Application of the Fourth-Order Runge-Kutta and Adam-Bashfort-Moulton Methods to Logistic Equations for the Prediction of Population Growth

Chusnul Chatimah Azis, Fadiah Hasna, and Sri Purwani

Abstract—**This study explores population growth prediction as an important element in planning and managing regional development, which is always governments' concern. Stability and prediction of population growth play an important role in decision-making related to infrastructure, education, health, and the economy. In this study, the logistic equation is used as a general model to describe the dynamics of population growth by taking into account factors such as intrinsic growth rate, environmental capacity, and initial population. The modeling and prediction process is solved using numerical methods, including the fourth-order Runge-Kutta (RK4) method and the fourth-order Adam-Bashfort-Moulton (ABM4) method. The RK4 method is a simple but accurate method that is used to obtain solutions to differential equations at several points in time, whereas the ABM4 method obtains solutions through a pairing of predictor-corrector methods that offer stability and a low truncation error. We apply both methods to solve the logistical model of predicting population growth in South Sulawesi, Indonesia. The results show that the predictions obtained using the RK4 method are more closely aligned with the actual data compared to the ABM4 method. This is demonstrated by the lower errors resulting from the RK4 method than those resulting from the ABM4 method. In addition, rounding the results obtained from the calculation of the growth rate (*k*) and the specification of the population carrying capacity (*K*) is also found to have a significant impact on the accuracy of the prediction results using the logistic model. In this case, specifying *k*=0.014991 and *K*=30,000,000 led to the best result in this study.**

Index Terms: **Runge-Kutta method, Adams-Bashforth-Moulton method, logistical equation, population growth**

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I. Introduction

POPULATION growth is an essential element in the planning and management of an area's development. The stability and forecasting of population growth play an important role in decision-making regarding infrastructure, education, health, and the economy of a particular area [1]. Therefore, the process of modeling and predicting population growth is an indispensable part of regional development planning [2]. Population growth is a continuous process and falls into the exponential and logistical categories.

The logistic equation is a general model used in population growth modeling, formulated as a nonlinear differential equation that describes the dynamics of population growth [3]. The logistic growth model has a solid application base and significant practical impact [4]. This equation provides an overview of the evolution of population growth over time, taking into account factors such as the intrinsic growth rate, environmental capacity, and the initial value of the population [5], [6].

Numerical methods are the main tool in the prediction of population growth models [7]. Typically, numerical integration for a rigid system of n ordinary differential equations is carried out using implicit numerical methods [8]. Numerical methods can help integrate differential equations to produce estimates of population growth at various points in time. Two such numerical methods that are often used include the fourth-order Runge-Kutta (RK4) method and the fourth-order Adam-Bashfort-Moulton (ABM4) method.

RK4 is a simple numerical approximation, but it provides a high degree of accuracy [9], [10]. This method is used to calculate the approximate solution of an ordinary differential equation (ODE) at certain points in time in an interval [11]. With its combination of precise results and calculation efficiency, the RK4 method is often aptly used in a variety of scientific and engineering applications that involve modeling and the simulation of dynamic systems using differential equations [12]. Some studies have used both the RK4 [13] and the ABM4 [14] methods as numerical methods for solving differential problems.

The ABM4 is a numerical approximation used to solve ordinary differential equations or systems of differential equations. The ABM4 belongs to the category of the multi-step methods or what is often referred to as the predictor-corrector method [15]. This approach entails the use of the Adams-Bashforth prediction method as an initial predictor step, followed by the Adams-Moulton correction method [16]. Using the ABM predictor-corrector method to solve first-order ordinary differential equations numerically provides stable results.

Several researchers have conducted studies related to this issue, such as [17]. This is a study that uses exponential growth models and logistic models, along with generalized logistic models, to forecast Turkey's population, international investment, and national income per capita in 2025. In addition, it reviews trends in home sales and the number of mobile phone subscribers in Turkey. Furthermore, [18] predicts China's population growth for 2016 and beyond using actual data from 1985 to 2015 by employing a logistic model approach. In [19], population growth in Bangladesh is modeled and designed to predict the population in Bangladesh from 2000 to 2050 using exponential growth models, logistic growth models, and actual data from 2000 to 2019.

