Adaptive Output-Feedback Prescribed Performance Control of Uncertain Switched Nonlinear Systems

Xinyu Ouyang, Feng Zhang, Nannan Zhao

***Abstract*—Adaptive Output-feedback prescribed performance control (PPC) scheme is investigated for a class of uncertain switched nonlinear systems. Compared with the existing methods, this paper designs a new type of error transformation function. Its advantage is that based solely on the properties of the function, the output error can be constrained within the preset band without any additional conditions. Neural networks are used to approach unknown nonlinearities in switched systems, and state observers are used to estimate unknown states. The stability analysis shows that the proposed method ensures that all closed-loop signals are bounded and the tracking error can converge to the adjustable constraint function under the switching condition of average dwell time. Finally, the effectiveness of the proposed algorithm is verified by simulation experiments.**

***Index Terms*—Adaptive neural control, average dwell time, prescribed performance control (PPC), switched nonlinear system, Output-Feedback**

I. Introduction

In recent years, the study of nonlinear systems has received widespread attention due to many practical engineering situations, and some excellent control algorithms, such as backstepping technology [1]–[6], neural network technology [7]–[11] and fuzzy technology [12]–[18], are widely used. But the practical controlled system often requires the proposed control scheme, which can not only make the system stable, but also consider the transient performance of the system. So, the PPC was addressed for the first time to solve this problem in [19], [20]. Due to the unknown uncertainties and external disturbances in the system, PPC issues are very challenging and difficult to achieve. To solve it, the traditional prescribed performance function was applied to the tracking control problem of nonlinear systems in [21]–[24]. On this basis, Wang et al. [25] constructed an improved predetermined performance function to avoid high-frequency chattering in the control input. More recently, Liu et al. [26] introduced a new constraint variable for system transformation, and studied the constrained control problem of strict feedback nonlinear systems. Different from the idea of the literatures [21]–[26], for a class of uncertain nonlinear systems with unknown control direction, another new error transformation function and a new update law were introduced into the controller, which can make the control structure simpler than the existing control technology, so that a fault-tolerant control scheme to guarantee the given tracking performance was proposed in the work [27]. Furthermore, for nonlinear systems with unknown control direction, Zhang et al. [28] proposed a low complexity PPC scheme without using the traditional Nussbaum gain techniques and any approximation technique. And Zhao et al. [29] designed and implemented event triggered control (ETC) and PPC simultaneously for a class of uncertain nonlinear systems with unknown control directions.

Meanwhile, from the point of view of engineering practice, system state variables are usually not observable. As a solution to this problem, observer-based output-feedback control scheme has be applied and many novel researches have been carried out. For example, by using backstepping technique, the authors in the result [30] addressed the problem of output-feedback adaptive fuzzy tracking control for a class of uncertain strict-feedback nonlinear systems, in which the input driven filter is designed to estimate the unknown state. By introducing fuzzy state observer, an adaptive fuzzy tracking controller for a class of single-input and single-output (SISO) uncertain nonstrict feedback nonlinear systems was proposed in [31]. For the nonlinear system with unknown control direction, Zhang and Yang [32] used a fuzzy adaptive state observer to estimate the unmeasured state, and proposed a low complexity adaptive fuzzy output feedback control scheme.

In addition, the actual nonlinear system model sometimes needs to be modeled by a hybrid system composed of multiple subsystems. Only one of these subsystems is active at a certain instant, and which subsystem is selected is determined by the switching rules. As an example of switched system, the control input of mass-spring-damper system needs to be switched between two specific candidate controllers. However, these mechanical systems switching rule are usually studied on the assumption that the system has unknown nonlinearity, because it is difficult to satisfy the assumption that the precise knowledge about the nonlinearity of the system is known. Therefore, it is meaningful to study the adaptive control of uncertain switching systems. Many robust adaptive control schemes have been successfully applied to switched systems [33]–[36]. Specifically, an adaptive output feedback neural tracking controller was designed for a class of strictly feedback nonlinear switched systems in [34], in which the proposed controller guaranteed the boundedness of all closed-loop signals under the switching condition of average dwell time. Long [35] proposed small-gain theorems based on multiple Lyapunov functions (MLFs) for switched nonlinear systems, which extended the small-gain technique from the original non-switched nonlinearity to the switched nonlinearity. A neural adaptive tracking control method is proposed for nonlinear nonlower-triangular switched systems

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Xinyu Ouyang is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 13392862@qq.com).

Feng Zhang is a doctoral student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO114051, China. (corresponding author, e-mail: 15941237106@163.com)

Nannan Zhao is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (corresponding author, e-mail: 723306003@qq.com)

[37], in which a backstepping-like recursive design process was established by using MLFs, and the radial basis function neural network was used to avoid the limitation of monotone increasing bounded function of non lower triangular system function.

Inspired by the aforementioned literatures, this paper intends to solve the PPC problem of SISO nonlinear switched systems. On the one hand, different from the conventional PPC introduced in [21]–[25], a more direct error transformation function (ETF) is given. Although the authors in [26] also provided an ETF, only by selecting design parameters reasonably can output error constraints be achieved. In addition, the proposed ETF is the summary of the algorithms in the literature [27]–[29], and the effectiveness of the proposed PPC scheme is proved in the subsequent stability analysis. From this point of view, this paper is an extension of the work of the literature [29]. On the other hand, the considered system is the form of a strict-feedback switched system with parameter uncertainties, unmeasurable states and unknown disturbances. The stability analysis of the proposed PPC scheme on the considered system needs to be reconstructed. Therefore, an adaptive output-feedback prescribed performance controller is proposed based on the above discussion.

II. System descriptions and basic knowledge

A. System descriptions

Consider a class of SISO uncertain switched nonlinear systems as follows

$$\begin{aligned} \dot{x}_i &= f_{i\eta(t)}(\bar{x}_i) + x_{i+1} + \lambda_{i\eta(t)}(t) \\ i &= 1, 2, \cdots, n-1 \\ \dot{x}_n &= f_{n\eta(t)}(\bar{x}_n) + u_{\eta(t)} + \lambda_{n\eta(t)}(t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$ $(i = 1, 2, \cdots, n)$ are the system state vectors, and $y \in \mathbb{R}$ is the system output. The function $\eta(t) : \mathbb{R}^+ \to M = \{1, 2, \cdots m\}$, which is assumed to be a piecewise right continuous function of time, represents a switching signal, where m is the number of subsystems. When $t \in [t_k, t_{k+1})$, $\eta(t) = j_k$, $(j_k \in M,\ k \in N)$. That is, the j_kth subsystem is active. Accordingly, for each subsystem j, $j \in M$, $f_{ij}(\cdot) : \mathbb{R}^i \to \mathbb{R}$ $(i = 1, 2, \cdots, n)$ are unknown smooth functions, $\lambda_{ij}(t)$ are unknown external disturbances satisfying $|\lambda_{ij}(t)| \leq \bar{\lambda}_{ij}^*$ with $\bar{\lambda}_{ij}^*$ being positive constants, and u_j is the control input of the jth subsystem. It is assumed that only output signal $y(t)$ can be measured in system (1), while other state variables are continuous and not measurable.

In order to simplify writing, the symbols in some functions have to be omitted. For example, denote $\eta(t)$, $f_{ij}(\bar{x}_i)$, $\lambda_{ij}(t)$ and $u_j(t)$ as η, f_{ij}, λ_{ij} and u_j, respectively. Next, let's introduce the definition of average dwell time, which has recently played a key role in the switched system research.

Definition 1: [34] If there exist positive constants C_0 and τ_0 such that

$$C_{\eta(t)}(T, t) \leq C_0 + \frac{T - t}{\tau_0},\ \forall\, T \geq t \geq 0, \tag{2}$$

where $C_{\eta(t)}(T, t)$ is the number of times the system has been switched in the time interval $[t, T)$. Then, the positive number τ_0 is called the average dwell time of the switching signal $\eta(t)$.

Remark 1: That is, for an initial time $t_0 := 0$ and an arbitrary time $T > 0$, $t_1, \cdots, t_{C_\eta(T,0)}$ are represented as the switching moment on the time interval $[0, T)$. In addition, assume that the state of system (1) will not jump instantaneously when the subsystem is switched, that is, the solution of system (1) is continuous everywhere. Also assume that j_k is not equal to j_{k+1} for all $k \in \{0, 1, \cdots, C_\eta(T, 0)\}$. It is worth mentioning that the above two assumptions are standard in the switched system literatures [33]–[35].

