Analysis of Carbon Dioxide Value with Extreme Value Theory Using Generalized Extreme Value Distribution

Pannawit Khamrot, Narin Phankhieo, Piyada Wachirawongsakorn, Supanaree Piros, Natthinee Deetae

Abstract—**This paper applies the generalized extreme value (GEV) distribution using maximum likelihood estimates to analyze extreme carbon dioxide data collected by the Provincial Energy Office of Phitsanulok from 2010 to 2023. The study aims to model return levels for carbon dioxide emissions for the periods of 5, 25, 50, and 100 years, utilizing data from various fuels—Gasohol E85, Gasohol E20, Gasohol 91, Gasohol 95, ULG95, and LPG. By fitting the GEV distribution, this research not only categorizes the behavior of emissions data under different subclasses of the GEV distribution but also confirms the suitability of the GEV model for this dataset. The findings indicate a trend of increasing return levels, suggesting rising peaks in carbon dioxide emissions over time. This model provides a valuable tool for forecasting and managing environmental risks associated with high emission levels.**

Index Terms—**Extreme values theory, Generalized extreme values distribution, Return level, Maximum likelihood estimation, Carbon dioxide value.**

I. Introduction

The air pollution problem has been long-standing. Additionally, an increasing release of carbon dioxide has caused climate change, as seen from an increase in the world's average temperature. In addition, based on the forecast regarding greenhouse gas emissions in Phitsanulok, it is expected that the average global temperature will undoubtedly increase if greenhouse gas emissions continue. Greenhouse gas emissions are caused by an increasing use of fuels, electricity, paper, tap water, and waste. As a result, a continuous increase in carbon dioxide in Phitsanulok exists (Environment and Pollution Control Office 3, Phitsanulok, 2010 to 2023.). Air pollution has caused concerns, and effective air pollution management is required. As a result, the government has stipulated policies to monitor and control the air pollution situation, and the amount of carbon dioxide measured last year decreased by 3.27 percent compared to that of the base year. However, this decrease is probably due to the COVID-19 outbreak, a higher cost of fuel, and less use of energy.

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In the future, the climate change situation will be more severe. Therefore, it is necessary to find ways to reduce the severity to a controllable level or at least slow down the climate change rate. In addition, activities in the energy sector must be improved to suit the current environmental situation, such as adapting to a low-carbon society practice, adopting a more effective form of air pollution forecast, and creating a model for monitoring air pollution. Furthermore, stipulating policies and developing work plans suitable to the area are necessary for efficient energy management [1], [2], [3], [4], [5], [6], [7], [8].

Selecting a more accurate forecasting model in system management leads to appropriate air pollution management planning. In 2022, Ben Clarke et al. [9] studied the sources of global climate change events to explain their connection to the current situation. Related to extreme temperatures, heavy rainfall, drought, power outages, and tropical cyclones cause damage to people around the world. Yonetani T. [10] studied the simulation of the change in the frequency of extreme values and regional characteristics of seasonal temperature and rainfall when CO_2 in the atmosphere increases. Daniel Cooley [11] presented severe climate conditions and used extreme value analysis statistics to estimate climate parameters and predict trends in temperature changes in extreme values. Therefore, it can be seen that air pollution is significant.

It is necessary to consider information regarding a changing trend of air pollution. This kind of data has an extreme value, as a model to predict air pollution data can be created by analyzing the occurrence of abnormally high or low values and employing the data to design a prediction model for events with the highest or lowest values. This has resulted in the formation of a good surveillance model. When an excellent preventive model is in place, it will also lead to the creation of good management policies. The application of extreme value theory was used by Fuller in 1914 [12]. Saraless Nadarajah and Dongseok Choi [13] also conducted research on the basic knowledge of extreme value theory and research that brings extreme value theory, which is used to analyze various aspects of data. For example, in hydrology, Piyapat Busabadin and Arun Kaewman [14] studied the statistics of extreme value for applying extreme value theory to real data in various fields of study. Their study also discussed the concept and development of extreme value theory and performed inferential statistics on extreme values. Distributions of extreme values include generalized extreme value distributions and the generalized Pareto distribution (GPD), and others, as well as checking the appropriateness of the extreme value model, recurrence period, and finding the level of recurrence in a particular year. In 2019, Faithful C. Onwuegbuche et al. [15] applied extreme value theory to predict climate change with extreme rainfall values in Kenya. Mathematical models were used to forecast using generalized extreme value distributions and the Pareto distribution to estimate parameters, period of recurrence, and level of recurrence. In 2015, Chikobvu and Chifurira [16] utilized the generalized extreme value (GEV) distribution to model extreme minimum rainfall patterns in Zimbabwe. Similarly, C. S. Withers and S. Nadarajah [17] observed trends in return levels of daily rainfall in New Zealand by analyzing annual rainfall maxima over time with the GEV distribution in 2000. In 2007, El Adlouni et al. [18] developed generalized maximum likelihood estimators for a nonstationary GEV model. More recently, in 2020, Samuel et al. [19] conducted studies on extreme rainfall in Kaduna, Nigeria, using data from the Nigeria Meteorological Agency. They fitted the monthly rainfall data to the GEV, generalized Pareto distribution (GPD) and calculated return levels. Such research underscores EVT's critical role in designing urban flood control systems and enhancing flood risk management [20], [21], [22], [23], [24], [25], [26], [27].

Based on the air pollution problem that is caused by carbon dioxide, as previously mentioned, the researchers are interested in analyzing and creating a probability model using the amount of carbon dioxide to model and forecast carbon dioxide production based on mathematical and statistical forecasting models. The results of the present study can be used to determine relevant policies and practices for efficiently and appropriately managing air pollution and controlling carbon dioxide emissions in Phitsanulok.