Moreover, several other studies have been conducted, such as [20], which is a study that uses hybrid numerical methods for exponential growth models. In [21], complex dynamics in predictor-corrector systems are discussed in terms of Watt-type functional responses and impulsive control strategies. However, based on these studies, no research has been conducted on the application of both the RK4 and ABM4 methods to predict population growth in South Sulawesi using logistic equations. Therefore, based on the advantages of the two methods and some of the research that has been discussed, this study was conducted.

This study analyzes the efficacy of the RK4 and ABM4 methods in predicting population growth using logistic equations, as well as the impact of parameter variations. The results from the application of these two methods are expected to provide valuable insights that can be used to support development planning and management in South Sulawesi.

II. Materials and Methods

This subsection details the population data for the province of South Sulawesi, in addition to explaining the population growth rate, population carrying capacity, logistic equations, and the RK4 and ABM4 methods.

A. Population Data

Population is the total number of individuals living in a region or country at a certain time, both on a local, national, and global scale. These populations consist of diverse age groups, ethnic backgrounds, religions, and social statuses, as well as diverse social and economic dynamics. Population is one of the important aspects of population studies, as it influences a country's governmental policies, economy, and infrastructure. Therefore, predicting population growth is essential for ascertaining an in-depth understanding of population change and its associated challenges, including sustainable development planning and resource equity. The population data of South Sulawesi is delineated in Table I.

TABLE I
Population of South Sulawesi Province

Year	Population
2010	8,034,776
2011	8,156,129
2012	8,250,018
2013	8,342,047
2014	8,432,163
2015	8,512,608
2016	8,598,604
2017	8,674,372
2018	8,748,052
2019	8,819,549
2020	9,073,509
2021	9,139,531
2022	9,225,747
2023	9,312,019

B. Population Growth Rate

Population growth rate is a measure that describes how quickly or slowly a population in a region or group grows over a period of time. To predict the rate of population growth, we can use the following basic formula:

$$k = \frac{1}{t}\ln\left(\frac{P}{P_0}\right)$$

where k is the rate of population growth; t is the period of time taken for the measurement of growth rate in years, months, or other time periods (which is dependent on the available data); P_0 is the number of initial inhabitants at the beginning of the period under study; and P is the number of inhabitants at a specified time.

A positive k value indicates population growth, while a negative k value denotes population decline. The estimation of population growth rate is based on a comparison of population numbers at two different times. The growth rate used in this study is as follows:

$$k = \frac{1}{1}\ln\left(\frac{8,156,129}{8,034,776}\right) \approx 0.014991 \approx 0.01$$

C. The Population Capacity of South Sulawesi

Population capacity in the context of population growth is the maximum number of individuals that can be accommodated in an area or environment in a given period of time without experiencing environmental deterioration or lack of resources. In some cases, trial and error can be used to estimate parameter values, including the capacity parameter of the population K.

The capacity of the population in this study was determined using trial and error that was based on assumption. The assumed value of the population capacity is typically determined to be greater than the population in the latest data available. Based on the data obtained in this study, the following assumptions are made: the population capacity values are K_1 = 20,000,000 and K_2 = 30,000,000; the growth rates are $k_1 = 0.01$ and $k_2 = 0.014991$; the initial population number is $P_0 = 8{,}034{,}776$; with the interval being [0,13], the step numbers being $n = 13$, and the step length being $h = (b-a)/n=1$.

D. Logistics Equation

Population growth refers to changes in the population of a region over a certain period of time. This process is influenced by three main factors: birth, death, and migration. Population growth can be modeled using a variety of mathematical methods, one of which is a logistic equation.

A logistic equation is a mathematical model used to describe population or population growth by accounting for the carrying capacity constraints of the population. The model was first introduced by Pierre-François Verhulst in 1838, and it is often used in the context of population biology and ecology. The logistic equation assumes that factors such as birth and death rates remain constant over the measurement period. The logistic equation is generally expressed as follows [22]:

$$\frac{dp}{dt} = kP\left(1 - \frac{P}{K}\right) \quad (1)$$

where P is the number of population at a given time t, k is the growth rate, K is the capacity of the population, and P_0 is the initial population at the beginning of the period under study t_0. By substituting the values of k and K herein, the logistical equation for this research is as follows:

$$\frac{dp}{dt} = k_i P\left(1 - \frac{P}{20{,}000{,}000}\right), i = 1, 2. \quad (2)$$

$$\frac{dp}{dt} = k_i P\left(1 - \frac{P}{30{,}000{,}000}\right), i = 1, 2. \quad (3)$$