B. Control objectives and preliminaries

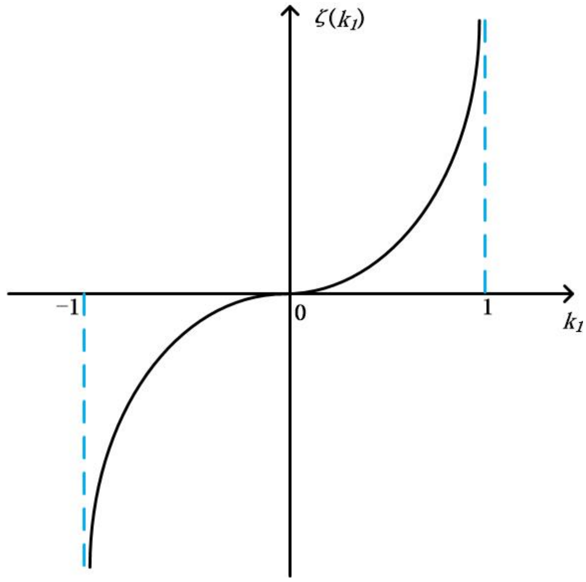

Fig. 1. Illustration of attribute of $\xi_1(\kappa)$.

The output error z_1 and the state errors z_i, $i = 2, \cdots, n$ are defined as

$$\begin{aligned} z_1 &= x_1 - y_r \\ z_i &= \hat{x}_i - \alpha_{i-1,j},\ i = 2, \cdots, n \end{aligned} \tag{3}$$

where $\alpha_{i-1,j}$, $j \in M$, are virtual control laws, and $y_r(t)$ is a desired trajectory.

In order to constrain the output error z_1, a boundary function $\varphi_1(t)$ is introduced as

$$\varphi_1(t) = (\varphi_{1_0} - \varphi_{1_\infty})e^{-\epsilon_1 t} + \varphi_{1_\infty}, \tag{4}$$

where φ_{1_0}, φ_{1_∞} and ϵ_1 are positive constants, $\varphi_{1_0} > \varphi_{1_\infty}$, φ_{1_0} is the initial value, φ_{1_∞} is the upper bound of steady-state error, and ϵ_1 denotes the convergence speed of exponential function.

Traditionally, for PPC scheme, the following equivalent unconstrained behaviors are usually acquired from the the constrained behavior $|z_1(t)| < \varphi_1(t)$ [21]–[25]:

$$z_1(t) = \varphi_1(t) K(\xi_1(t)) \tag{5}$$

where $K(\xi_1(t)) = \frac{e^{\xi_1} - e^{-\xi_1}}{e^{\xi_1} + e^{-\xi_1}}$. And the error transformation ξ_1 and its derivation are

$$\xi_1(t) = K^{-1}\left(\frac{z_1(t)}{\varphi_1(t)}\right) = \frac{1}{2}\ln\left(\frac{K+1}{1-K}\right) \tag{6}$$

and

$$\dot{\xi}_1(t) = \Lambda_1 \left(\dot{z}_1 - \frac{z_1 \dot{\varphi}_1}{\varphi_1} dt \right) \tag{7}$$

with $\Lambda_1 = \frac{1}{2\varphi_1}(\frac{1}{K+1} - \frac{1}{K-1})$ respectively.

Here, we introduces an auxiliary smooth function $\xi_1(\kappa)$ which can help to realize PPC and satisfies

$$\lim_{\kappa \to -1} \xi_1(\kappa) = -\infty, \quad \lim_{\kappa \to 1} \xi_1(\kappa) = \infty.$$

The tend of $\xi_1(\kappa)$ is shown in Fig. 1. For instance, $\xi_1(\kappa) = 2\tanh^{-1}(\kappa) = \ln(\frac{1+\kappa}{1-\kappa})$ or $\xi_1(\kappa) = \tan(\frac{\pi}{2}\kappa)$ can be a candidate for $\xi_1(\kappa)$. Based on $\xi_1(\kappa)$, a new error transformation is designed as:

$$\xi_1(t) = \xi_1(\kappa) = \xi_1(\frac{z_1(t)}{\varphi_1(t)}) \tag{8}$$

And $\dot{\xi}_1(t)$ is

$$\dot{\xi}_1(t) = 2\Gamma_1(\dot{z}_1 - \frac{\dot{\varphi}_1 z_1}{\varphi_1}) \tag{9}$$

where $\Gamma_1 = \frac{1}{\varphi_1}\frac{\partial \xi_1(\kappa)}{\partial \kappa}$, and $\dot{\xi}_1(t)$ is here ready for later use.

Remark 2: It can be seen from Fig. 1 that if $\xi_1(\kappa)$ is bounded, then $|\kappa| < 1$ holds. Naturally, let $\kappa = \frac{z_1(t)}{\varphi_1(t)}$. So, for $|z_1(0)| < |\varphi_1(0)|$, the overshoot of $z_1(t)$ $(t > 0)$ can be limited to less than $\varphi_1(t)$. The next step is to build the appropriate objective function $V_n(\xi_1^2, \cdot)$.

The aim of the work is to design a switched adaptive neural network output-feedback controller of the subsystems, and ensure that all closed-loop signals are uniformly bounded and the system output error does not violate the prescribed constraint function. To this end, the following assumption and lemma are introduced.

Assumption 1: The high-order time derivatives $y_r^{(i)}(t)$, $i = 0, 1, 2, \cdots, n$, of the reference signal $y_r(t)$ are continuous and bounded.

Lemma 1: [7]–[11] RBF neural network (RBFNN) $\theta^T S(\Upsilon)$ can approximate a smooth continuous function $G(\Upsilon)$ defined on a compact set Ω_Υ such that

$$G(\Upsilon) = \theta^T S(\Upsilon) + \varepsilon(\Upsilon), \ |\varepsilon(\Upsilon)| \le \bar{\varepsilon}^* \tag{10}$$

where $\varepsilon(\Upsilon)$ is the approximation error with $\bar{\varepsilon}^* > 0$ being an unknown constant, $\Upsilon \subset \mathbb{R}^n$ is the input vector, $S(\Upsilon) = [S_1(\Upsilon), S_2(\Upsilon), \cdots, S_l(\Upsilon)]^T$ are the basic function vector, $\theta = [\theta_1, \theta_2, \ldots, \theta_l]^T$ are the constant weight vector, and l is the number of neuron nodes. The following Gauss function is usually chosen as the basis function:

$$S_i(\Upsilon) = \exp\left[\frac{-(\Upsilon - r_i)^T(\Upsilon - r_i)}{\omega^2}\right], \quad i = 1, 2, \cdots, l \tag{11}$$

where $r_i = [r_{i1}, r_{i2}, \cdots, r_{in}]^T$ is the center vector, and ω is Gaussian function's width.

By Lemma 1, the optimal weight θ^* can be obtained by

$$\theta^* := \arg\min_{\theta \in \mathbb{R}^l} \left\{ \sup_{\Upsilon \in \Omega_\Upsilon} |G(\Upsilon) - \theta^T S(\Upsilon)| \right\}$$

Remark 3: In fact, the bounded properties of $y_r^{(i)}(t)$ $(i = 0, 1, \cdots, n)$ described in Assumption 1 is a common assumption in nonlinear tracking control [3]–[5], [7], [10]–[13], [16], [17].

III. Main Results

In this part, for uncertain switched nonlinear systems with disturbances, an adaptive output-feedback tracking control method with prescribed performance is proposed by using backstepping technology, in which a suitable switched input-driven filter is designed to estimate the unmeasured state.

A. Design of switched input-driven filter

Since the states of the system (1) are not available, the following switched input-driven filter is established to estimate the system states.

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} - d_{ij}\hat{x}_1, \\ \dot{\hat{x}}_n &= u_j - d_{nj}\hat{x}_1, \\ i &= 1, 2, \cdots, n-1, \ j \in M \end{aligned} \tag{12}$$

where $\hat{x}_i$ is the estimated value of each state x_i, d_{ij} are the designed parameters of the filter, and u_j is the actual input of the jth subsystem. Define $e_i = x_i - \hat{x}_i$, from (1) and (12), one has

$$\begin{aligned} \dot{e}_i &= e_{i+1} - d_{ij}e_1 + f_{ij}(\bar{x}_i) + d_{ij}x_1 + \lambda_{ij} \\ \dot{e}_n &= -d_{nj}e_1 + f_{nj}(\bar{x}_n) + d_{nj}x_1 + \lambda_{nj} \\ i &= 1, 2, \cdots, n-1, \ j \in M \end{aligned} \tag{13}$$

In order to get a compact expression, denote $e = [e_1, e_2, \cdots, e_n]^T$, $G_{0j}(\bar{x}_n) = [f_{1j}(\bar{x}_1) + d_{1j}x_1, \cdots, f_{n-1,j}(\bar{x}_{n-1}) + d_{n-1,j}x_1, f_{nj}(\bar{x}_n) + d_{nj}x_1]^T$, $\Lambda_j(t) = [\lambda_{1j}, \cdots, \lambda_{n-1,j}, \lambda_{nj}]^T$ and

$$C_j = \begin{bmatrix} -d_{1j} & \\ \vdots & I_{n-1} \\ -d_{nj} & 0 \cdots 0 \end{bmatrix}, \ j \in M.$$

It's easy to see that $||\Lambda_j||^2 \le ||[\bar{\lambda}_{1j}^*, \cdots, \bar{\lambda}_{n-1j}^*, \bar{\lambda}_{nj}^*]^T||^2 \le \bar{\Lambda}_j^{*2}$, where $\bar{\Lambda}_j^*$, $j \in M$, are unknown positive constants. Then, it follows from (1), (12) and (13) that

$$\begin{aligned} \dot{e} &= C_j e + G_{0j}(\bar{x}_n) + \Lambda_j \\ \dot{x}_1 &= \hat{x}_2 + e_2 + f_{1j} + \lambda_{1j} \\ \dot{\hat{x}}_2 &= \hat{x}_3 - d_{2j}\hat{x}_1 \\ &\cdots \\ \dot{\hat{x}}_n &= u_j - d_{nj}\hat{x}_1 \end{aligned} \tag{14}$$

It is also noted that d_{ij} is chosen such that the matrix C_j is Hurwitz, which implies that for any positive symmetric matrix Q_j, there is a positive symmetric matrix P_j such that $C_j^T P_j + P_j C_j = -Q_j, \ j \in M$.