II. Extreme Value Theory

In the extreme value theory, we generalized the extreme values distribution, the details of which are as follows:

A. Generalized Extreme Value (GEV) Distribution

A random variable X follows the GEV distribution, $GEV(x;\mu,\sigma,\xi)$ if its cumulative distribution function (CDF) is given by

$$\exp\{-[1+\xi(x-\mu)/\sigma]^{-1/\xi}\}, \quad (1)$$

for $\xi \neq 0$, and

$$\exp\{-\exp[-(x-\mu)/\sigma]\}, \quad (2)$$

for $\xi \to 0$,

which is defined in the set $\{x : 1+\xi(x-\mu)/\sigma > 0\}$, where $\mu \in \mathbb{R}$ is a location parameter; $\sigma > 0$ is a scale parameter; and $\xi \in \mathbb{R}$ is a shape parameter. The condition for a distribution to belong to any of the distributions of the extreme values is given as follows: $\xi = 0$ for Gumbel distribution; $\xi > 0$ for Fréchet distribution; and $\xi < 0$ for Weibull distribution, as shown in Figure 1.

The corresponding probability density function (PDF) of GEV distribution, $gev(x;\mu,\sigma,\xi)$, is then obtained as

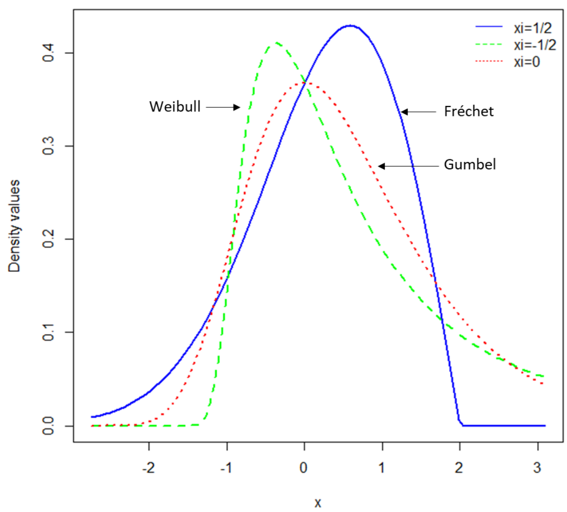

Fig. 1. Examples of generalized extreme value distributions for $\xi = 1/2$ (Fréchet); $\xi = 0$ (Gumbel); and $\xi = -1/2$ (Weibull)

$$\sigma^{-1}[1+\xi(x-\mu)/\sigma]^{-(1/\xi)-1}\exp\{-[1+\xi(x-\mu)/\sigma]^{-1/\xi}\}, \quad (3)$$

for $\xi \neq 0$, and

$$\sigma^{-1}\exp[-(x-\mu)/\sigma]\exp\{-\exp[-(x-\mu)/\sigma]\}, \quad (4)$$

for $\xi \to 0$.

The estimates of extreme quantiles x_p of the GEV distribution are then obtained by inverting $GEV(x;\mu,\sigma,\xi)$,

$$x_p = \begin{cases} \mu + \dfrac{\sigma}{\xi}\left\{[-\log(p)^{-\xi} - 1]\right\}, & \xi \neq 0, \\ \mu - \sigma\log[-\log(p)], & \xi \to 0. \end{cases} \quad (5)$$

Estimating the unknown parameters of the GEV distribution above leads to finding the maximum likelihood estimation (MLE) of GEV distribution, which can be written as:

$$\begin{aligned} L(\mu,\sigma,\xi) =& \prod_{i=1}^{n}\left(\frac{1}{\sigma}\left(1+\xi\left(\frac{z-\mu}{\sigma}\right)\right)\right)^{-1/\xi+1} \\ & \exp\left(-\left(1+\xi\left(\frac{z-\mu}{\sigma}\right)^{-1/\xi}\right)\right) \\ =& \frac{1}{\sigma^n}\prod_{i=1}^{n}\left(1+\xi\left(\frac{z-\mu}{\sigma}\right)\right)^{-1/\xi+1} \\ & \exp\left(-\left(1+\xi\left(\frac{z-\mu}{\sigma}\right)^{-1/\xi}\right)\right), \end{aligned} \quad (6)$$

and

$$\begin{aligned} l(\mu,\sigma,\xi) =& -n\log\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\log\left(1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right) \\ & -\sum_{i=1}^{n}\log\left(1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right)^{1/\xi}, \end{aligned} \quad (7)$$

where $1+\xi\left(\dfrac{z_i-\mu}{\sigma}\right) > 0$ and $\xi \neq 0$.

Return periods can calculate the return level R_T at T of GEV as follows:

$$R_T = \mu - \frac{\sigma}{\xi}\left(1 - \left(-\log\left(1 - \frac{1}{T}\right)\right)^{-\xi}\right), \qquad (8)$$

for $T = N \times n_y$ when n_y is one observation per year, and N is a number of years.

The confidence interval of the return level for GEV distribution is derived by utilizing the Delta method as follows:

$$Var(R_T) \approx \nabla R_T^t V \nabla R_T, \qquad (9)$$

where V is a covariance matrix of $(\mu, \sigma, \xi)^t$ and

$$\nabla R_T^t = [\frac{\partial R_T}{\partial \mu}, \frac{\partial R_T}{\partial \sigma}, \frac{\partial R_T}{\partial \xi}].$$

Where

$$\begin{aligned}\frac{\partial R_T}{\partial \mu} =& 1,\\ \frac{\partial R_T}{\partial \sigma} =& -\frac{1}{\xi}(1 - T^{-\xi}),\\ \frac{\partial R_T}{\partial \xi} =& \sigma\xi^{-2}(1 - T^{-\xi}) - \sigma\xi^{-1}T^{-\xi}\log T.\end{aligned}$$

The Wald method is commonly used to construct confidence intervals for parameters of interest. In the context of extreme value analysis with the GEV distributions, the Wald confidence interval for a parameter θ is also implemented as $\theta \pm Z_{\alpha/2} \times \sqrt{Var(R_T)}$, where $Var(R_T)$ is the estimated variance from the Delta method.

