Adaptive Fuzzy Sliding Mode Control for Nonlinear Systems with Unknown Dead-zone

Rui Chen, Zhangping You, Wenhui Zhang

Abstract—**A new adaptive fuzzy control scheme is proposed to solve the uncertainty and input dead-time problems of nonlinear systems. Firstly, based on the system error signal, the state equation combining the input dead zone and the nonlinear model is established. The complex calculation. The nonlinear dead zone is approximated to a time-varying system by means of the mean value theorem. Then, on the basis of this approximation, the differential evolution algorithm (DEA) is used to identify the optimal parameters of the fuzzy model, so that the adaptive fuzzy model can approximate the unknown uncertain part and the dead zone of the nonlinear system. In addition, an adaptive term is introduced into the controller structure to compensate the approximation error of the fuzzy system. Finally, based on Lyapunov stability theory, the proposed control scheme guarantees the boundedness of all closed-loop signals and the convergence of tracking errors to zero.**

Index Terms—**Nonlinear systems, Differential evolution algorithm, Input dead-zone, Adaptive fuzzy control, Sliding mode control, Fuzzy logic**

I. Introduction

ISSUES such as dead-zone nonlinearity, parameter uncertainties, and external disturbances are prevalent in nonlinear systems [1]-[6]. These factors significantly impacted system performance, posing challenges for control. In recent years, many scholars devoted themselves to mitigating the effects of these nonlinearities on control systems, achieving significant progress [7]-[12]. Addressing such issues: [13] proposed a neural network event-triggered finite-time consensus control method for uncertain nonlinear multi-agent systems with dead-zone inputs and actuator failures. The paper established an input dead-zone model and utilized backstepping and radial basis function neural networks to construct a compensating controller to offset the effects of dead-zone input, aiming to eliminate the adverse impact of dead-zone input. However, the modeling of some nonlinear functions was overly idealized and did not fully reflect the complex situations of actual systems. [14] investigated the output feedback robust stabilization problem of nonlinear systems with asymmetric dead-zone in actuators and uncertain nonlinearities. A robust control scheme was proposed to replace constructing a dead-zone inverse, involving the creation of an input-driven observer that employed scaling gains to control the nonlinear terms. Based on the non-separation principle and backstepping method, a time-varying smooth output feedback controller was derived to ensure global asymptotic stability of the closed-loop system. [15] studied the adaptive state feedback quantized control problem of a class of switching nonlinear systems with unknown asymmetric actuator dead-zone and multiple inputs multiple outputs (MIMO). A new approximation model was proposed to handle the coupling between quantizers and dead-zone, and corresponding robust adaptive control laws were designed to eliminate these nonlinear terms. Additionally, the paper adopted a direct neural control scheme to significantly reduce the number of adaptive control laws and proposed an adaptive control scheme based on the backstepping method to ensure system performance.

Because the radial basis function neural networks could be used to approximate unknown functions and utilized input-driven filters to estimate unmeasurable states [16]-[22]. [23] primarily investigated the neural network adaptive output feedback control problem of nonlinear systems with dead-zone outputs and unmeasurable states. The use of Nussbaum functions addressed the uncertainty in virtual control coefficients caused by dead-zone in the output mechanism. [24] addressed the design of a quantized controller for uncertain nonlinear systems with unknown disturbances and unknown dead-zone nonlinearity. By estimating the maximum upper bound of disturbances instead of each disturbance individually, the designed controller could simultaneously handle system uncertainties, unknown disturbances, and unknown dead-zone nonlinearity. The stability of the closed-loop system was finally proven. [25] proposed an adaptive dynamic surface control scheme based on interval type-2 fuzzy logic systems (IT2FLS) for uncertain nonlinear systems with dead-zone inputs and unknown gains. The dead-zone nonlinearities were represented as time-varying systems with bounded disturbances. The approach utilized fuzzy systems to approximate unknown nonlinear dynamics and introduced adaptive terms to compensate for the effects of disturbance-like terms in dead-zone constraints. A dynamic surface control (DSC) scheme based on IT2FLS and dead-zone models was designed, and adaptive laws for parameters were obtained. [26] dealt with a class of fractional-order multiple inputs multiple outputs nonlinear dynamic systems with dead-zone inputs. It combined backstepping dynamic surface control with fractional-order adaptive type-2 fuzzy technology to construct a control scheme. Interval type-2 fuzzy logic

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systems were used to approximate unknown nonlinear functions in uncertain multiple inputs multiple outputs systems. To achieve better control performance in reducing tracking errors, particle swarm optimization was employed to tune controller parameters.