B. Controller Design

First, an unknown constant is defined as

$$\theta = \max_{1 \le i \le n, \ 1 \le j \le m} \{||\theta_{ij}||^2\} \tag{15}$$

where θ_{ij} will be specified later. $\hat{\theta}$ is the estimation of θ, there has

$$\begin{aligned} \dot{\hat{\theta}}(t) =& \frac{\gamma}{2a_{1j}^2}\xi_1^2 S_{1j}(\Upsilon_1)^T S_{1j}(\Upsilon_1) \\ &+ \sum_{i=2}^{n} \frac{\gamma}{2a_{ij}^2} z_i^2 S_{ij}(\Upsilon_i)^T S_{ij}(\Upsilon_i) - \sigma\hat{\theta} \end{aligned} \tag{16}$$

with $a_{ij} > 0$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, $\gamma > 0$ and $\sigma > 0$ being design parameters, where $S_{ij}(\Upsilon_i)$ will be defined later. $\tilde{\theta} = \theta - \hat{\theta}$ is the estimated error.

Step 1: A positive definite Lyapunov function is constructed as:

$$V_{1j} = \frac{1}{\gamma} e^T P_j e + \frac{1}{4}\xi_1^2 + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{17}$$

By (9) and $\dot{z}_1 = f_{1j} + e_2 + \hat{x}_2 + \lambda_{1j} - \dot{y}_r$, one has

$$\begin{aligned}\dot{V}_{1j} =& \frac{2}{\gamma} e^T P_j (C_j e + G_{0j} + \Lambda_j) - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \\ &+ \xi_1 \Gamma_1 \left(f_{1j} + e_2 + \hat{x}_2 + \lambda_{1j} - \dot{y}_r - \frac{\dot{\varphi}_1 z_1}{\varphi_1} \right)\end{aligned} \tag{18}$$

For the unknown term $G_{0j}(\bar{x}_n)$, RBFNN $\theta_{0j}^T S_0(\bar{x}_n)$ is invoked. It can be deduced that $f_{ij} + d_{ij}x_1 = \theta_{0ij}^T S_0(\bar{x}_n) + \varepsilon_{0ij}(\bar{x}_n)$, $i = 1, 2, \cdots, n$ and $j \in M$, where $\varepsilon_{0ij}(\bar{x}_n)$ are the approximation errors satisfying $|\varepsilon_{0ij}(\bar{x}_n)| \le \bar{\varepsilon}_{0ij}^*$ with $\bar{\varepsilon}_{0ij}^*$ being positive constants. Furthermore, rewrite $G_{0j}(\bar{x}_n)$ as the following compact form:

$$G_{0j}(\bar{x}_n) = \theta_{0j}^T S_0(\bar{x}_n) + \varepsilon_{0j}(\bar{x}_n), \ ||\varepsilon_{0j}(\bar{x}_n)|| \le \bar{\varepsilon}_0^*$$

where $\theta_{0j} = [\theta_{01j}^T, \cdots, \theta_{0nj}^T]^T$, $\varepsilon_{0j}(\bar{x}_n) = [\varepsilon_{01j}(\bar{x}_n), \ \cdots, \ \varepsilon_{0nj}(\bar{x}_n)]^T$ and $\bar{\varepsilon}_0^*$ is the upper bound of $||\varepsilon_{0j}(\bar{x}_n)||$. Assume that $0 < S_0^T(\bar{x}_n) S_0(\bar{x}_n) \le s$ for a given constant $s > 0$, and define a positive constant $\theta_0 = \max_{j \in M}\{||\theta_{0j}||^2\}$. Next, by invoking the completion of square, the following inequalities hold:

$$\frac{2}{\gamma} e^T P_j G_{0j} \le \frac{1}{\gamma}\left(2||e||^2 + ||P_j||^2 \theta_0 s + ||P_j||^2 \bar{\varepsilon}_0^{*2}\right) \tag{19a}$$

$$\frac{2}{\gamma} e^T P_j \Lambda_j \le \frac{1}{\gamma}\left(||e||^2 + ||P_j||^2 \bar{\Lambda}_j^{*2}\right) \tag{19b}$$

$$\xi_1 \Gamma_1 e_2 \le \frac{1}{\gamma}||e||^2 + \frac{\gamma}{4}\xi_1^2 \Gamma_1^2 \tag{19c}$$

$$\xi_1 \Gamma_1 \lambda_{1j} \le \frac{\xi_1^2 \Gamma_1^2}{2} + \frac{\bar{\lambda}_{1j}^{*2}}{2} \tag{19d}$$

Now, let $G_{1j}(\Upsilon_1) = \Gamma_1(f_{1j} - \dot{y}_r - \frac{\dot{\varphi}_1 z_1}{\varphi_1} + \frac{\xi_1 \Gamma_1}{2} + \frac{\gamma \xi_1 \Gamma_1}{4}) + \frac{1}{2}\xi_1$, where $\Upsilon_1 = [x_1, y_r, \dot{y}_r, \varphi_1, \dot{\varphi}_1]^T \in \mathbb{R}^5$. And then substituting (19) into (18) yields

$$\begin{aligned}\dot{V}_{1j} \le& -\frac{1}{\gamma}\left(\lambda_{min}(Q_j) - 4\right)||e||^2 \\ &+ \xi_1 \Gamma_1 (z_2 + \alpha_{1j}) + \xi_1 G_{1j} - \frac{\xi_1^2}{2} + \frac{\bar{\lambda}_{1j}^{*2}}{2} - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \\ &+ \frac{1}{\gamma}||P_j||^2\left(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}\right)\end{aligned} \tag{20}$$

From Lemma 1, the RBFNN can be used to approximate $G_{1j}(\Upsilon_1)$ such that

$$G_{1j}(\Upsilon_1) = \theta_{1j}^T S_{1j}(\Upsilon_1) + \varepsilon_{1j}(\Upsilon_1), \ |\varepsilon_{1j}(\Upsilon_1)| \le \bar{\varepsilon}_{1j}^* \tag{21}$$

where $\varepsilon_{1j}(\Upsilon_1)$, $j \in M$, denote the approximation errors with $\bar{\varepsilon}_{1j}^* > 0$ being constants. It follows from the completion of square, (15) and (21) that

$$\begin{aligned}\xi_1 G_{1j}(\Upsilon_1) &\le \frac{1}{2a_{1j}^2}\xi_1^2 ||\theta_{1j}||^2 S_{1j}^T S_{1j} + \frac{a_{1j}^2}{2} + \frac{\xi_1^2}{2} + \frac{\bar{\varepsilon}_{1j}^{*2}}{2} \\ &\le \frac{1}{2a_{1j}^2}\xi_1^2 \theta S_{1j}^T S_{1j} + \frac{a_{1j}^2}{2} + \frac{\xi_1^2}{2} + \frac{\bar{\varepsilon}_{1j}^{*2}}{2}\end{aligned} \tag{22}$$

Next, choose the virtual control law α_{1j} as

$$\alpha_{1j} = -\frac{\xi_1}{\Gamma_1}\left(c_{1j} + \frac{1}{2a_{1j}^2}\hat{\theta} S_{1j}^T(\Upsilon_1) S_{1j}(\Upsilon_1)\right) \tag{23}$$

where c_{1j}, $j \in M$, are design positive constants. Combining (22) and (23) with (20) produces

$$\dot{V}_{1j} \le \Delta_{1j}^* - c_{1j}\xi_1^2 + \Gamma_1 \xi_1 z_2 + \frac{1}{\gamma}\tilde{\theta}\left(\frac{\gamma \xi_1^2 S_{1j}^T S_{1j}}{2a_{1j}^2} - \dot{\hat{\theta}}\right) \tag{24}$$

where $\Delta_{1j}^* = -\frac{1}{\gamma}(\lambda_{min}(Q_j) - 4)||e||^2 + \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \frac{a_{1j}^2}{2} + \frac{\bar{\lambda}_{1j}^{*2}}{2} + \frac{\bar{\varepsilon}_{1j}^{*2}}{2}$. The term $\Gamma_1 \xi_1 z_2$ will be processed in the next step.