E. The RK4 Method

The RK4 method is one of the most popular numerical methods for solving ODE. It strikes a good balance between computational accuracy and efficiency, making it a common choice in numerical simulations. Its advantage lies in its ability to achieve a fourth-order method without having to involve high derivatives. This method is a valuable tool in the modeling and simulation of various dynamical systems that involve differential equations. The general equation for the RK4 method is as described in the following equation (4) [23]:

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (4)$$

where

$$k_1 = hf(x_r, y_r)$$

$$k_2 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1)$$

$$k_3 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2)$$

$$k_4 = hf(x_r + h, y_r + k_3)$$

F. The ABM4 Method

The ABM4 method is a numerical approximation employed to solve ODE. This method is a combination of the Adams-Bashforth method, which is used as a predictor, and the Adams-Moulton method, which is used as a corrector.

The ABM4 method is one that can be implemented to improve the accuracy and stability of solutions, making it a common choice in the simulation of population dynamics and its related mathematical problems. In this study, the ABM4 method uses numerical solutions obtained from the RK4 method as initial values [24]. The formula of the ABM4 method that is used as the predictor is as follows:

$$y_{n+1}^{p} = y_n + \frac{h}{24}\left(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3}\right)$$

The predictor value is then corrected using the corrector AM method of fourth-order as follows:

$$y_{n+1}^{c} = y_n + \frac{h}{24}\left(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}\right)$$

Therefore this combination is called the predictor-corrector ABM4.

III. Results and Discussion

Based on the methods described above, the following numerical solution using the RK4 method and the ABM4 method, as well as error calculation to predict population growth in South Sulawesi, are presented in the form of tables and graphs. The error is represented as a relative error, which is the ratio of absolute errors to actual data.

A. Solutions from the RK4 and ABM4 Methods (k = 0.01and K = 20,000,000)

Table II details the prediction results using the RK4 and ABM4 methods from 2014 to 2023, which shows that both provide fairly accurate estimates with relatively small relative errors. For example, in 2014, the actual population was 8,432,163. Thus, the predictions using the RK4 method have a relative error of about 0.024, while the results from the ABM4 method have a relative error of approximately 0.026.

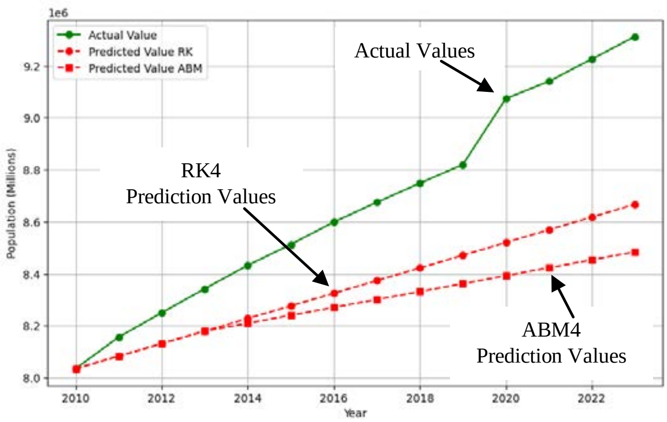

Fig. 1. Plots of population growth using RK4 and ABM4 (k=0.01 and K = 20.000.000).

TABLE II
SOLUTIONS AND ERRORS OF RK4 AND ABM4 (K=0.01AND K=20,000,000)S

Year	Population	4th order RK Solution	4th order RK Error	4th order ABM Solution	4th order RK Error
2010	8,034,776	8,034,776	0.000000	8,034,776	0.000000
2011	8,156,129	8,082,892	0.008979	8,082,892	0.008979
2012	8,250,018	8,131,100	0.014414	8,131,100	0.014414
2013	8,342,047	8,179,398	0.019497	8,179,398	0.019497
2014	8,432,163	8,227,785	0.024238	8,209,662	0.026387
2015	8,512,608	8,276,257	0.027765	8,239,973	0.032027
2016	8,598,604	8,324,813	0.031841	8,270,330	0.038178
2017	8,674,372	8,373,450	0.034691	8,300,731	0.043074
2018	8,748,052	8,422,166	0.037252	8,331,176	0.047654
2019	8,819,549	8,470,960	0.039525	8,361,664	0.051917
2020	9,073,509	8,519,828	0.061022	8,392,193	0.075088
2021	9,139,531	8,568,768	0.062450	8,422,764	0.078425
2022	9,225,747	8,617,778	0.065899	8,453,376	0.083719
2023	9,312,019	8,666,857	0.069283	8,484,028	0.088916

Although there is a small variation in the degree of accuracy between the two methods, both show good ability in modeling population growth.