III. Data and Research Methodology

The study focused on Phitsanulok, a province situated either in the upper central or lower northern region of Thailand. It encompasses an area of 10,815.8 square kilometers (6,759,909 rai), which constituting 6.4 percent of the northern region and 2.1 percent of Thailand's total area. It is flanked by Uttaradit province to the north, Phichit to the south, Phetchabun to the east, and Kamphaeng Phet and Sukhothai to the west. The climate of Phitsanulok is marked by a hot and windless rainy season that is often overcast, followed by a hot, humid, and partly cloudy dry season. Temperatures generally range between 19°C and 37°C throughout the year, and it rarely falls below 15°C or rising above 39°C.

The present study employed data regarding the annual carbon dioxide emissions caused by the use of various fuels, including Gasohol E85, Gasohol E20, Gasohol 91, Gasohol 95, ULG 95, and LPG. The data were collected by the Provincial Energy Office of Phitsanulok in the past 14 years (from 2010 to 2023). Employing the data regarding carbon dioxide emissions provided by the Provincial Energy Office of Phitsanulok, an extreme value analysis was conducted by fitting the generalized extreme value distribution using the maximum likelihood estimation method. This analysis considered various families of distributions within the generalized extreme value framework.

The researchers employed the R package for extremes, developed by Gilleland et al. [28], and designed to conduct parametric inferential analysis on the GEV distribution. The findings of the present research are presented in four sections, each of which focuses on analyzing the generalized extreme value distribution as follows:

A. Analysis of the generalized extreme value distribution

This section will explain the fundamental principles of conducting normality tests on GEV distributions. The analysis utilizes carbon dioxide data collected by the Provincial Energy Office of Phitsanulok from 2010 to 2023, which represents the most reliable annual data available. Table I provides a statistical summary of these data.

TABLE I
A STATISTICAL SUMMARY OF THE ANNUAL CARBON EMISSION DIOXIDE DATA (10^6 UNIT) OF PHITSANULOK FROM 2010 TO 2023

Type	min	max	mean	sd	sk	ku
Gasohol E85	0.28	11.35	6.75	3.55	-0.62	-0.90
Gasohol E20	22.83	53.08	39.77	10.06	-0.33	-1.48
Gasohol91	33.88	102.19	70.67	21.67	-0.19	-1.31
Gasohol95	31.93	154.45	88.54	36.82	0.14	-1.14
ULG95	0.57	11.08	0.38	3.19	-0.59	-0.70
LPG	40.90	194.48	96.73	35.44	1.16	1.66

The annual accumulation of carbon dioxide in Phitsanulok shows an increasing trend. Figures 2 and 3 illustrate the carbon dioxide accumulation trends of each fuel type.

B. Estimating parameters with the GEV distribution.

Next, the researchers calculated the estimated parameters (μ, σ, ξ) and standard errors (SE) for Gasohol E85, Gasohol E20, Gasohol 91, Gasohol 95, ULG95, and LPG using the GEV distribution in Table II.

TABLE II
ESTIMATED PARAMETERS (10^6 UNIT) FOR GASOHOL E85, GASOHOL E20, GASOHOL 91, GASOHOL 95, ULG95, AND LPG USING THE GEV DISTRIBUTION

Type	Dist.	μ(SE)	σ(SE)	ξ(SE)
Gasohol E85	Weibull	6.34(1.31)	3.89(1.23)	−0.74(0.32)
Gasohol E20	Weibull	38.64(3.84)	11.27(3.73)	−0.75(0.35)
Gasohol91	Gumbel	66.10(7.15)	23.08(6.12)	−0.55(0.30)
Gasohol95	Gumbel	75.75(10.65)	34.36(7.97)	−0.27(0.25)
ULG95	Weibull	7.44(0.00)	3.90(0.00)	−1.07(0.00)
LPG	Gumbel	26.85(7.94)	27.08(5.49)	−0.03(0.15)
Total_{CO_2}	Weibull	82.09(0.00)	10.53(0.00)	−1.13(0.00)

Table II indicates the results of the GEV distribution of three subclasses (Fréchet ($\xi > 0$), Gumbel ($\xi = 0$), and Weibull ($\xi < 0$) distribution) with a consideration of 95% confidence intervals (CI) of ξ, whose details are as follows:

- Gasohol E85 is $(\mu, \sigma, \xi) = (6.34, 3.89, -0.74)$ with standard errors (1.31,1.23,0.32), and so it is the Weibull distribution with $(-1.3672, -0.1128)$ of ξ.
- Gasohol E20 is $(\mu, \sigma, \xi) = (38.64, 11.27, -0.75)$ with standard errors (3.84,3.73,0.35), and so it is the Weibull distribution with $(-1.4360, -0.0640)$ of ξ.
- Gasohol91 is$(\mu, \sigma, \xi) = (66.10, 23.08, -0.55)$ with standard errors (7.15,6.12,0.30), and so it is the Gumbel distribution with $(-1.1380, 0.0380)$ of ξ.
- Gasohol95 is $(\mu, \sigma, \xi) = (75.75, 34.36, -0.27)$ with standard errors (10.65,7.97,0.25), and so it is the Gumbel distribution with $(-1.1380, 0.0380)$ of ξ.
- ULG95 is $(\mu, \sigma, \xi) = (7.44, 3.90, -1.07)$ with standard errors all with zero uncertainty, and so it is the Weibull distribution with $(-1.0700, -1.0700)$ of ξ.

