# Dihedral Group Contra Mean Cordial Labeling for Path, Cycle, Ladder and Dumbbell Graphs

A. Sudha Rani, H.A. Bhavithra and S. Sindu Devi

*Abstract*—**Let G be a simple undirected graph. Consider the Dihedral group with eight elements, let us define a function from vertex set to elements in dihedral group in such a way that for each edge $uv$ assign the labels for conditions in contra mean for order of element. Here we shall discuss the dihedral group contra mean labeling for some standard graphs like path and cyclic.**

*Index Terms*—**dihedral group, order of an element, contra mean cordial labeling.**

## I. Introduction

LABELING Labeling plays vital role in coding theory, signal processing, networks, radar and so on. Graph labelling is an assignment of positive integers to vertices or edges or both which satisfy certain conditions. All graphs consider here are undirected graphs without loops or multiple edges. For graph theory concept, we refer bondy and murty [1] for group theory concept we refer Dummit and Footy [2]. Cordial labeling was introduced by Cahit [3] who found that it was weaker version of graceful and harmonious labelling, in which he introduce the function $S : V(G) \to \{0, 1\}$ in such a way, the edge uv is labelled as $|S(u)-S(v)|$. $S$ is called cordial labeling if the difference between the number of vertices labelled 0 and the number of vertices labelled 1 is atmost 1,the difference between the number of vertices labelled 0 and the number of vertices labelled 1 is atmost 1. The above author again proved in [4] that the trees, complete graphs, complete bipartite graphs whose order are not more than three are cordial. Later ramanjanyulu et al. [5] showed that two classes of planar graphs for complete graph and complete bipartite graphs are cordial. Somasundaram et al. [6] introduced the concept of mean labeling of graphs who described an injective function from vertex set of graph to $\{0, 1, \ldots n\}$ such that for each edge uv they labelled with smallest integer function of $+(f(u) + f(v))/2+$ and they found that edges are labelled with distinct numbers." Later the same author in [7], [8] proved that all paths $P_n$, cycles $C_n$, complete bipartite graph $K_{2,n}$, disjoint union of cycles and paths say $C_m \cup P_n$, Cartesian product of paths say $P_m \times P_n$, Cartesian product of path with cycle $P_m \times C_n$ are also mean labelled graph. Barrientos et al. [9] introduced the concept of $\alpha$ labeling and they proved that if two graphs are $\alpha$ mean labeling then their amalgamation is also $\alpha$ mean labeling. They also proved that all quadrilateral snakes are also $\alpha$ mean graphs. Ponraj et al. [10] defined the mapping from vertex set of a graph to $\{0, 1, 2\}$ and applied mean cordial property and proved that complete graph $K_n$ is mean cordial iff n is atmost 2. Motivated by these definitions we introduce the concept of dihedral group $D_4$ contra mean cordial labelling. In this paper we are going to deal with dihedral group $D_4$ with 8 elements. The elements of $D_4$ be $\{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$.

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## II. Preliminaries

### A. Scientific Definition [1]

Let $G$ be a group. An element a in $G$ is said to have order $m$ if $a^m = e$, where $m$ is the least positive integer.

### B. Scientific Definition [2]

Let $S : V(G) \to \{0, 1\}$ in such a way, the edge $uv$ is labeled as $|S(u) - S(v)|$. $S$ is called cordial labeling if the difference between the number of vertices labeled 0 and the number of vertices labeled 1 is atmost 1 and the difference between the number of vertices labeled 0 and the number of vertices labeled 1 is atmost 1.

### C. Scientific Definition [3]

Let $\phi : V(G) \to \{0, 1, 2\}$ for each edge $uv$ we assign the label $\lfloor(\phi(u) + \phi(v))/2\rfloor$. $\phi$ is called mean cordial labeling if $|I_\phi(a) - I_\phi(b)| \leq 1$ & $|J_\phi(a) - J_\phi(b)| \leq 1$ where $a, b \in \{0, 1\}$ and $I_\phi(a)$ denotes the number of vertices labeled as $a$ and $J_\phi(a)$ denote the number of edges labeled as $a$.