Step 2: Select Lyapunov function candidate as follows

$$V_{2j} = V_{1j} + \frac{1}{2}z_2^2 \tag{25}$$

By $z_2 = \hat{x}_2 - \alpha_{1j}$, $\dot{z}_2 = \hat{x}_3 - d_{2j}\hat{x}_1 - \dot{\alpha}_{1j}$ and $\dot{\alpha}_{1j} = \frac{\partial \alpha_{1j}}{\partial x_1}(f_{1j} + \hat{x}_2 + e_2 + \lambda_{1j}) + \frac{\partial \alpha_{1j}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial \varphi_1^{(k)}}\varphi_1^{(k+1)} + \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial y_r^{(k)}} y_r^{(k+1)}$, the time derivative of V_{2j} is given by

$$\begin{aligned}\dot{V}_{2j} =& \dot{V}_{1j} + z_2 \Bigg(\hat{x}_3 - d_{2j}\hat{x}_1 - \frac{\partial \alpha_{1j}}{\partial x_1}(f_{1j} + \hat{x}_2 \\ &+ e_2 + \lambda_{1j}) - \frac{\partial \alpha_{1j}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial \varphi_1^{(k)}}\varphi_1^{(k+1)} \\ &- \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial y_r^{(k)}} y_r^{(k+1)} \Bigg)\end{aligned} \tag{26}$$

For the terms $-z_2 \frac{\partial \alpha_{1j}}{\partial x_1} e_2$ and $-z_2 \frac{\partial \alpha_{1j}}{\partial x_1}\lambda_{1j}$, $j \in M$, the following inequalities hold

$$\begin{cases} -z_2 \frac{\partial \alpha_{1j}}{\partial x_1} e_2 & \le \frac{1}{\gamma}||e||^2 + \frac{\gamma}{4}\left(\frac{\partial \alpha_{1j}}{\partial x_1}\right)^2 z_2^2 \\ -z_2 \frac{\partial \alpha_{1j}}{\partial x_1}\lambda_{1j} & \le \frac{1}{2}\left(\frac{\partial \alpha_{1j}}{\partial x_1}\right)^2 z_2^2 + \frac{\bar{\lambda}_{1j}^{*2}}{2} \end{cases} \tag{27}$$

Denote $G_{2j}(\Upsilon_2) = \Gamma_1 \xi_1 - d_{2j}\hat{x}_1 - \frac{\partial \alpha_{1j}}{\partial x_1}(f_{1j} + \hat{x}_2) + (\frac{\gamma}{4} + \frac{1}{2}) z_2 (\frac{\partial \alpha_{1j}}{\partial x_1})^2 - \frac{\partial \alpha_{1j}}{\partial \hat{\theta}}\frac{\gamma}{2a_{1j}^2}\xi_1^2 S_{1j}^T S_{1j} - \frac{\partial \alpha_{1j}}{\partial \hat{\theta}}\frac{\gamma}{2a_{2j}^2} z_2^2 S_{2j}^T S_{2j} + \frac{\partial \alpha_{1j}}{\partial \hat{\theta}}\sigma\hat{\theta} - \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial \varphi_1^{(k)}}\varphi_1^{(k+1)} - \sum_{k=0}^{1}\frac{\partial \alpha_{1j}}{\partial y_r^{(k)}} y_r^{(k+1)} + \frac{z_2}{2}$ with $\Upsilon_2 = [x_1, \hat{x}_1, \hat{x}_2, \hat{\theta}, y_r, \dot{y}_r, \ddot{y}_r, \varphi_1, \dot{\varphi}_1, \ddot{\varphi}_1]^T \in \mathbb{R}^{10}$. Then, the substitution of (27) into (26) results in

$$\begin{aligned}\dot{V}_{2j} \le& \Delta_{1j}^* - c_{1j}\xi_1^2 + \frac{1}{\gamma}\tilde{\theta}\left(\frac{\gamma \xi_1^2 S_{1j}^T S_{1j}}{2a_{1j}^2} - \dot{\hat{\theta}}\right) \\ &- \frac{\partial \alpha_{1j}}{\partial \hat{\theta}} z_2 \sum_{\ell=3}^{n}\frac{\gamma}{2a_{\ell j}^2} z_\ell^2 S_{\ell j}^T S_{\ell j} \\ &+ z_2(z_3 + \alpha_{2j} + G_{2j}) - \frac{z_2^2}{2} + \frac{||e||^2}{\gamma} + \frac{\bar{\lambda}_{1j}^{*2}}{2}\end{aligned} \tag{28}$$

Similar to Step 1, $G_{2j}(\Upsilon_2)$ can be approximated by RBFNN $\theta_{2j}^T S_{2j}(\Upsilon_2)$ as follows

$$G_{2j}(\Upsilon_2) = \theta_{2j}^T S_{2j}(\Upsilon_2) + \varepsilon_{2j}(\Upsilon_2), \ |\varepsilon_{2j}(\Upsilon_2)| \le \bar{\varepsilon}_{2j}^* \tag{29}$$

with $\bar{\varepsilon}_{2j}^*$, $j \in M$, being positive constants. Also similar to Step 1, it can be obtained

$$z_2 G_{2j} \le \frac{z_2^2 \theta S_{2j}^T S_{2j}}{2a_{2j}^2} + \frac{a_{2j}^2}{2} + \frac{z_2^2}{2} + \frac{\bar{\varepsilon}_{2j}^{*2}}{2} \tag{30}$$

Now, take the virtual control law α_{2j} as

$$\alpha_{2j} = -c_{2j}z_2 - \frac{1}{2a_{2j}^2}z_2\hat{\theta}S_{2j}^T(\Upsilon_2)S_{2j}(\Upsilon_2) \tag{31}$$

where c_{2j}, $j \in M$, are design positive constants. Then, it follows from (28)-(31) that

$$\begin{aligned}\dot{V}_{2j} \leq& \Delta_{2j}^* - c_{1j}\xi_1^2 - c_{2j}z_2^2 + z_2z_3\\ &+ \frac{1}{\gamma}\tilde{\theta}(\frac{\gamma\xi_1^2S_{1j}^TS_{1j}}{2a_{1j}^2} + \frac{\gamma z_2^2S_{2j}^TS_{2j}}{2a_{2j}^2} - \dot{\hat{\theta}})\\ &- \frac{\partial\alpha_{1j}}{\partial\hat{\theta}}z_2\sum_{\ell=3}^{n}\frac{\gamma}{2a_{\ell j}^2}z_\ell^2S_{\ell j}^TS_{\ell j}\end{aligned} \tag{32}$$

where $\Delta_{2j}^* = -\frac{1}{\gamma}(\lambda_{min}(Q_j)-5)||e||^2 + \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \sum_{k=1}^{2}(\frac{a_{kj}^2}{2} + \frac{\bar{\varepsilon}_{kj}^{*2}}{2}) + \bar{\lambda}_{1j}^{*2}$.

Step i ($i = 3, \cdots, n-1$): Choose Lyapunov function as follows

$$V_{ij} = V_{i-1,j} + \frac{1}{2}z_i^2 \tag{33}$$

By combining $z_i = \hat{x}_i - \alpha_{i-1,j}$, $\dot{z}_i = \hat{x}_{i+1} - d_{ij}\hat{x}_1 - \dot{\alpha}_{i-1,j}$, and $\dot{\alpha}_{i-1,j} = \frac{\partial\alpha_{i-1,j}}{\partial x_1}(f_{1j} + \hat{x}_2 + e_2 + \lambda_{1j}) + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{x}_k}\dot{\hat{x}}_k + \frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\dot{\hat{\theta}} + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\varphi_1^{(k)}}\varphi_1^{(k+1)} + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial y_r^{(k)}}y_r^{(k+1)}$, we have

$$\begin{aligned}\dot{V}_{ij} =& \dot{V}_{i-1,j} + z_i\Bigg(\hat{x}_{i+1} - d_{ij}\hat{x}_1 - \frac{\partial\alpha_{i-1,j}}{\partial x_1}(f_{1j} + \hat{x}_2\\ &+e_2 + \lambda_{1j}) - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{x}_k}\dot{\hat{x}}_k - \frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\dot{\hat{\theta}}\\ &- \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\varphi_1^{(k)}}\varphi_1^{(k+1)} - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial y_r^{(k)}}y_r^{(k+1)}\Bigg)\end{aligned} \tag{34}$$