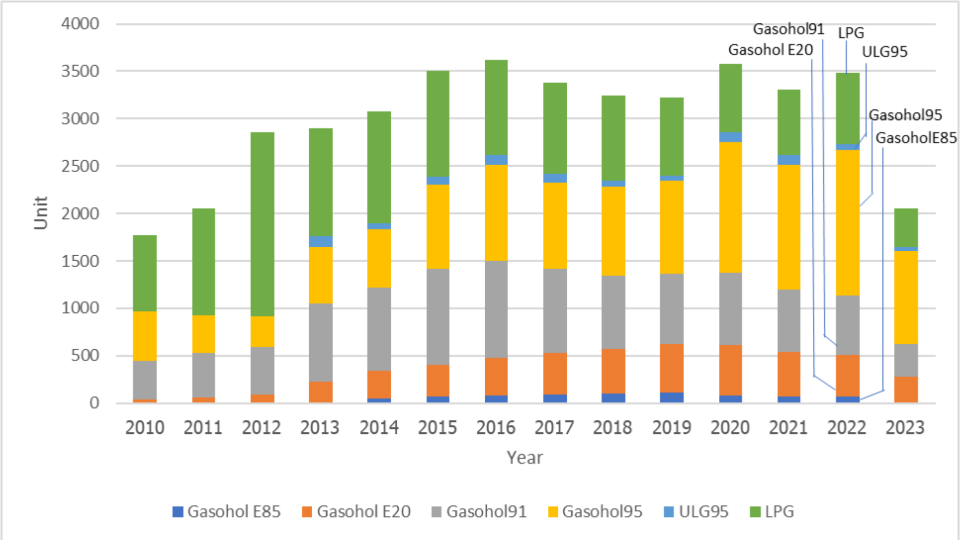

Fig. 2. The accumulation of annual carbon dioxide in Phitsanulok Province

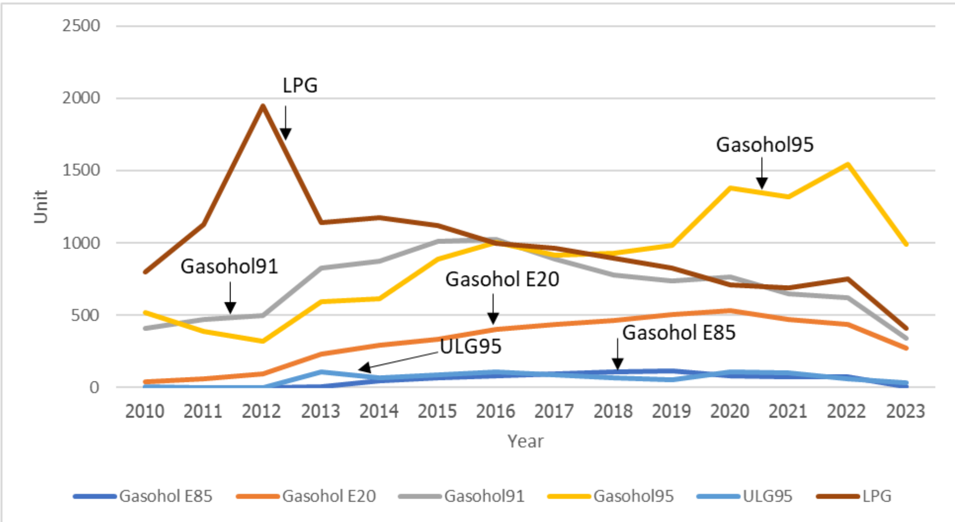

Fig. 3. The trends of annual carbon dioxide type in Phitsanulok Province

- LPG is (μ, σ, ξ) = $(26.85, 27.08, -0.03)$ with standard errors (7.94,5.49,0.15), and so it is the Gumbel distribution with $(-0.3240, 0.2640)$ of ξ.
- Total$_{CO_2}$ is an overview of all types of carbon dioxide formation, (μ, σ, ξ) = $(82.09, 10.53, -1.13)$ with standard errors all with zero uncertainty, and so it is the Weibull distribution with $(-1.13, -1.13)$ of ξ (10^7 UNIT).

These parameter estimates indicate the variability and distribution characteristics of the different fuel types as modeled under the generalized extreme value distribution from the Total$_{Co_2}$ the following:

$$Total_{CO_2} = \exp\left\{-\left[1 - \frac{1.13(x - 82.09)}{10.53}\right]^{\frac{1}{1.13}}\right\}.$$

C. Goodness-of-fit tests

The researchers conducted rigorous goodness-of-fit tests to determine the distribution that most closely matches the data. To do so, the researcher employed the Kolmogorov-Smirnov (KS) test and the Anderson-Darling (AD) test, where smaller statistic values suggest a better fit of the distribution to the data. Let X be a continuous random variable with the distribution function $F(x)$, and let $X_1, X_2, X_3, ..., X_n$ be a random sample from X with order statistics $X_{(1)}, X_{(2)}, X_{(3)}, ..., X_{(n)}$. The researchers calculated the KS and AD statistics as follows:

$$KS^2 = (\max_{1\leq i\leq n}[\max\{\frac{i}{n} - F_0(X_{(i)}), F_0(X_{(i)} - \frac{i-1}{n}\}])^2$$

and

$$AD^2 = -n - S,$$

where $S = \sum_{i=1}^{n} \frac{2i-1}{n}[\ln(F(X_{(i)})) + \ln(X_{n+1-i})].$

The goodness-of-fit tests for the GEV distribution applied to carbon dioxide data are summarized in Table III. The tests include the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests, which assess the fit of the GEV distribution across various fuel types. The results suggest that the GEV distribution generally fits all data types well.

TABLE III
GOODNESS-OF-FIT TEST

Type	KS	p-value	AD	p-value
Gasohol E85	0.177	0.824	0.317	0.923
Gasohol E20	0.132	0.978	0.249	0.971
Gasohol91	0.121	0.970	0.261	0.964
Gasohol95	0.152	0.857	0.336	0.908
ULG95	0.190	0.713	3.301	0.020
LPG	0.161	0.807	0.353	0.892
Total$_{CO_2}$	0.201	0.556	2.829	0.034

The Kolmogorov-Smirnov test, less sensitive for normal distributions, evaluates the empirical cumulative distribution function to determine if samples are derived from the hypothesized continuous distribution. Results indicate a p-value of all data over 0.05, suggesting the proposed distribution is appropriate. However, in ULG95 and Total$_{CO_2}$, the p-value of the AD test is significant. These findings support the assumption that the data are independent and identically distributed (i.i.d) with non-significant monotonic trends in the data.