## III. Main Result

### A. Scientific Definition

Let $f : V(G) \to D_4$ in such a way that for each edge uv assign the label 1 if $\lfloor(o(u)^2 + o(v)^2)/(o(u) + o(v))\rfloor$ is even & 0 if $\lfloor(o(u)^2 + o(v)^2)/(o(u) + o(v))\rfloor$ is odd, where $\lfloor x\lfloor$ denotes the greatest integer function less than or equal to $x$. let $v_f(a)$ represents the number of vertices of $G$ having label a under the mapping $f$ & $m_f(a)$ represents the number of edges of $G$ having label a under the mapping $f$. $f$ is called $D_4$ contra mean cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ for all $a, b$ in $D_4$ and $|m_f(0) - m_f(1)| \leq 1$ where $m_f(0)$ is the number of edges labelled as 0 and $m_f(1)$ is the number of edges labelled as 1. The graph which admits $f$ is called Dihedral group $D_4$ Contra Mean cordial labeling graph.

### B. Remark

Consider the dihedral group $D_4$. Let the elements of $D_4$ be $\{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ where

$$e = (1)(2)(3)(4), a = (1234), a^2 = (12)(34), a^3 = (1432),$$
$$b = (13)(24), ab = (13), a^2b = (24), a^3b = (14)(23).$$

Order of each element is given below

$$O(e) = 1, O(a) = O(a^3) = 4,$$
$$O(a^2) = O(b) = O(ab) = O(a^2b) = O(a^3b) = 2$$

*Axiom 1:* All path $P_m$, $m \geq 2$ are Dihedral group $D_4$ Contra Mean cordial labeling graph.

*Proof:* Let $v_1, v_2, \ldots v_m$ be the vertices of $P_m$ and the edges are given by

$$E(P_m) = v_i v_{i+1}, 1 \leq i \leq m-1$$

For $2 \leq m \leq 7$ we have

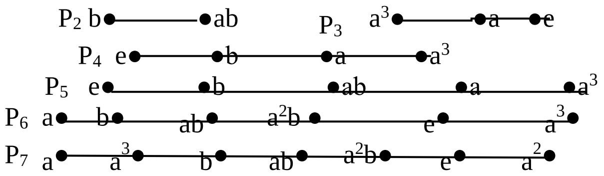


Fig. 1. Path $P_m$: $2 \leq m \leq 7$

Figure 1 for $P_m$: $2 \leq m \leq 7$.
For $m \geq 8$ we discuss eight conditions. ■

*Condition 1:* $m = 8p$

Let $m = 8p, p \geq 1$. We assign the label to $f(V_i)$ as $e$ when $i = 8p+1$, $a^2$ when $i = 8p+2$, a when $i = 8p+3$, $a^3$ when $i = 8p+4$, $b$ when $i = 8p+5$, $ab$ when $i = 8p+6$, $a^2b$ when $i = 8p+7$, $a^3b$ when $i = 8p$ and $i$ varies from 1 to $8p$. In this case each vertex label will appear p times in the dihedral group $D_4$ Contra Mean cordial labeling and each edge labeled as 0 will appear $4p-1$ times and 1 will appear 4p times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 2:* $m = 8p + 1$

Let m = 8p+1, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1. In this case each vertex label will appear p times in the dihedral group $D_4$ Contra Mean cordial labeling except {a} which will appear p+1 times, and each edge label will appear 4p times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} Contra Mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 3:* $m = 8p + 2$

Let m = 8p+2, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1 and $a^3$when i is 8p+2. In this case each vertex label will appear p times from dihedral group $D_4$ except {a,$a^3$} which will appear p+1 times, and each edge labeled as 0 will appear 4p times and 1 will appear 4p+1 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 4:* $m = 8p + 3$

Let m = 8p+3, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1,$a^3$ when i is 8p+2 and $a^2$ when i is 8p+3 In this case each vertex label will appear p times from dihedral group $D_4$except {a,$a^2$,$a^3$} which will appear p+1 times, and each edge label will appear 4p+1 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 5:* $m = 8p + 4$