Recursively, there has

$$\begin{aligned}\dot{V}_{i-1,j} \leq& \Delta_{i-1,j}^* - c_{1j}\xi_1^2 - \sum_{k=2}^{i-1}c_{kj}z_k^2 + z_{i-1}z_i\\ &+ \frac{1}{\gamma}\tilde{\theta}(\frac{\gamma\xi_1^2S_{1j}^TS_{1j}}{2a_{1j}^2} + \sum_{k=2}^{i-1}\frac{\gamma z_k^2S_{kj}^TS_{kj}}{2a_{kj}^2} - \dot{\hat{\theta}})\\ &- \sum_{k=1}^{i-2}\frac{\partial\alpha_{kj}}{\partial\hat{\theta}}z_{k+1}\sum_{\ell=i}^{n}\frac{\gamma}{2a_{\ell j}^2}z_\ell^2S_{\ell j}^TS_{\ell j}\end{aligned} \tag{35}$$

where $\Delta_{i-1,j}^* = -\frac{1}{\gamma}(\lambda_{min}(Q_j)-2-i)||e||^2 + \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \sum_{k=1}^{i-1}(\frac{a_{kj}^2}{2} + \frac{\bar{\varepsilon}_{kj}^{*2}}{2}) + \frac{i-1}{2}\bar{\lambda}_{1j}^{*2}$.

For the terms $-z_i\frac{\partial\alpha_{i-1,j}}{\partial x_1}e_2$ and $-z_i\frac{\partial\alpha_{i-1,j}}{\partial x_1}\lambda_{1j}$, we have

$$\begin{cases}-z_i\frac{\partial\alpha_{i-1,j}}{\partial x_1}e_2 & \leq \frac{1}{\gamma}||e||^2 + \frac{\gamma}{4}\left(\frac{\partial\alpha_{i-1,j}}{\partial x_1}\right)^2z_i^2\\ -z_i\frac{\partial\alpha_{i-1,j}}{\partial x_1}\lambda_{1j} & \leq \frac{1}{2}\left(\frac{\partial\alpha_{i-1,j}}{\partial x_1}\right)^2z_i^2 + \frac{\bar{\lambda}_{1j}^{*2}}{2}\end{cases} \tag{36}$$

Denoting $G_{ij}(\Upsilon_i) = z_{i-1} - d_{ij}\hat{x}_1 - \frac{\partial\alpha_{i-1,j}}{\partial x_1}(f_{1j} + \hat{x}_2) - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\hat{x}_k}\dot{\hat{x}}_k + (\frac{\gamma}{4} + \frac{1}{2})z_i(\frac{\partial\alpha_{i-1,j}}{\partial x_1})^2 - \frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\frac{\gamma}{2a_{1j}^2}\xi_1^2S_{1j}^TS_{1j} - \frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\sum_{k=2}^{i}\frac{\gamma}{2c_{kj}^2}z_k^2S_{kj}^TS_{kj} + \frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\sigma\hat{\theta} - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial\varphi_1^{(k)}}\varphi_1^{(k+1)} - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1,j}}{\partial y_r^{(k)}}y_r^{(k+1)} + \frac{z_i}{2} - \frac{\gamma}{2a_{ij}^2}z_iS_{ij}^TS_{ij}\sum_{k=1}^{i-2}\frac{\partial\alpha_{kj}}{\partial\hat{\theta}}z_{k+1}$, where $\Upsilon_i = [x_1, \hat{\bar{x}}_i^T, \hat{\theta}, \bar{y}_r^{(i)T}, \bar{\varphi}_1^{(i)T}]^T \in \mathbb{R}^{(3i+4)}$ with $\hat{\bar{x}}_i = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i]^T$, $\bar{y}_r^{(i)} = [y_r, \dot{y}_r, \cdots, y_r^{(i)}]^T$, $\bar{\varphi}_1^{(i)} = [\varphi_1, \dot{\varphi}_1, \cdots, \varphi_1^{(i)}]^T$. Further, it is easy to obtain that

$$\begin{aligned}\dot{V}_{ij} \leq& \Delta_{i-1,j}^* + \frac{1}{\gamma}||e||^2 - c_{1j}\xi_1^2 - \sum_{k=2}^{i-1}c_{kj}z_k^2\\ &+ \frac{1}{\gamma}\tilde{\theta}\left(\frac{\gamma\xi_1^2S_{1j}^TS_{1j}}{2a_{1j}^2} + \sum_{k=2}^{i-1}\frac{\gamma z_k^2S_{kj}^TS_{kj}}{2a_{kj}^2} - \dot{\hat{\theta}}\right)\\ &- \sum_{k=1}^{i-1}\frac{\partial\alpha_{kj}}{\partial\hat{\theta}}z_{k+1}\sum_{\ell=i+1}^{n}\frac{\gamma}{2a_{\ell j}^2}z_\ell^2S_{\ell j}^TS_{\ell j}\\ &+ z_i(z_{i+1} + \alpha_{ij} + G_{ij}) - \frac{z_i^2}{2} + \frac{\bar{\lambda}_{1j}^{*2}}{2}\end{aligned} \tag{37}$$

Using RBFNNs again, for the given positive constants $\bar{\varepsilon}_{ij}^*$, $j \in M$, we have

$$G_{ij}(\Upsilon_i) = \theta_{ij}^TS_{ij}(\Upsilon_i) + \varepsilon_{ij}(\Upsilon_i),\ |\varepsilon_{ij}(\Upsilon_i)| \leq \bar{\varepsilon}_{ij}^*$$

For the term z_iG_{ij}, invoking the completion of square leads to

$$z_iG_{ij} \leq \frac{z_i^2\theta S_{ij}^TS_{ij}}{2a_{ij}^2} + \frac{a_{ij}^2}{2} + \frac{z_i^2}{2} + \frac{\bar{\varepsilon}_{ij}^{*2}}{2} \tag{38}$$

Next, the virtual control law of Step i is constructed as

$$\alpha_{ij} = -c_{ij}z_i - \frac{1}{2a_{ij}^2}z_i\hat{\theta}S_{ij}^T(\Upsilon_i)S_{ij}(\Upsilon_i) \tag{39}$$

where c_{ij}, $j \in M$, are design positive constants. Then, following the same procedure in Steps 1 and 2, one has

$$\begin{aligned}\dot{V}_{ij} \leq& \Delta_{ij}^* - c_{1j}\xi_1^2 - \sum_{k=2}^{i}c_{kj}z_k^2 + z_iz_{i+1}\\ &+ \frac{1}{\gamma}\tilde{\theta}(\frac{\gamma\xi_1^2S_{1j}^TS_{1j}}{2a_{1j}^2} + \sum_{k=2}^{i}\frac{\gamma z_k^2S_{kj}^TS_{kj}}{2a_{kj}^2} - \dot{\hat{\theta}})\\ &- \sum_{k=1}^{i-1}\frac{\partial\alpha_{kj}}{\partial\hat{\theta}}z_{k+1}\sum_{\ell=i+1}^{n}\frac{\gamma}{2a_{\ell j}^2}z_\ell^2S_{\ell j}^TS_{\ell j}.\end{aligned} \tag{40}$$

where $\Delta_{ij}^* = -\frac{1}{\gamma}(\lambda_{min}(Q_j) - 3 - i)||e||^2 + \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \sum_{k=1}^{i}(\frac{a_{kj}^2}{2} + \frac{\bar{\varepsilon}_{kj}^{*2}}{2}) + \frac{i}{2}\bar{\lambda}_{1j}^{*2}$.

Remark 4: Inspired by the literature [38], there is only one adaptive update parameter $\dot{\hat{\theta}}$ in (16). This method does not need to design adaptive laws $\theta_1,\ \theta_2,\ \cdots,\ \theta_n$ in all recursive steps, thus reducing the complexity of controller. However, the difficulty is that the nonlinear term $\dot{\alpha}_{i-1,j}$ in (39) includes $\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\dot{\hat{\theta}}$, which is a function of Υ_n rather than Υ_i, so it can not be directly approximated by RBFNN $\theta_{ij}^TS_{ij}(\Upsilon_i)$. Therefore, the term $\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}\dot{\hat{\theta}}$ needs to be decomposed into the sum of $\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}(\frac{\gamma}{2a_{1j}^2}\xi_1^2S_{1j}^TS_{1j} + \sum_{k=2}^{i}\frac{\gamma}{2a_{kj}^2}z_k^2S_{kj}^TS_{kj} - \sigma\hat{\theta})$ that can be merged into the composite function $G_{ij}(\Upsilon_i)$ and $\frac{\partial\alpha_{i-1,j}}{\partial\hat{\theta}}(\sum_{k=i+1}^{n}\frac{\gamma}{2a_{kj}^2}z_i^2S_{kj}^TS_{kj})$ that will be processed by step $i+1$.