However, it can be seen that from 2019 to 2023, there is an increase in relative error values for both methods. In addition, the relative error difference between the two methods became more noticeable over time. In 2023, the ABM4 method shows a relative error of about 0.089, while it is approximately 0.069 using the RK4 method. Figure 1 shows plots of the solutions derived from both methods.

Figure 1 compares the actual population data and the prediction results obtained using the RK4 and ABM4 methods, which shows that the prediction results tend to follow the increase in the overall population. At the beginning of the period, the predictions were relatively close to the actual data, but there were significant differences over time. For example, from 2019 to 2020, there was a significant increase in the actual data, which caused errors in the prediction results. This is due to these methods no longer aligning with the actual data. There results from these two methods also differ.

B. The Solutions from the RK4 and ABM4 Methods (k = 0.01499 and K = 20,000,000)

From 2014 to 2023, the prediction of the population using the RK4 method is fairly accurate compared to the actual data (see Table III). Despite slight deviations or errors, the relative errors for each year remain within an acceptable range, ranging from 0.012752 to 0.034927. However, in 2020, there was a significant increase in relative errors, reaching 0.034027, which indicates greater inaccuracies in the year's predictions.

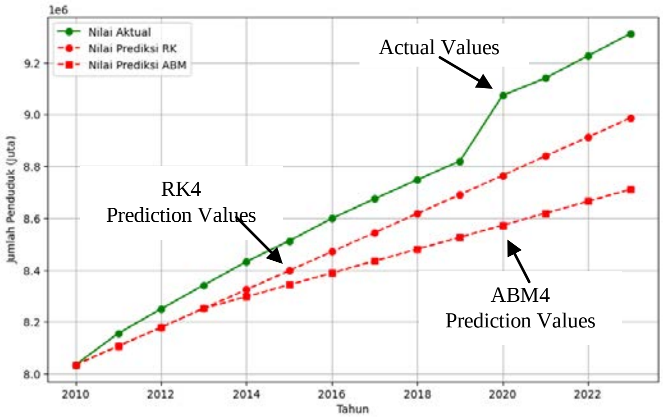

Fig. 2. Plots of population growth using RK4 and ABM4 (k=0,01499 and $K = 20.000.000$).

The ABM4 method provides results that are comparable to the RK4 method, with the relative errors ranging from 0.015982 to 0.064508. This method also provides an estimate of the number of inhabitants that closely aligns with the actual data. However, it should be noted that in 2020 and

TABLE III
SOLUTIONS AND ERRORS OF RK4 AND ABM4 (K=0,01499 AND K=20,000,000)S

Year	Population	4th order RK Solution	4th order RK Error	4th order ABM Solution	4th order RK Error
2010	8,034,776	8,034,776	0.000000	8,034,776	0.000000
2011	8,156,129	8,106,941	0.006031	8,106,941	0.006031
2012	8,250,018	8,179,311	0.008570	8,179,311	0.008570
2013	8,342,047	8,251,879	0.010809	8,251,879	0.010809
2014	8,432,163	8,324,638	0.012752	8,297,404	0.015982
2015	8,512,608	8,397,579	0.013513	8,343,030	0.019921
2016	8,598,604	8,470,696	0.014875	8,388,756	0.024405
2017	8,674,372	8,543,980	0.015032	8,434,575	0.027644
2018	8,748,052	8,617,425	0.014932	8,480,485	0.030586
2019	8,819,549	8,691,022	0.014573	8,526,483	0.033229
2020	9,073,509	8,764,764	0.034027	8,572,568	0.055209
2021	9,139,531	8,838,642	0.032922	8,618,737	0.056983
2022	9,225,747	8,912,649	0.033937	8,664,987	0.060782
2023	9,312,019	8,986,777	0.034927	8,711,316	0.064508