Let m = 8p+4, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1,$a^3$ when i is 8p+2 $a^2$ when i is 8p+3 e when i is 8p+4.In this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,$a^3$} which will appear p+1 times, and each edge labeled as 0 will appear 4p+2 times and 1 will appear 4p+1 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 6:* $m = 8p + 5$

Let m = 8p+5, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as e for f($V_i$) when i is 8p+1,a when i is 8p+2 , $a^2$ when i is 8p+3, b when i is 8p+4 and ab when i is 8p+5 In this case each vertex label will appear p times from dihedral group $D_4$contra mean cordial labeling except {e,a,$a^2$,b,ab} which will appear p+1 times, and each edge label will appear 4p+2 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 7:* $m = 8p + 6$

Let m = 8p+6, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as e for f($V_i$) when i is 8p+1,a when i is 8p+2 $a^2$ when i is 8p+3, b when i is 8p+4, ab when i is 8p+5,$a^2$b when i is 8p+6.n this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,b,ab,$a^2$b} which will appear p+1 times, and each edge labeled as 0 will appear 4p+2 times and 1 will appear 4p+3 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling

*Condition 8:* $m = 8p + 7$

Let m = 8p+7, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as e for f($V_i$) when i is 8p+1,a when i is 8p+2 , $a^2$ when i is 8p+3, b when i is 8p+4 and ab when i is 8p+5, $a^2$b when i is 8p+6 and $a^3$ when i is 8p+7. In this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,$a^3$,b,ab,$a^2$b} which will appear p+1 times, and each edge label will appear 4p+3 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling

From the below Tables I and II it is clear that all paths $P_n$ are $D_4$ Contra Mean cordial labeling graph.

Hence all paths $P_m$ are Dihedral group $D_4$ Contra Mean cordial labeling graph.

TABLE I
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF PATH GRAPH $P_m$

| Nature of $n$ & $p \geq 1$ | $v_f(e)$ | $v_f(a)$ | $v_f(a^2)$ | $v_f(a^3)$ | $v_f(b)$ |
|---|---|---|---|---|---|
| $8p$ | $p$ | $p$ | $p$ | $p$ | $p$ |
| $8p+1$ | $p$ | $p+1$ | $p$ | $p$ | $p$ |
| $8p+2$ | $p$ | $p+1$ | $p$ | $p+1$ | $p$ |
| $8p+3$ | $p$ | $p+1$ | $p+1$ | $p+1$ | $p$ |
| $8p+4$ | $p+1$ | $p+1$ | $p+1$ | $p+1$ | $p$ |
| $8p+5$ | $p+1$ | $p+1$ | $p+1$ | $p$ | $p+1$ |
| $8p+6$ | $p+1$ | $p+1$ | $p+1$ | $p$ | $p+1$ |
| $8p+7$ | $p+1$ | $p+1$ | $p+1$ | $p+1$ | $p+1$ |

TABLE II
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF PATH GRAPH $P_m$

| Nature of $n$ & $p \geq 1$ | $v_f(ab)$ | $v_f(a^2b)$ | $v_f(a^3b)$ | $m_f(0)$ | $m_f(1)$ |
|---|---|---|---|---|---|
| $8p$ | $p$ | $p$ | $p$ | $4p-1$ | $4p$ |
| $8p+1$ | $p$ | $p$ | $p$ | $4p$ | $4p$ |
| $8p+2$ | $p$ | $p$ | $p$ | $4p$ | $4p+1$ |
| $8p+3$ | $p$ | $p$ | $p$ | $4p+1$ | $4p+1$ |
| $8p+4$ | $p$ | $p$ | $p$ | $4p+2$ | $4p+1$ |
| $8p+5$ | $p+1$ | $p$ | $p$ | $4p+2$ | $4p+2$ |
| $8p+6$ | $p+1$ | $p+1$ | $p$ | $4p+2$ | $4p+3$ |
| $8p+7$ | $p+1$ | $p+1$ | $p$ | $4p+3$ | $4p+3$ |

*Axiom 2:* All cycles $C_m$ $m \geq 3$ are Dihedral group $D_4$ Contra Mean cordial labeling graph.