In summary, this paper explores an adaptive fuzzy sliding mode control (AFC) approach for a category of uncertain nonlinear single input single output (SISO) systems with dead-zone input. The proposed method exhibits enhanced performance and greater accuracy in effectively managing nonlinear and uncertain single input single output systems characterized by dead-zone input, exemplified by the nonlinear single-joint robotic arm system studied. The contributions of this paper are outlined as follows:

1.The proposed control method addresses the tracking control problem of single input single output uncertain nonlinear systems with unknown dead-zone. Unlike most AFC algorithms that focus solely on tracking control of nonlinear systems without considering dead-zone input, this paper addresses the dead-zone input issue on the basis of uncertain nonlinear systems, making the designed controller more practical and universally applicable.

2.The fuzzy model with optimal parameter selection is identified using the DEA, which is used to approximate unknown uncertainties and functions of the studied nonlinear single input single output system. The adaptive laws of the AFC algorithm are redesigned based on input dead zones and nonlinear systems.

3. The DEA is employed to optimize the fuzzy system for better approximation in accordance with the designed controller.

The organization of this paper is as follows. Section 2 presents the problem statement, section 3 describes nonlinear dead-zone and their characteristics, section 4 proposes a new AFC controller, including the optimization of fuzzy approximation of unknown functions using the DEA, adaptive laws of the AFC algorithm, and stability analysis. To validate the propositions, numerical examples and simulation results are provided in section 5. Finally, conclusions are drawn in section 6.

II. Problem Statement

The general expression for a nonlinear SISO system of order n is as follows:

$$\begin{cases} x^{(n)} = f(x,t) + g(x,t)u(t) \\ y = x \end{cases} \tag{1}$$

Here, $g(x,t)$ and $f(x,t)$ are unknown but bounded non-linear functions; $g(x,t)$ is nonzero function; y is output of investigated system, $u(t)$ is control input; $x = [x, \dot{x}, \ddot{x} ... x^{(n-1)}]^T$ is state vector of the system.

The control goal is to design a stable control law so that state x can stably track the reference signal x_d. The tracking error is defined as follows:

$$e = x - x_d = [e, \dot{e}, ... e^{(n-1)}] \in R \tag{2}$$

The sliding surface is defined as:

$$s = c_1 e + c_2 \dot{e} + ... c_{n-1} e^{n-2} + e^{n-1} \tag{3}$$

Here, $c = [c_1, c_2, c_3 ... c_{n-1}]^T$ represents the coefficients that pass the Routh-Hurwitz stability condition.

Take the derivative of equation (3):

$$\begin{aligned} \dot{s} &= c_1 \dot{e} + c_2 \ddot{e} + ... + c_{n-1} e^{n-1} + e^n \\ &= \sum_i^{n-1} c_i e^{(i)} + e^{(n)} \\ &= \sum_i^{n-1} c_i e^{(i)} + x^{(n)} - x_d{}^{(n)} \end{aligned} \tag{4}$$

For satisfying Lyapunov stability theory, the definition is as follows:

$$\dot{s} = -\eta |s|^{\alpha} \, sign(s) \tag{5}$$

Here, $f(x,t)$, $g(x,t)$ are assumed to be known, the sliding mode control law is given by: $\dot{s} = -\eta |s|^{\alpha} \, sign(s)$, $0 < \eta$, $0 < \alpha < 1$, the stable sliding mode control law can be described as:

$$u(t) = \frac{1}{g(x,t)} \left[-\sum_i^{n-1} c_i e^{(i)} - f(x,t) + x_d^{(n)} - \eta |s|^{\alpha} \, sign(s) \right] \tag{6}$$

In practical applications, $f(x,t)$ and $g(x,t)$ are often unknown nonlinear functions, making it challenging for sliding mode controllers (SMC) to achieve stable control in systems with unknown functions. To address this issue, the following proposes an AFC to resolve such problems.

III. Nonlinear Dead-zone and Its Characteristics

As shown in Fig. 1, $u(t)$ is the output signal of the following dead-zone nonlinearity.

$$ut = G(v(t)) = \begin{cases} g_r(v) & if \quad v(t) \geq b_r \\ 0 & if \quad b_l \leq v(t) \leq b_r \\ g_l(v) & if \quad v(t) \leq b_l \end{cases} \tag{7}$$

Here, $v(t)$ denotes the dead-zone input (actual control signal), $g_r(v)$, $g_l(v)$ denotes the unknown smooth nonlinear function, and b_l, b_r represents the unknown dead-zone width parameter, without loss of generality, assuming $b_l < 0, b_r > 0$, the input-output characteristics of the dead-zone are depicted in the figure below.