TABLE IV
SOLUTIONS AND ERRORS OF RK4 AND ABM4 (K=0,01499 AND K=30,000,000)S

Year	Population	4th order RK Solution	4th order RK Error	4th order ABM Solution	4th order RK Error
2010	8,034,776	8,034,776	0.000000	8,034,776	0.000000
2011	8,156,129	8,123,272	0.004028	8,123,272	0.004028
2012	8,250,018	8,212,379	0.004562	8,212,379	0.004562
2013	8,342,047	8,302,092	0.004790	8,302,092	0.004790
2014	8,432,163	8,392,407	0.004715	8,358,681	0.008715
2015	8,512,608	8,483,321	0.003440	8,415,594	0.011397
2016	8,598,604	8,574,829	0.002765	8,472,830	0.014627
2017	8,674,372	8,666,927	0.000858	8,530,378	0.016600
2018	8,748,052	8,759,609	0.001321	8,588,239	0.018268
2019	8,819,549	8,852,871	0.003778	8,646,409	0.019631
2020	9,073,509	8,946,708	0.013975	8,704,889	0.040626
2021	9,139,531	9,041,114	0.010768	8,763,677	0.041124
2022	9,225,747	9,136,084	0.009719	8,822,772	0.043679
2023	9,312,019	9,231,612	0.008635	8,882,173	0.046160

2021, the ABM4 method shows a higher relative error increase compared to the RK4 method.

Based on the prediction results obtained using the RK4 and ABM4 methods (see Figure 2), both methods provide estimates of population numbers that closely match the actual data from 2014 to 2018. However, from 2019 to 2020, there was a significant increase in population, and the differences between the predictions and the actual data become more striking.

In 2019, the actual population reached 8,819,549, while the prediction provided by the RK4 method is 8,691,022.31, and the prediction from the ABM4 method is 8,526,483.36. Therefore, both methods produced lower predictions compared to the actual data. In addition, 2020 marked a significant increase in the number of residents to 9,073,509, but the RK4 method's prediction of 8,764,763.98 and the ABM4 method's prediction of 8,572,568.39 do not capture this sharp increase.

C. The Solutions from the RK4 and ABM4 Methods (k = 0,01499 and K = 30,000,000)

Table IV delineates the population data and prediction results obtained using the RK4 and ABM4 methods. It shows that both methods provide results that are relatively similar to the actual population values in certain years. Despite the differences, the errors produced by these two methods are relatively small and practically acceptable. For 2014, the RK4 method results in an approximate error of 0.0047, while the ABM4 method's error is approximately 0.0087. Furthermore, an analysis of the errors in subsequent years was similar.

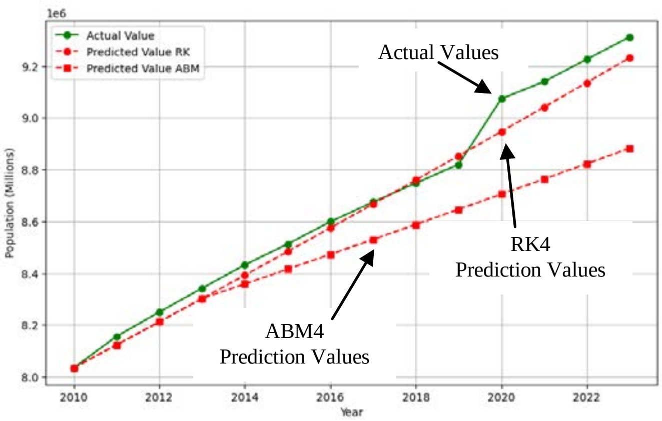

Fig. 3. Plots of population growth using RK4 and ABM4 (k=0,01499 and K = 30.000.000)).

Figure 3 compares the prediction results from the RK4 and ABM4 methods to the actual population data. It demonstrates that both methods provide relatively accurate predictions compared to the actual data. However, there are differences between the predictions and actual data in certain years.

A significant increase in the population from 2019 to 2020 in the actual data is the focus of this analysis. The RK4 method gives higher predictions than the actual data, while the ABM4 method provides lower predictions. Therefore, neither method fully captured the actual population spike that occurred in that period.

D. Comparison of the Errors in the RK4 and ABM4 Methods with Three Parameter Variations

Using the RK4 method and the population growth error calculation data with the parameter of k = 0.01 or 0.014991, K remains 20,000,000 or 30,000,000 (see Table V). Therefore, the prediction results have varying error rates.

