Dihedral Group Divisor Cordial Labeling For Path, Cycle Graph, Star Graph, Jelly Fish And Wheel Graphs

A. Sudha Rani and S. Sindu Devi

***Abstract*—Let G=(V,E) be a graph. In the D_3,the order of any element x in D_3 is denoted by O(x).We define a function χ:V(G)→D_3 such that for each edge xy we assign the label 0 if o(u)/o(v) or o(v)/o(u) assign the label 1 if o(u) not divides o(v). Let us define the new function $\mu\alpha(\chi)$ which represent number of edges of G having label α under the mapping χ. Now χ is called D3 divisor cordial labeling if $|\mu 1(\chi) - \mu 0(\chi)| \leq 1$ and $|V\chi(a) - V\chi(b)| \leq 1$ where $V\chi(a)$ represents number of vertices having label a under χ .The graph which satisfies above condition is calledD$_3$divisor cordial labeling graph. Here we discuss the Dihedral group divisor Cordial Labeling of Path, Cycle , star graph, jelly fish graph and wheel graphs.**

***Index Terms*—D_3, order of an element, divisor Cordial Labeling, path, cycle, star graphs, jelly fish graph, wheel graph.**

I. Introduction

A graph G (V, E) is a finite, simple, undirected graph. Graph labeling is the process of assigning integers to a graph's vertices, edges, or both under particular circumstances. The majority of graph labeling issues share the following three traits: Vertex labels are assigned using a set of integers, each edge is given a label according to a rule, and these labels must meet certain requirements. The application of graph labeling innumerous academic disciplines makes it significant. Coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, database management, secret sharing methods, and models for constraint programming over finite domains are a few fields where graph labeling is frequently used. Labeling of graphs can be done in a variety of sectors, but it mostly concentrates on essential ones like datamining, image processing, cryptography, software testing, information security, communication networks, etc. Engineering studies cover a variety of topics that are more effectively utilized in a variety of areas, such as the government and business. Each of these subjects has a notion and a method that relate to graph labeling. Future developments in cloud computing, signal processing, etc. should be applied to graph labeling.We adhere to Gross and Yellen [1]'s nomenclature and notational guidelines. We refer Beineke and Hegde [2]'s strongly multiplicative graphs. For further graphs, we refer Gallian [3]'s "Dynamic Survey of Graph Labeling." We also refer Cahit [4], "Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs for cordial labeling. Numbers and graph structure have a tight relationship thanks to graph labeling. When the vertices of the graph are assigned values according to specific conditions, it is referred to a as graph Labeling. The following three common conditions are used in most of the graph labeling problems. 1. a set of numbers used for assigning vertex labels; 2. a rule that assigns a label to every edge; 3. some condition(s) that these labels must satisfy. Labeled graphs have applications in many diverse fields." A detailed study on variety of applications of graph labeling is reported in Bloom et al. [15]. they make it easier for integers to be encoded in the best nonstandard ways. They have also been used to solve additive number theorem difficulties, create a communication network addressing system, determine the best circuit layouts, and resolve ambiguities in X-ray crystallographic research.

We refer M.Sundaram, Ponrajand Somasundram [5] for prime cordial labeling of graphs, and also refer many journals based on prime cordial labeling of graphs. In this paper we introduced a concept Matrix Cordial Labeling. Vaidya and vihol [7] show that the square graph of Bn,n is a prime cordial graph while middle graph of P_n is a prime cordial graph for $n \geq 4$. Vaidya and Shah [8] demonstrate that the center graph of is a prime cordial graph for and that the square graph is a prime cordial graph. Also they provide an approach for expanding a given prime cordial graph using other prime cordial graphs. Additionally, we have looked into the degree splitting and double fan graphs of path and bistar's prime cordial labeling. Kuo, Chang and Kwong [9] determines all m and n for which mKn is cordial Hei-Yenlee, Hsun-Minglee and Gearardj. Chang [10] prove that acomplete n-partite graph G is cordial if and only if there are at most three parts of G having an odd number of vertices, that the Cartesian product of any n paths is cordial, and that the Cartesian product of any n cycles is cordial if there is a cycle of length 4m or if there are two even cycles. Vaidhya et al. [6] proved prime cordial labeling for some graphs. AmitH.Rokad, KalpeshM.Patadiya [11] they proved some splitting graphs are cordial labeling. We refer Umamaheswari et al. [12] they used appropriate examples to demonstrate how the graph (G) cycle with zigzag chords is a vertex even mean graph, vertex odd mean graph, square sum graph, and square difference graph. Additionally, they demonstrated with relevant instances that the graph, cycle with zigzag chords, admits cube sum labeling, cube difference labeling, strongly multiplicative labeling, and strongly *labeling. Abdel-Aal [13] demonstrate that the odd harmonic graphs formed by the Cartesian product of cycle graph Cm and path P_n for each $n \geq 2m \equiv 0$ (mod)4. We refer Vidhya et al. [14]