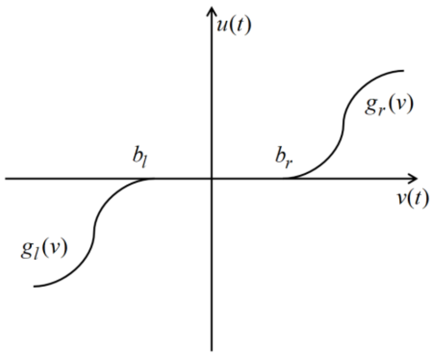

Fig. 1. Nonlinear dead-zone

The literature suggests that dead-zone nonlinearity may encompass diverse scenarios, including linear, symmetric, and asymmetric dead-zones. Moreover, the dead-zone function $g_l(v)$ and $g_r(v)$ is continuous. Here, the characteristic parameter b_l, b_r signifies the break point of the input dead-zone. To streamline controller design, this section initially introduces the following assumptions.

Assumption 1: For the smooth function $g_l(v)$ and $g_r(v)$, there exist unknown positive constants $g_{l0}(v)$, $g_{l1}(v)$, $g_{r0}(v)$ and $g_{r1}(v)$ such that: $0 < g_{l0} \le g'_l \le g_{l1}$, $\forall v \in (-\infty, b_l]$ and $0 < g_{r0} \le g'_r \le g_{r1}$, $\forall v \in [b_r, +\infty)$.

Here, $g'_l(v) = dg_l(z)/dz|_{z=v}$ and $g'_r(v) = dg_r(z)/dz|_{z=v}$.

Due to $g_l(b_l) = g_r(b_r) = 0$, according to the mean value theorem, there exist $\xi_l \in (-\infty, b_l)$ and $\xi_r \in (b_r, +\infty)$ such that

$$g_l(v) = g_l(v) - g_l(b_l) = g'_l(\xi_l)(v - b_l), \forall v \in (-\infty, b_l] \tag{8}$$

$$g_r(v) = g_r(v) - g_r(b_r) = g'_r(\xi_r)(v - b_r), \forall v \in [b_r, +\infty) \tag{9}$$

From the above equation, it can be derived that

$$g_l(v) = g'_l(\xi_l)(v - b_l), \quad \forall v \in (-\infty, b_r] \tag{10}$$

Here, $\xi'_l \in (-\infty, b_l]$

$$g_r(v) = g'_r(\xi_r)(v - b_r), \quad \forall v \in (b_r, +\infty] \tag{11}$$

Here, $\xi'_l \in (-\infty, b_l]$

From the above equation and Assumption 1, the dead-zone can be rewritten as:

$$u(t) = \varphi(t)v(t) + \rho(t), \quad \forall t \ge 0 \tag{12}$$

Here, $|\rho(t)| \le \rho_N$, $\rho_N = (g_{r1} + g_{l1})\max\{b_r, -b_l\} > 0$.

$$\varphi(t) = \varphi_r(t) + \varphi_l(t) \tag{13}$$

Here,

$$\varphi_r(t) = \begin{cases} g'_r(\xi_r) & if \quad v(t) \ge b_l \\ 0 & if \quad v(t) < b_l \end{cases} \tag{14}$$

$$\varphi_l(t) = \begin{cases} g'_l(\xi_l) & if \quad v(t) \le b_r \\ 0 & if \quad v(t) > b_r \end{cases} \tag{15}$$

$$\rho(t) = \begin{cases} -g'_r(\xi_r)b_r & if \quad v(t) \ge b_r \\ -[g'_r(\xi_r) + g'_l(\xi_l)]v(t) & if \quad b_l < v(t) < b_r \\ -g'_l(\xi_l)b_l & if \quad v(t) \le b_l \end{cases} \tag{16}$$

The system state equation can be rewritten as:

$$\begin{cases} x^{(n)} = f(x,t) + g(x,t)[\varphi(t)v(t) + \rho(t)] \\ y = x \end{cases} \tag{17}$$

IV. Proposed Adaptive Fuzzy Sliding Mode Control

This section presents an AFC design tailored for nonlinear systems with input dead-zone. The proposed controller scheme, depicted in Fig. 3, employs the DEA to optimize parameters of the fuzzy model, facilitating approximation of unknown functions such as $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$. Additionally, by incorporating fuzzy logic, an adaptive law has been designed to minimize the impact of the error term and striving to eliminate this error to achieve the system's asymptotic stability.

A. Optimization of T-S Fuzzy Approximation of Unknown Functions Using Differential Evolution Algorithm

To implement the proposed algorithm, it is essential to identify the functions $\hat{f}(x,t)$, $\hat{g}(x,t)$, $\hat{\rho}(x,t)$ and $\hat{\varphi}(x,t)$ in advance. In this study, the fuzzy model was employed to represent the functions $\hat{f}(x,t)$, $\hat{g}(x,t)$, $\hat{\rho}(x,t)$ and $\hat{\varphi}(x,t)$ with the parameters of the fuzzy model optimized using the DEA.