TABLE V
ERRORS COMPARISON OF RK4 WITH 3 PARAMETER VARIATIONS

Year	k=0.01and K=20,000,000	k=0,01499 and K=20,000,000	k=0,01499 and K=30,000,000
2010	0.000000	0.000000	0.000000
2011	0.008979	0.006031	0.004028
2012	0.014414	0.008570	0.004562
2013	0.019497	0.010809	0.004790
2014	0.024238	0.012752	0.004715
2015	0.027765	0.013513	0.003440
2016	0.031841	0.014875	0.002765
2017	0.034691	0.015032	0.000858
2018	0.037252	0.014932	0.001321
2019	0.039525	0.014573	0.003778
2020	0.061022	0.034027	0.013975
2021	0.062450	0.032922	0.010768
2022	0.065899	0.033937	0.009719
2023	0.069283	0.034927	0.008635

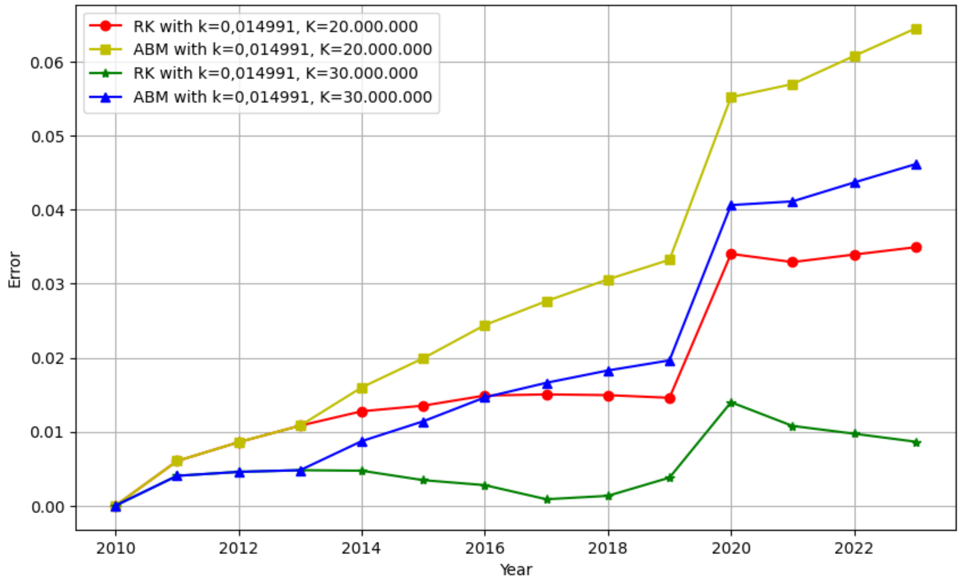

Fig. 4. Error Comparison of RK4 and ABM4 with parameter variations (k=0,01499; $K = 20.000.000$ and $K = 30.000.000$)

Where the values of k and K are 0.01 and 20,000,000, respectively, the lowest error occurred in 2014, with a value of 0.024238. However, the error increases with each subsequent year. Meanwhile, when k and K have the values of 0.014991 and 20,000,000 and 0.014991 and 30,000,000, respectively, the initial error in 2014 is higher. However, the error growth was more stable until 2023.

Using the ABM4 method and the population growth error calculation data with the parameter of k between 0.01 and 0.014991, K remains 20,000,000 or 30,000,000 (see Table VI). This demonstrates that the prediction results have varying error rates. In the case of a k and K of 0.01 and 20,000,000, respectively, the lowest error occurred in 2014, with a value of 0.026387. However, for each year after this, the error increases. Meanwhile, for a k and K with values of 0.014991 and 20,000,000 and 0.014991 and 30,000,000, respectively, the initial error in 2014 is higher. However, the error growth was more stable until 2023.

Table VII details the error calculation data for population growth using the RK4 and ABM4 methods from 2014 to 2023. It shows that the RK4 method has a lower error rate than that of the ABM4 method. For example, in 2023, the error value of the RK4 method when k = 0.014991 and K = 30,000,000 is 0.008635. Moreover, when k = 0.014991 and K = 20,000,000, the error value is 0.034927. Conversely, the ABM4 has a slightly higher error, in that when k = 0.014991