Step n: The actual controller $v_j(t)$ of the jth subsystem will be provided at the end. Select the Lyapunov function as follows

$$V_{nj} = V_{n-1,j} + \frac{1}{2}z_n^2 \tag{41}$$

and

$$\dot{V}_{nj} \leq \dot{V}_{n-1,j} + z_n \left(u_j - d_{nj}\hat{x}_1 - \frac{\partial \alpha_{n-1,j}}{\partial x_1}(f_{1j} + \hat{x}_2 + e_2 + \lambda_{1j}) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{x}_k}\dot{\hat{x}}_k - \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \varphi_1^{(k)}}\varphi_1^{(k+1)} - \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_r^{(k)}} y_r^{(k+1)} \right) \tag{42}$$

Similar to the previous $n-1$ steps, denote $G_{nj}(\Upsilon_n) = -\frac{\gamma}{2a_{nj}^2} z_n S_{nj}^T S_{nj} \sum_{k=1}^{n-2} \frac{\partial \alpha_{kj}}{\partial \hat{\theta}} z_{k+1} + z_{n-1} - d_{nj}\hat{x}_1 - \frac{\partial \alpha_{n-1,j}}{\partial x_1}(f_{1j} + \hat{x}_2) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{x}_k}\dot{\hat{x}}_k + (\frac{\gamma}{4} + \frac{1}{2}) z_n (\frac{\partial \alpha_{n-1,j}}{\partial x_1})^2 - \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \varphi_1^{(k)}}\varphi_1^{(k+1)} - \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_r^{(k)}} y_r^{(k+1)} + \frac{z_n}{2}$, where $\Upsilon_n = [x_1, \hat{\bar{x}}_n^T, \hat{\theta}, \bar{y}_r^{(n)T}, \bar{\varphi}_1^{(n)T}]^T \in \mathbb{R}^{(3n+4)}$ with $\hat{\bar{x}}_n = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n]^T$, $\bar{y}_r^{(n)} = [y_r, \dot{y}_r, \cdots, y_r^{(n)}]^T$, $\bar{\varphi}_1^{(n)} = [\varphi_1, \dot{\varphi}_1, \cdots, \varphi_1^{(n)}]^T$. We then have

$$\dot{V}_{n,j} \leq \Delta_{n-1,j}^* + \frac{1}{\gamma}||e||^2 - c_{1j}\xi_1^2 - \sum_{k=2}^{n-1} c_{kj} z_k^2 + \frac{1}{\gamma}\tilde{\theta}\left(\frac{\gamma \xi_1^2 S_{1j}^T S_{1j}}{2a_{1j}^2} + \sum_{k=2}^{n-1} \frac{\gamma z_k^2 S_{kj}^T S_{kj}}{2a_{kj}^2} - \dot{\hat{\theta}} \right) + z_n(u_j + G_{nj}) - \frac{z_n^2}{2} + \frac{\bar{\lambda}_{1j}^{*2}}{2} \tag{43}$$

By using RBFNN, for $\bar{\varepsilon}_{nj}^*$, $j \in M$, we have

$$G_{nj}(\Upsilon_n) = \theta_{nj}^T S_{nj}(\Upsilon_n) + \varepsilon_{nj}(\Upsilon_n), \ |\varepsilon_{nj}(\Upsilon_n)| \leq \bar{\varepsilon}_{nj}^*$$

Similar to (38), one has

$$z_n G_{nj} \leq \frac{1}{2a_{nj}^2} z_n^2 \theta S_{nj}^T S_{nj} + \frac{a_{nj}^2}{2} + \frac{z_n^2}{2} + \frac{\bar{\varepsilon}_{nj}^{*2}}{2} \tag{44}$$

The actual controller is chosen as

$$u_j = -c_{nj} z_n - \frac{1}{2a_{nj}^2} z_n \hat{\theta} S_{nj}^T(\Upsilon_n) S_{nj}(\Upsilon_n) \tag{45}$$

with c_{nj}, $j \in M$, are design positive constants. Substitute (44) and (45) into (43) leads to

$$\dot{V}_{nj} \leq -c_{1j}\xi_1^2 - \sum_{k=2}^{n} c_{kj} z_k^2 + \frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta} + \Delta_{nj}^* \tag{46}$$

where $\Delta_{nj}^* = -\frac{1}{\gamma}(\lambda_{min}(Q_j) - 3 - n)||e||^2 + \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \sum_{k=1}^{n}(\frac{a_{kj}^2}{2} + \frac{\bar{\varepsilon}_{kj}^{*2}}{2}) + \frac{n}{2}\bar{\lambda}_{1j}^{*2}$. Because $\frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta} \leq \frac{\sigma}{\gamma}(-\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2)$, rewriting (46) gets

$$\dot{V}_{nj}(\chi(t)) \leq -\mu_0 V_{nj}(\chi(t)) + \nu_0 \tag{47}$$

where $V_{nj}(\chi(t)) = \frac{1}{\gamma}e^T P_j e + \frac{1}{4}\xi_1^2 + \sum_{i=2}^{n}\frac{1}{2}z_i^2 + \frac{1}{2\gamma}\tilde{\theta}^2$, $\chi(t) = [e^T, \xi_1, z_2, \cdots, z_n, \tilde{\theta}]^T$, and μ_0 and ν_0 are constants given by

$$\mu_0 = \min_{j \in M}\left\{ \frac{\lambda_{min}(Q_j) - 3 - n}{\lambda_{max}(P_j)}, 4c_{1j}, 2c_{ij}, i = 2, \cdots, n, \sigma \right\}$$

$$\nu_0 = \max_{j \in M}\left\{ \frac{1}{\gamma}||P_j||^2(\theta_0 s + \bar{\varepsilon}_0^{*2} + \bar{\Lambda}_j^{*2}) + \sum_{i=1}^{n}(\frac{a_{ij}^2}{2} + \frac{\bar{\varepsilon}_{ij}^{*2}}{2}) + \frac{n}{2}\bar{\lambda}_{1j}^{*2} + \frac{\sigma}{2\gamma}\theta^2 \right\}.$$

C. Stability analysis

Theorem 1: For the system (1) under Assumption 1, assuming the unknown functions $G_{ij}(\Upsilon_i)$ can be approached by RBFNNs $\theta_i^T S_i(\Upsilon_i)$ with a bounded error $\varepsilon_{ij}(\Upsilon_i)$, $i = 1, 2, \cdots, n$, $j \in M$. By using the designed PPC (23), (31), (39), (45) and adaptive parameter update rate (16), if a positive number $\tau_0 > \frac{\log(a_0)}{\mu_0}$ is chosen as the average dwell time of the switching signal $\eta(t)$ such that

$$a_0 = \max_{j,\ \ell \in M}\left\{ \frac{\lambda_{max}(P_j)}{\lambda_{min}(P_\ell)} \right\} \tag{48}$$

then all signals can be semi-globally bounded in the closed-loop system. Furthermore, $y(t)$ can follow $y_r(t)$ in the sense that $z_1(t)$ is constrained by $\varphi_1(t)$, $|z_1(0)| < \varphi_1(0)$.

Proof: Note that the proof of the semi-global stability is similar to the one in [34], while the proof of the output tracking error $z_1(t)$ constrained by preset function $\varphi_1(t)$ is different from other similar documents [21]–[26] because of the proposed new error transformation function $\xi_1(t)$.

1) For the Lyapunov function V_{nj}, it can be easily deduced that there is a $\mathcal{K}$-class function $\mathcal{K}(\chi)$ such that $V_{nj}(\chi) \leq \mathcal{K}(||\chi||)$. With (48), one has $V_{nj} \leq a_0 V_{n\ell}$, $\forall\ j, \ell \in M$. Next, we construct an auxiliary function $\Phi(t) = e^{\mu_0 t}V_{n\eta}(\chi(t))$ that is piecewise differentiable. On each time interval $t \in [t_k, t_{k+1})$, invoking (47), one has

$$\begin{aligned} \dot{\Phi}(t) &= e^{\mu_0 t}\dot{V}_{n\eta(t)}(\chi(t)) + \mu_0 e^{\mu_0 t}V_{n\eta(t)}(\chi(t)) \\ &\leq \nu_0 e^{\mu_0 t}, \qquad t \in [t_k, t_{k+1}) \end{aligned} \tag{49}$$

Integrating (49) with interval $[t_k, t_{k+1})$ yields

$$\Phi(t_{k+1}^-) \leq \Phi(t_k) + \int_{t_k}^{t_{k+1}} \nu_0 e^{\mu_0 t} dt \tag{50}$$