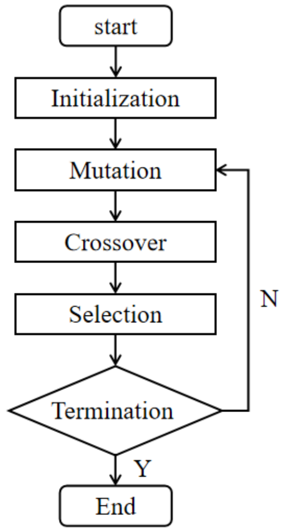

Fig. 2. Flowchart of the differential evolution algorithm

The fuzzy models utilized for approximating the function $\hat{f}(x,t)$, $\hat{g}(x,t)$, $\hat{\rho}(x,t)$ and $\hat{\varphi}(x,t)$ are concurrently trained. The objective is to ascertain the function $\hat{x}_*^{(n)}$ that best matches $x^{(n)}$. Subsequently, the cost function is defined as:

$$J = \frac{1}{N}\sum_{n=1}^{N}(x^{(n)} - \hat{x}^{(n)}) \tag{18}$$

In this study, the DEA is employed to accurately determine the parameters of the fuzzy model. The flowchart illustrating the DEA is presented in Fig. 2.

Initialization

The initial vector consists of NP randomly selected D-dimensional elements, ensuring comprehensive coverage across the parameter space

$$X_{i,G} = [x_{1,i,G}, x_{1,i,G} \ldots x_{D,i,G}] \tag{19}$$

Here, G is the number of generations, $G = 0,1\ldots,G_{\max}$, $i = 1,2,\ldots,NP$.

Mutation

the differential evolution generates new parameter vectors by employing a mutation operation, which involves adding the weighted difference between two population vectors to a third vector. For each target vector $x_{i,G}$, a mutant vector is generated in accordance with this operation.

$$v_{i,G+1} = x_{r1,G} + F(x_{r2,G} - X_{r3,G}) \tag{20}$$

Here, $r_1, r_2, r_3 \in 1,2,\ldots NP$.

The randomly chosen values r_1, r_2, r_3 are distinct from the running index i. With $F \in [0,2]$ representing a real and constant coefficient.

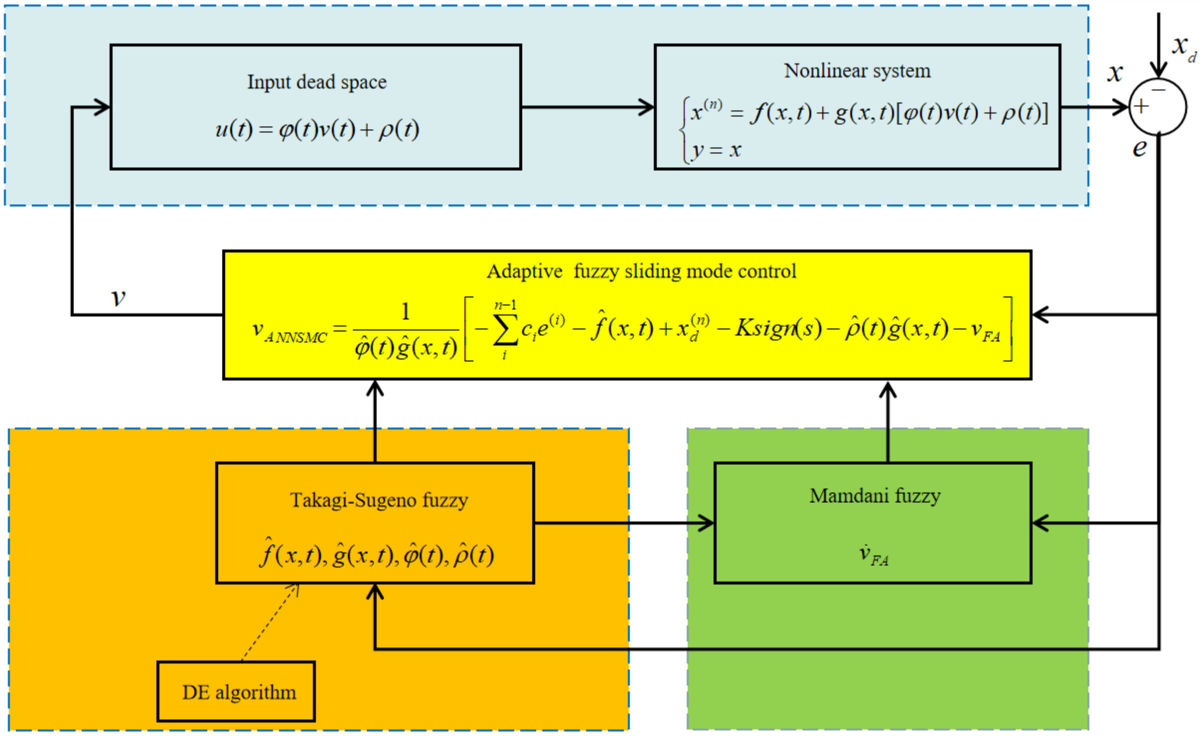

Fig. 3. Scheme of the proposed adaptive fuzzy sliding mode control adaptive fuzzy sliding mode control system