Manuscript received October 12, 2023; revised August 17, 2024.

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they demonstrate that product cordial labeling is permitted on the graph formed by linking two copies of Cn(Cn) with a path of any length. We refer Shee et al. [16] they proved how the solution of a system containing an equation and two inequalities is related to the cordiality of the path-union of n copies of a graph and provide some necessary conditions for that path-union. It is demonstrated that the path-unions of several graphs, including some cliques, cycles, wheels, fans, and compositions of some graphs, are amicable. Sundaram et al. [17] generate some product cordial graphs from a gear graph that has had its vertices switched.

II. PRELIMINARIES

This section contains basic definitions that are needed through out this paper.

Scientific Definition 2.1

A Symmetric triangle group D_3, is the group of symmetries of a regular polygon, which includes rotations and reflections.

Scientific Definition 2.2

In a group for any element x, the order of an element a is denoted by $O(x)$, which is defined by the least positive integer α such that x^{α} = identity element.

Scientific Definition 2.3

Let $\eta : (G) \to \{0, 1\}$ in such a way, the edge $\alpha\beta$ is labeled as $|\eta(\alpha) - \eta(\beta)|$. S is called cordial labelling if the difference between the number of vertices labelled 0 and the number of vertices labelled 1 is ≤ 1 and the difference between the number of vertices labelled 0 and the number of vertices labelled 1 is ≤ 1.

III. MAIN RESULT

Scientific Definition 3.1

Let $\chi : (G) \to D_3$ be a mapping such that for each edge ab we assign the label 0 if $o(u)/o(v)$ and assign the label 1 if $o(u)$ not divides $o(v)$. Let us define the new function $\mu(\chi)$ which represent number of edges of G having label α under the mapping χ. Now χ is called Dihedral group divisor cordial labeling if $|\mu_1(\chi) - \mu_0(\chi)| \leq 1$ and $|V_{\chi}(a) - V_{\chi}(b)| \leq 1$ where $V_{\chi}(a)$ represents number of vertices having label a under χ. The graph which satisfies above condition is called Dihedral group divisor cordial labeling graph.

Scientific Definition 3.2

Consider the dihedral group D_3. Let the elements of D_3 are $I, R_1, R_2, F_1, F_2, F_3$ where $I = (1)(2)(3)$, $R_1 = (123), R_2 = (132), F_1 = (12), F_2 = (13), F_3 = (23)$. Order of each element is given below $O(I) = 1, O(R_1) = 3, O(R_2) = 3, O(F_1) = 2, O(F_2) = 2, O(F_3) = 2$.

Axiom 3.1.1: All path graph P_n is Dihedral divisor cordial labeling graph.

Proof: Let $G = P_n$ be a path graph whose vertices are given by $x_1, x_2, \ldots x_n$ and edges are

$$E(P_n) = x_i x_{i+1},\ 1 \leq i \leq n-1$$

For $2 \leq m \leq 5$

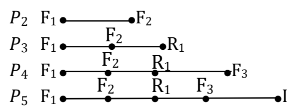

Fig. 1. Path P_m: $2 \leq m \leq 5$

Figure 1 discuss with path graph for P_2, P_3, P_4, P_5. For $m \geq 6$ we discuss six conditions. ■

Condition 1: $m = 6q$

Let m = 6q, q $\geq$ 1. We allotted the label to $\chi(x_i)$ as $F_1$when i = 6q+1, R_1 when i = 6q+2, F_2 when i = 6q+3, F_3 when i = 6q+4, I when i = 6q+5, R_2 when i = 6q, and i varies from 1 to 6q. In this case each vertex label will appear a times in the dihedral group D_3divisor cordial labeling and each edge labeled as 0 will appear 3q - 1 times and 1 will appear 3q times respectively. Hence in this case we get as Dihedral group D_3 divisor cordial labeling.