*Proof:* Let $G = C_m$ be a cycle whose vertices are given by $v_1, v_2, \ldots v_m$ and edges are $E(C_m) = \{v_i v_{i+1},\ 1 \leq i \leq m-1\} \cup \{v_m v_1\}$.

For $3 \leq m \leq 7$ we have

Figure 2 for $P_m$: $3 \leq m \leq 7$.

For $m \geq 8$ we discuss eight conditions. ∎

*Condition 1:* $m = 8p$

Let m = 8p, p = 1. We assign the label to f($V_i$) as b when $i = 8p+1$, ab when $i = 8p+2$, $a^2$b when $i = 8p+3$, a when $i = 8p+4$, $a^3$ when $i = 8p+5$, $a^3$b when $i = 8p+6$, $a^2$ when $i = 8p+7$, e when i =8p, i varies from 1 to 8k. In this case each vertex label will appear p times in the dihedral group $D_4$Contra Mean cordial labeling and each edge label will appear 4p times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 2:* $m = 8p+1$

Let m = 8p+1, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1. In this case each vertex label will appear p times in the dihedral group $D_4$ Contra Mean cordial

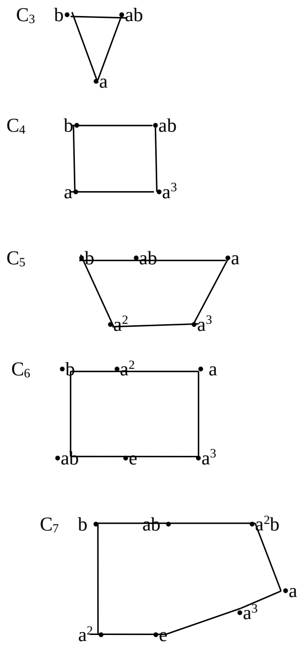


Fig. 2. Cycle $P_m$: $3 \leq m \leq 7$

labeling except {a} which will appear p+1 times, and each edge labeled as 0 will appear 4p+1 times and 1 will appear 4p times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 3:* $m = 8p+2$

Let m = 8p+2, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as e for f($V_i$) when i is 8p+1 and bwhen i is 8p+2. In this case each vertex label will appear p times from dihedral group $D_4$except {e,b} which will appear p+1 times, and each edge label will appear 4p+1 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 4:* $m = 8p+3$

Let m = 8p+3, kp = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1,ab when i is 8p+2 and a when i is 8p+3 In this case each vertex label will appear p times from dihedral group $D_4$except {b,ab,a} which will appear p+1 times, and each edge labeled as 0 will appear 4p+2 times and 1 will appear 4k+1 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 5:* $m = 8p+4$

Let m = 8p+4, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1,ab when i is 8p+2 a when i is 8p+3 $a^3$ when i is 8p+4.In this case each vertex label will appear p times from dihedral group $D_4$ except {b,a,ab,$a^3$} which will appear p+1 times, each edge label will appear 4p+2 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 6:* $m = 8p + 5$

Let m = 8p+5, p= 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1,ab when i is 8p+2 , $a^2$b when i is 8p+3,$a^3$b when i is 8p+4 and a when i is 8p+5 In this case each vertex label will appear p times from dihedral group $D_4$ contra mean cordial labeling except {a,b,ab,$a^2$b,$a^3$b} which will appear p+1 times, and each edge labeled as 0 will appear 4p+2 times and 1 will appear 4p+3 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 7:* $m = 8p + 6$

Let m = 8p+6, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as a for f($V_i$) when i is 8p+1,$a^3$ when i is 8p+2, b when i is 8p+3, ab when i is 8p+4, e when i is 8p+5,$a^2$b when i is 8p+6.n this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^3$,b,ab,$a^2$b} which will appear p+1 times, and each edge label will appear 4p+3 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 8:* $m = 8p + 7$

Let m = 8p+7, k = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1,a when i is 8p+2 , $a^2$ when i is 8p+3, e when i is 8p+4 and ab when i is 8p+5, ,$a^2$b when i is 8p+6 and ,$a^3$b when i is 8p+7. In this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,$a^3$b,b,ab,$a^2$b} which will appear p+1 times, and each edge labeled as 0 will appear 4p+4 times and 1 will appear 4p+3 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

From the below Tables III and IV it is clear that all cycles $C_m$ are $D_4$ Contra Mean cordial labeling graph.