Crossover

Following vector generation through mutation, the crossover step is executed to enrich the diversity within the population. During this step, the donor vector exchanges its components with the target vector $\vec{X}_{i,G}$ to produce the trial vector $\vec{U}_{i,G} = [u_{1,i,g}, u_{2,i,g} ... u_{D,i,g}]$. The DEA commonly employs the binomial crossover method, which can be outlined as follows:

$$u_{j,i,G} = \begin{cases} v_{j,i,G} & if\,(rand_{j,i}[0,1] < C) \\ v_{j,i,G} & otherwise \end{cases} \tag{21}$$

The selection process determines whether the target vector $X_{i,G}$ should be included in the next generation $(G+1)$. This determination is made by comparing the target vector $X_{i,G}$ with the trial vector $U_{i,G}$, and the one exhibiting a lower function value is retained for advancement to the subsequent generation. The selection operation is outlined as follows:

$$\vec{X}_{i,G+1} = \begin{cases} \vec{U}_{i,G} & if \quad f(\vec{U}_{i,G}) < f(\vec{X}_{i,G}) \\ \vec{X}_{i,G} & otherwise \end{cases} \tag{22}$$

Termination

The termination criterion for DEA is as follows: The algorithm ceases operation when any of the following conditions are met: the maximum generation count is reached, a best fitness value lower than the specified target fitness is achieved, or there is no improvement in the best fitness value over an extended period.

B. Proposed Adaptive Fuzzy Sliding Mode Control

To address these issues, certain methodologies have advocated for the implementation of an adaptive sliding mode controller through the approximation of the unknown functions $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$. Then, the state-space model can be formulated as follows:

$$\begin{cases} \hat{x}_*^{(n)} = \hat{f}(x,t)+\hat{g}(x,t)[\hat{\varphi}(t)v(t)+\hat{\rho}(t)] \\ \hat{y} = \hat{x} \end{cases} \tag{23}$$

The fuzzy sliding mode control law can be designed as follows:

$$v_{FSMC} = \frac{1}{\hat{\varphi}(t)\hat{g}(x,t)}\left[-\sum_i^{n-1} c_i e^{(i)} - \hat{f}(x,t) + x_d^{(n)} - K\tanh(s) - \hat{\rho}(t)\hat{g}(x,t)\right] \tag{24}$$

Here, $\hat{f}(x,t)$, $\hat{g}(x,t)$, $\hat{\rho}(x,t)$ and $\hat{\varphi}(x,t)$ are estimated by fuzzy-based model.

As a strategy to mitigate the occurrence of chattering phenomena, the utilization of the *sign*( ) function in the control law is substituted with a saturation function

$$\tanh(s) = \frac{e^s - e^{-s}}{e^s + e^{-s}} \tag{25}$$

The hyperbolic tangent function possesses the following property: $|\tanh(s)| \le 1$ use the hyperbolic tangent function enables boundedness of control inputs.

In practical applications, the estimation functions $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$ based on fuzzy models cannot accurately approximate the functions $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$ due to external disturbances, uncertainties, and the precision of the identification models. The modeling errors are defined as follows:

$$e_m^* = x^{(n)} - \hat{x}_*^{(n)} \tag{26}$$

Based on the above equation, the fuzzy sliding mode control law can be rewritten as follows:

$$v_{FSMC} = \frac{1}{\varphi(x,t)g(x,t)}\left[-\sum_i^{n-1} c_i e^{(i)} - f(x,t) + x_d^{(n)} - K\tanh(s) - \rho(t)g(x,t) - e_m^*\right] \tag{27}$$

The fuzzy sliding mode control law incorporates an additional term denoted as e_m^* in its formulation, under the assumption that $\|e_m^*\| < \varepsilon$. Here, ε is sufficiently small to ensure the stability of the closed-loop control system, the system may exhibit stability, albeit with the possibility that the tracking error may not asymptotically converge to zero. To establish the asymptotic stability of the closed-loop system, the elimination of the term e_m^* is warranted.

To address these problems, the AFC algorithm is introduced, which leverages the Takagi-Sugeno fuzzy logic system for the approximation of $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$ combined with Mamdani fuzzy logic for adaptive law.