Considering $V_{nj} \leq a_0 V_{n\ell}$, $\forall\ j, \ell \in M$ and (50), it can be obtain that

$$\begin{aligned} \Phi(t_{k+1}) &= e^{\mu_0 t_{k+1}}V_{n\eta(t_{k+1})}(\chi(t_{k+1})) \\ &\leq a_0 e^{\mu_0 t_{k+1}}V_{n\eta(t_k)}(\chi(t_{k+1})) \\ &\leq a_0 \Phi(t_{k+1}^-) \\ &\leq a_0\left(\Phi(t_k) + \int_{t_k}^{t_{k+1}} \nu_0 e^{\mu_0 t} \right) \end{aligned} \tag{51}$$

Next, for an arbitrary $T > 0$ (according to convention, $t_0 = 0$), iterating the inequality (51) from $k = 0$ to $k = C_\eta(T, 0)$ gets

$$\begin{aligned} \Phi(T^-) \leq & a_0^{C_\eta(T,0)}\Phi(0) + \int_{t_{C_\eta(T,0)}}^{T} \nu_0 e^{\mu_0 t} dt \\ & + \sum_{k=0}^{C_\eta(T,0)-1} a_0^{C_\eta(T,0)-k}\int_{t_k}^{t_{k+1}} \nu_0 e^{\mu_0 t} dt \end{aligned} \tag{52}$$

In addition, by observing, for each $k \in \{0, 1, \cdots, C_\eta(T, 0)\}$, one has

$$C_\eta(T, 0) - k \leq C_\eta(T, t_{k+1}) + 1 \tag{53}$$

Note that $\tau_0 > \frac{\log a_0}{\mu_0}$, there exists a constant $\varrho \in (0, \mu_0 - \frac{\log a_0}{\tau_0})$, such that $\tau_0 > \frac{\log a_0}{\mu_0 - \varrho}$. This, together with (2), means that

$$C_\eta(T, t) \leq C_0 + \frac{(\mu_0 - \varrho)(T - t)}{\log a_0}, \ \forall\, T \geq t \geq 0 \tag{54}$$

Combining (54) with (53) obtains

$$a_0^{C_\eta(T,0)-k} \leq a_0^{1+C_0} e^{(\mu_0-\varrho)(T-T_{k+1})} \tag{55}$$

By $\varrho < \mu_0$, it holds that $e^{-\mu_0(t_{k+1}-t_k)} < e^{-\varrho(t_{k+1}-t_k)}$. Further, one has

$$\int_{t_k}^{t_{k+1}} \nu_0 e^{\mu_0 t} dt \leq e^{(\mu_0-\varrho)t_{k+1}} \int_{t_k}^{t_{k+1}} \nu_0 e^{\varrho t} dt \tag{56}$$

Substituting (56) into (52) leads to

$$\Phi(T^-) \leq a_0^{C_\eta(T,0)} \Phi(0) + a_0^{1+C_0} e^{(\mu_0-\varrho)T} \int_0^T \nu_0 e^{\varrho t} dt \tag{57}$$

which implies that

$$\begin{aligned} V_{n\eta(T^-)}(\chi(T^-)) \leq\ & e^{C_0 \log a_0} e^{(\frac{\log a_0}{\tau_0} - \mu_0)T} \mathcal{K}(||\chi(0)||) \\ & + a_0^{1+C_0} \frac{\nu_0}{\varrho}(1 - e^{-\varrho T}) \\ \leq\ & e^{C_0 \log a_0} e^{(\frac{\log a_0}{\tau_0} - \mu_0)T} \mathcal{K}(||\chi(0)||) \\ & + a_0^{1+C_0} \frac{\nu_0}{\varrho}, \quad \forall\, T > 0. \end{aligned} \tag{58}$$

By (58), if $\frac{\log a_0}{\tau_0} - \mu_0 < 0$, then $V_{n\eta(T^-)}(\chi(T^-))$ is bounded, so is ξ_1, z_i, $i = 2, 3, \cdots, n$ and $\tilde{\theta}$. θ is a constant that deduces $\hat{\theta}$ to be bounded. According to the definition of (3), it can be inferred that x_i, $i = 1, 2, \cdots, n$ are bounded. Therefore, all signals of the closed-loop system (1), (16), (23), (31), (39) and (45) are bounded.

2) From (58) and $V_{nj}(\chi(t)) = \frac{1}{\gamma} e^T P_j e + \frac{1}{4}\xi_1^2 + \sum_{i=2}^n \frac{1}{2} z_i^2 + \frac{1}{2\gamma}\tilde{\theta}^2$, it means that

$$\begin{aligned} \frac{\xi_1^2(T)}{4} =& \frac{1}{4} \ln^2 \left(\frac{1 + \kappa(T)}{1 - \kappa(T)} \right) \\ \leq & e^{C_0 \log a_0} e^{(\frac{\log a_0}{\tau_0} - \mu_0)T} \mathcal{K}(||\chi(0)||) + a_0^{1+C_0} \frac{\nu_0}{\varrho} \\ \leq & \Delta_0, \ \forall\, T > 0, \end{aligned} \tag{59}$$

where $\kappa(T) = \frac{z_1(T)}{\varphi_1(T)}$, and $\Delta_0 > 0$ is a constant. It naturally holds that, for any given constant $\Delta_0 > 0$, if $\frac{1}{4}\ln^2(\frac{1+\kappa(T)}{1-\kappa(T)}) \leq \Delta_0$, then $|\kappa(T)| < 1$. Hence, for all $T > 0$, the inequality $|z_1| < \varphi_1$ holds under the initial condition $|z_1(0)| < \varphi_1(0)$.

IV. Simulation

Consider a second-order nonlinear system as follows:

$$\begin{aligned} \dot{x}_1 &= f_{1j}(x_1) + x_2 + \lambda_{1j}(t) \\ \dot{x}_2 &= f_{2j}(\bar{x}_2) + u_j + \lambda_{2j}(t) \\ y &= x_1 \end{aligned} \tag{60}$$

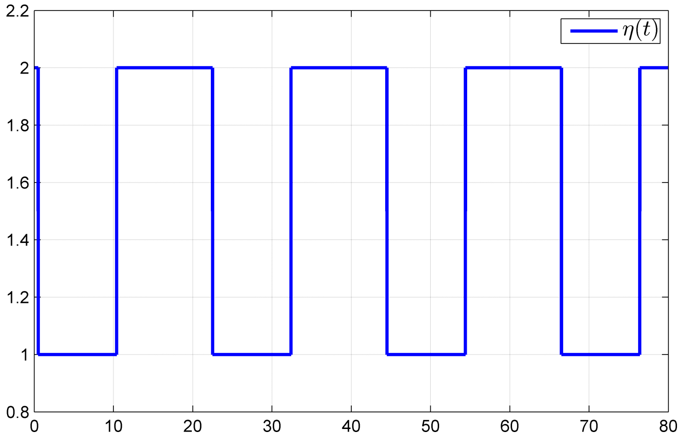

Fig. 2. Switching signal $\eta(t)$.

where $f_{11}(x_1) = x_1 e^{-0.5x_1}$, $f_{21}(\bar{x}_2) = x_1 \sin(x_2^2)$, $\lambda_{11} = 0.2\sin(t)$, $\lambda_{21} = \sin(2t)$, $f_{12}(x_1) = 0.3x_1 \sin(2x_1)$, $f_{22}(\bar{x}_2) = 0.5(0.8x_2 + e^{-x_1^2})$, $\lambda_{12} = -0.1\cos(t)$, $\lambda_{22} = 0.5\sin(t)$, and $j \in M = \{1, 2\}$, that is, the number of subsystems to be switched is 2. Next, the simulation parameters are chosen as $d_{11} = 15$, $d_{21} = 11$, $d_{12} = 10$, $d_{22} = 15$, $c_{11} = 10$, $c_{21} = 1$, $c_{12} = 10$, $c_{22} = 1$, $a_{11} = 0.3$, $a_{21} = 0.2$, $a_{12} = 0.35$, $a_{22} = 0.15$, $\sigma = 0.5$ andp $\gamma = 20$. The reference signal $y_r(t) = 0.5\sin(t)$. Meanwhile, the prescribed performance function is $\varphi_1 = (0.3 - 0.02)e^{-1.5t} + 0.02$. We set $Q_1 = [10, 0; 0, 10]$, $Q_2 = [9, 0; 0, 9]$. Hence, it follows from d_{ij} and Q_j, $i \in \{1, 2\}$, $j \in \{1, 2\}$, that $P_1 = [4, -5; -5, 7.1818]$, $P_2 = [7.2, -4.5; -4.5, 3.48]$. Furthermore, it can be calculated that $\mu_0 = 0.3691$, $a_0 = 31.5137$ and $\tau_0 > \frac{\ln(a_0)}{\mu_0} = 9.3488$.