Hence all cycles $C_m$ are Dihedral group $D_4$ Contra Mean cordial labeling graph.

TABLE III

DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF PATH GRAPH $C_m$

| Nature of $n$ & $p \geq 1$ | $v_f(e)$ | $v_f(a)$ | $v_f(a^2)$ | $v_f(a^3)$ | $v_f(b)$ |
|---|---|---|---|---|---|
| $8p$ | $p$ | $p$ | $p$ | $p$ | $p$ |
| $8p+1$ | $p$ | $p+1$ | $p$ | $p$ | $p$ |
| $8p+2$ | $p+1$ | $p$ | $p$ | $p$ | $p+1$ |
| $8p+3$ | $p$ | $p+1$ | $p$ | $p$ | $p+1$ |
| $8p+4$ | $p$ | $p+1$ | $p$ | $p+1$ | $p+1$ |
| $8p+5$ | $p$ | $p+1$ | $p$ | $p$ | $p+1$ |
| $8p+6$ | $p+1$ | $p+1$ | $p$ | $p+1$ | $p+1$ |
| $8p+7$ | $p+1$ | $p+1$ | $p+1$ | $p$ | $p+1$ |

*Axiom 3:* The Ladder graph Lm is Dihedral group $D_4$ Contra Mean cordial labeling graph.

*Proof:* Let the vertices of Ln be $\{v_1, v_2, \ldots v2_m\}$ and edge set is given by $\{v_{2i+1}v_{2i}\} \cup \{v_{2i-1}v_{2i+1}\} \cup \{v_{2i}v_{2i+2}\}$ i varies from 1 to n.

For $2 \leq m \leq 3$ we have

TABLE IV

DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF PATH GRAPH $C_m$

| Nature of $n$ & $p \geq 1$ | $v_f(ab)$ | $v_f(a^2b)$ | $v_f(a^3b)$ | $m_f(0)$ | $m_f(1)$ |
|---|---|---|---|---|---|
| $8p$ | $p$ | $p$ | $p$ | $4p$ | $4p$ |
| $8p+1$ | $p$ | $p$ | $p$ | $4p+1$ | $4p$ |
| $8p+2$ | $p$ | $p$ | $p$ | $4p+1$ | $4p+1$ |
| $8p+3$ | $p+1$ | $p$ | $p$ | $4p+2$ | $4p+1$ |
| $8p+4$ | $p+1$ | $p$ | $p$ | $4p+2$ | $4p+2$ |
| $8p+5$ | $p+1$ | $p+1$ | $p+1$ | $4p+2$ | $4p+3$ |
| $8p+6$ | $p+1$ | $p+1$ | $p$ | $4p+3$ | $4p+3$ |
| $8p+7$ | $p+1$ | $p+1$ | $p+1$ | $4p+4$ | $4p+3$ |

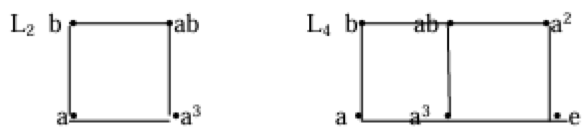


Fig. 3. Ladder $L_n$: $2 \leq m \leq 3$

Figure 3 for $L_n$: $2 \leq m \leq 3$.