An adaptive fuzzy law(v_{FA}) is proposed to ensure that the error of the fuzzy model approaches zero within predefined bounded conditions. The state-space model added with adaptive fuzzy law is described as:

$$\begin{cases} \hat{x}^{(n)} = \hat{f}(x,t) + \hat{g}(x,t)[\hat{\varphi}(t)v(t) + \hat{\rho}(t)] + v_{FA} \\ \hat{y} = \hat{x} \end{cases} \tag{28}$$

Consider the control law of the nonlinear system presented . Here, $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$ are estimated by fuzzy-based model; v_{FA} is adaptive function. The proposed AFC law can be defined as:

$$v_{AFSMC} = \frac{1}{\hat{\varphi}(t)\hat{g}(x,t)}\Big[-\sum_{i}^{n-1} c_i e^{(i)} - \hat{f}(x,t) + x_d^{(n)} - Ksign(s) - \hat{\rho}(t)\hat{g}(x,t) - v_{FA}\Big] \tag{29}$$

The error of model is defined as:

$$e_m = x^{(n)} - \hat{x}^{(n)} = x^{(n)} - \hat{f}(x,t) - \hat{g}(x,t)[\hat{\varphi}(t)v(t) + \hat{\rho}(t)] - v_{FA} \tag{30}$$

$$e_m = x^{(n)} - \hat{x}^{(n)} = x^{(n)} - \hat{x}_*^{(n)} - u_{FA} = e_m^* - v_{FA} \tag{31}$$

Here, $e_m^* = x^{(n)} - \hat{x}_*^{(n)}$ with $\dot{e}_m$ representing the derivative of e_m . It can be rewritten as:

$$\dot{e}_m = \dot{e}_m^* - \dot{v}_{FA} \tag{32}$$

Choosing the derivative of e_m to satisfy the Lyapunov stability concept is proposed:

$$\dot{e}_m = -K_A sign(e_m) \tag{33}$$

Here, K_A is positive value.

Substitute the equations (32) into (33):

$$\dot{e}_m^* - \dot{v}_{FA} = -K_A sign(e_m) \tag{34}$$

Assumption 2: Assuming both $|\dot{v}_{FA}| > |\dot{e}_m^*|$ and $sign(\dot{v}_{FA}) = sign(e_m)$ hold true.

Theorem: In addressing the control problem posed by nonlinear system (1), $f(x,t)$, $g(x,t)$, $\rho(x,t)$ and $\varphi(x,t)$ propose a control law v_{AFSMC}. This approach involves the estimation process facilitated by fuzzy modeling and optimization employing evolutionary algorithms, operating under the assumption of $|\dot{v}_{FA}| > |\dot{e}_m^*|$ and $sign(\dot{v}_{FA}) = sign(e_m)$. Consequently, it is anticipated that the signals of the closed-loop system will remain bounded, with the tracking error asymptotically converging to zero.

Proof: Assume that the $\dot{v}_{FA}$ is chosen as $|\dot{v}_{FA}| > |\dot{e}_m^*|$ and $sign(\dot{v}_{FA}) = sign(e_m)$. From equations (34), the following results are obtained:

$$-sign(\dot{v}_{FA})|\dot{e}_m^* - \dot{v}_{FA}| = -K_A sign(e_m) \tag{35}$$

$$|\dot{e}_m^* - \dot{v}_{FA}| = K_A \tag{36}$$

Therefore, assuming the hypothesis is valid, $\dot{e}_m = -K_A sign(e_m)$ satisfies the condition. e_m has different sign with $\dot{e}_m$.

The state-space model is designated as v_{AFSMC}. Taking the derivative with respect to the sliding surface yields:

$$\begin{aligned}
\dot{s} &= \sum_{i}^{n-1} c_i e^{(i)} + x^{(n)} - x_d^{(n)} \\
&= \sum_{i}^{n-1} c_i e^{(i)} + f(x,t) + g(x,t)[\varphi(t)v(t) + \rho(t)] - x_d^{(n)} \\
&= \sum_{i}^{n-1} c_i e^{(i)} + f(x,t) + g(x,t)[\varphi(t)v(t) + \rho(t)] - \sum_{i}^{n-1} c_i e^{(i)} \\
&\quad - \hat{f}(x,t) - \hat{\rho}(t)\hat{g}(x,t) - \hat{g}(x,t)\hat{\varphi}(t)v(t) - v_{FA} - ksign(s) \\
&= (f(x,t) - \hat{f}(x,t)) + (g(x,t)\varphi(t) - \hat{g}(x,t)\hat{\varphi}(t))v(t) \\
&\quad + (g(x,t)\rho(t) - \hat{\rho}(t)\hat{g}(x,t)) - v_{FA} - ksign(s) \\
&= e_m^* - v_{FA} - ksign(s) \\
&= e_m - ksign(s)
\end{aligned} \tag{37}$$

Consider the Lyapunov function candidate:

$$V = \frac{1}{2}s^2 + \frac{1}{2}e_m^2 \tag{38}$$

The time derivative of V gives:

$$\begin{aligned}
V &= s\dot{s} + e_m\dot{e}_m \\
&= s(e_m - Ksign(s)) - e_m K_A sign(e_m) \\
&= se_m - K|s| - K_A|e_m|
\end{aligned} \tag{39}$$

Since $\dot{e}_m = -K_A sign(e_m)$, it follows that $e_m \to 0$ when $t \to \infty$ .Then, $\dot{V} \le 0$ when $t \to \infty$. The closed-loop system signals will be bounded and the tracking error will converge to zero asymptotically.