Condition 2: $m = 6q + 1$

Let m=6q+1, q $\geq$ 1, for 1 $\geq$ i $\geq$ 6q ,we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_1 for $\chi(x_i)$ when i is 6q + 1.In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except $\{F_1\}$ which will appear q + 1 times, and each edge label 1 will appear 3q times and label 0 will appear 3q times in the dihedral group $\{I, R_1, R_2, F_1, F_2, F_3\}$ divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 3: $m = 6q + 2$

Let m=6q+2, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_1 for $\chi(x_i)$ when i is 6q + 1, labeled as F_2 for $\chi(x_i)$ when i is 6q+ 2. In this case each vertex label will appear q + 1 times in the dihedral group D_3 divisor cordial labeling except $\{F_1, F_2\}$ which will appear q times, and each edge label 1 will appear 3q times and label 0 will appear 3q+1 times in the dihedral group $\{I, R_1, R_2, F_1, F_2, F_3\}$ divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 4: $m = 6q + 3$

Let m=6q+3, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1. The remaining vertices are labeled as $F_2$for (x_i) when i is 6q + 1, labeled as R_1 for $\chi(x_i)$ when i is 6q + 2 and also we assign the label R_2 for $\chi(x_i)$ when i is 6q + 3. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except $\{F_2, R_1, R_2\}$ which will appear q+1 times, and each edge label 1 will appear 3q + 1times and label 0 will appear 3q + 1 times in the dihedral group $\{I, R_1, R_2, F_1, F_2, F_3\}$ divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 5: $m = 6q + 4$

Let m=6q+4, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1. The remaining vertices are labeled as F_1 for $\chi(x_i)$ when i is 6q + 1, labeled as F_3 for $\chi(x_i)$ when i is 6q + 2, labeled R_1 for $\chi(x_i)$ when i is 6q + 3 and also we assign R_2 when i is 6q + 4. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_2,R_1,R_2,F_3} which will appear q+1 times, and each edge label 1 will appear 3q + 1 times and label 0 will appear 3q + 2 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 6: $m = 6q + 5$

Let m = 6q + 5, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_2 for $\chi(x_i)$ when i is 6q + 1, labeled as F_3 for $\chi(x_i)$ when i is 6q + 2,labeled $R_1$for $\chi(x_i)$ when i is 6q + 3 and also we assign $F_1$when i is 6q + 4 ,we assign I when I is 6q+5 .In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_2,F_3,R_1,F_1,I} which will appear q+1 times, and each edge label 1 will appear 3q + 2 times and label 0 will appear 3q + 2 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Table I it is clear that all graphs P_m are Dihedral group divisor cordial labeling graph.

Hence all path graphs P_m are Dihedral group divisor cordial labeling graph.

For P_5 let us see Figure 2

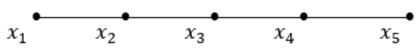

Fig. 2. Path P_5

The number of edges labeled 1 is 2 and the number of edges labeled 0 is 2. Thus $|\mu_1(\chi) - \mu_0(\chi)| = |2 - 2| = |0| = 0 \leq 1|$ and $V_\chi(a) - V_\chi(b)| \leq 1$. Hence the path graph P_5 is Dihedral group divisor cordial labeling graph.

Axiom 3.1.2: All cycles $C_m \geq 3$ are Dihedral group divisor cordial labeling graph.

Proof: Let $G = C_m$ be a cycle whose vertices are given by $x_1, x_2, \ldots x_m$ and edges are

$$(C_m) = \{x_i x_{i+1}, 1 \leq i \leq m-1\} \cup \{x_m x_1\}.$$

For $3 \leq m \leq 5$,
Figure 3 discuss with path graph for C_3, C_4, C_5.

