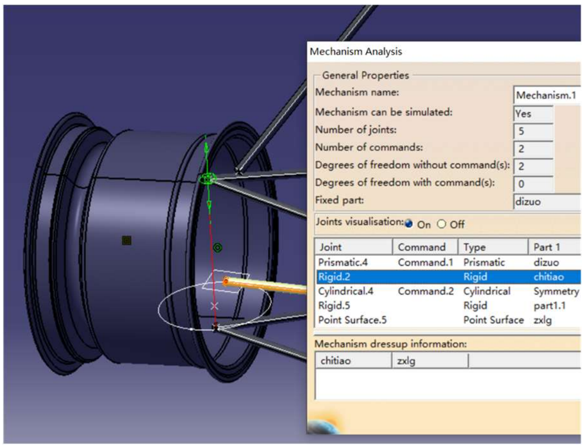


Fig.16. Suspension and steering assembly model

Open the collision analysis, let the rim rotate around the kingpin to the maximum design angle, and perform Dmu interference check. As shown in Fig.17, when the rim rotated around the kingpin to the maximum angle, the minimum distance between the wheel and the suspension rod system was 11.385mm, and there was no interference with the rim. That is, the steering angle and hardpoint design met the requirements.

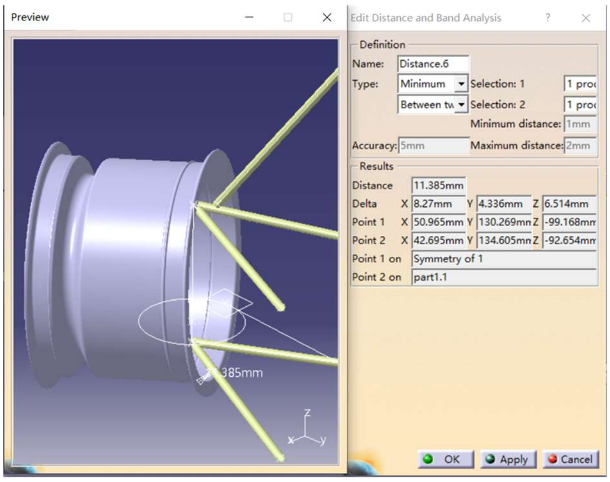


Fig. 17. Dmu interference analysis

VII. Conclusion

This research offers a comprehensive design approach for the FSAE racing steering system. During the design phase, we examined the slip deflection attributes of tires, ensuring they provide ample lateral force for the racing car in high-speed cornering scenarios. The Ackermann correction coefficient and steering trapezoid parameters were revamped, considering factors like load redistribution and slip angle. Using the "three-center theorem," we identified the disconnection point, optimized the Ackermann percentage through a kinetic analysis with Adams Car, and employed Catia Dmu for interference analysis. This streamlined the car steering system's performance, leading to the subsequent conclusions:

(1) An examination of tire slip deflection characteristics revealed correlations among tire cornering force, slip angle, vertical load, and camber angle. The findings indicated that at 1.4g of lateral acceleration and a wheel camber of -1.35°, the wheel delivers an adequate lateral force for high-speed

cornering: the outer wheel's slip angle being 15° and the inner wheel's at 16°.

(2) The Ackermann correction coefficient underwent a redesign to incorporate the slip angle's impact. A weighted factor analysis determined that the actual inner wheel rotation angle θ_1 aligns closely with the inner wheel rotation θ_{wl} considering the slip angle when the Ackermann correction coefficient K is set at 46%. Through optimized design in Adams Car, the maximum absolute discrepancy in the Ackermann percentage during actual operation decreased from 8.92% to 4.54%. Such adjustments ensure the FSAE car's steering geometry offers the ideal conditions for tires to excel during high-speed cornering.

(3) Leveraging the "three-center theorem," the design of the steering rod system was harmonized with the suspension rod system. This ensures toe angle variations remain within a specific limit during the FSAE car's operation. Validation confirmed that during actual movement, the FSAE car experienced a mere 0.01104° variation in parallel wheel jump in toe angle and 0.011022° in the opposite wheel jump. This minimized bump steer, enhancing the FSAE car's stability throughout its drive.

REFERENCES

[1] China University Student Formula Car Competition Rules Committee. Chinese University Formula Car Competition Rules[S]. Beijing: China Automotive Engineering Society, 2022.

[2] Zeng Jinhua, Jin Jiachen, “Optimization of turning point design based on FSCC,” *Proceedings of the 2020 China SAE Annual Conference*, vol.8, pp. 5, 2020.

[3] Song Xueqian, Ding Huafeng, Jing Wenqian, and et al, “Design and optimization of FSAE car steering system,” *Journal of Chongqing University of Technology (Natural Science)*, vol.33, no.2, pp. 38-44, 2019.

[4] Sun Jiuzeng, Xiang Huiyu, Zhang Yong, “Simulation of optimization and design of FASE racing steering system,” *Computer Simulation*, vol.39, no.2, pp. 126-130+144, 2022.

[5] Pei Xinzhe, Yao Quan, Jian Haohong, and et al, “Study on the steering geometry design method of FSAE racing car based on the tyre cornering characteristics,” *Proceedings of the 2020 China SAE Annual Conference*, vol.8, pp. 102-107, 2020.

[6] Yu Zhisheng, *Automobile theory* (Book style). Beijing: China Machine Press, 2018, pp. 188-196.

[7] Zhao Cong, “FASE racing car steering system design and performance analysis,” M.S. thesis, Hefei University of Technology, Hefei, China, 2015.

[8] Hou Jie, “The optimization design and analysis of suspension and steering for Formula SAE racing car,” M.S. thesis, Hefei University of Technology, Hefei, China, 2017.