D. Return levels

In this section, the researchers explored the estimation of return levels using the maximum likelihood method. This method was used to predict the probability of carbon dioxide emission scenarios over the next 5, 25, 50, and 100 years, as detailed in Table IV. These return level estimates help quantify the expected maximum emission levels for each period under the GEV distribution.

In Table IV, these return level estimates help quantify the expected maximum emission levels for each period under the GEV distribution. These values are crucial for assessing emissions' risk and potential impact on environmental strategies as follows:

- Gasoline E85, Gasoline E20, Gasoline 91, Gasoline 95, ULG95, and LPG gradually increase with the return period, indicating that higher emission levels are expected over longer intervals. For instance, Gasohol E85 has an estimated return level of 9.86 million for five years and increases to 11.40 million for 100 years. This pattern suggests that emissions might increase, or rare high-emission events are more likely as the time horizon extends.
- Estimates of Total$_{CO_2}$ show a significant increase with an extended payback period. It starts at 349.05 million for five years and increases to 361.20 million for 100 years. This indicates the trend of climate change, variability, and a large amount of greenhouse gas emissions over time.

The return level is available for environmental planning and policy-making, as it helps predict future emission scenarios and facilitates the development of strategies to mitigate the highest risks associated with these emissions. An overview of the return level is shown in Figure 4.

Next, the researchers present the diagnostic plots of the GEV distribution, consisting of a probability plot, a quantile plot, a return level plot, and a density plot for various fuels—Gasoline E85, Gasoline E20, Gasoline 91, Gasoline 95, ULG95, LPG, and Total$_{Co_2}$, as shown Figures 5 to 11.

Figures 5 to 11 display the diagnostic plots for the GEV distribution. The probability and quantile plot points lie close to the diagonal unit, indicating that the generalized extreme value distribution function fits well, as shown in Table III. The return level plot demonstrates that the empirical return levels closely match those from the fitted distribution function, as seen in Table IV. Additionally, the density plot reveals a good agreement between the fitted GEV distribution function and the empirical density from Table II, which includes subclasses with Weibull (Figures 5, 6, 9, and 11) and Gumbel distributions (Figures 7, 8, and 10), whose graphs also provide an overview of the efficiency of the GEV distribution for each type of carbon dioxide.

IV. CONCLUSION

This study provides insights into the behavior of high carbon dioxide emissions using extreme value theory, based on annual carbon dioxide data from 2010 to 2023 in Phitsanulok obtained from the Provincial Energy Office of Phitsanulok. The analysis, conducted using the GEV distribution, informs governmental and non-governmental organizations and enables them to make informed decisions, and prepare strategies to address the impacts of peak carbon dioxide levels. The results offer parameter estimates for return levels at 5, 25, 50, and 100 years, revealing an upward trend in carbon dioxide emissions. This information is crucial for preparing the public for environmental changes driven by increased carbon dioxide emissions.

For decision-makers, the advantages of this analysis include developing early warning systems, enhancing strategies for climate change mitigation, improving public health measures, and bolstering disaster and risk management. Additionally, by applying extreme value theory, an understanding can be expanded, and the efficiency of various theoretical models across these domains can be increased.

APPENDIX

Figures 5 to 11 show the diagnostic plots for the goodness-of-fit test for each type of carbon dioxide value.

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TABLE IV
RETURN LEVEL ESTIMATES (10^6 UNIT) AT SELECTED RETURN PERIODS FOR THE GEV DISTRIBUTIONS

Type	Return levels (years) and 95% CI			
	5	25	50	100
Gasohol E85	9.86(8.29,11.43)	11.09(10.37,11.80)	11.29(10.54,12.03)	11.40(10.53,12.27)
Gasohol E20	48.80(44.06,53.55)	52.34(50.27,54.41)	52.91(50.70,55.12)	53.24(49.53,55.87)
Gasohol91	89.68(79.05,100.32)	100.86(92.02,109.70)	103.19(92.15,114.22)	104.75(90.53,118.23)
Gasohol95	118.27(95.85,140.69)	149.83(115.61,184.04)	159.26(114.06,204.45)	167.03(113.53,224.57)
ULG95	10.33(10.33,10.33)	10.95(10.95,10.95)	11.02(11.02,11.02)	11.05(10.53,11.57)
LPG	121.76(97.62,145.90)	164.82(117.13,212.51)	182.06(118.76,245.36)	198.84(113.53,281.01)
Total$_{CO_2}$	349.05(349.05,349.05)	359.65(359.65,359.65)	360.71(360.71,360.71)	361.20(361.20,361.20)