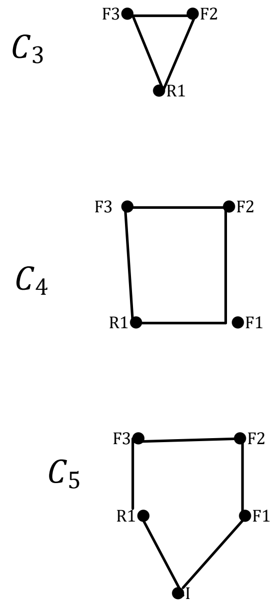

Fig. 3. Cycle C_m: $3 \leq m \leq 5$

For $m \geq 6$ we discuss six conditions. ■

Condition 1: $m = 6q$

Let m = 6q, q $\geq$ 1. We allotted the label to $\chi(x_i)$ as F_2 when i is 6q+1, R_2 when i is 6q+2, F_3 when i is 6q+3, F_1 when i is 6q+4, R_1 when i is 6q+5, I when i is 6q, and i varies from 1 to 6q. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling and each edge labeled as 0 will appear 3q times and 1 will appear 3q times respectively. Hence in this case we get as Dihedral group D_3 divisor cordial labeling.

Condition 2: $m = 6q + 1$

Let m = 6q + 1, q $\geq$ 1, for 1 $\leq$ i $\leq$ 6q ,we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_3 for $\chi(x_i)$ when i is 6q + 1 . In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_3} which will appear q + 1 times, and each edge label 1 will appear 3q+1 times and label 0 will appear 3q times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3divisor cordial labeling.

TABLE I
REFERS NUMBER OF EDGES LABELED 0 AND 1, NUMBER OF TIMES THE VERTICES WILL APPEAR

Real m & $q \geq 1$	$\mu_1(\chi)$	$\mu_0(\chi)$	$V_\chi(F_1)$	$V_\chi(R_1)$	$V_\chi(F_2)$	$V_\chi(F_3)$	$V_\chi(I)$	$V_\chi(R_2)$
$6q$	$3q$	$3q-1$	q	q	q	q	q	q
$6q+1$	$3q$	$3q$	$q+1$	q	q	q	q	q
$6q+2$	$3q$	$3q+1$	q	$q+1$	q	$q+1$	$q+1$	$q+1$
$6q+3$	$3q+1$	$3q+1$	q	$q+1$	$q+1$	q	q	$q+1$
$6q+4$	$3q+1$	$3q+2$	q	$q+1$	$q+1$	$q+1$	q	$q+1$
$6q+5$	$3q+2$	$3q+2$	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	q

TABLE II
REFERS NUMBER OF EDGES LABELED 0 AND 1, NUMBER OF TIMES THE VERTICES WILL APPEAR

Real m & $q \geq 1$	$\mu_1(\chi)$	$\mu_0(\chi)$	$V_\chi(F_1)$	$V_\chi(R_1)$	$V_\chi(F_2)$	$V_\chi(F_3)$	$V_\chi(I)$	$V_\chi(R_2)$
$6a$	$3q$	$3q$	q	q	q	q	q	q
$6q+1$	$3q+1$	$3q$	q	q	q	$q+1$	q	q
$6q+2$	$3q+1$	$3q+1$	$q+1$	q	q	q	q	$q+1$
$6q+3$	$3q+1$	$3q+2$	q	$q+1$	q	$q+1$	q	$q+1$
$6q+4$	$3q+2$	$3q+2$	q	$q+1$	$q+1$	$q+1$	q	$q+1$
$6q+5$	$3q+3$	$3q+2$	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	q

Condition 3: $m = 6q + 2$

Let m = 6q + 2, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_2 for $\chi(x_i)$ when i is 6q + 1, labeled as F_1 for $\chi(x_i)$when i is 6q+ 2 .In this case each vertex label will appear q +1 times in the dihedral group D_3 divisor cordial labeling except {R_2, F_1} which will appear q times, and each edge label 1 will appear 3q+1 times and label 0 will appear 3q+1 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 4: m=6q+3