TABLE VI
ERRORS COMPARISON OF ABM4 WITH 3 PARAMETER VARIATIONS

Year	k=0.01and K=20,000,000	k=0,01499 and K=20,000,000	k=0,01499 and K=30,000,000
2010	0.000000	0.000000	0.000000
2011	0.008979	0.006031	0.004028
2012	0.014414	0.008570	0.004562
2013	0.019497	0.010809	0.004790
2014	0.026387	0.015982	0.008715
2015	0.032027	0.019921	0.011397
2016	0.038178	0.024405	0.014627
2017	0.043074	0.027644	0.016600
2018	0.047654	0.030586	0.018268
2019	0.051917	0.033229	0.019631
2020	0.075088	0.055209	0.040626
2021	0.078425	0.056983	0.041124
2022	0.083719	0.060782	0.043679
2023	0.088916	0.064508	0.046160

TABLE VII
ERRORS COMPARISON OF RK4 AND ABM4 WITH 2 PARAMETER VARIATIONS
(K=0.014991 AND (K=20,000,000 OR K=30,000,000))

Year	Population	RK4 k=0.014991 and K=20,000,000	ABM4 k=0.014991 and K=20,000,000	RK4 k=0.014991 and K=30,000,000	ABM4 k=0.014991 and K=30,000,000
2010	8,034,776	0.000000	0.000000	0.000000	0.000000
2011	8,156,129	0.006031	0.006031	0.004028	0.004028
2012	8,250,018	0.008570	0.008570	0.004562	0.004562
2013	8,342,047	0.010809	0.010809	0.004790	0.004790
2014	8,432,163	0.012752	0.015982	0.004715	0.008715
2015	8,512,608	0.013513	0.019921	0.003440	0.011397
2016	8,598,604	0.014875	0.024405	0.002765	0.014627
2017	8,674,372	0.015032	0.027644	0.000858	0.016600
2018	8,748,052	0.014932	0.030586	0.001321	0.018268
2019	8,819,549	0.014573	0.033229	0.003778	0.019631
2020	9,073,509	0.034027	0.055209	0.013975	0.040626
2021	9,139,531	0.032922	0.056983	0.010768	0.041124
2022	9,225,747	0.033937	0.060782	0.009719	0.043679
2023	9,312,019	0.034927	0.064508	0.008635	0.046160

and K = 30,000,000 the error value is 0.046160, and when k = 0.014991 and K = 20,000,000, the error value is 0.064508. Thus, the RK4 method is more stable and accurate in the modeling of population growth compared to the ABM4. However, these results must be interpreted by considering the assumptions and parameters used in both methods.

Figure 4 shows the results of the error calculations in the population growth predictions derived from the RK4 and ABM4 methods from 2014 to 2023. These two numerical methods are used to estimate population growth by comparing the prediction results with the actual data. The figure indicates that, in each year, the error for the RK4 method is lower than that for the ABM4 method. This indicates that the RK4 method provides more accurate predictions in this case. Even though there are fluctuations in the error rates over several years, the overall trend is consistent, in that the RK4 method provides results that are more closely aligned to the actual data compared to the ABM4 method. An analysis of these error differences can be the basis for choosing a more effective numerical method for modeling population growth in this study. In addition to method selection, parameter selection is also very influential and important in this case. The results obtained also remain dependent on the selection of parameters that are appropriate to the characteristics of the observed population growth.

IV. Conclusion

The results of this study on the prediction of population growth using logistic equations demonstrate that the ABM4 method has a fairly good level of stability compared to the RK4 method. However, in terms of the equations' levels of accuracy, the RK4 method shows better performance than the ABM4 method.

In addition, based on the parameter variations, namely the growth rate k and population carrying capacity K, the prediction results using the RK4 and ABM4 methods produce different numerical solutions. In the first case, when k = 0.01 and K = 20,000,000, both models give predictions that are relatively similar to the actual data, but they both have considerable errors. In the second case, when k = 0.014991 and K = 20,000,000, the prediction results from both methods are more similar to the actual data than the results in the first case. In addition, in the third case, when k = 0.014991 and K = 30,000,000, the prediction results are the most similar to the actual data. Therefore, the selection of parameters such as k and K plays an important role in the accuracy of the prediction results using logistic models.

This study successfully shows that the RK4 and ABM4 methods can be used to forecast future population numbers effectively. However, it is important to closely pay attention to the selection of appropriate parameters, as this will have an impact on the accuracy of the prediction results. In addition, the RK4 method was proven to be more effective than the ABM4 method in predicting population growth in South Sulawesi. Therefore, in the same order, the Runge-Kutta method performs better than the Adam-Bashfort-Moulton method in solving differential equations [23].

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