In simulation, select the error transformation function as

$$\xi_1(t) = \ln(\frac{\varphi_1 + z_1}{\varphi_1 - z_1}). \tag{61}$$

and the time derivation of $\xi_1(t)$ is

$$\dot{\xi}_1(t) = 2\Gamma_1(\dot{z}_1 - \frac{\dot{\varphi}_1 z_1}{\varphi_1}), \tag{62}$$

where $\Gamma_1 = \varphi_1/(\varphi_1^2 - z_1^2)$.

The initial conditions $[x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0), \hat{\theta}(0)]^T = [0.1, -0.2, 0.1, 0.3, 0]^T$. In addition, 61 neuron nodes distributed in [-15:0.5:15] and [-30:1:30] are used to construct the basis function vectors S_{1j} and S_{2j}, $j = 1, 2$, respectively. The width ω of Gaussian function is designed as $\sqrt{2}$. The simulation time is set to 80 seconds, and the simulation results of the proposed algorithm are shown in Figs. 2-8. Concretely, the switching signal are shown in Fig. 2. It is clear from Figs. 3 and 4 that the system output $y(t)$ can well track the reference signal $y_r(t)$. And the tracking error $z_1(t)$ does not violate the constraint function $\varphi_1(t)$. Fig. 5 shows the state x_2 and its estimated value $\hat{x}_2$, and Fig. 8 shows the adaptive parameter $\hat{\theta}$. More especially, for each subsystem j, the actual control input signal $u_j(t)$ are shown on Figs 6 and 7 respectively. From the above simulation results, the PPC method is implemented for the considered system. Also, in the closed-loop system, all signals, especially the adaptive estimation curve $\hat{\theta}$, are all bounded.

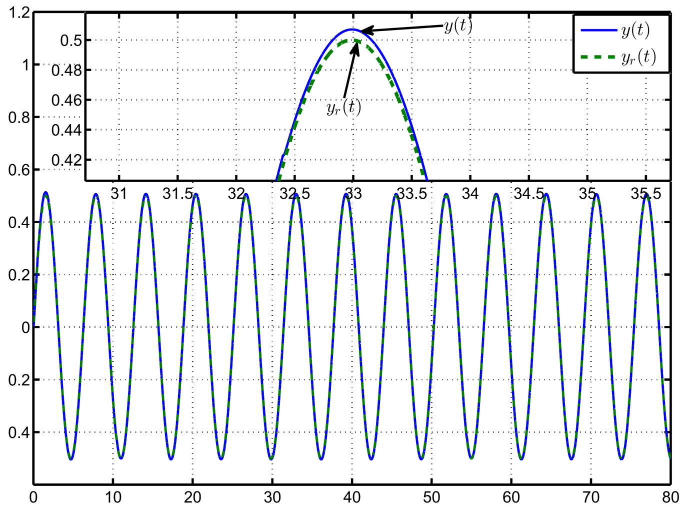

Fig. 3. System output $y(t)$ and reference signal $y_r(t)$.

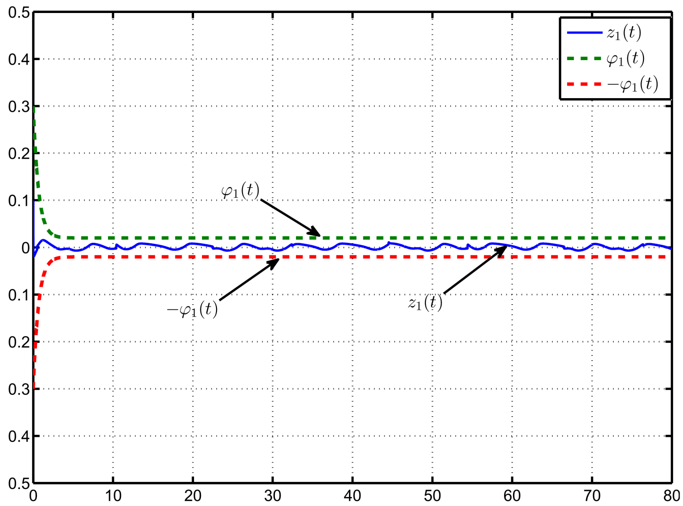

Fig. 4. Tracking error z_1 under the prescribed performance constraint $\varphi_1(t)$.

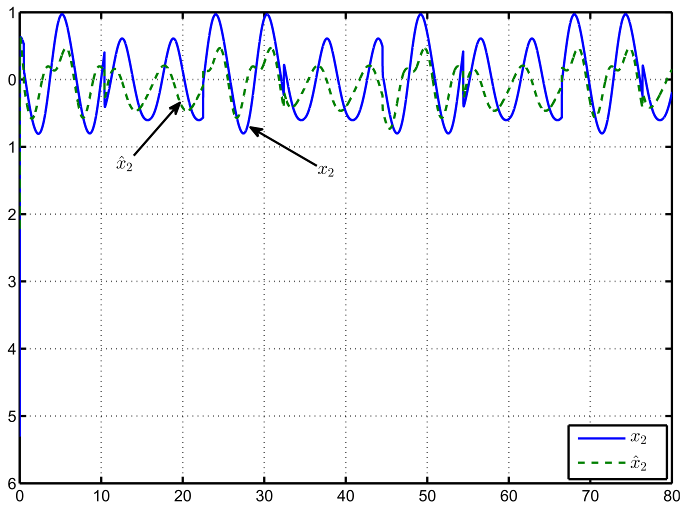

Fig. 5. System state x_2.

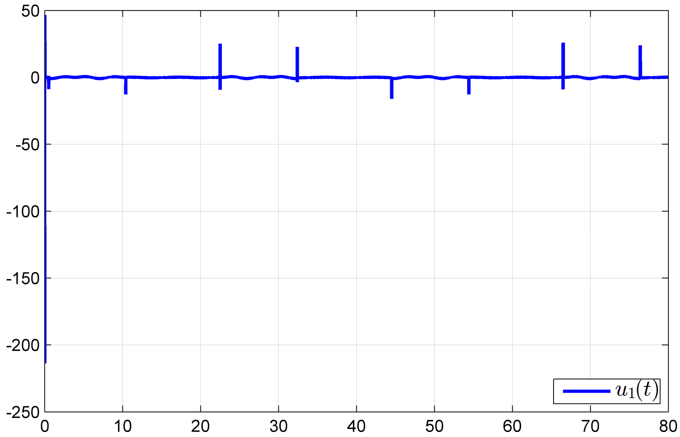

Fig. 6. Actual control signal u_1 of subsystem 1.

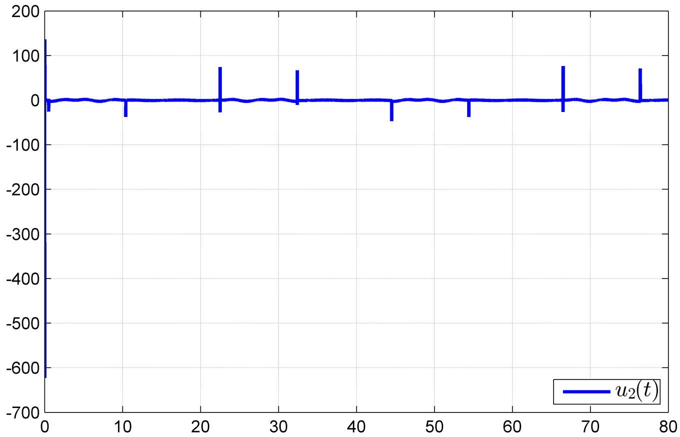

Fig. 7. Actual control signal u_2 of subsystem 2.

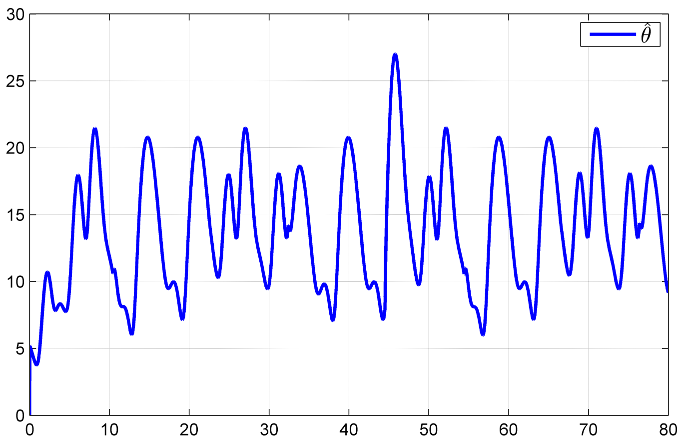

Fig. 8. Adaptive laws $\hat{\theta}(t)$.

V. Conclusion

In this paper, a state observer based adaptive PPC strategy is proposed for uncertain strictly feedback switched nonlinear systems including external disturbances. Different from other similar PPC algorithms, a novel error transformation function is proposed to realize the performance constraint on the output error. In addition, the uncertain disturbance and uncertain nonlinearity of the system are compensated by using RBFNNs. In future research, the authors intend to apply the proposed algorithm to the ETC problem of switched systems.

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