Let m = 6q + 3, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_3 for $\chi(x_i)$ when i is 6q + 1, labeled as R_1 for $\chi(x_i)$when i is 6q + 2 and also we assign the label $R_2$for $\chi(x_i)$when i is 6q + 3. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_3,R_1,R_2} which will appear q+1 times, and each edge label 1 will appear 3q + 1 times and label 0 will appear 3q + 2 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 5: $m = 6q + 4$

Let m = 6q + 4, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_2 for $\chi(x_i)$when i is 6q + 1, labeled as $F_3$for $\chi(x_i)$when i is 6q + 2 , labeled $R_2$for $\chi(x_i)$ when i is 6q + 3 and also we assign R_1 when i is 6q + 4. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_2,F_3,R_2,R_1} which will appear q+1 times, and each edge label 1 will appear 3q + 2 times and label 0 will appear 3q + 2 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 6: $m = 6q + 5$

Let m = 6q + 5, q $\geq$ 1, For 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as F_3 for $\chi(x_i)$ when i is 6q + 1, labeled as F_2 for $\chi(x_i)$ when i is 6q + 2, labeled R_1 for $\chi(x_i)$ when i is 6q + 3 and also we assign F_1 when i is 6q + 4, we assign I when I is 6q+5. In this case each vertex label will appear q times in the dihedral group D_3 divisor cordial labeling except {F_3,F_2,R_1,F_1,I} which will appear q+1 times, and each edge label 1 will appear 3q + 3 times and label 0 will appear 3q + 2 times in the dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Table II it is clear that all graphs C_m are Dihedral group divisor cordial labeling graph.

It is clear that χ is Dihedral group divisor cordial Labeling graph. Hence all cycles $C_m \geq 3$ are Dihedral group divisor cordial labeling graph.

For C_5 let us see Figure 4.

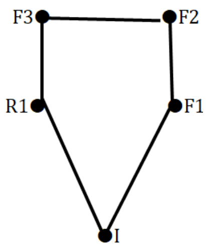

Fig. 4. Cycle C_5

The number of edges labeled 1 is 3 and the number of edges labeled 0 is 2. Thus $\mu_1(\chi)-\mu_0(\chi)| = |3-2| = |1| \leq 1$ and $|V_\chi(a) - V_\chi(b)| \leq 1$. Hence the cycles C_5 is Dihedral group divisor cordial labeling graph.

Axiom 3.1.3: All star graph K_1, is Dihedral group divisor cordial labeling graph.

Proof: Let $G = K_1$, be a star graph whose vertices are given by $x_0, x_1, x_2, \ldots x_m$ and edges are $E(K_1, m) = x_0, x_i$, $1 \leq i \leq m$.

Let us define the function $\chi : V(K_{1,m}) \to D_3$ such that $\chi(x_i) = R_2$ and

$$\chi(x_i) = \begin{cases} F_1, \ i = 6m+1 \\ F_2, \ i = 6m+2 \\ R_1, \ i = 6m+3 \ , \quad m \geq 0 \\ F_3, \ i = 6m+4 \\ R_2, \ i = 6m+5 \end{cases}$$

It is clear that the distance between number of vertices label I and number of vertices label 0 and no. of edges 1 is atmost 1. Here χ is D_3 divisor cordial labeling. ■

TABLE III
DIHEDRAL DIVISOR CORDIAL LABELING OF JELLY FISH GRAPH $J_{m,n}$ REFERS NUMBER OF EDGES LABELED 0 AND 1, NUMBER OF TIMES THE VERTICES WILL APPEAR

Real m & $q \geq 1$	$\mu_1(\chi)$	$\mu_0(\chi)$	$V_\chi(F_1)$	$V_\chi(R_1)$	$V_\chi(F_2)$	$V_\chi(F_3)$	$V_\chi(I)$	$V_\chi(R_2)$
3q	3q+3	3q+2	q	q+1	q+1	q+1	q	q
3q+1	3q+4	3q+3	q+1	q+1	q+1	q+1	q+1	q+1
3q+2	3q+5	3q+4	q+1	q+2	q+2	q+1	q+1	q+1

Axiom 3.1.4: A Jelly fish graph $J_{m,n}$ with, $n \geq 3$ are Dihedral group divisor cordial labeling graph.