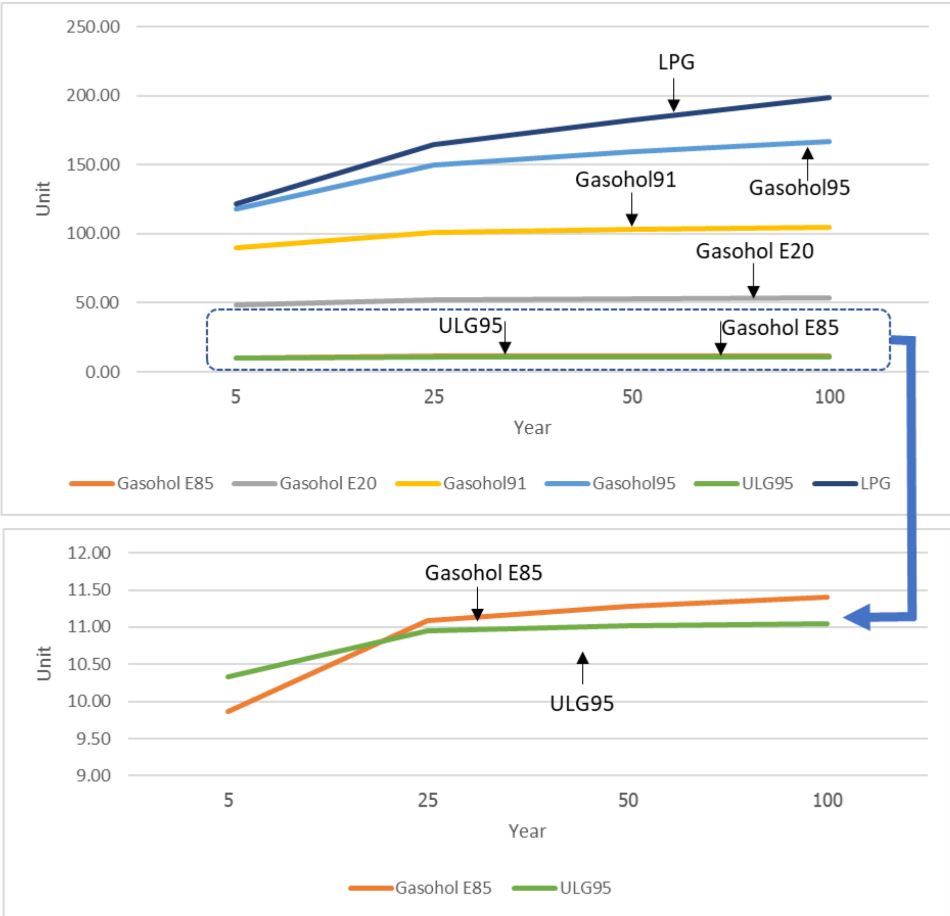

Fig. 4. The trends of return level estimates of carbon dioxide types

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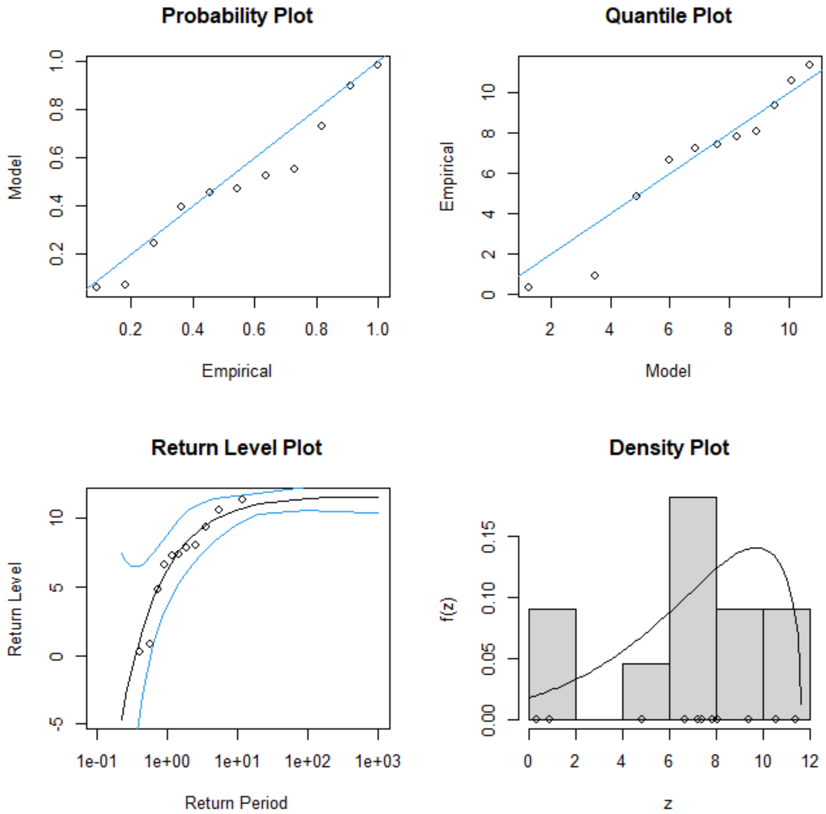

Fig. 5. Diagnostic plots of GEV distribution, applying to annual Gasohol E85 for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

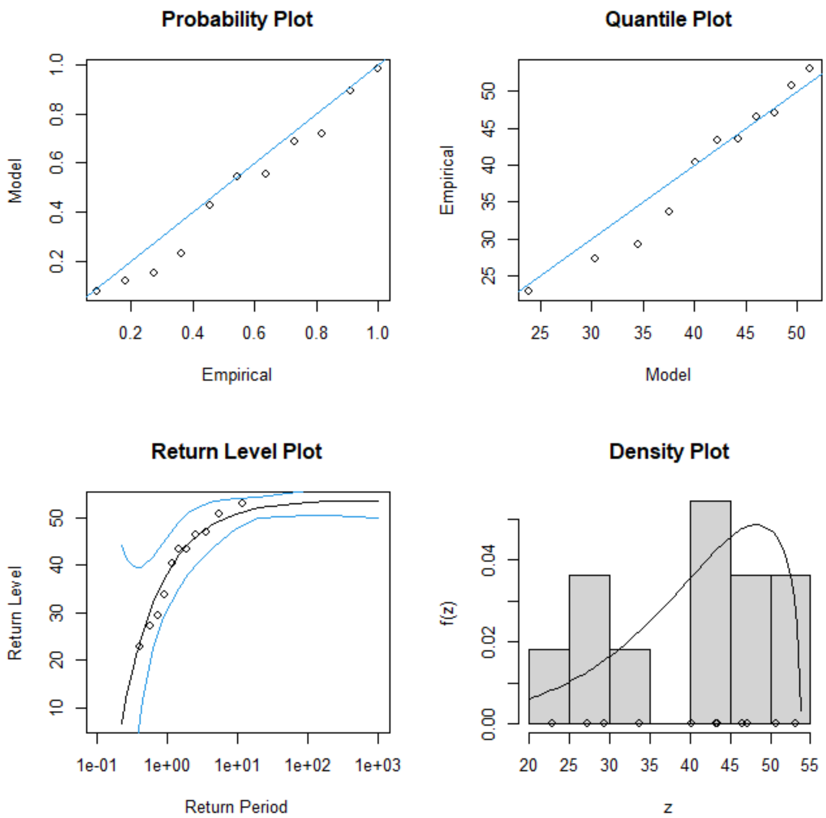

Fig. 6. Diagnostic plots of GEV distribution, applying to annual Gasohol E20 for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