TABLE I
ADAPTIVE FUZZY RULES

	e_m		
	N	ZO	P
$\dot{v}_{FA}$	N	ZO	P

To approximate the adaptive function $\dot{v}_{FA}$, a fuzzy model will be utilized due to the complexity arising from multiple derivative terms and intricate higher-order derivatives. The proposed fuzzy function must maintain relative simplicity while ensuring Lyapunov stability. Utilizing e_m as input and $\dot{v}_{FA}$ as output, it functions as the primary fuzzy rule for the AFC. The selection of fuzzy rules is delineated according to Table 1. (N = negative, ZO = zero, P = positive), ensuring the validity of $sign(\dot{v}_{FA}) = sign(e_m)$. The determination of positive (P) and negative (N) values adheres to the constraint $|\dot{v}_{FA}| > |\dot{e}_m^*|$.

V. Simulation and Discussion

After completing the research and stability analysis of the fuzzy sliding mode control algorithm, to further demonstrate the effectiveness and feasibility of the control algorithm, numerical simulations were conducted using a single-joint robotic arm model. The mathematical state-space equations for the single-joint robotic arm system are as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{I}(dx_2 + mgl\cos x_1) + \frac{1}{I}(1 + x_1 + x_2)\tau \\ y = x \end{cases} \tag{40}$$

Here, $x_1 = \theta$, $x_2 = \dot{\theta}$, $m = 2$, $d = 4$, $l = 0.4$,the initial state of the system is $\left[\frac{\pi}{10}, 0\right]$, the initial value of θ is taken as $0, k_1 = 1$, $k_2 = 8$, $I = \frac{4}{3}ml^2$.

A. Fuzzy Sliding Mode Control System Simulation Analysis Based on Differential Evolution Algorithm

In this section, simulation experiments were conducted to evaluate the performance of the proposed control algorithm. The results are depicted in Fig. 4 through 7.

The position trajectory tracking curve is illustrated in Fig. 4, while the control input signals are depicted in Fig. 5. The output curve with dead-zone constraints is presented in Fig. 6, and the tracking error curve is shown in Fig. 7.

From Fig. 4, it can be observed that, with non-zero initial values, the designed control algorithm ensures accurate tracking of the joint actual trajectory to the desired trajectory within 2.3s. Compared to a traditional sliding mode control system without DEA optimization, which typically achieves similar alignment in about 3.5s, the improvement in response speed is notable. This demonstrates the enhanced efficiency of the DEA-optimized fuzzy sliding mode control system in handling initial state discrepancies.

Fig. 5 shows that the joint torque output presents periodic fluctuations, with the maximum torque output occurring approximately every 3.4s. Each cycle includes a vertical pulse signal output of about 10 N.m. The maximum joint torque output from the AFC based on the DEA is slightly lower than that of the fuzzy control system. Although there are some fluctuations, the curve demonstrates overall stability.

As can be seen from Fig. 6, the overall torque output presents a periodic amplitude change. Due to input dead zone constraints, the vertical pulse signal in Fig. 5 transforms into a smoother curve, resulting in a stable overall torque output.

Furthermore, Fig. 7 illustrates that despite the presence of uncertainties, input saturation, and frictional nonlinearity, the AFC base on DEA exhibits considerable approximation to the modeling error. It effectively compensates for most of the unmodeled nonlinearities, leading to the joint gradually approaching zero within 3s, with minimal oscillation in the latter half. This signifies high control precision. In comparison, a conventional fuzzy control system without DEA optimization demonstrates slower error convergence and higher oscillations, reflecting inefficiency in dealing with complex uncertainties.