For $m \geq 4$ we discuss four conditions. ■

*Condition 1:* $m = 4p$

Let m = 4p, p = 1. We assign the label to f($V_i$) as b when i =8p+1, a when i =8p+2 , $a^2$ when i =8p+3 , $a^3$ when i =8p+4 , ab when i =8p+5 , e when i =8p+6 , $a^2$b when i =8p+7 , $a^3$b when i =8p and i varies from 1 to 8p. In this case each vertex label will appear p times in the dihedral group $D_4$ Contra Mean cordial labeling and each edge label will appear 6p-1 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 2:* $m = 4p + 1$

Let m = 4p+1, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1 and a when i is 8p+2. In this case each vertex label will appear p times from dihedral group $D_4$ except {b,a} which will appear p+1 times, and each edge labeled as 0 will appear 6p+1 times and 1 will appear 6p times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling. Cordial labeling.

*Condition 3:* $m = 4p + 2$

Let m = 4p+2, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$) when i is 8p+1,ab when i is 8p+2 a when i is 8p+3 e when i is 8p+4.In this case each vertex label will appear p times from dihedral group $D_4$ except {b,ab,a,e} which will appear p+1 times, and each edge label will appear 6p+2 times in the dihedral group {e,a,$a^2$,$a^3$,b,ab,$a^2$b,$a^3$b} contra mean cordial labeling. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 4:* $m = 4p + 3$

Let m = 4p+3, p = 1. For 1 = i = 8p we assign the same labeling as in case 1. The remaining vertices are labeled as b for f($V_i$)when i is 8p+1,$a^2$when i is 8p+2 a when i is 8p+3, e when i is 8p+4, $a^3$ when i is 8p+5,ab when i is 8p+6.In this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,b,ab,$a^3$} which will appear p+1 times, and each edge labeled as 0 will appear 6p+4 times and 1 will appear 6p+3 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

From the below Tables V and VI it is clear that all Ladder graphs $L_n$ are $D_4$ Contra Mean cordial labeling graph. Hence all Ladder graphs Ln are Dihedral group $D_4$ Contra Mean cordial labeling graph.

TABLE V
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF LADER GRAPH $L_n$

| Nature of n & p $\geq$ 1 | $v_g$(e) | $v_g$(a) | $v_g$($a^2$) | $v_g$($a^3$) | $v_g$(b) |
|---|---|---|---|---|---|
| 4p | p | p | p | p | p |
| 4p+1 | p | p+1 | p | p | p+1 |
| 4p+2 | p+1 | p+1 | p | p | p+1 |
| 4p+3 | p+1 | p+1 | p+1 | p+1 | p+1 |

TABLE VI
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF LADER GRAPH $L_m$

| Nature of n & p $\geq$ 1 | $v_g$(ab) | $v_g$($a^2$b) | $v_g$($a^3$b) | $n_g$(0) | $n_g$(1) |
|---|---|---|---|---|---|
| 4p | p | p | p | 6p-1 | 6p-1 |
| 4p+1 | p | p | p | 6p+1 | 6p |
| 4p+2 | p+1 | p | p | 6p+2 | 6p+2 |
| 4p+3 | p+1 | p | p | 6p+4 | 6p+3 |

*Axiom 4:* The Dumbbell graph Dbm, m $\geq$ 3 are Dihedral group $D_4$ Contra Mean cordial labeling graph.

*Proof:* The dumbbell graph $D_{bn}$ is obtained by joining two disjoint cycles by an edge. Let $v_1$,$v_2$,… $v_m$,$u_1$,$u_2$,… $u_n$ be the vertices of dumbbell graph $D_{bn}$ whose edge set is given by $E(D_{bn}) = \{v_i v_{i+1}$, 1 $\leq$ i $\leq$ n-1$\} \cup \{u_i u_{i+1}$, 1 $\leq$ i $\leq$ n-1$\} \cup \{v_1 u_1\}$

For m = 3, we have

Figure 4 for $D_{bm}$: $m = 3$.