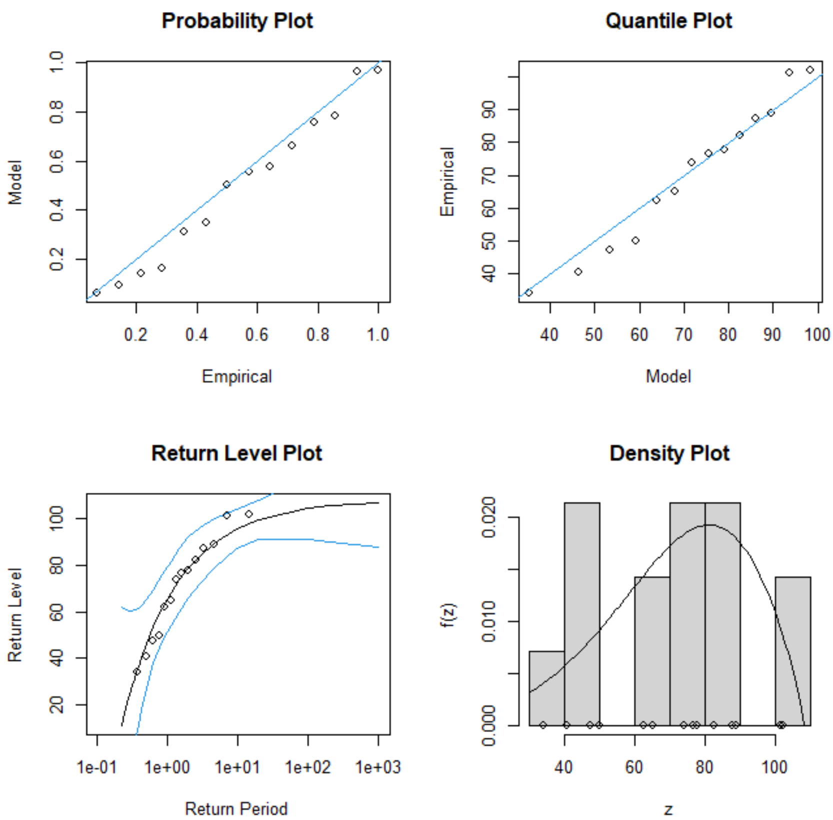

Fig. 7. Diagnostic plots of GEV distribution, applying to annual Gasohol91 for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

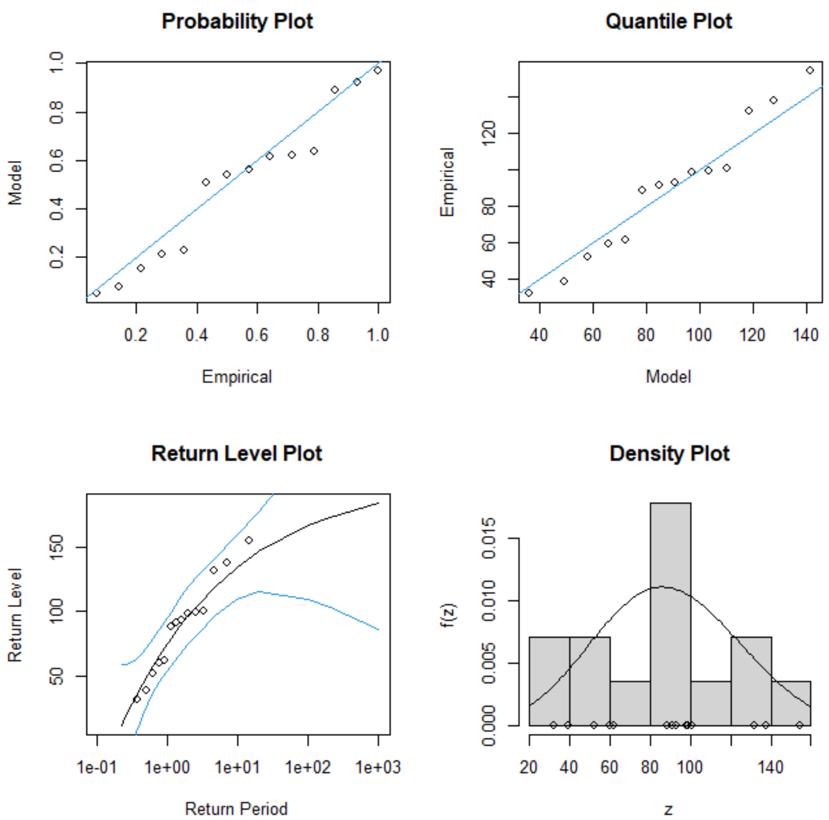

Fig. 8. Diagnostic plots of GEV distribution, applying to annual Gasohol95 for Phitsanulok for the 14-year period from 2010 to 2023for the GEV