Proof: Let G = $J_{m,n}$, be any jelly fish graph. Jelly fish graph is a 4 - cycle graph with vertices x, y, u, v, including the prime edge conneting to x and y and also by appending m pendent edges to u and n pendent edges to v.

Let u_i (1 ≤ i ≤ m) be m vertices which are connected to u and v_j (1 ≤ j ≤ n) be n vertices which are connectes to v. i.e. V(G) = {x, y, u, v, u_i , v_j/1 ≤ i ≤ m, 1 ≤ j ≤ n} and E(G) = {xy, ux, vx, uy, vy, uu_i , vv_j/1 ≤ i ≤ m, 1 ≤ j ≤ n} Here, |V(G)|= p = m + n + 4 and |E(G)|= q = m + n + 5

Here we discuss 3 cases for n≥3 ■

Condition 1: $m = 3q$

Let m = 3q, q ≥ 1. We allotted the label to $\chi(x_i)$as F_2 when i is 3q+1, label to $\chi(x_i)$as R_1 when i is 3q+2 and label to $\chi(x_i)$as I when i is 3q and also we assign the label to $\chi(y_i)$as F_3 when i is 3q+1 , label to $\chi(y_i)$as R_2 when i is 3q+2 and label $F_1$when i is 3q and i varies from 1 to 6q. In this case each vertex label will appear a times in the Dihedral group D_3divisor cordial labelling except $\{R_1, F_2, F_3\}$ will appear q+1 times and each edge labeled as 0 will appear 3q+2 times and 1 will appear 3q+3 times respectively. Hence in this case we get χ as Dihedral group D_3divisor cordial labeling.

Condition 2: $m = 3q + 1$

Let m = 3q+1, q ≥ 1, for 1 ≤ i ≤ 3q we allotted the same labeling as in case 1.

The remaining vertices are labeled as I for $\chi(x_i)$ when i is 3q+1 and also we assign the label $F_1$for $\chi(y_i)$ when i is 3q+1. In this case each vertex label will appear q+1 times in the Dihedral group D_3divisor cordial labeling and each edge label 1 will appear 3q+4 times and label 0 will appear 3q+3 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 3: $m = 3q + 2$

Let m = 3q+2, q ≥ 1, for 1 ≤ i ≤ 3q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 3q+1 and also we assign the label I for $\chi(x_i)$ when i is 3q+2, we assign the label $R_2$for $\chi(x_i)$ when i is 3q+1, F_1 for $\chi(x_i)$ when i is 3q+2. In this case each vertex label will appear q+1 times in the Dihedral group D_3divisor cordial labeling except $\{R_2, R_1\}$ will appear q+2 times and each edge label 1 will appear 3q+5 times and label 0 will appear 3q+4 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3divisor cordial labeling.

Table III it is clear that all graphs $J_{m,n}$ are Dihedral group divisor cordial labeling graph.

Axiom 3.1.5

A wheel graph w_n with, n ≥ 6 are Dihedral group divisor cordial labeling graph.

Proof: Let $G = w_n$ be a graph whose vertices are given by $v_1, v_2, v_3, \ldots v_n$ and edges are defined by $E(w_n) = v_i v_{i+1}$ $1 \leq i \leq n-1$.

The apex vertex is F_3

For $m \geq 6$ we discuss six conditions. ■

Condition 1: $m = 6q$

Let m = 6q, q ≥ 1. We allotted the label to $\chi(x_i)$as F_2 when i is 6q+1, R_1 when i is 6q+2 , $F_1$when i is 6q+3,I when i is 6q+4,F_3 when i is 6q+5, R_2 when i is 6q and i varies from 1 to 6q. In this case each vertex label will appear q times in the Dihedral group D_3divisor cordial labeling and each edge labeled as 0 will appear 6q times and 1 will appear 6a times respectively. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 2: $m = 6q + 1$

Let m = 6q+1, q ≥ 1, for 1 ≤ i ≤ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 6q+1. In this case each vertex label will appear q times in the Dihedral group D_3divisor cordial labeling except $\{R_1\}$ which will appear q+1 times, and each edge label 1 will appear 6q+1 times and label 0 will appear 6q+1 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3divisor cordial labeling.