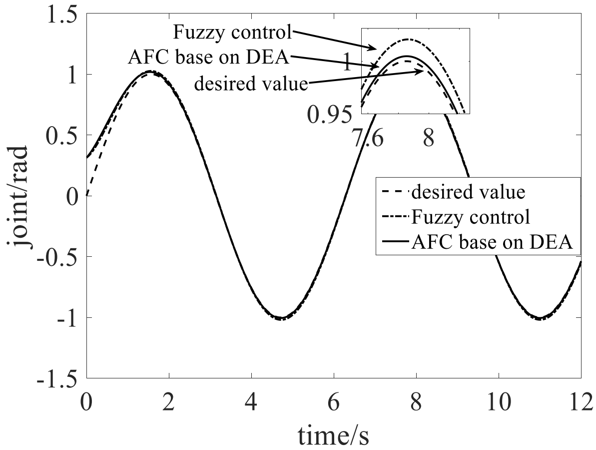

Fig. 4. Position trajectory tracking curves

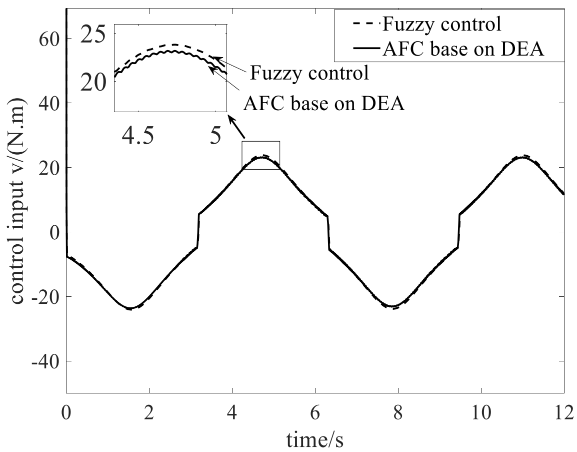

Fig. 5. Controller output curve

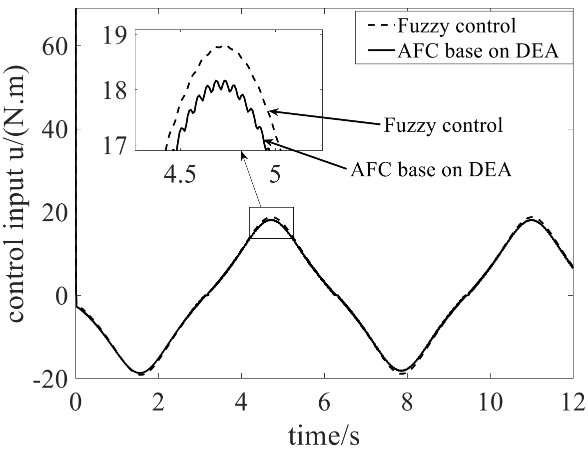

Fig. 6. Output curve with dead-zone limitation

These findings illustrate that relying exclusively on a fuzzy sliding mode controller without incorporating the DEA and adaptive fuzzy compensation leads to diminished control effectiveness. Moreover, they underscore the significance of integrating DEA and adaptive fuzzy compensation to enhance control performance. Simulation results indicate that the AFC system, optimized by the DEA, effectively manages complex nonlinear systems. Although the controller retains the capability to track nonlinear systems in the absence of DEA and adaptive fuzzy compensation, there is a notable reduction in control precision. This observation confirms the effectiveness of the DEA-optimized AFC in managing complex nonlinear systems. Importantly, the reduced control precision without the DEA and adaptive fuzzy adjustments further highlights the necessity of integrating these techniques to achieve optimal control accuracy in challenging nonlinear environments.

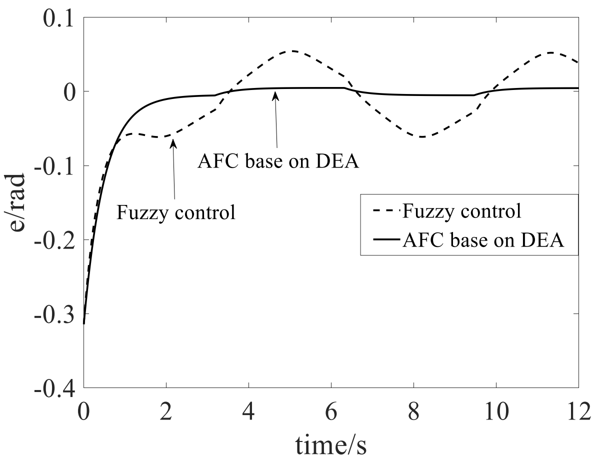

Fig. 7. Tracking error curve

The experimental results validate the effectiveness of the proposed control algorithm in tracking joint trajectories accurately and mitigating the effects of nonlinearities, uncertainties, and input saturation.

VI. CONCLUSIONS

An AFC based on DEA optimization is proposed for nonlinear systems with uncertainties and input dead zones. By applying the mean value theorem, the nonlinear dead zone is linearly approximated as a simple time-varying system. DEA is used to optimally identify the parameters of the fuzzy model, which then approximates the unknown uncertainties and dead zones. Finally, Lyapunov stability theory ensures that all signals in the closed-loop system remain bounded. The efficacy of this approach is demonstrated through simulation examples.

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