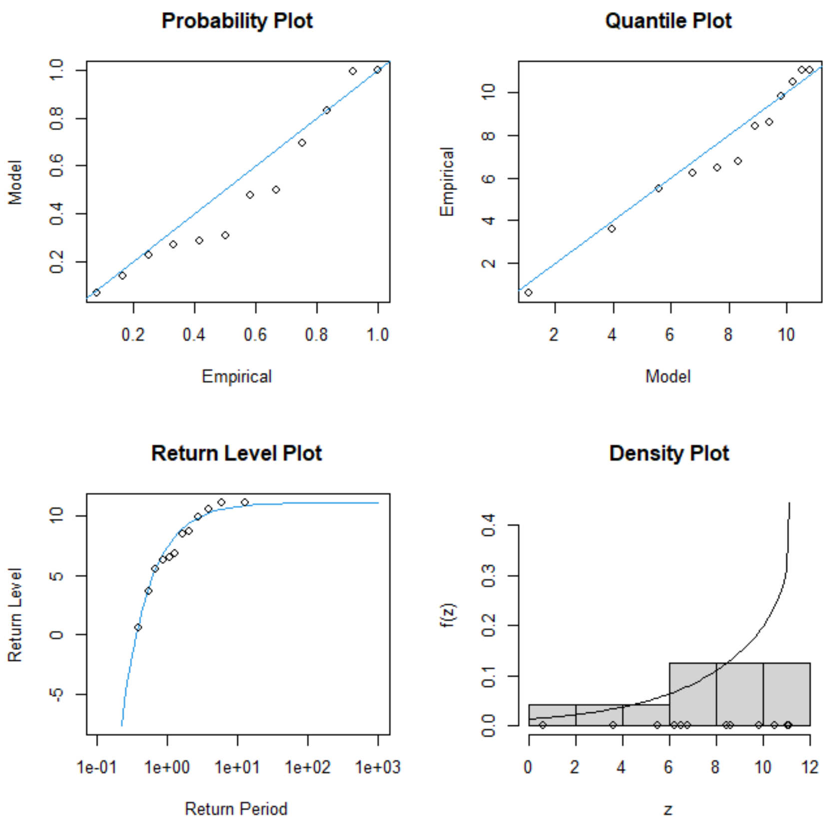

Fig. 9. Diagnostic plots of GEV distribution, applying to annual ULG95 for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

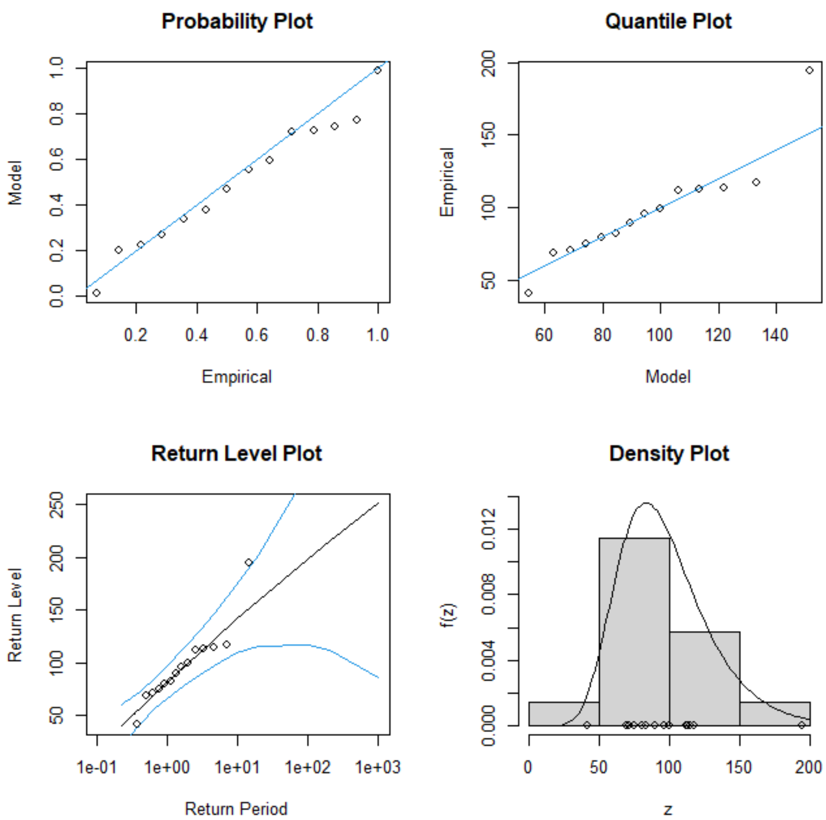

Fig. 10. Diagnostic plots of GEV distribution, applying to annual LPG for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

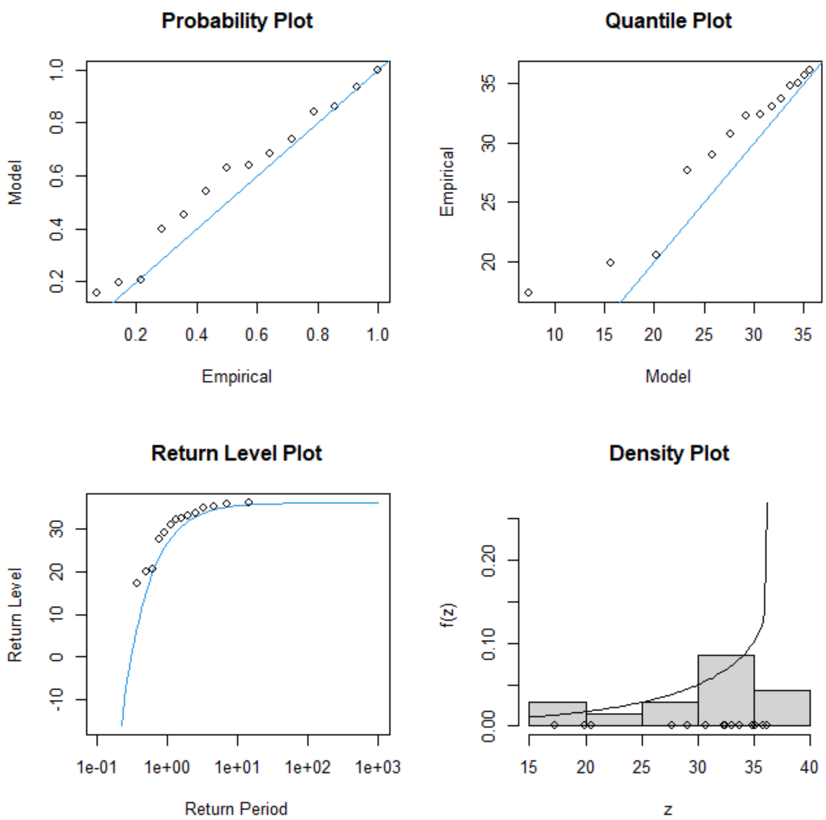

Fig. 11. Diagnostic plots of GEV distribution, applying to annual Total$_{CO_2}$ for Phitsanulok for the 14-year period from 2010 to 2023 for the GEV

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