Condition 3: $m = 6q + 2$

Let m = 6q+2, q ≥ 1, for 1 ≤ i ≤ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 6q+1 and labeled as F_1 for $\chi(x_i)$ when i is 6q+2.In this case each vertex label will appear a times in the Dihedral group D_3 divisor cordial labeling except $\{R_1, F_1\}$ for which will appear q+1 times, and each edge label 1 will appear 6q+2 times and label 0 will appear 6q+2 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

TABLE IV
DIHEDRAL DIVISOR CORDIAL LABELING OF WHEEL w_n GRAPH REFERS NUMBER OF EDGES LABELED 0 AND 1, NUMBER OF TIMES THE VERTICES WILL APPEAR

Real m & $q \geq 1$	$\mu_1(\chi)$	$\mu_0(\chi)$	$V_\chi(F_1)$	$V_\chi(R_1)$	$V_\chi(F_2)$	$V_\chi(F_3)$	$V_\chi(I)$	$V_\chi(R_2)$
6q	6q	6q	q	q+1	q	q+1	q+1	q
6q+1	6q+1	6q+1	q	q+1	q	q	q	q
6q+2	6q+2	6q+2	q+1	q+1	q	q	q	q
6q+3	6q+3	6q+3	q+1	q+1	q	q+1	q	q+1
6q+4	6q+4	6q+4	q+1	q+1	q	q+1	q	q+1
6q+5	6q+5	6q+5	q+1	q+1	q+1	q+1	q	q+1

Condition 4: $m = 6q + 3$

Let m = 6q+3, q $\geq$ 1, for 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 6q+1 , labeled as $R_2$for $\chi(x_i)$ when i is 6q+2 and labeled as F_3 for $\chi(x_i)$ when i is 6q+3. In this case each vertex label will appear q times in the Dihedral group D_3 divisor cordial labeling except $\{R_1, R_2, F_3\}$ will appear q+1 times and each edge label 1 will appear 6q+3 times and label 0 will appear 6q+3 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 5: $m = 6q + 4$

Let m = 6q+4, q $\geq$ 1, for 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 6q+1 , labeled as $F_1$for $\chi(x_i)$ when i is 6q+2 , labeled as F_3 for $\chi(x_i)$ when i is 6q+3 and labeled as R_2 for $\chi(x_i)$ when i is 6q+4.In this case each vertex label will appear q times in the Dihedral group D_3divisor cordial labeling except $\{R_1, F_1, F_3, R_2\}$ which will appear q +1 times and each edge label 1 will appear 6q+4 times and label 0 will appear 6q+4 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Condition 6: $m = 6q + 5$

Let m = 6q+5, q $\geq$ 1, for 1 $\leq$ i $\leq$ 6q we allotted the same labeling as in case 1.

The remaining vertices are labeled as R_1 for $\chi(x_i)$ when i is 6q+1, labeled as F_2 for $\chi(x_i)$ when i is 6q+2, labeled as $R_2$for $\chi(x_i)$ when i is 6q+3 , labeled as F_3 for $\chi(x_i)$ when i is 6q+4 and labeled as F_1 for $\chi(x_i)$ when i is 6q+5.In this case each vertex label will appear q times in the Dihedral group D_3 divisor cordial labeling except $\{R_1, R_2, F_1, F_2, F_3\}$ which will appear q +1 times and each edge label 1 will appear 6q+5 times and label 0 will appear 6q+5 times in the Dihedral group {I,R_1,R_2,F_1,F_2,F_3} divisor cordial labeling. Hence in this case we get χ as Dihedral group D_3 divisor cordial labeling.

Table IV it is clear that all graphs w_m are Dihedral group divisor cordial labeling graph.

IV. CONCLUSION

This paper preserves our findings indicate that the path, Cycle, star, jelly fish and wheel graph are Dihedral group divisor cordial labeling graph. In future we can investigate the concept for duplication graphs.

ACKNOWLEDGMENT

We thank SRM Institute of science and technology for their continuous support.

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