For m $\geq$ 4 we discuss four conditions. ∎

*Condition 1:* $m = 4p$

Let m = 4p, p = 1. We assign the label to f($V_i$) as b when i =4p+1, $a^2$ when i =4p+2 , a when i =4p+3 , $a^3$ when i =4p , and for f($U_i$) we assign the label as e when i =4p+1 , ab when i =4p+2 , $a^2$b when i =4p+3 , $a^3$b when i =4p and i varies from 1 to 4p. In this case each vertex label will appear p times in the dihedral group $D_4$ Contra Mean cordial labeling and each edge labeled as 0 will appear 4p+1 times and 1 will appear 4p times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

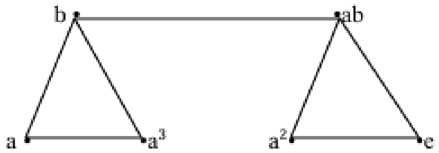


Fig. 4. Dumbbell $D_{bm}$: $m = 3$

*Condition 2:* $m = 4p + 1$

Let m = 4p+1, p = 1. For 1 = i = 4p we assign the same labeling as in case 1. For the remaining vertices f($V_i$) should label b when i is 4p+1 and f($U_i$) should label $a^2$ when i is 4p+1. In this case each vertex label will appear p times from dihedral group $D_4$ except {b,$a^2$} which will appear p+1 times, and each edge labeled as 0 will appear 4p+1 times and 1 will appear 4p+2 times respectively.Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

*Condition 3:* $m = 4p + 2$

Let m = 4p+2, p= 1. For 1 = i = 4p we assign the same labeling as in case 1. For the remaining vertices f($V_i$) should label a when i is 4p+1, $a^2$ when i is 4p+2 and f($U_i$) should label $a^3$ when i is 4p+1 and e when i is 4p+2. In this case each vertex label will appear k times from dihedral group $D_4$except {e,a,$a^2$,$a^3$} which will appear p+1 times, and each edge labeled as 0 will appear 4p+3 times and 1 will appear 4p+2 times respectively. Hence in this case we get f as Dihedral group $D_4$Contra Mean cordial labeling.

*Condition 4:* $m = 4p + 3$

Let m = 4p+3, p = 1. For 1 = i = 4p we assign the same labeling as in case 1. For the remaining vertices f($V_i$) should label a when i is 4p+1, $a^2$ when i is 4p+2, e when i is 4p+3 and f($U_i$) should label b when i is 4p+1 and ab when i is 4p+2, $a^3$ when i is 4p+2 In this case each vertex label will appear p times from dihedral group $D_4$ except {e,a,$a^2$,$a^3$,b,ab} which will appear p+1 times, and each edge labeled as 0 will appear 4p+4 times and 1 will appear 4p+3 times respectively. Hence in this case we get f as Dihedral group $D_4$ Contra Mean cordial labeling.

From the below Tables VII and VIII it is clear that all Dumbbell graph Dbn are $D_4$ Contra Mean cordial labeling graph. Hence all Dumbbell graph Dbn are Dihedral group $D_4$ Contra Mean cordial labeling graph.

TABLE VII
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF DUMBBELL GRAPH $D_{bm}$

| Nature of n & p $\geq$ 1 | $v_g$(e) | $v_g$(a) | $v_g$($a^2$) | $v_g$($a^3$) | $v_g$(b) |
|---|---|---|---|---|---|
| 4p | p | p | p | p | p |
| 4p+1 | p | p | p+1 | p | p+1 |
| 4p+2 | p+1 | p+1 | p+1 | p+1 | p |
| 4p+3 | p+1 | p+1 | p+1 | p+1 | p+1 |

TABLE VIII
DIHEDRAL GROUP $D_4$ CONTRA MEAN CORDIAL LABELING OF DUMBBELL GRAPH $D_{bm}$

| Nature of n & p $\geq$ 1 | $v_g$(ab) | $v_g$(a$^2$b) | $v_g$(a$^3$b) | $n_g(0)$ | $n_g(1)$ |
|---|---|---|---|---|---|
| 4p | p | p | p | 4p+1 | 4p |
| 4p+1 | p | p | p | 4p+1 | 4p+2 |
| 4p+2 | p | p | p | 4p+3 | 4p+2 |
| 4p+3 | p+1 | p | p | 4p+4 | 4p+3 |

## IV. CONCLUSION

This paper preserves our findings indicate that the path, cycle,ladder and dumbbell graphs are $D_4$ Contra Mean cordial labeling. In future we can investigate the concept for duplication and splitting of graphs.

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